Lehrstuhl für
Angewandte Mechanik

A Domain Decomposition interface solver with multiple directions of descent for heterogeneous problems

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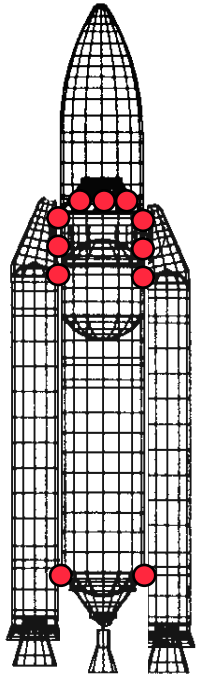
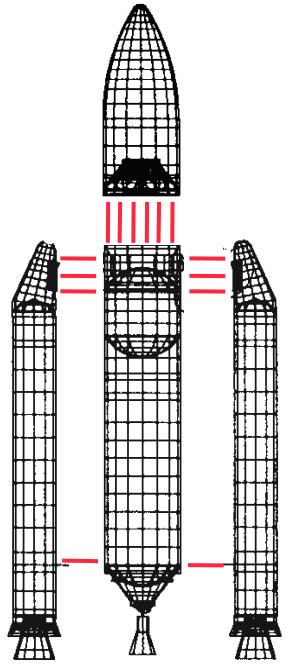
What to expect ....

1. A crash course on Domain Decomposition and FETI
2. Hard problem for FETI
3. A new FETI-like idea: the FETI-S
4. Preliminary numerical examples and discussion

Domain Decomposition: Primal / Dual Assembly

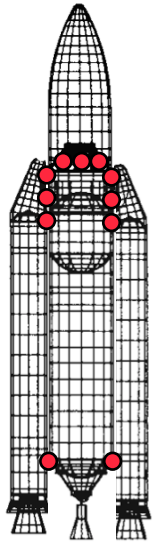


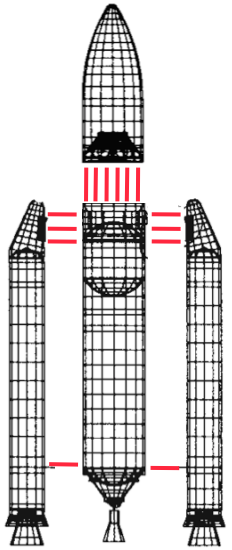


$$\begin{bmatrix} \mathbf{K}^{(1)} & & 0 \\ & \ddots & \\ 0 & & \mathbf{K}^{(N)} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{(1)} \\ \vdots \\ \mathbf{u}^{(N)} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^{(1)} \\ \vdots \\ \mathbf{f}^{(N)} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{K}^{(1)} & & 0 & \mathbf{B}^{(1)^T} \\ & \ddots & & \\ 0 & & \mathbf{K}^{(N)} & \mathbf{B}^{(N)^T} \\ \mathbf{B}^{(1)} & \dots & \mathbf{B}^{(N)} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{u}^{(1)} \\ \vdots \\ \mathbf{u}^{(N)} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^{(1)} \\ \vdots \\ \mathbf{f}^{(N)} \\ 0 \end{bmatrix}$$

Iterative solution of interface problem





Iterate on interface dofs $\mathbf{u}_b$

| solve for internal dofs $\mathbf{u}_i$

end

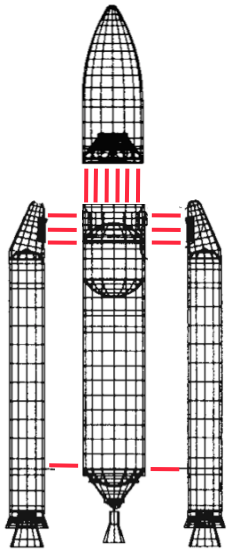
Iterate on interface forces $\boldsymbol{\lambda}$

| solve for domain dofs $\mathbf{u}$

end

FETI
(Finite Element Tearing and Interconnecting) [Farhat-Roux]

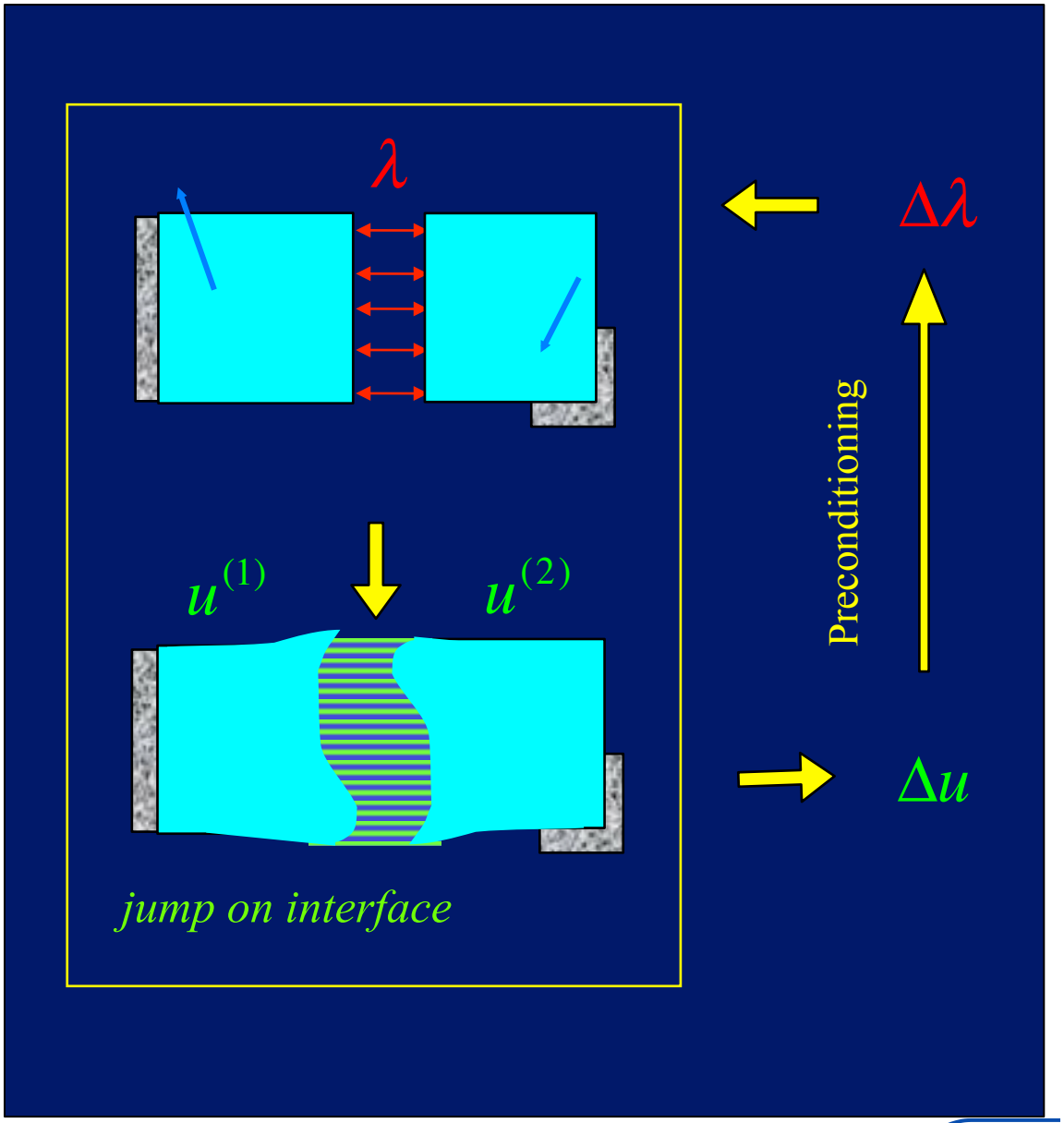
Finite Element Tearing and Interconnecting - FETI



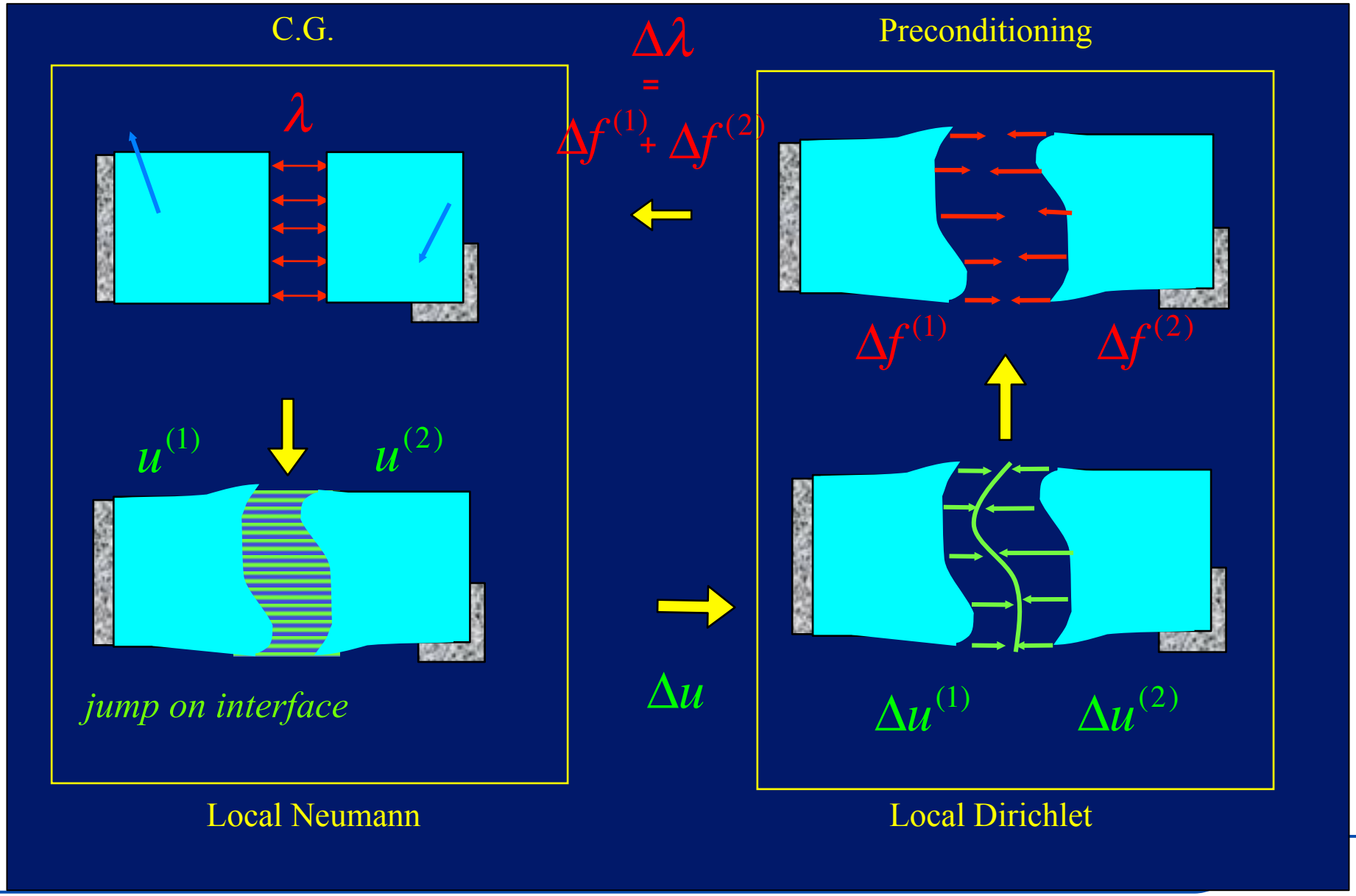
Iterate on interface forces $\boldsymbol{\lambda}$

| solve for domain dofs $\mathbf{u}$

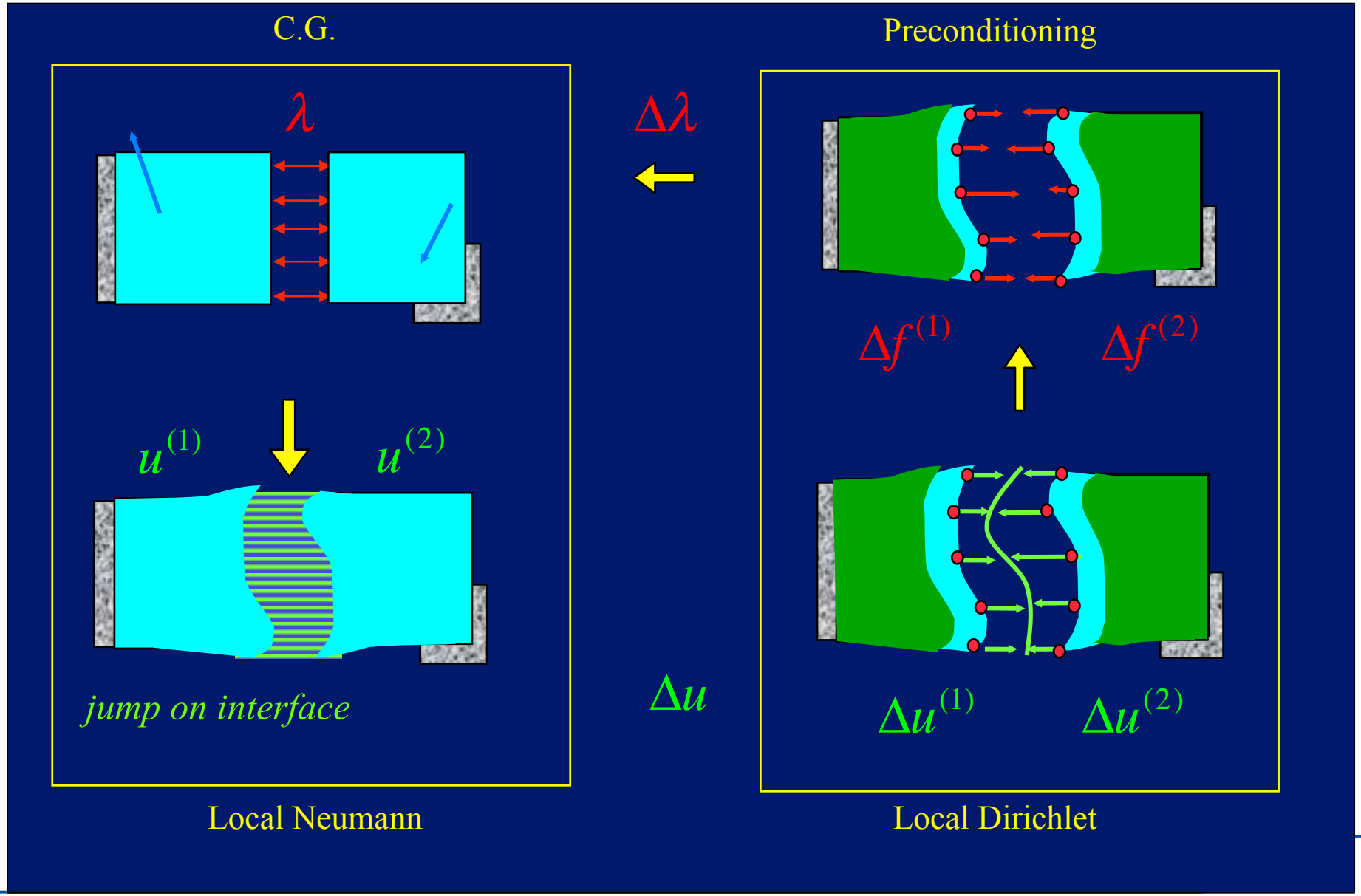
end



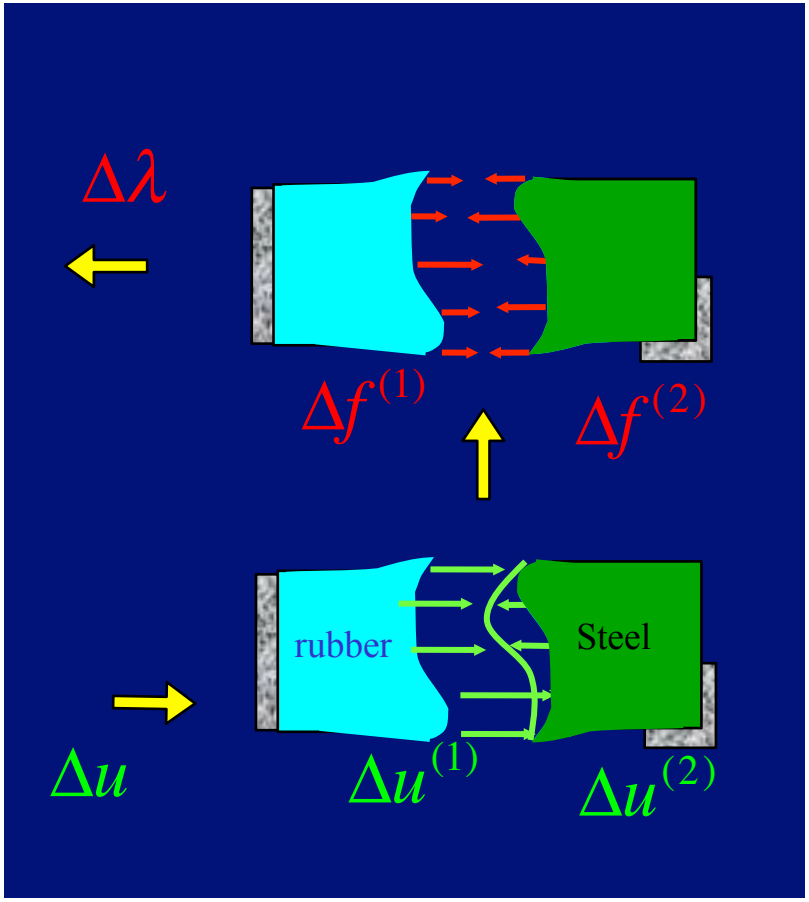
FETI iterations in a nutshell



FETI with lumped preconditioner



Preconditioning heterogeneous problems



$$\tilde{\mathbf{F}}_I^{-1} = \mathbf{W}\mathbf{B} \begin{bmatrix} \mathbf{S}_{bb}^{(1)} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{S}_{bb}^{(N)} \end{bmatrix} \mathbf{B}^T\mathbf{W}^T$$

scaling according to local stiffness

Note: when domains are floating, need to introduce a coarse space in FETI

Example

1 000 000 d.o.f
Highly heterogeneous

FETI with scaling:

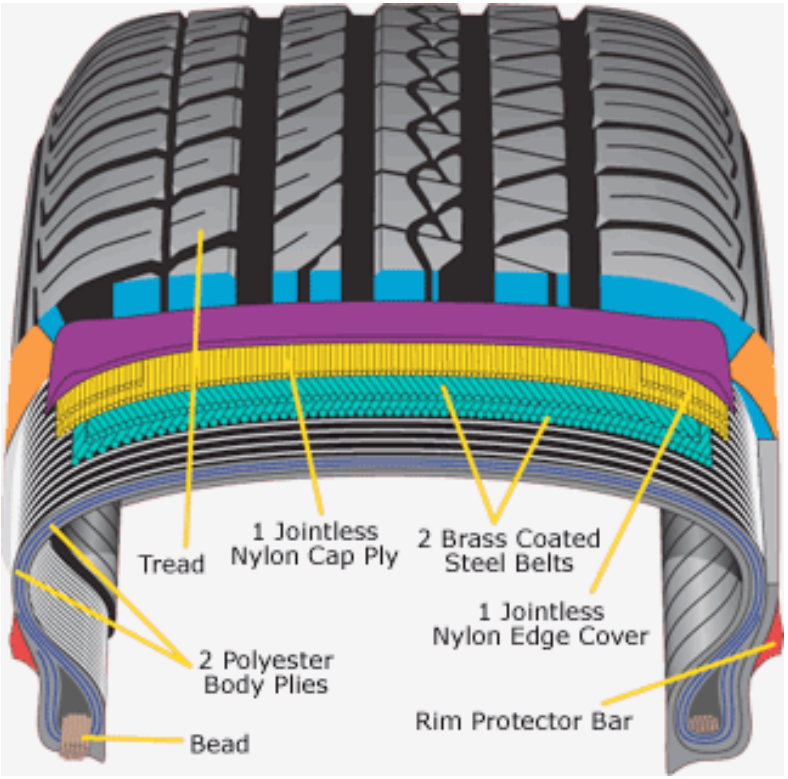
250 CPU: 370 sec

500 CPU: 160 sec

Reentry vehicle (SANDIA)

[Bhardwaj, Day, Farhat,
Lesoinne, Pierson, Rixen], 2000

But then .... the hard problems:



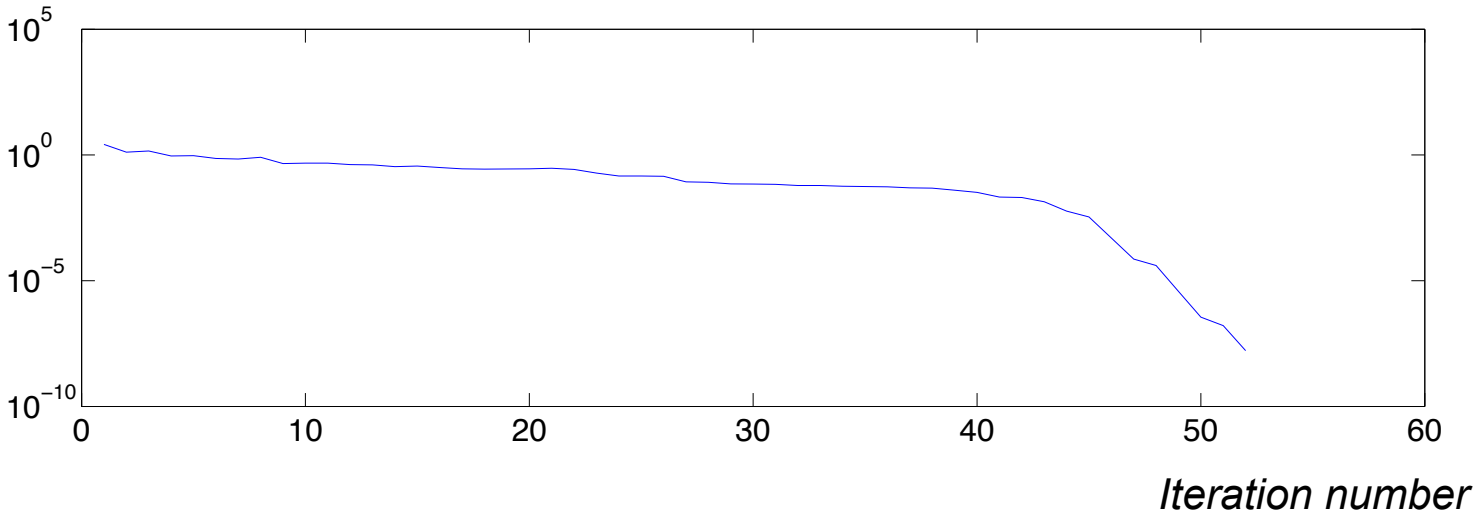
Steel cables, rubber, thin structures .....

But then .... the hard problems:

When decomposed into slices,
we have the classical „Schwarz -Wälder kirsch“ problem

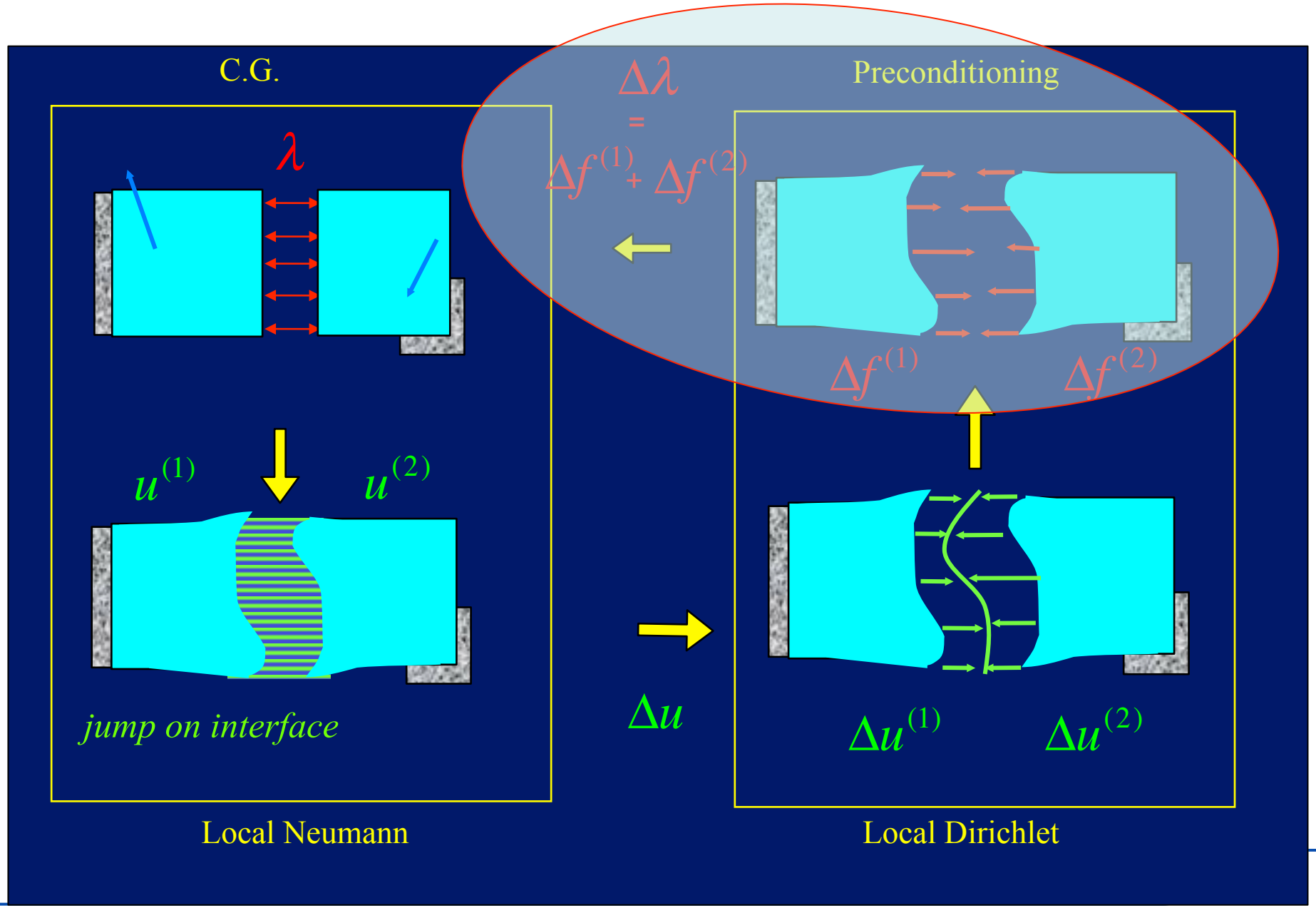
High heterogeneties ALONG the interface ! (scaling does not help)

Typical convergence of FETI on heterogeneous interface ALONG the interface

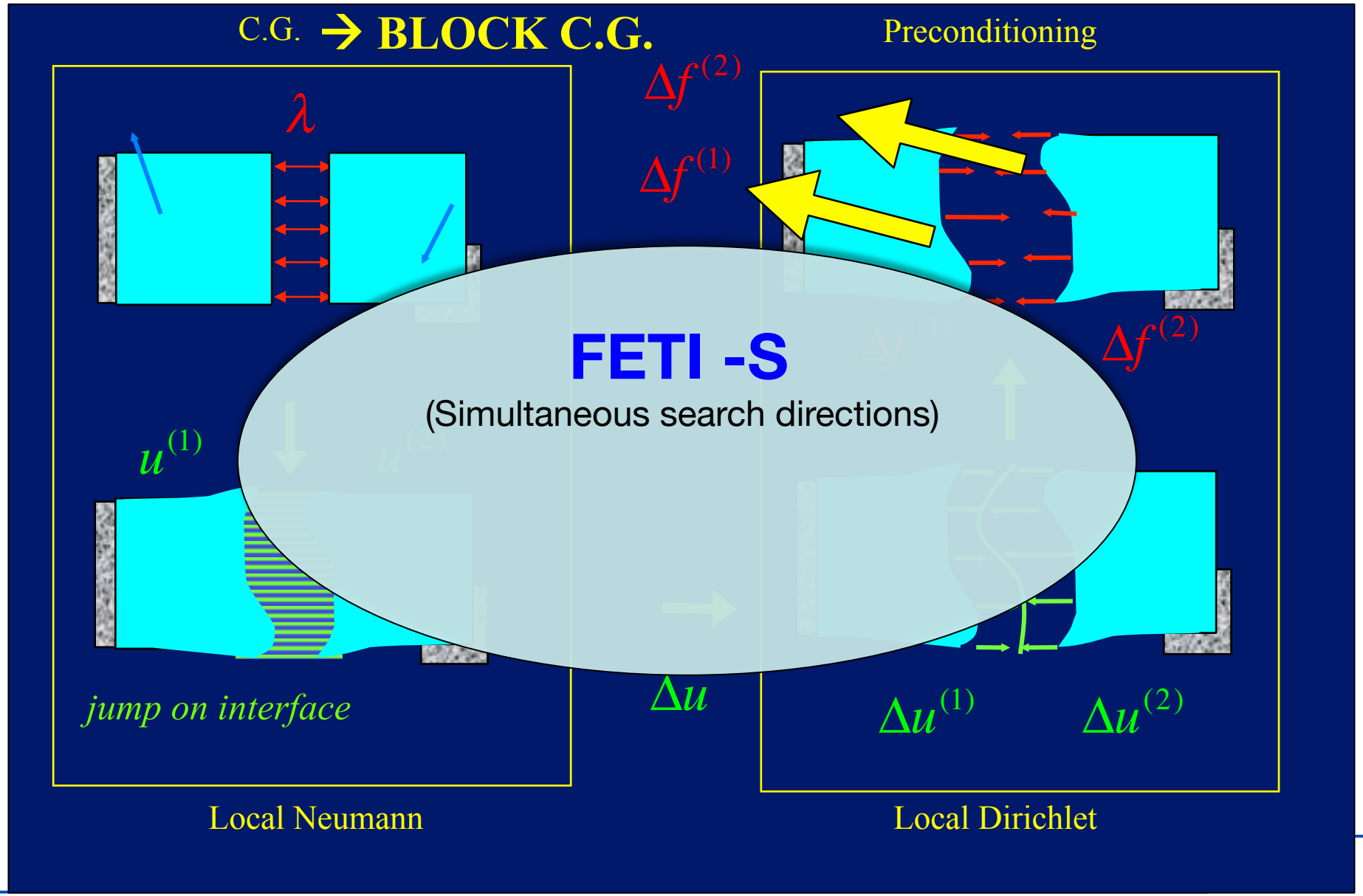


BAD !!!

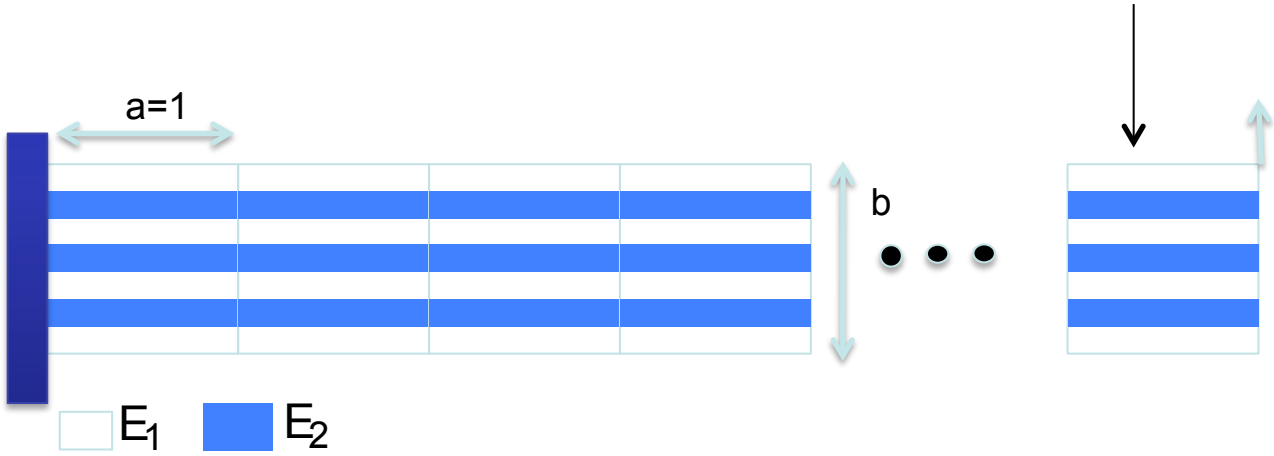
Where does it go wrong ??



IDEA: Recycle the maximum information computed in the preconditioner

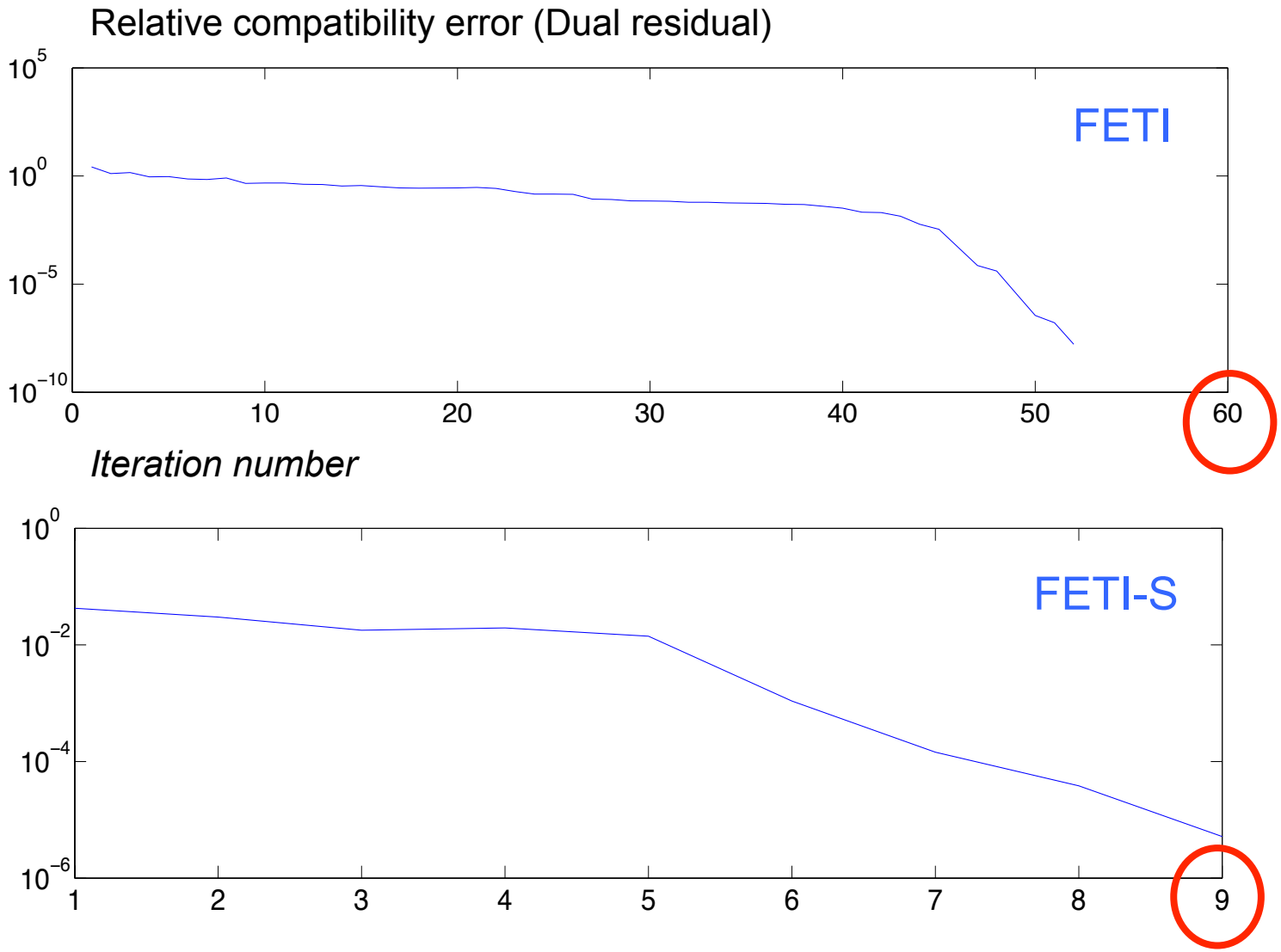


The „Schwarz – wälder kirsch“ problem

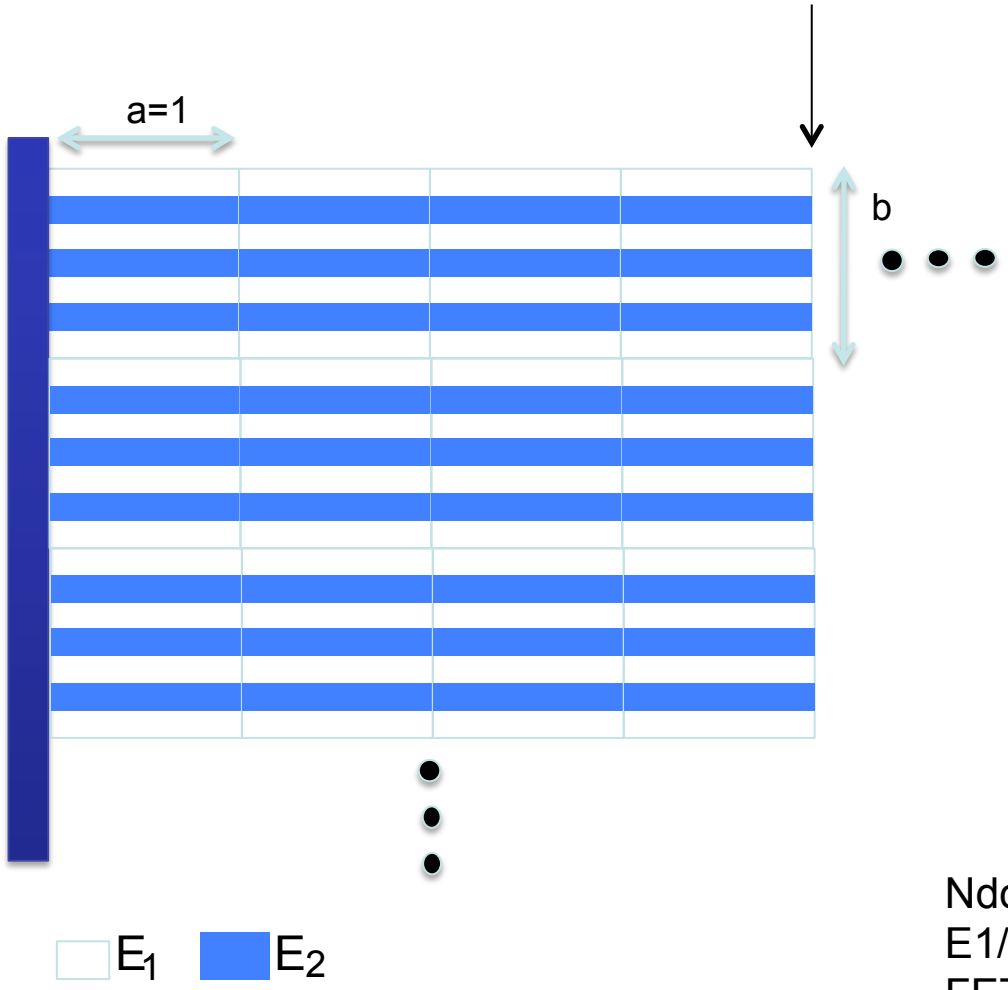


N domains = 8,
E1/E2 = 1e-5
FETI
FETI-S
Preconditioner: Dirichlet

Convergenge of FETI and FETI-S



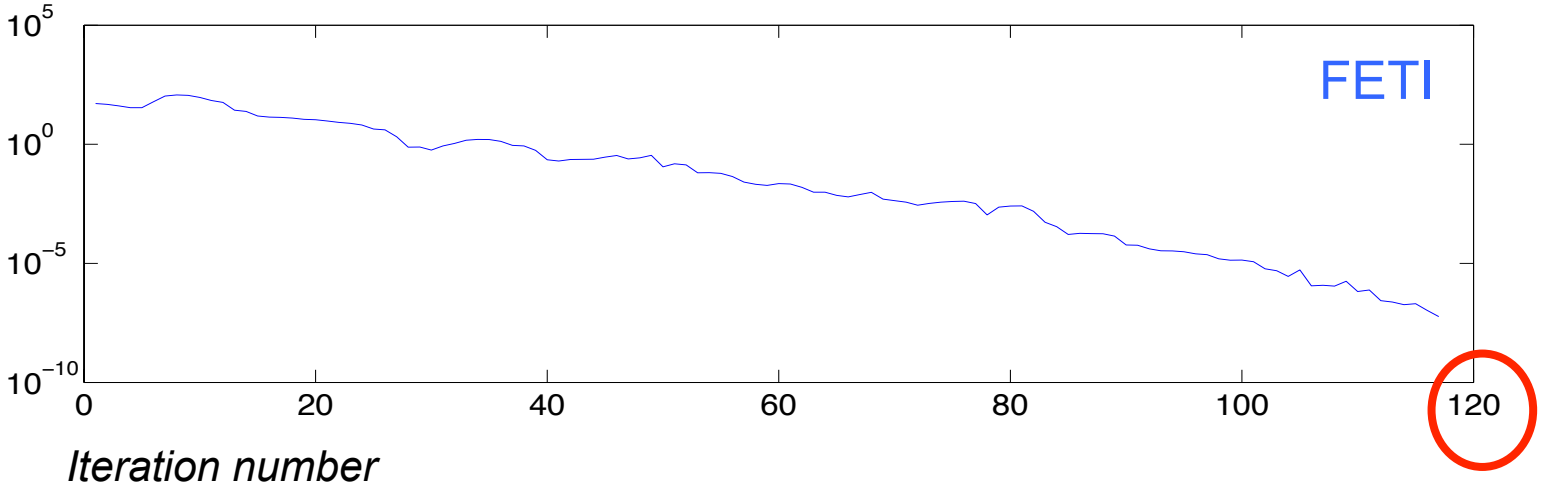
The „Schwarz – wälder kirsch“ problem
No piece of cake ....



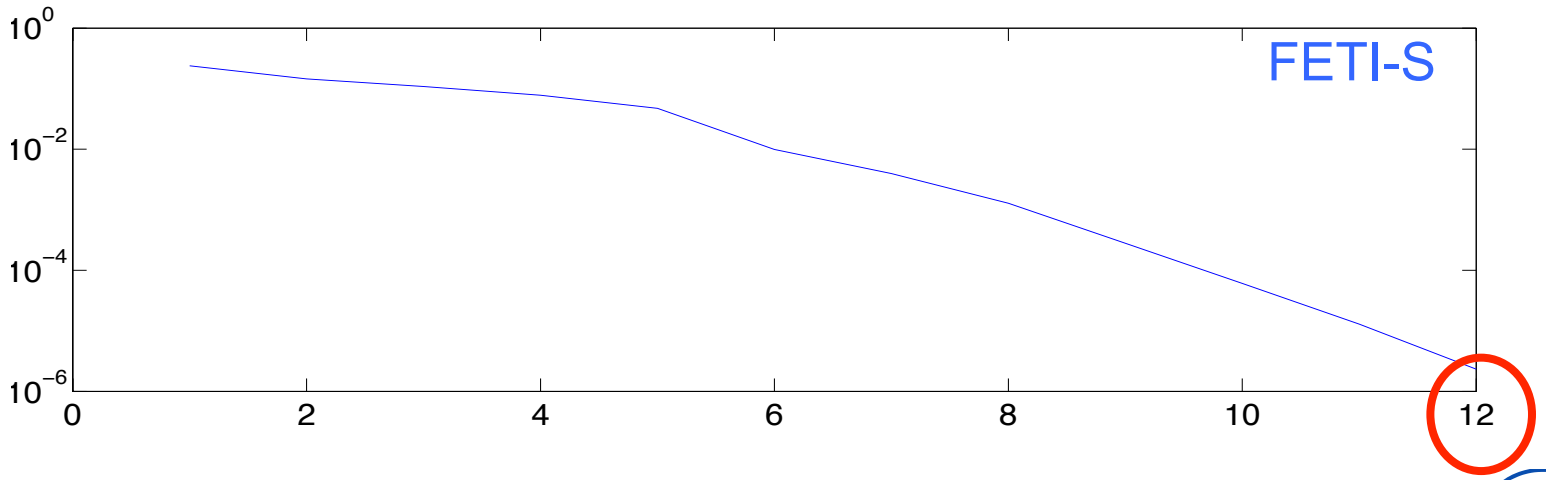
Ndomains = 4 X 8 = 32,
E1/E2 = 1e-5
FETI
FETI-S
Prec: Dirichlet

Convergence of interface problem

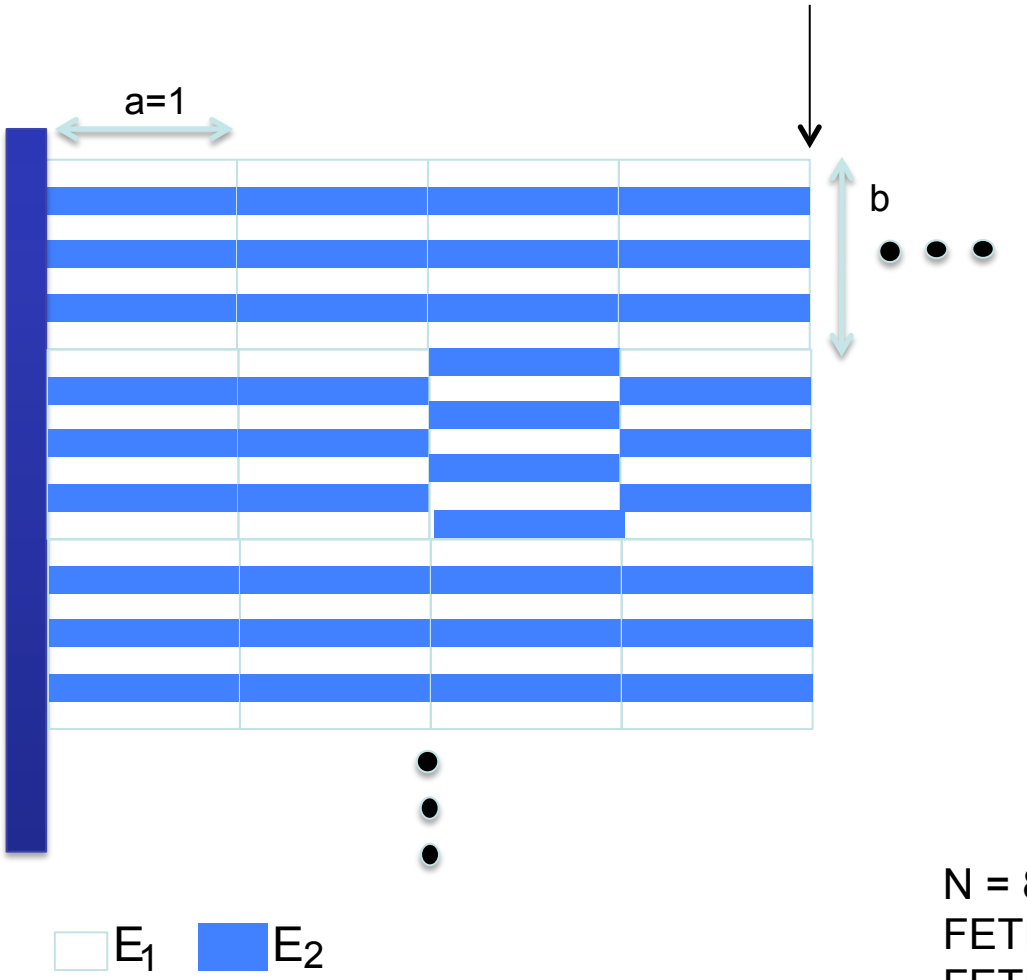
Relative compatibility error (Dual residual)



Iteration number

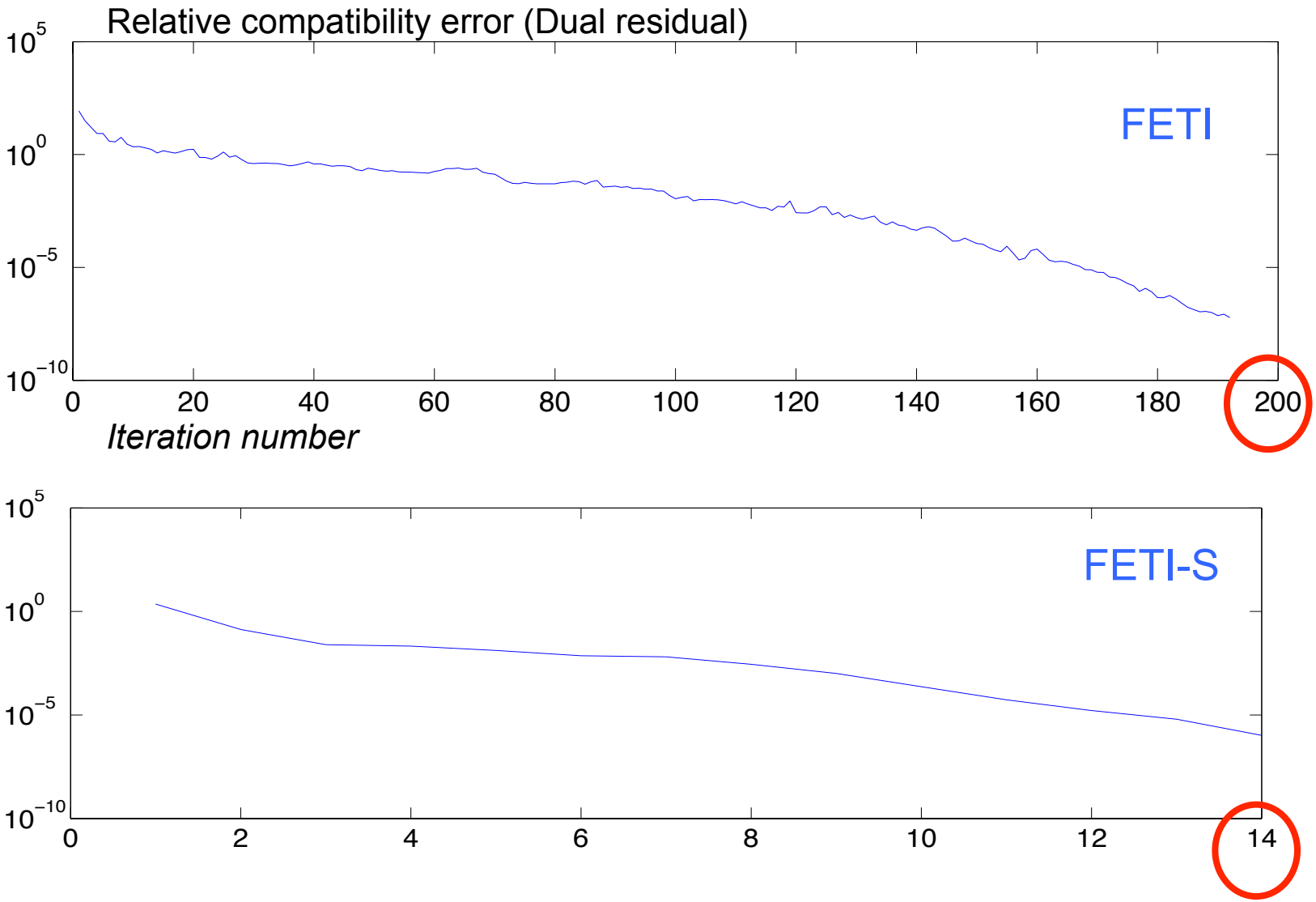


Test 2:
heterogeneities along and across interface



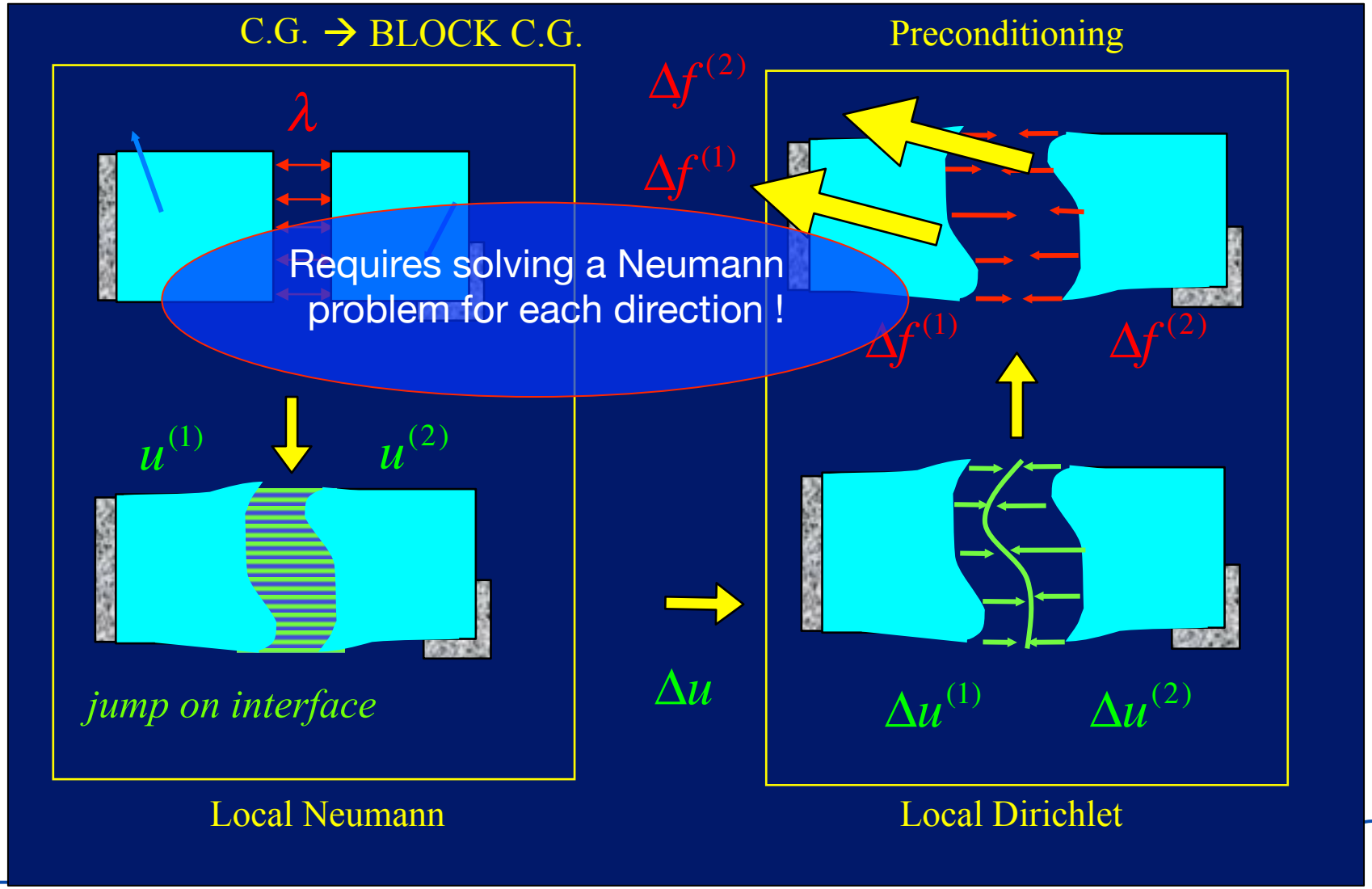
N = 8, E1/E2 = 1e-5
FETI
FETI-S
Prec: Dirichlet

Convergence of interface problem



But you might ask ..... [A. Patera, March 2013]

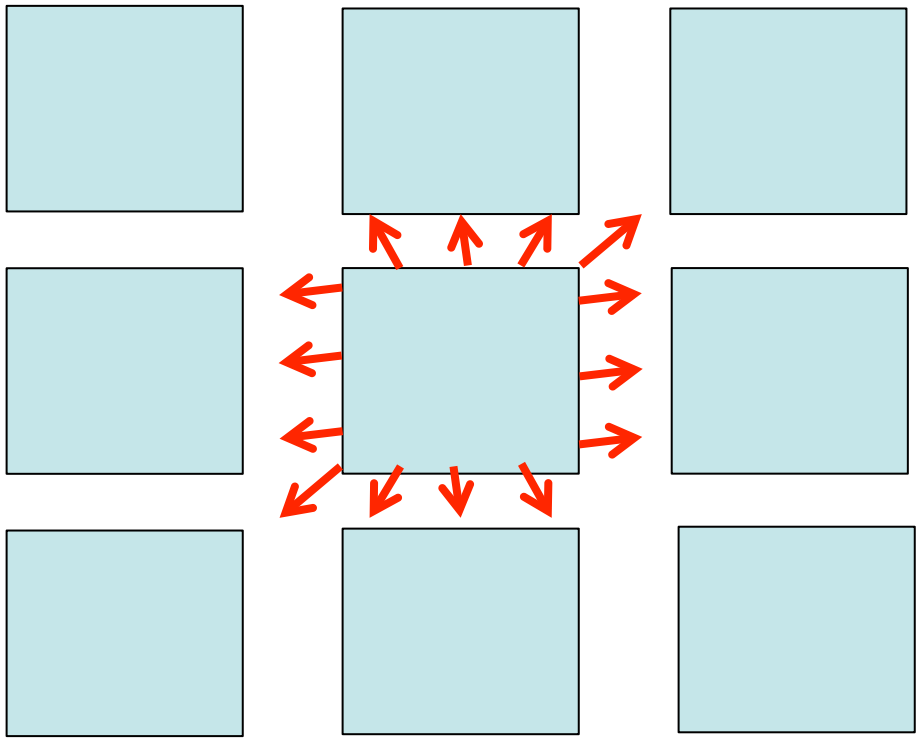
„Is the global number of operations really reduced with the FETI-S ?“



Cost ...

But

- Cost of Dirichlet preconditioner identical for one iteration
- Per direction of descent, the Neumann Problem is local (me and my neighbors)



Cost ...

So

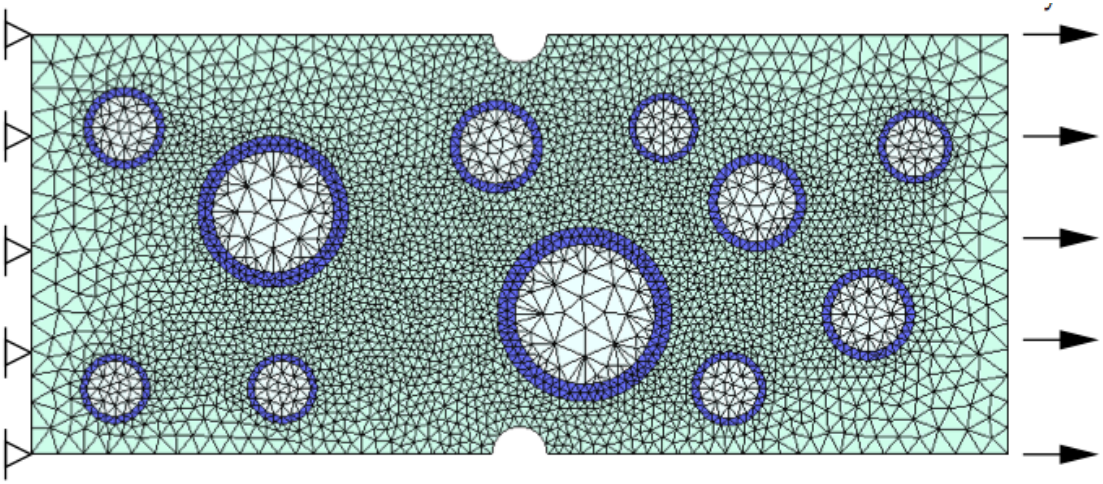
cost of 1 FETI-S iteration $\Longleftrightarrow$ 5 X cost of 1 FETI iteration

$\Longrightarrow$ Potentially about 50% gain in number of operations

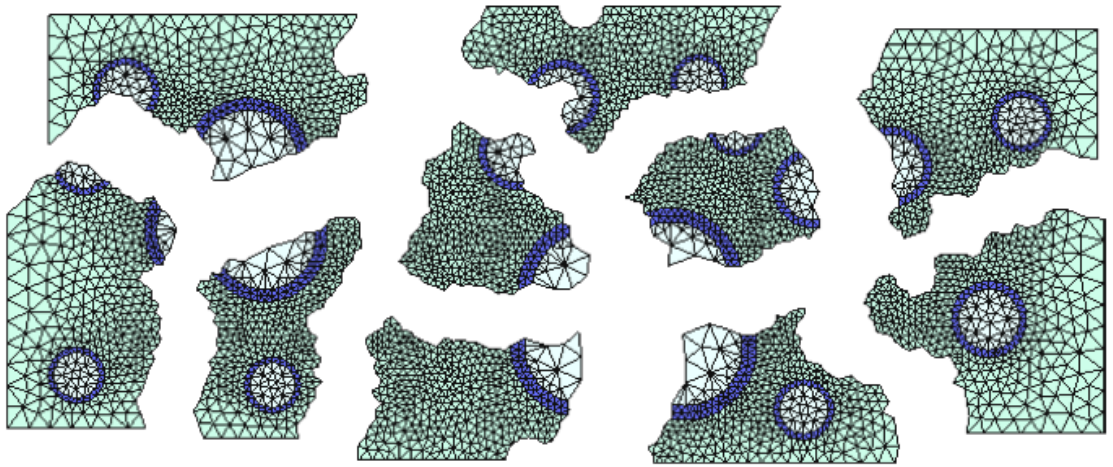
Further possible improvements:

- Use FETI-DP to reduce the number of neighbors
- Use only most important directions of descent (how to find them?)
- Make efficient use of block structure of operations.

Test case we are currently working on: Granular quasi-Brittle material



Decomposition in 10 domains using a graph partitioner



On micro-to-macro connections in domain decomposition multiscale methods,
O. Lloberas-Valls, D. J. Rixen, A. Simone and L. J. Sluys
Comput. Methods Appl. Mech. Engrg., vol 225–228, pp. 177–196, 2012

Conclusion

- FETI-S: simple extension of FETI
- Looks like one can significantly reduce the global number of operations
- Makes FETI robust against
  - heterogeneities along the interface
  - bad aspect ratios (not shown here)

Note:

similarities with the new FETI-Geneo to appear in

„Automatic spectral coarse spaces for robust FETI and BDD algorithms“, N.Spillane, D. Rixen, IJNME 2013