

(1) Calculate equilibrium solution with regards to rigid body modes

$$\lambda_{\mathbf{N}0} = \mathbf{A}\mathbf{G}(\mathbf{G}^T\mathbf{A}\mathbf{G})^{-1}\mathbf{e}$$

(2) Projector to natural subspace

$$\mathbf{P} = \mathbf{I} - \mathbf{A}\mathbf{G}(\mathbf{G}^T\mathbf{A}\mathbf{G})^{-1}\mathbf{G}^T$$

(3) Preconditioner

$$\mathbf{S} = \sum_s \mathbf{S}^{[s]}$$

(4) Coarse space equilibrium solution

$$\lambda_{\mathbf{C}0} = \mathbf{C}(\mathbf{C}^T\mathbf{F}\mathbf{C})^{-1}\mathbf{C}^T(\mathbf{d} - \mathbf{F}\lambda_{\mathbf{N}0})$$

(5) Coarse space projector

$$\mathbf{P}_c = \mathbf{I} - \mathbf{C}(\mathbf{C}^T\mathbf{F}\mathbf{C})^{-1}\mathbf{C}^T\mathbf{F}$$

(6) Calculate residual(gap) in natural subspace

$$\mathbf{r}_0 = \mathbf{P}^T(\mathbf{d} - \mathbf{F}\lambda_{\mathbf{N}0})$$

(7) Calculate resulting forces (preconditioning)

$$\mathbf{z} = \mathbf{S}\mathbf{r}_0$$

(8) Remove rigid body-components of forces

$$\mathbf{w}_0 = \mathbf{P}\mathbf{z}_0$$

(9) Initialize

$$\lambda_{\mathbf{F}0} = \mathbf{0}, i = 0$$

(10) While not converged

$$\sqrt{\mathbf{r}_i^T \mathbf{z}_i} > \epsilon$$

(11) Remove coarse space components of gap-change due to forces \mathbf{w}_i

$$\mathbf{q} = \mathbf{P}_c^T \mathbf{F} \mathbf{w}_i$$

(12) Auxiliary parameter

$$\delta_i = \mathbf{q}_i^T \mathbf{w}_i$$

(13) Auxiliary parameter

$$\gamma_i = \mathbf{r}_i^T \mathbf{z}_i$$

(14) Step in the new direction

$$\lambda_{\mathbf{F}i+1} = \lambda_{\mathbf{F}i} + (\gamma_i/\delta_i)\mathbf{w}_i$$

(15) Gap after force step

$$\mathbf{r}_{i+1} = \mathbf{r}_i - (\gamma_i/\delta_i)\mathbf{P}^T \mathbf{q}_i$$

(16) Calculate resulting forces (preconditioning)

$$\mathbf{z}_{i+1} = \mathbf{S}\mathbf{r}_{i+1}$$

(17) Remove rigid body-components of forces

$$\mathbf{w}_{i+1} = \mathbf{P}\mathbf{z}_{i+1}$$

(18) Loop over previous iterations

$$\text{for: } 0 \leq j \leq i$$

(19) Compute factor

$$\phi_{i,j} = \mathbf{q}_j^T \mathbf{w}_{i+1}$$

(20) Orhogonalize to direction j

$$\mathbf{w}_{i+1} \leftarrow \mathbf{w}_{i+1} - (\phi_{i,j}/\delta_j)\mathbf{w}_j$$

(21) Increase iteration counter

$$i \leftarrow i + 1$$

(22) Compute total interface forces

$$\lambda = \lambda_{\mathbf{N}0} + \lambda_{\mathbf{C}0} + \mathbf{P}_c \lambda_{\mathbf{F}i}$$