

(0) Calculate equilibrium solution with regards to rigid body modes

$$\lambda_{N0} = \mathbf{AG}(\mathbf{G}^T \mathbf{AG})^{-1} \mathbf{e}$$

(0) Projector to natural subspace

$$\mathbf{P} = \mathbf{I} - \mathbf{AG}(\mathbf{G}^T \mathbf{AG})^{-1} \mathbf{G}^T$$

(0) Preconditioner

$$\tilde{\mathbf{S}} = \sum_s \tilde{\mathbf{B}}^{(s)} \tilde{\mathbf{S}}^{(s)} \tilde{\mathbf{B}}^{(s)T}$$

(1) Calculate residual(gap) in natural subspace

$$\mathbf{r}_0 = \mathbf{P}^T (\mathbf{d} - \mathbf{F} \lambda_{N0})$$

(2) Calculate resulting forces (preconditioning)

$$\mathbf{z} = \tilde{\mathbf{S}} \mathbf{r}_0,$$

(3) Remove rigid body-components of forces

$$\mathbf{w}_0 = \mathbf{P} \mathbf{z}_0$$

(4) Initialize

$$\lambda_{F0} = \mathbf{0}, i = 0$$

(5) While not converged

$$\sqrt{\mathbf{r}_i^T \mathbf{z}_i} > \epsilon$$

(6) Compute gap-change due to forces \mathbf{w}_i

$$\mathbf{q} = \mathbf{F} \mathbf{w}_i$$

(7) Energy of???

$$\delta_i = \mathbf{q}_i^T \mathbf{w}_i$$

(8) Energy of???

$$\gamma_i = \mathbf{r}_i^T \mathbf{z}_i$$

(9) Step in the new direction

$$\lambda_{Fi+1} = \lambda_{Fi} + (\gamma_i / \delta_i) \mathbf{w}_i$$

(10) Gap after force step

$$\mathbf{r}_{i+1} = \mathbf{r}_i - (\gamma_i / \delta_i) \mathbf{P}^T \mathbf{q}_i$$

(11) Calculate resulting forces (preconditioning)

$$\mathbf{z}_{i+1} = \tilde{\mathbf{S}} \mathbf{r}_{i+1}$$

(12) Remove rigid body-components of forces

$$\mathbf{w}_{i+1} = \mathbf{P} \mathbf{z}_{i+1}$$

(13) Loop over previous iterations

$$\text{for: } 0 \leq j \leq i$$

(14) Compute factor

$$\phi_{i,j} = \mathbf{q}_j^T \mathbf{w}_{i+1}$$

(15) Orthogonalize to direction j

$$\mathbf{w}_{i+1} \leftarrow \mathbf{w}_{i+1} - (\phi_{i,j} / \delta_j) \mathbf{w}_j$$

(16) Increase iteration counter

$$i \leftarrow i + 1$$

(17) Compute total interface forces

$$\lambda = \lambda_{N0} + \lambda_{Fi}$$