

(x) Calculate equilibrium solution with regards to rigid body modes

$$\lambda_{\mathbf{N}0} = \mathbf{A}\mathbf{G}(\mathbf{G}^T\mathbf{A}\mathbf{G})^{-1}\mathbf{e}$$

(x) Projector to natural subspace

$$\mathbf{P} = \mathbf{I} - \mathbf{A}\mathbf{G}(\mathbf{G}^T\mathbf{A}\mathbf{G})^{-1}\mathbf{G}^T$$

(x) Preconditioner

$$\tilde{\mathbf{S}} = \sum_s \tilde{\mathbf{B}}^{(s)} \tilde{\mathbf{S}}^{(s)} \tilde{\mathbf{B}}^{(s)T}$$

(x) Coarse space equilibrium solution

$$\underline{\lambda_{\mathbf{C}0} = \mathbf{C}(\mathbf{C}^T\mathbf{F}\mathbf{C})^{-1}\mathbf{C}^T(\mathbf{d} - \mathbf{F}\lambda_{\mathbf{N}0})}$$

(x) Coarse space projector

$$\underline{\mathbf{P}_c = \mathbf{I} - \mathbf{C}(\mathbf{C}^T\mathbf{F}\mathbf{C})^{-1}\mathbf{C}^T\mathbf{F}}$$

(x) Calculate residual(gap) in natural subspace

$$\mathbf{r}_0 = \mathbf{P}^T(\mathbf{d} - \mathbf{F}\lambda_{\mathbf{N}0})$$

(x) Calculate resulting forces (preconditioning)

$$\mathbf{z} = \tilde{\mathbf{S}}\mathbf{r}_0,$$

(x) Remove rigid body-components of forces

$$\mathbf{w}_0 = \mathbf{P}\mathbf{z}_0$$

(x) Initialize

$$\lambda_{\mathbf{F}0} = \mathbf{0}, i = 0$$

(x) While not converged

$$\sqrt{\mathbf{r}_i^T \mathbf{z}_i} > \epsilon$$

(6) Remove coarse space components of gap-change due to forces \mathbf{w}_i

$$\mathbf{q} = \underline{\mathbf{P}_c}^T \mathbf{F} \mathbf{w}_i$$

(x) Energy of???

$$\delta_i = \mathbf{q}_i^T \mathbf{w}_i$$

(x) Energy of???

$$\gamma_i = \mathbf{r}_i^T \mathbf{z}_i$$

(x) Step in the new direction

$$\lambda_{\mathbf{F}i+1} = \lambda_{\mathbf{F}i} + (\gamma_i/\delta_i)\mathbf{w}_i$$

(x) Gap after force step

$$\mathbf{r}_{i+1} = \mathbf{r}_i - (\gamma_i/\delta_i)\mathbf{P}^T \mathbf{q}_i$$

(x) Calculate resulting forces (preconditioning)

$$\mathbf{z}_{i+1} = \tilde{\mathbf{S}}\mathbf{r}_{i+1}$$

(x) Remove rigid body-components of forces

$$\mathbf{w}_{i+1} = \mathbf{P}\mathbf{z}_{i+1}$$

(x) Loop over previous iterations

$$\text{for: } 0 \leq j \leq i$$

(x) Compute factor

$$\phi_{i,j} = \mathbf{q}_j^T \mathbf{w}_{i+1}$$

(x) Orhorgonalize to direction j

$$\mathbf{w}_{i+1} \leftarrow \mathbf{w}_{i+1} - (\phi_{i,j}/\delta_j)\mathbf{w}_j$$

(x) Increase iteration counter

$$i \leftarrow i + 1$$

(x) Compute total interface forces

$$\lambda = \lambda_{\mathbf{N}0} + \underline{\lambda_{\mathbf{C}0}} + \underline{\mathbf{P}_c}\lambda_{\mathbf{F}i}$$