

(1) Calculate equilibrium solution with regards to rigid body modes

$$\lambda_{\mathbf{N}0} = \mathbf{AG}(\mathbf{G}^T \mathbf{AG})^{-1} \mathbf{e}$$

(2) Projector to natural subspace

$$\mathbf{P} = \mathbf{I} - \mathbf{AG}(\mathbf{G}^T \mathbf{AG})^{-1} \mathbf{G}^T$$

(3) Calculate residual(gap) in natural subspace

$$\mathbf{r}_0 = \mathbf{P}^T (\mathbf{d} - \mathbf{F} \lambda_0)$$

(4) Preconditioning

$$\underline{\mathbf{Z}}_0 = \mathbf{S} \mathbf{r}_0$$

(5) Rigid body-components of forces

$$\mathbf{W}_0 = \mathbf{P} \mathbf{Z}_0$$

(6) Initialize

$$\lambda_{\mathbf{F}0} = 0, i = 0$$

While not converged

$$\sqrt{\mathbf{r}^T \mathbf{Z} \mathbf{I}} > \epsilon$$

(7) Compute gap-changes due to forces \mathbf{W}_i

$$\mathbf{Q}_i = \mathbf{F} \mathbf{W}_i$$

(8) Auxiliary parameter

$$\Delta_i = \mathbf{Q}_i^T \mathbf{W}_i$$

(9) Auxiliary parameter

$$\gamma_i = \mathbf{Z}_i^T \mathbf{r}_i$$

(10) Auxiliary parameter

$$\alpha_i = \Delta_i^+ \gamma_i$$

(11) Step in new direction

$$\lambda_{\mathbf{F}i+1} = \lambda_{\mathbf{F}i} + \mathbf{W}_i \alpha_i$$

(12) Gap change due to force step

$$\mathbf{r}_{i+1} = \mathbf{r}_i - \mathbf{P}^T \mathbf{Q}_i \alpha_i$$

(13) New

$$\underline{\mathbf{Z}}_{i+1} = \mathbf{S} \mathbf{r}_{i+1}$$

Loop over all substructures

for $s = 1, \dots, N$ do

(14) Compute convergence indicator

$$t_i^s = \frac{\langle \mathbf{W}_i \alpha_i, \mathbf{F}^{[s]} \mathbf{W}_i \alpha_i \rangle}{\mathbf{r}_{i+1}^T \mathbf{S}^{[s]} \mathbf{r}_{i+1}}$$

(15) τ - test

$t_i^s < \tau$?

true

false

(16) Add direction to search space

$$\mathbf{Z}_{i+1} = [\mathbf{Z}_{i+1} | \mathbf{S}^{[s]} \mathbf{r}]$$

(17) Do nothing

(18) Remove rigid body-components of forces

$$\mathbf{W}_{i+1} = \mathbf{P} \mathbf{Z}_{i+1}$$

Loop over previous iterations

for: $0 \leq j \leq i$

(19) Compute orthogonalization factor

$$\phi_{i,j} = \mathbf{Q}_j^T \mathbf{W}_{i+1}$$

(20) Orthogonalize to direction j

$$\mathbf{W}_{i+1} \leftarrow \mathbf{W}_{i+1} - \mathbf{W}_j \Delta_j^+ \phi_{i,j}$$

(21) Increase iteration counter

$$i \leftarrow i + 1$$

(22) Compute total interface forces

$$\lambda = \lambda_{\mathbf{N}0} + \lambda_{\mathbf{F}0}$$