Cryptography

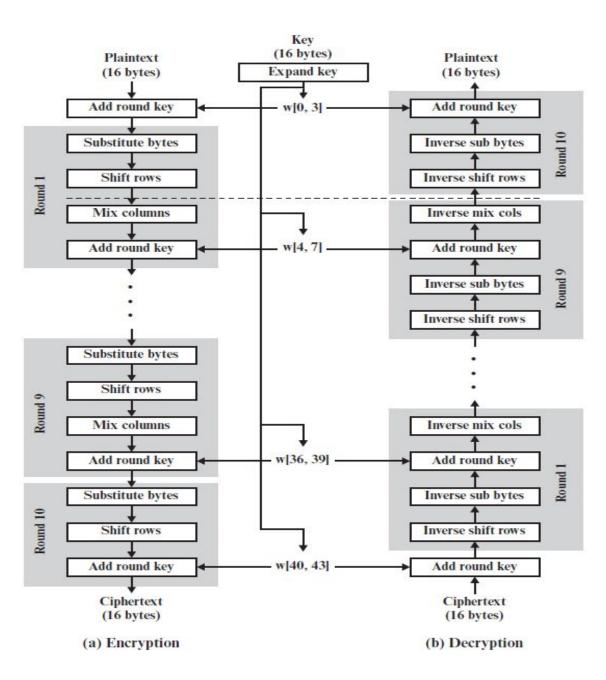
AES, Decryption, Key Expansion

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Topics

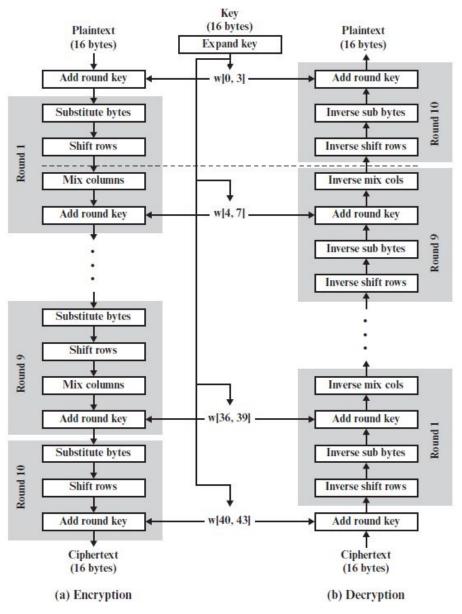
- Finite Field Arithmetic
- AES Structure
 - AES transformation structure
 - Fix row transformation
 - Mixed row transformation
 - Addround transformation
- AES Key Expansion
- Avalanche effect
- Relationship between Rijndael and AES

AES Encryption and Decryption



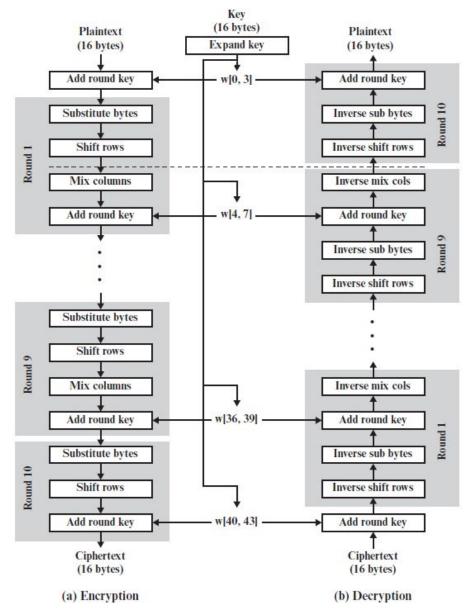
AES Encryption and Decryption

- Each stage is easily reversible. For the Substitute Byte, ShiftRows, and MixColumns stages, an **inverse function is used** in the decryption algorithm.
- For the AddRoundKey stage, the **inverse** is achieved by XORing the **same round key to the block, using the result that A B B B A**
- As with most block ciphers, the decryption algorithm makes use of the expanded key in reverse order.
- However, the decryption algorithm is not identical to the encryption algorithm. This is a consequence of the particular structure of AES.



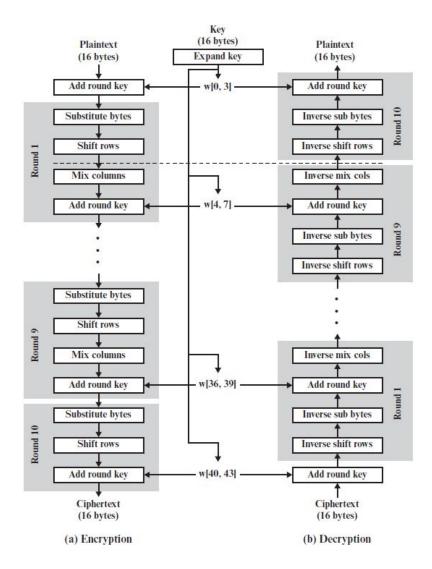
AES Encryption and Decryption

- Once it is established that all four stages are reversible, it is easy to verify that decryption does recover the plaintext
- The final round of both encryption and decryption consists of only three stages.
- this is a consequence of the particular structure of AES and is required to make the cipher reversible.



AES Encryption and Decryption

- AES decryption cipher is **not identical to the encryption** cipher. That is, the sequence of transformations for decryption differs from that for encryption.
- the form of the key schedules for encryption and decryption is the same.
- This has the disadvantage that two separate software or firmware modules are needed for applications that require both encryption and decryption.



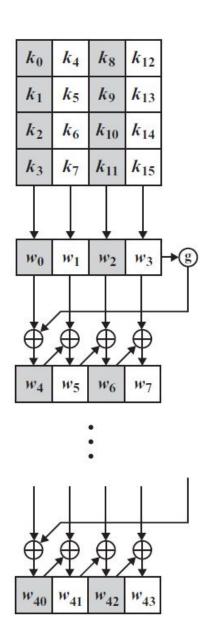
AES Analysis

AES Analysis

- In present day cryptography, AES is widely adopted and supported in both hardware and software.
- Till date, no practical cryptanalytic attacks against AES has been discovered.
- Additionally, **AES has built-in flexibility of key length**, which allows a degree of 'future-proofing' against progress in the ability to perform exhaustive key searches.
- However, just as for DES, the AES security is assured only if it is **correctly implemented**, and good **key management** is employed.

Key Expansion Algorithm

- The AES key expansion algorithm takes as input a four-word (16-byte) key and produces a linear array of 44 words (176 bytes).
- This is sufficient to provide a four-word round key for the initial AddRoundKey stage and each of the 10 rounds of the cipher.
- The key is copied into the first four words of the expanded key.
- The remainder of the expanded key is filled in four words at a time.
- Each added word w[i] depends on the **immediately preceding** word, w[i 1], and the word four positions back, w[i 4]. In three out of four cases, a simple XOR is used.
- For a word whose position in the w array is a multiple of 4, a more complex function is used.



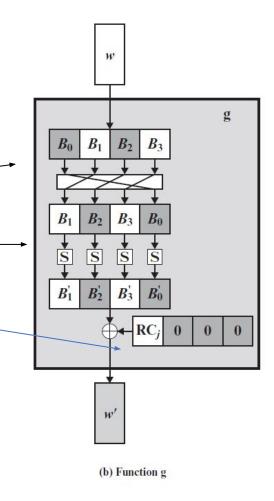
Key Expansion: How algorithm works?

- Initial Key The algorithm makes use of an initial key. Depending on the level of security needed, this key's length can range from 128 to 256 bits.
- Round Constants The approach makes use of a set of round constants, which are predefined values used in the key expansion process.
- Word Size The key divides words into individual blocks. A word typically has 32 bits in it. For example, four 32-bit words are created from a 128-bit key.
- **Key Schedule** The term "key schedule" refers to a **set of round keys generated using the key expansion process**. The initial round key and the extra round keys that were derived from it are both included in this schedule.

expansion of the 16-byte key into 10 round keys. As previously explained, this process is performed word by word, with each four-byte word occupying one column of the word round-key matrix.

Key Expansion : How algorithm work

- Expansion Rounds The algorithm performs several tasks in each expansion round, such as –
- Complex function. The function g consists of the following subfunctions.
 - RotWord This function rotates the bytes in a word.
 - SubWord Applies a substitution operation using a predetermined S-box.
 - Rcon XORs the word using a round constant.
- Round Keys The order of round keys that are still in place following all expansion rounds defines the key schedule.
 Each round key is used in the corresponding round of AES encryption or decryption.



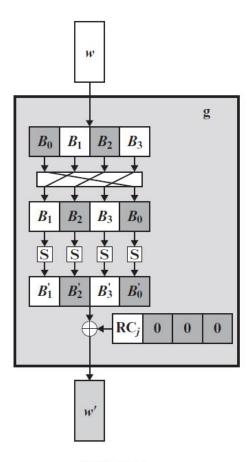
Overall, the key expansion method increases security and prevents cryptographic attacks by ensuring that each AES encryption and decryption round has a unique round key.

Key Expansion: How algorithm works'

Complex function. The **function g** consists of the following subfunctions.

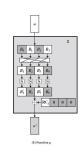
The result of steps 1 and 2 is XORed with a round constant, Rcon[j].

- The **round constant** is a word in which the three rightmost bytes are always 0.
- Thus, the effect of an XOR of a word with Rcon is to only perform an XOR on the leftmost byte of the word.
- The round constant is different for each round and is defined as Rcon[j] = (RC[j], 0, 0, 0), with
- RC[1] = 1, RC[j] = 2 . RC[j 1]
- and with multiplication defined over the field GF(28).



(b) Function g

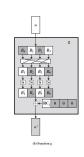
How algorithm works?



- Rationale
- The Rijndael developers **designed the expansion** key algorithm to be **resistant to known cryptanalytic attacks**.
- The inclusion of a round-dependent round constant eliminates the symmetry, or similarity, between the ways in which round keys are generated in different rounds.

• specific criteria ...

How algorithm works?



The **specific criteria** that were used are [DAEM99]

- Knowledge of a part of the cipher key or round key does not enable calculation of many other round-key bits.
- An **invertible transformation** [i.e., knowledge of any Nk consecutive words of the expanded key enables regeneration of the entire expanded key (Nk = key size in words)].
- **Speed** on a wide range of processors.
- Usage of round constants to eliminate symmetries.
- Diffusion of cipher key differences into the round keys; that is, each key bit affects many round key bits.
- **Enough nonlinearity** to prohibit the full determination of round key differences from cipher key differences only.
- Simplicity of description.

AES Example

- Table shows the progression of State through the AES encryption process.
- The **first column shows** the value of State at the start of a round. For the first row, State is just the matrix arrangement of the plaintext.
- The **second, third, and fourth columns** show the value of State for that round after the SubBytes, ShiftRows, and MixColumns transformations, respectively.
- The **fifth column** shows the round key.
- 55
- The first column shows the value of State resulting from the bitwise XOR of State after
- the preceding MixColumns with the round key for the preceding round.

AES Example

Start of Round	After SubBytes	After ShiftRows	After MixColumns	Round Key
01 89 fe 76	*			Of 47 Oc af
23 ab dc 54				15 d9 b7 7f
45 cd ba 32				71 e8 ad 67
67 ef 98 10				c9 59 d6 98
0e ce f2 d9	ab 8b 89 35	ab 8b 89 35	b9 94 57 75	dc 9b 97 38
36 72 6b 2b	05 40 7f f1	40 7f f1 05	e4 8e 16 51	90 49 fe 81
34 25 17 55	18 3f f0 fc	f0 fc 18 3f	47 20 9a 3f	37 df 72 15
ae b6 4e 88	e4 4e 2f c4	c4 e4 4e 2f	c5 d6 f5 3b	b0 e9 3f a7
65 Of c0 4d	4d 76 ba e3	4d 76 ba e3	8e 22 db 12	d2 49 de e6
74 c7 e8 d0	92 c6 9b 70	c6 9b 70 92	b2 f2 dc 92	c9 80 7e ff
70 ff e8 2a	51 16 9b e5	9b e5 51 16	df 80 f7 c1	6b b4 c6 d3
75 3f ca 9c	9d 75 74 de	de 9d 75 74	2d c5 1e 52	b7 5e 61 c6
5c 6b 05 f4	4a 7f 6b bf	4a 7f 6b bf	b1 c1 0b cc	c0 89 57 b1
7b 72 a2 6d	21 40 3a 3c	40 3a 3c 21	ba f3 8b 07	af 2f 51 ae
b4 34 31 12	8d 18 c7 c9	c7 c9 8d 18	f9 1f 6a c3	df 6b ad 7e
9a 9b 7f 94	b8 14 d2 22	22 b8 14 d2	1d 19 24 5c	39 67 06 c0
71 48 5c 7d	a3 52 4a ff	a3 52 4a ff	d4 11 fe Of	2c a5 f2 43
15 dc da a9	59 86 57 d3	86 57 d3 59	3b 44 06 73	5c 73 22 8c
26 74 c7 bd	f7 92 c6 7a	c6 7a f7 92	cb ab 62 37	65 0e a3 dd
24 7e 22 9c	36 f3 93 de	de 36 f3 93	19 b7 07 ec	f1 96 90 50
f8 b4 0c 4c	41 8d fe 29	41 8d fe 29	2a 47 c4 48	58 fd 0f 4c
67 37 24 ff	85 9a 36 16	9a 36 16 85	83 e8 18 ba	9d ee cc 40
ae a5 c1 ea	e4 06 78 87	78 87 e4 06	84 18 27 23	36 38 9b 46
e8 21 97 bc	9b fd 88 65	65 9b fd 88	eb 10 0a f3	eb 7d ed bd
72 ba cb 04	40 f4 1f f2	40 f4 1f f2	7b 05 42 4a	71 8c 83 cf
1e 06 d4 fa b2 20 bc 65	72 6f 48 2d	6f 48 2d 72 65 4d 37 b7	1e d0 20 40 94 83 18 52	c7 29 e5 a5 4c 74 ef a9
00 6d e7 4e	37 b7 65 4d 63 3c 94 2f	2f 63 3c 94	94 83 18 52 94 c4 43 fb	c2 bf 52 ef
	67 a7 78 97	67 a7 78 97		37 bb 38 f7
0a 89 c1 85 d9 f9 c5 e5	35 99 a6 d9	99 a6 d9 35	ec 1a c0 80 0c 50 53 c7	14 3d d8 7d
d8 f7 f7 fb	61 68 68 Of	68 Of 61 68	3b d7 00 ef	93 e7 08 a1
56 7b 11 14	b1 21 82 fa	fa b1 21 82	b7 22 72 e0	48 f7 a5 4a
db a1 f8 77	b9 32 41 f5	b9 32 41 f5	b1 1a 44 17	48 f3 cb 3c
18 6d 8b ba	ad 3c 3d f4	3c 3d f4 ad	3d 2f ec b6	26 1b c3 be
a8 30 08 4e	c2 04 30 2f	30 2f c2 04	0a 6b 2f 42	45 a2 aa 0b
ff d5 d7 aa	16 03 0e ac	ac 16 03 0e	9f 68 f3 b1	20 d7 72 38
f9 e9 8f 2b	99 le 73 fl	99 1e 73 f1	31 30 3a c2	fd 0e c5 f9
1b 34 2f 08	af 18 15 30	18 15 30 af	ac 71 8c c4	0d 16 d5 6b
4f c9 85 49	84 dd 97 3b	97 3b 84 dd	46 65 48 eb	42 e0 4a 41
bf bf 81 89	08 08 0c a7	a7 08 08 0c	6a 1c 31 62	cb 1c 6e 56
cc 3e ff 3b	4b b2 16 e2	4b b2 16 e2		b4 ba 7f 86
a1 67 59 af	32 85 cb 79	85 cb 79 32		8e 98 4d 26
04 85 02 aa	f2 97 77 ac	77 ac f2 97		f3 13 59 18
a1 00 5f 34	32 63 cf 18	18 32 63 cf		52 4e 20 76
ff 08 69 64			*	
0b 53 34 14				
84 bf ab 8f				
4a 7c 43 b9				

Mathematical Representation

- Let us denote –
- K is the first key, having N bits in its length.
- The word count (Nk) of the key is its total word count (e.g., 4 for a 128-bit key, 6 for a 192-bit key, and 8 for a 256-bit key).
- Nr, the AES round identifier, is 10 for AES-128, 12 for AES-192, and 14 for AES-256.

Key Expansion Process

- Step 1: Make the round keys first. Establish the first word K in a word array W.
- Iterate to generate (Nr + 1) round keys -
- Step 2: Finalize Round Keys: After the loop, the array W has all round keys.

Notations

- RotWord(w) Rotate the word w's bytes in a manner that is cyclic.
- SubWord(w) Use the AES S-box to swap out each byte in the word w.
- Rcon(i) Produce the current round's i 's round constant, or Rcon.

Example of AES Key Expansion

- Now we are going to discuss the AES (Advanced Encryption Standard) key expansion algorithm with an example. For this example, we will be using AES-128, so our initial key will be 128 bits (16 bytes).
- The first key can be considered as a set of bytes –
- 2b 7e 15 16 28 ae d2 a6 ab f7 97 66 76 15 13 1
- This key is 128 bits long.
- Let's look at every step of the key expansion process –
- Initial Key

2b 7e 15 16

28 ae d2 a6

ab f7 97 66

76 15 13 1

Example of AES Key Expansion

- Expansion Rounds
- We start by adding the first key to our list of round keys. Following that, we iteratively generate more round keys until we reach a total of 11.
- Round 1
- In the RotWord, SubWord, and XOR operations, we use the round constant —

RotWord: 7e 15 16 2b SubWord: 63 cb e7 8c

Rcon: 01 00 00 00

Round 1 Key: 63 cb e7 8c 09 cf 4f 3c 3b a9 82 fb 11 13 d8 2c

Round 2

The word from the previous round key is XORed four positions back —

Round 2 Key: a0 fa fe 17 88 54 2c b1 23 a3 39 39 2a 6c 76 05

Example of AES Key Expansion

- Subsequent Rounds
- Round keys are generated in this way until a total of 11 keys have been generated.
- Final Round Keys

Round 0 (Initial Key): 2b 7e 15 16 28 ae d2 a6 ab f7 97 66 76 15 13 1

Round 1: 63 cb e7 8c 09 cf 4f 3c 3b a9 82 fb 11 13 d8 2c

Round 2: a0 fa fe 17 88 54 2c b1 23 a3 39 39 2a 6c 76 05

...

Round 10: 3d 47 0e 52 77 37 2e 10 1f 7e 0e 20 6a 51 7f a7

In this example, an original key was expanded into a number of round keys using the AES key expansion technique. Every round key is created from the one before it using round constants and operations like XOR, RotWord, and SubWord. These round keys are then used in each round of AES encryption to increase security and prevent cryptographic attacks.

Key Expansion in AES

- Key expansion, also known as the AES key schedule, is a critical process in AES
 (Advanced Encryption Standard) that generates multiple round keys from the original
 encryption key.
- These round keys are used in each round of AES encryption and decryption.
- Overview of Key Expansion:
- AES uses keys of different lengths depending on the variant:
 - AES-128 uses a 128-bit key (16 bytes).
 - AES-192 uses a 192-bit key (24 bytes).
 - AES-256 uses a 256-bit key (32 bytes).
- The key expansion algorithm takes the initial key and derives a series of round keys from it. Each round of AES (which is either 10, 12, or 14 rounds depending on the key size) uses a unique round key.

1. Initial Key:

- The **original key** is the starting point of the key schedule.
- It's divided into **4-byte words** (W[0], W[1], ..., W[N-1]), where N depends on the key size.
- For AES-128, the original key is split into 4 words (W[0] to W[3]).

2. Word Generation:

- The key expansion generates a total of **44 words** (for AES-128), **52 words** (for AES-192), or **60 words** (for AES-256), depending on the key length.
- Each round uses 4 words (16 bytes) as the round key, which is why the number of words is directly linked to the number of rounds in AES.

3. RotWord:

- Every four words, a transformation called RotWord is applied to the last word. This operation takes a word and performs a cyclic left shift on its bytes. For example:
 - Input: W[3] = {0x1a, 0x2b, 0x3c, 0x4d}
 - RotWord result: {0x2b, 0x3c, 0x4d, 0x1a}

4. SubWord:

- After RotWord, each byte of the word undergoes a substitution using the AES S-Box, a non-linear substitution box used to provide confusion.
- Each byte is replaced with its corresponding value from the S-Box table.

5. Rcon (Round Constant):

- A round constant (Rcon) is XORed with the first byte of the word after SubWord.
- The round constant is a pre-defined set of values that depend on the round number.
- For example, for the **first round, Rcon might be 0x01**, and it **increases exponentially** for each subsequent round.

6. XOR Operations:

The result of the SubWord and Rcon operations is XORed with the word from four positions back.
 This is the key expansion's recursive part, ensuring that each round key is dependent on the previous one.

7. Final Round Keys:

- The process continues until enough round keys are generated for the encryption process.
- For AES-128, this means generating 44 words, which correspond to 10 rounds + 1 initial round key (because the round key is added at the start).

Initial Key

128 bit

Key Expansion for AES-128 (Example)

Assume the original key for AES-128 is:

Key = [2b7e151628aed2a6abf7158809cf4f3c]

W(0) W(1) W(2) W(3)

- This 16-byte (128-bit) key will be expanded into 44 words (176 bytes).
- Here's an outline of the process:
- Initial words (W[0] to W[3]): The first four words are simply taken from the key itself.
- W[0] = 2b7e1516
- W[1] = 28aed2a6
- W[2] = abf71588
- W[3] = 09cf4f3c

Key Expansion for AES-128 (Example)

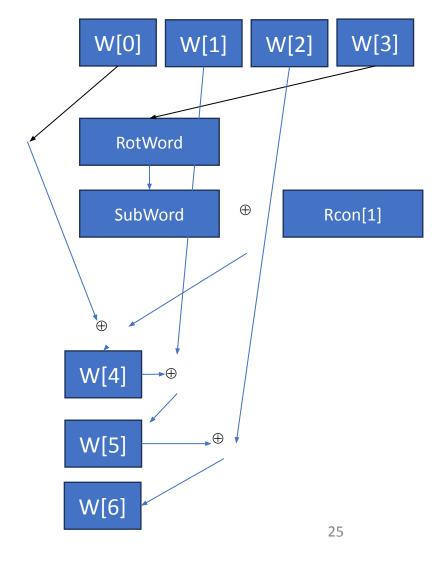
- Next words (W[4] to W[43]):
- These words are generated using the process of RotWord,
 SubWord, Rcon, and XORing with the previous words.
- Here's a simple breakdown:

```
W[4] = W[0] ⊕ SubWord( RotWord(W[3]) ) ⊕ Rcon[1]
W[5] = W[1] ⊕ W[4]
W[6] = W[2] ⊕ W[5]
W[7] = W[3] ⊕ W[6]
```

• This process continues until 44 words are generated for AES-128, providing one round key for each of the 10 rounds and the initial round key.

Initial Key

128 bit



Avalanche Effect

- If a small change in the key or plaintext were to produce a corresponding small change in the ciphertext, this might be used to effectively reduce the size of the plaintext (or key) space to be searched.
- What is desired is the avalanche effect, in which a small change in plaintext or key produces a large change in the ciphertext.

Table 6.5 Avalanche Effect in AES: Change in Plaintext

Round		Number of Bits that Differ
	0123456789abcdeffedcba9876543210	1
	0023456789abcdeffedcba9876543210	
0	0e3634aece7225b6f26b174ed92b5588	1
	0f3634aece7225b6f26b174ed92b5588	
1	657470750fc7ff3fc0e8e8ca4dd02a9c	20
	c4a9ad090fc7ff3fc0e8e8ca4dd02a9c	
2	5c7bb49a6b72349b05a2317ff46d1294	58
200	fe2ae569f7ee8bb8c1f5a2bb37ef53d5	
3	7115262448dc747e5cdac7227da9bd9c	59
	ec093dfb7c45343d689017507d485e62	
4	f867aee8b437a5210c24c1974cffeabc	61
	43efdb697244df808e8d9364ee0ae6f5	
5	721eb200ba06206dcbd4bce704fa654e	68
	7b28a5d5ed643287e006c099bb375302	SYST.
6	0ad9d85689f9f77bc1c5f71185e5fb14	64
	3bc2d8b6798d8ac4fe36a1d891ac181a	
7	db18a8ffa16d30d5f88b08d777ba4eaa	67
	9fb8b5452023c70280e5c4bb9e555a4b	
8	f91b4fbfe934c9bf8f2f85812b084989	65
	20264e1126b219aef7feb3f9b2d6de40	
9	cca104a13e678500ff59025f3bafaa34	61
	b56a0341b2290ba7dfdfbddcd8578205	4:
10	ff0b844a0853bf7c6934ab4364148fb9	58
	612b89398d0600cde116227ce72433f0	440

Avalanche Effect

- Change in Plaintext.
- Table shows the result when the **eighth bit of the plaintext is changed.**
- The **second column** of the table shows the value of the State matrix at the **end of each round** for the two plaintexts.
- just one round, 20 bits of the State vector differ.
- After two rounds, close to half the bits differ.
- This magnitude of difference **propagates through** the remaining rounds.
- A bit difference in **approximately half the positions** in the most **desirable outcome**.

Table 6.5 Avalanche Effect in AES: Change in Plaintext

Round		Number of Bits that Differ
	0123456789abcdeffedcba9876543210	1
	0023456789abcdeffedcba9876543210	
0	0e3634aece7225b6f26b174ed92b5588	1
	0f3634aece7225b6f26b174ed92b5588	
1	657470750fc7ff3fc0e8e8ca4dd02a9c	20
	c4a9ad090fc7ff3fc0e8e8ca4dd02a9c	
2	5c7bb49a6b72349b05a2317ff46d1294	58
	fe2ae569f7ee8bb8c1f5a2bb37ef53d5	
3	7115262448dc747e5cdac7227da9bd9c	59
	ec093dfb7c45343d689017507d485e62	
4	f867aee8b437a5210c24c1974cffeabc	61
	43efdb697244df808e8d9364ee0ae6f5	
5	721eb200ba06206dcbd4bce704fa654e	68
S-2	7b28a5d5ed643287e006c099bb375302	37492
6	0ad9d85689f9f77bc1c5f71185e5fb14	64
	3bc2d8b6798d8ac4fe36a1d891ac181a	
7	db18a8ffa16d30d5f88b08d777ba4eaa	67
***	9fb8b5452023c70280e5c4bb9e555a4b	
8	f91b4fbfe934c9bf8f2f85812b084989	65
	20264e1126b219aef7feb3f9b2d6de40	
9	cca104a13e678500ff59025f3bafaa34	61
	b56a0341b2290ba7dfdfbddcd8578205	
10	ff0b844a0853bf7c6934ab4364148fb9	58
	612b89398d0600cde116227ce72433f0	887

Avalanche Effect

Change in Key

- Table shows the change in State matrix values when the same plaintext is used and the two keys differ in the eighth bit.
- . Again, one round produces a **significant change**, **and the magnitude of change** after all subsequent rounds is roughly half the bits.
- Thus, based on this example, AES exhibits a very strong avalanche effect.

Table 6.6 Avalanche Effect in AES: Change in Key

Round		Number of Bits that Differ
	0123456789abcdeffedcba9876543210	0
	0123456789abcdeffedcba9876543210	
0	0e3634aece7225b6f26b174ed92b5588	1
	0f3634aece7225b6f26b174ed92b5588	
1	657470750fc7ff3fc0e8e8ca4dd02a9c	22
	c5a9ad090ec7ff3fc1e8e8ca4cd02a9c	
2	5c7bb49a6b72349b05a2317ff46d1294	58
	90905fa9563356d15f3760f3b8259985	
3	7115262448dc747e5cdac7227da9bd9c	67
	18aeb7aa794b3b66629448d575c7cebf	
4	f867aee8b437a5210c24c1974cffeabc	63
	f81015f993c978a876ae017cb49e7eec	
5	721eb200ba06206dcbd4bce704fa654e	81
	5955c91b4e769f3cb4a94768e98d5267	3
6	0ad9d85689f9f77bc1c5f71185e5fb14	70
	dc60a24d137662181e45b8d3726b2920	
7	db18a8ffa16d30d5f88b08d777ba4eaa	74
	fe8343b8f88bef66cab7e977d005a03c	
8	f91b4fbfe934c9bf8f2f85812b084989	67
	da7dad581d1725c5b72fa0f9d9d1366a	
9	cca104a13e678500ff59025f3bafaa34	59
	0ccb4c66bbfd912f4b511d72996345e0	
10	ff0b844a0853bf7c6934ab4364148fb9	53
	fc8923ee501a7d207ab670686839996b	

Relationship between Rijndael and AES

- The **relationship between Rijndael and AES** is that AES (Advanced Encryption Standard) is a **standardized version** of the Rijndael cipher. In 2001, the **National Institute of Standards and Technology (NIST)** selected the Rijndael algorithm, designed by cryptographers **Vincent Rijmen** and **Joan Daemen**, as the algorithm to become the new Advanced Encryption Standard (AES) for secure encryption. Here's how they are connected and what differentiates them:
- AES as a Subset of Rijndael:
- While **Rijndael supports** a variety of block sizes (128, 192, and 256 bits) and key lengths (in multiples of 32 bits), **AES is a standardized subset** with a fixed block size of **128 bits**.
- AES supports three key sizes: **128 bits, 192 bits, and 256 bits**, corresponding to 10, 12, and 14 rounds, respectively.
- This fixed block size (128 bits) and limited key sizes (128, 192, 256 bits) were chosen by NIST to make the encryption standard simpler, more uniform, and compatible across various platforms.

Relationship between Rijndael and AES

• Structural Similarity:

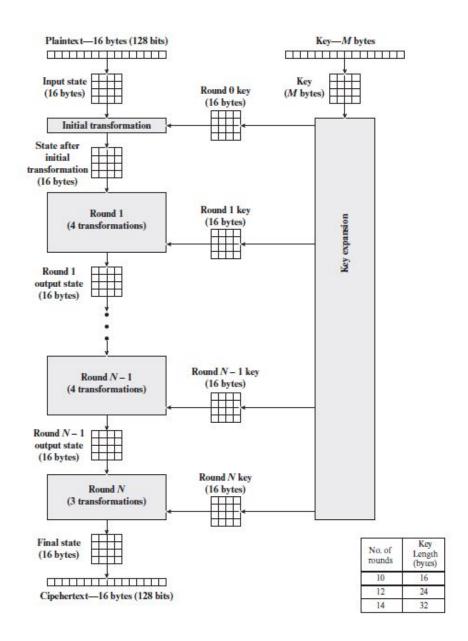
- The core structure and steps of AES and Rijndael are the same: both use a combination of SubBytes, ShiftRows, MixColumns, and AddRoundKey operations in a Substitution-Permutation Network (SPN).
- The only difference is that Rijndael's design allows for more flexibility in block and key sizes, while AES restricts these to a specific standard.

• Purpose and Implementation:

- AES, as a standardized form of Rijndael, was widely adopted in government, industry, and technology for secure data encryption.
- Rijndael can still theoretically be implemented with different block sizes (e.g., 192 or 256 bits), but **AES is the globally recognized and widely used implementation** of Rijndael.

Thank You

Encryption Process



Encryption Process

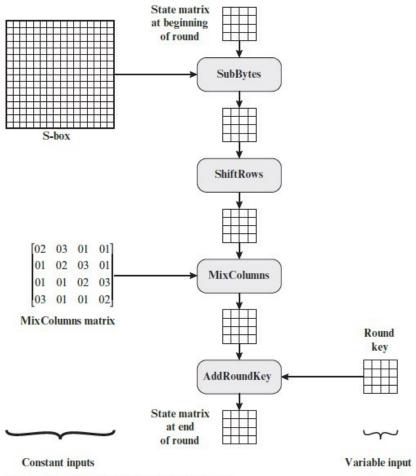


Figure 6.8 Inputs for Single AES Round