

## Motivation and Approach

Objective: **Investigate** the **functionality** of high-dimensional battleships. **Realize** and **compare** possible shooting **strategies** based on discretization methods. The general idea for this topic comes from [1].

- Adapt classic battleship to a **high-dimensional hypercube**.
- Redefine terms like **ship**, **fleet** and shooting **strategies**.
- Evaluation of strategies requires many sample fleets and even more ships.
- A **Monte-Carlo approach** reduces the computation cost significantly while still allowing a decent impression of the quality of the strategy.
- Firstly we use an algorithm to compute the number of shots  $v$  to sink a random ship.
- With this algorithm we can then calculate the **distribution function**  $P(\mathbf{T}_{\text{strat}}^{\Omega_L} \leq v)$  of the strategy for a sample  $\Omega_L$ .
- The Monte-Carlo approach now assumes that this distribution resembles the distribution for all possible ships.

$$P(\mathbf{T}_{\text{strat}}^{\Omega_L} \leq v) \approx P(\mathbf{T}_{\text{strat}}^{L_{\text{all}}} \leq v) \quad (1)$$

- Note: Careful when discretizing ships as shown in [1], there are many continuous ships that represent the same discrete ship as seen in figure 1.

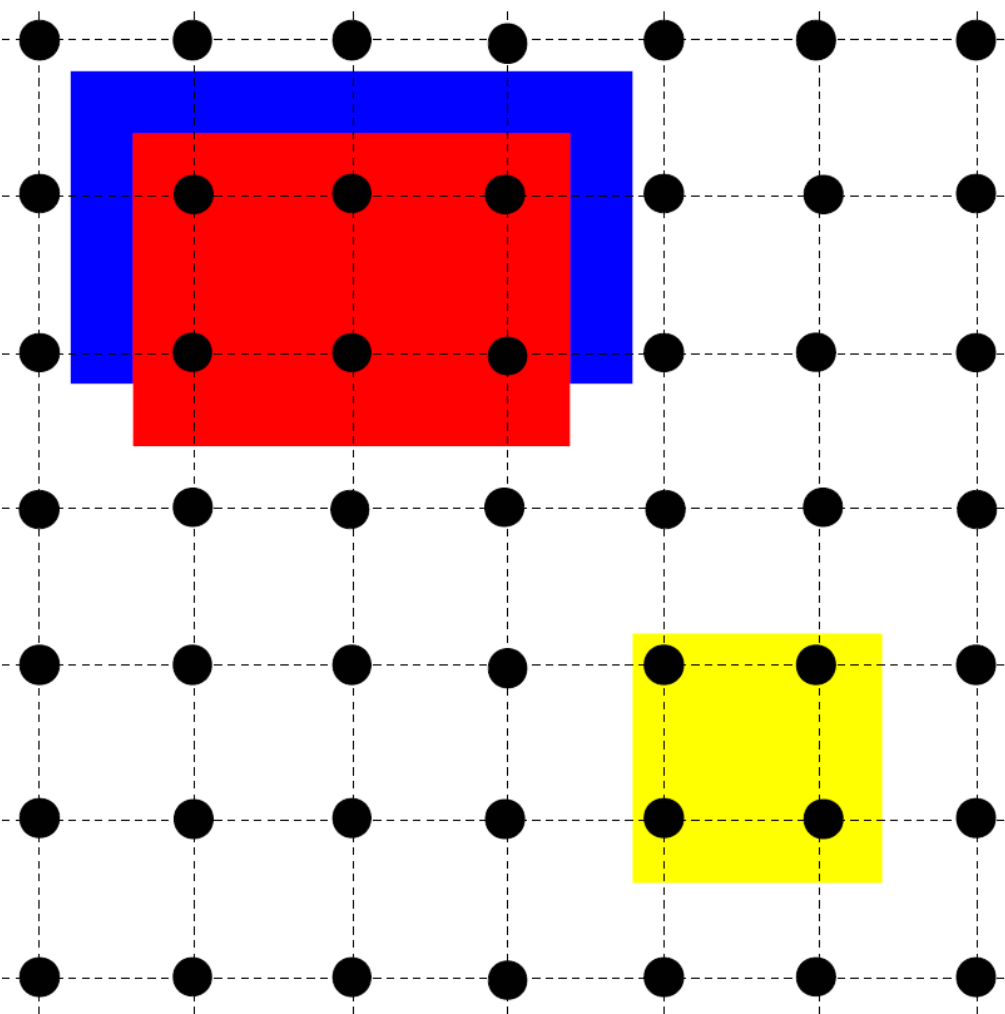


Figure 1: Difference continuous to discrete ships

- The red and blue continuous ships in figure 1 assign to the same discrete ship represented by the six dots.

## Strategies

For shooting strategies we want to orientate us at **discretization methods** since they are easily adapted to high dimensions. For that matter we came up with **five different strategies**. In general grid strategies use the following partitioning:

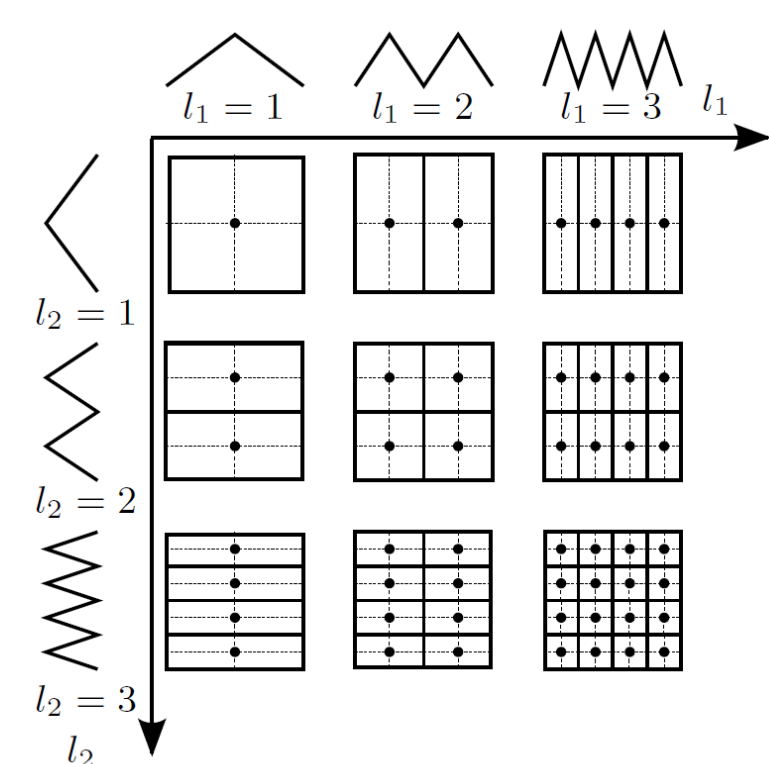


Figure 2: Halving of the grid coordinates

- **Random Strategy**: Shoots at a random cell of the hypercube. This is the closest strategy to the reality since most battleship players target random cells.
- **Full Grid Strategy**: Shoots cells according to the discretization given by a full grid.

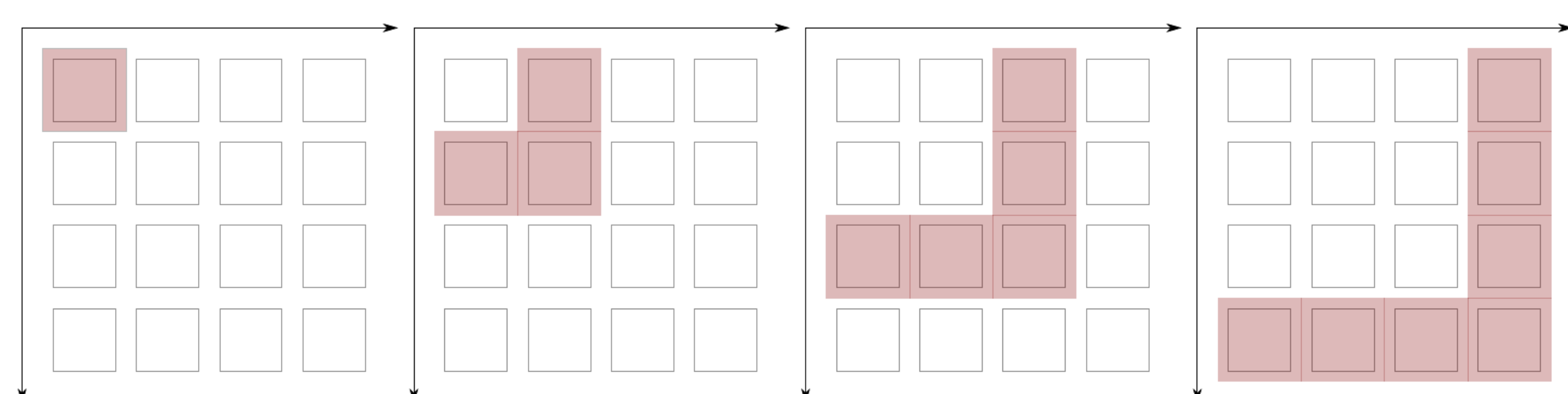


Figure 3: Structure of Full Grid Strategy

- **Sparse Grid Strategy**: Shoots cells according to the discretization given by a sparse grid.

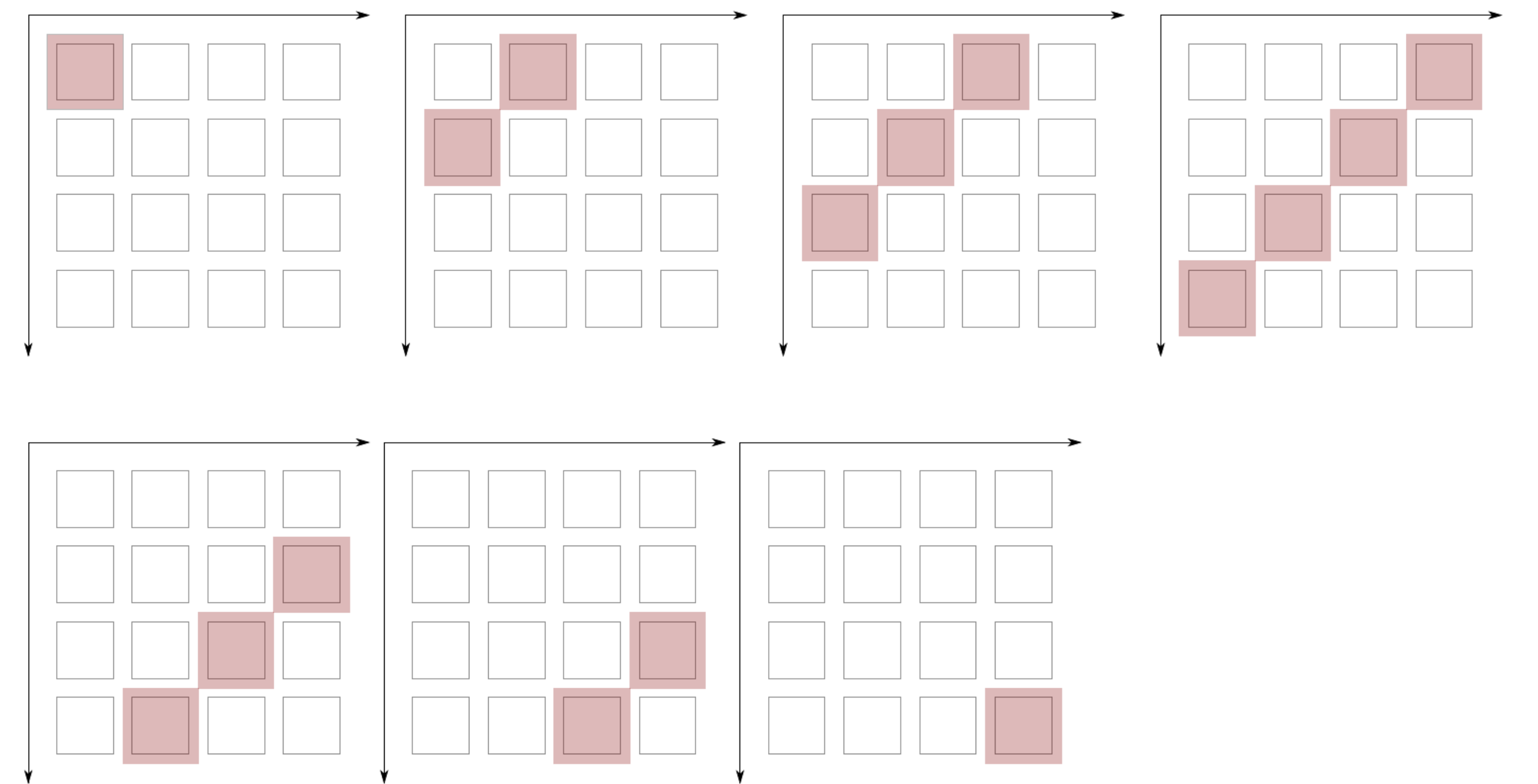


Figure 4: Structure of Sparse Grid Strategy

- **Sobol Strategy**: This strategy shoots according to the pseudorandom Sobol number sequence which is based on polynomials in  $\mathbb{Z}_2$ . The definitions and algorithms are based on [2].
- **Halton Strategy**: Here the strategy is based on onedimensional number sequences of fractions of primes which we combine to multidimensional coordinates.

## Results

After computing the results for the different strategies with **varying dimensions** and **sample sizes** we can compare them now.

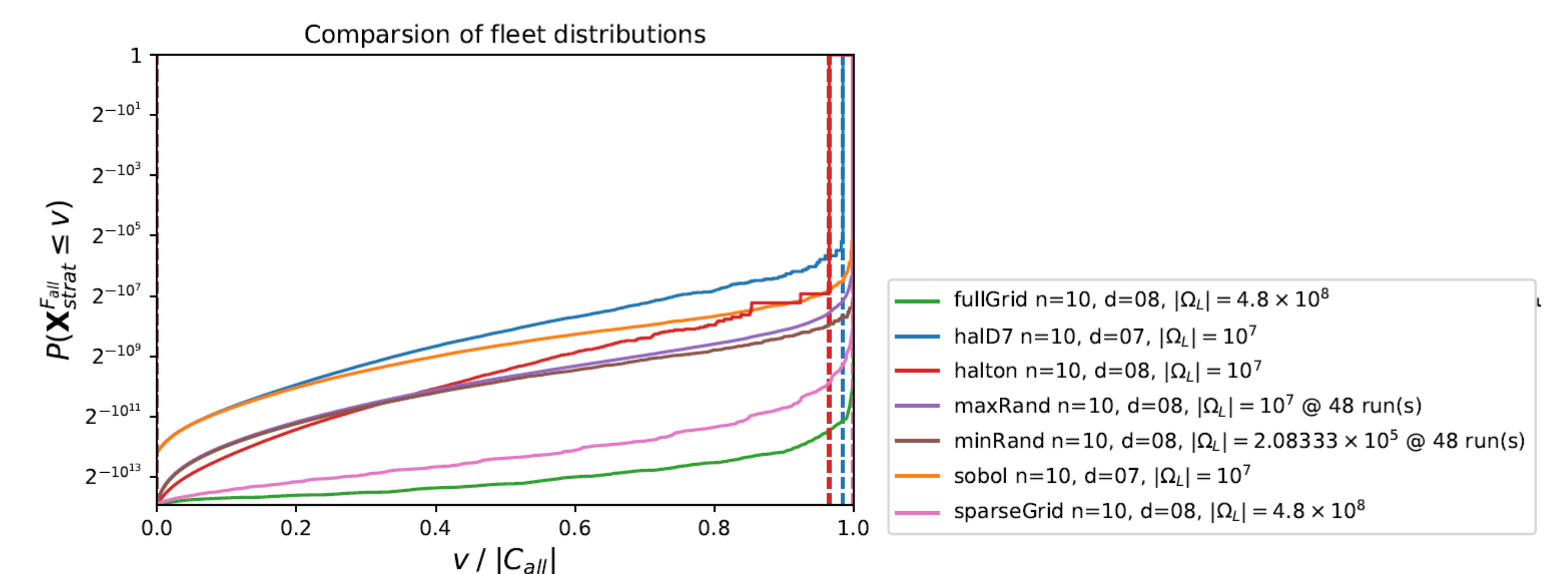


Figure 5: Comparison Strategies for random fleet

In figure 5 we see, that the Halton Strategy for 7 dimensions overtakes Sobol. This implies the same for 8 dimensions. Which is why we conclude that the **Halton Strategy performs the best** out of all the tested strategies. Note: The goal of the curves is to reach the top as fast as possible. This conclusion is also confirmed by the expected value of shots that a strategy needs in which Halton generally needs  $3 \cdot 10^6$  less shots in 8 dimensions than other strategies.

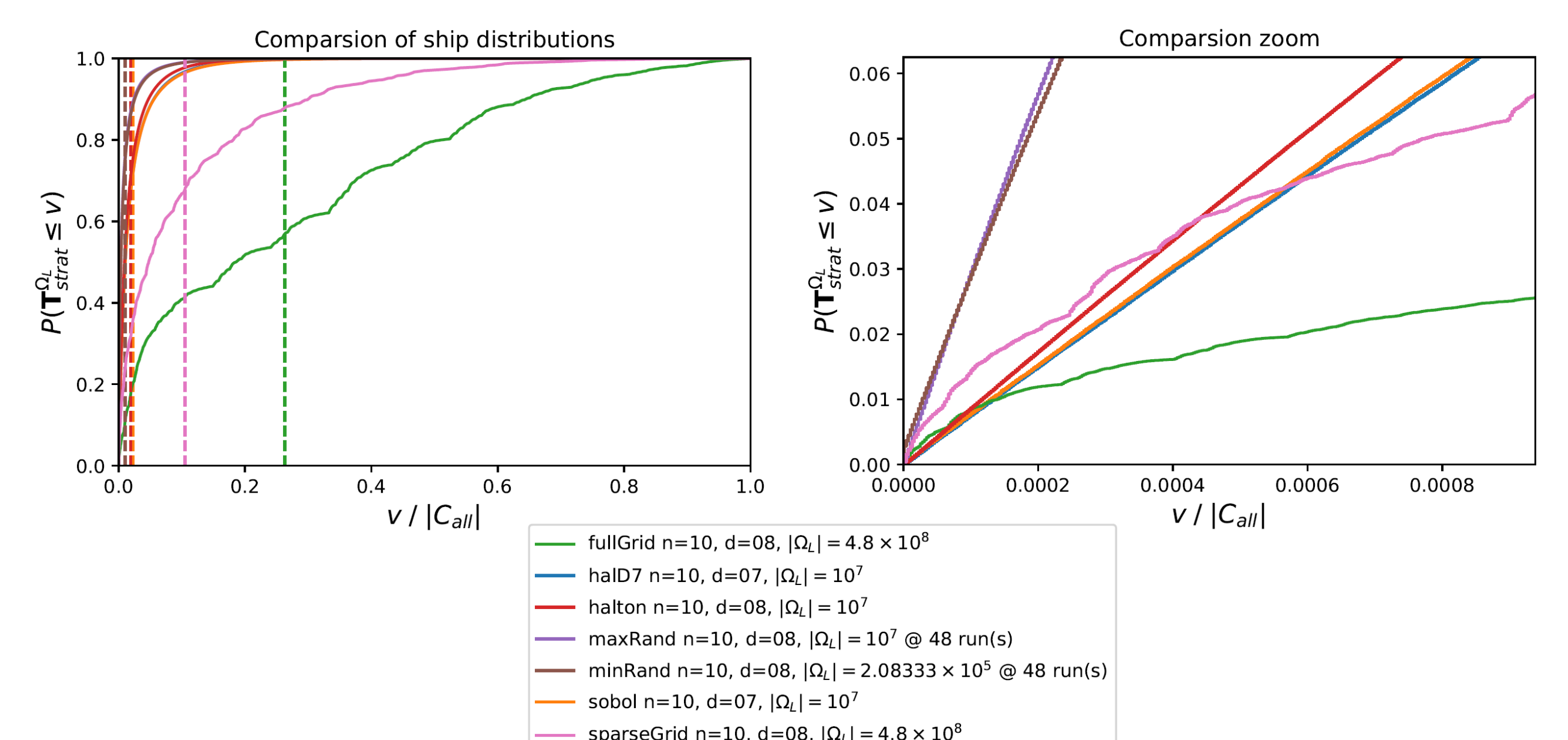


Figure 6: Comparison Strategies for random ship distribution

## References

- [1] [Mehlbeer F.](#)  
Hierarchische methoden am beispiel von schiffe versenken, 06 2013.
- [2] [Joe S. and Kuo F. Y.](#)  
Remark on algorithm 659: Implementing sobol's quasirandom sequence generator.  
*ACM Transactions on Mathematical Software*, 29:49–57, 2003.