2020-08-14_Testing_Equation_Numbers_v01

August 17, 2020

Run the following 2 cells to export this document as PDF, without input cells. Currently, the "Save as" option from the menu bar doesn't work as well.

naming conventions:

- v_ = numerical value or result
- res_ = symbolic result
- m_ = markdown
- subs_ = equation with values substituted in
- eq_ = equation
- set_ = solution set
- lab_ = label reference to first form of equation
- lab2_ = label reference to second form of equation

- xbar_A = symbol
- eq_xbar_A = symbolic equation
- sym_xbar_A = symbolic result for xbar_A

Mark notebook as "Trusted" to render completely.

Remember to click the "reset equation numbering" button on the toolbar after modifying equations or references.

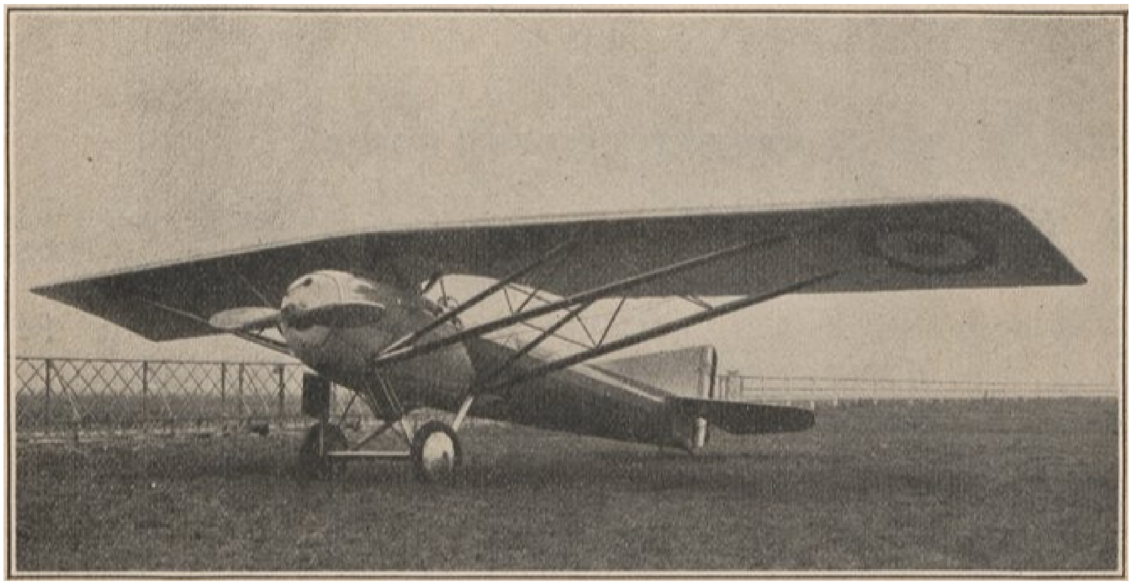
1 Setup

2 Stress Report

This document presents a stress analysis for the **Super Cool Gizmo2**.

2.1 Description of Model

The Super Cool Gizmo, (Figure 1), provides dynamic loading during flight by rotating a slotted cylinder. The slots in the cylinder deflect the air stream causing a cyclical dynamic load. The frequency and the force are adjustable.



(1)

Figure 1: SCG.

2.2 Aerodynamic Data

Blah.

Blah blah.

The following is a summary of the measured aerodynamic data used in this analysis:

- **Static Lift Coefficient,** {{m_C_LS}}
  - From wind tunnel test data – DEI Report D-277. 0.4 to 1.1 Mach
  - at AOA = 15 degrees
  - reference area equals planform area, airfoil plus cylinder
- **Dynamic Root Bending Moment Coefficient,** {{m_C_MB}}
  - From flight test data – Gates Learjet Model 36, 0.5 to 0.8 Mach, 2 to 30 Hz
  - reference area equals planform area, airfoil plus cylinder

For this analysis, the lift and normal force coefficients are assumed to be the same value.

2.3 Loads Data

Design loads are based on flight conditions for the maximum allowable fluid dynamic pressure, {{m_q}}, and the stall angle of attack (AOA = 15 degrees).

Static and dynamic loads act on the SCG.

- The static load is lift primarily generated by the airfoil.
  - The static lift on the cylinder is considered negligible.
- The dynamic load is the lift generated by the rotating cylinder.
  - Dynamic load is assumed to be split between the airfoil and the cylinder.

2.3.1 Airfoil Static Load

Static lift load, L_s is defined in equation (2).

$$
\begin{aligned}
L_s &= C_{LS} S q && (2)\\
\text{where,} &\\
L_s &= \text{static lift load (lbf)},\\
C_{LS} &= \text{coefficient of lift, static (lbf)},\\
S &= \text{airfoil surface area } (\text{in}^2), \text{and}\\
q &= \text{fluid dynamic pressure (psi)}.
\end{aligned}
$$

References:

- https://en.wikipedia.org/wiki/Lift_coefficient accessed 2019-11-26
  - Abbott, Ira H., and Doenhoff, Albert E. von: Theory of Wing Sections. Section 1.2
  - Clancy, L. J.: Aerodynamics. Section 8.11

Airfoil surface area, S is defined in equation (8) from dimensions illustrated in Figure 9. Overall airfoil area is the area of the rectangular portion plus the area of the triangular portion.

$$
\begin{aligned}
S &= b\ C_T + \frac{b\ (C_R - C_T)}{2} && (3)\\
&= b\ C_T + \frac{b\ C_R - b\ C_T}{2} && (4)\\
&= \frac{2\ b\ C_T}{2} + \frac{b\ C_R - b\ C_T}{2} && (5)\\
&= \frac{2\ b\ C_T + b\ C_R - b\ C_T}{2} && (6)\\
&= \frac{b\ C_T + b\ C_R}{2} && (7)\\
&= \frac{b\ (C_R + C_T)}{2} && (8)\\
\text{where,} &\\
S &= \text{airfoil surface area } (\text{in}^2),\\
b &= \text{airfoil span (in)},\\
C_R &= \text{airfoil root chord (in)}, \text{and}\\
C_T &= \text{airfoil tip chord (in)}.
\end{aligned}
$$

$$(9)$$

Figure 9: SCG Chord Dimensions.

$$L_s = C_{LS} S q \tag{10}$$

$$\tag{11}$$

$$S = \frac{b\,(C_R + C_T)}{2} \tag{12}$$

$$\tag{13}$$

Substitute airfoil size values from **Interface Control Document** and solve for S.

C_R = {{v_C_R}} in
C_T = {{v_C_T}} in
b = {{v_b}} in

Substitute dimensional values to get, {{m_S}}.

Therefore, airfoil static lift load, {{m_L_s}}.

2.3.2 Airfoil Dynamic Load

Dynamic lift load can be obtained from dynamic root bending moment. Dynamic lift load is per equation (14).

$$L_d = C_{Ld} S q \tag{14}$$

where,

$$\begin{aligned}
L_d &= \text{dynamic lift load (lbf)},\\
C_{Ld} &= \text{coefficient of lift, dynamic},\\
S &= \text{airfoil surface area (in}^2\text{)},\\
q &= \text{fluid dynamic pressure (psi)}
\end{aligned}$$

From flight test we have an estimate for dynamic root bending coefficient, C_{MB}. Dynamic root bending moment, M, about the airfoil root is defined in terms of C_{MB} per equation (17).

$$\begin{aligned}
M &= (F)\ d && (15)\\
&= (F)\ b && (16)\\
&= (C_{MB}\ S\ q)\ b && (17)
\end{aligned}$$

where,

$$\begin{aligned}
M &= \text{moment (lbf-in)},\\
F &= \text{force (lbf)},\\
d &= \text{distance from line of action of force F to root (in)},\\
b &= \text{airfoil span (in)},\\
C_{MB} &= \text{dynamic root bending moment coefficient},\\
S &= \text{airfoil surface area (in}^2\text{)},\\
q &= \text{fluid dynamic pressure (psi)}
\end{aligned}$$

Dynamic lift load from equation (14) can be related to bending moment about the airfoil root, M, by equation (20).

$$\begin{aligned}
M &= (F)\ d && (18)\\
&= (L_d)\ d && (19)\\
&= (L_d)\ X_0 && (20)
\end{aligned}$$

where,

$$X_0 = \text{distance from root to centroid (in)}$$

solving for L_d,

$$L_d = \frac{M}{X_0} \tag{21}$$

Deriving the distance from root to centroid, X_0 by considering the airfoil as a combination of a rectangular shape and a triangle shape:

$$X_0 = \frac{A_A \bar{x}_A + A_B \bar{x}_B}{A_A + A_B}$$
$$\bar{x}_A = \frac{b}{3}$$
$$\bar{x}_B = \frac{b}{2}$$
$$A_A = \frac{b\left(C_R - C_T\right)}{2}$$
$$A_B = C_T b$$

$$X_0 = \frac{\frac{C_T b^2}{2} + \frac{b^2(C_R - C_T)}{6}}{C_T b + \frac{b(C_R - C_T)}{2}} \tag{22}$$
$$\tag{23}$$

Simplifying, the distance from root to centroid, X_0, is shown in equation (24).

$$X_0 = \frac{b\left(C_R + 2C_T\right)}{3\left(C_R + C_T\right)} \tag{24}$$
$$\tag{25}$$

Therefore, distance from root to centroid, {{m_X_0}}.

Set dynamic lift load from equation (14) equal to that from (21), replace M per equation (17), then solve for C_{Ld} to get equation (28).:

$$C_{Ld} S q = \frac{M}{X_0} \tag{26}$$
$$C_{Ld} S q = \frac{\left(C_{MB}\ S\ q\right)\ b}{X_0} \tag{27}$$
$$C_{Ld} = \frac{C_{MB}\ b}{X_0} \tag{28}$$
$$\tag{29}$$

$$C_{Ld} S q = \frac{C_{MB} S b q}{X_0}$$
$$C_{Ld} = \frac{C_{MB} b}{X_0}$$

Therefore, coefficient of lift, dynamic, {{m_C_Ld}}.

Now substitute values into equation (14) to get L_d

$$L_d = C_{Ld} S q \tag{30}$$
$$\tag{31}$$

Therefore, airfoil dynamic lift load, {{m_L_d}}.

2.3.3 Total Airfoil Lift

Total lift load on airfoil, L_A is defined in equation (32).

$$L_A = \frac{L_d}{2} + L_s \tag{32}$$
$$\tag{33}$$

Substituting values into equation (32) to get {{m_L_A}}.

2.3.4 Total Cylinder Lift

Total lift load on cylinder, L_{Cyl} is defined in equation (34).

$$L_{Cyl} = \frac{L_d}{2} \tag{34}$$
$$\tag{35}$$

Substituting values into equation (34) to get {{m_L_Cyl}}.

2.3.5 Airfoil Bending Moment

Loads into the airfoil are due to L_A which acts at the center of pressure, and, assuming that the cylinder is a simply supported beam, $\frac{L_{Cyl}}{2}$.

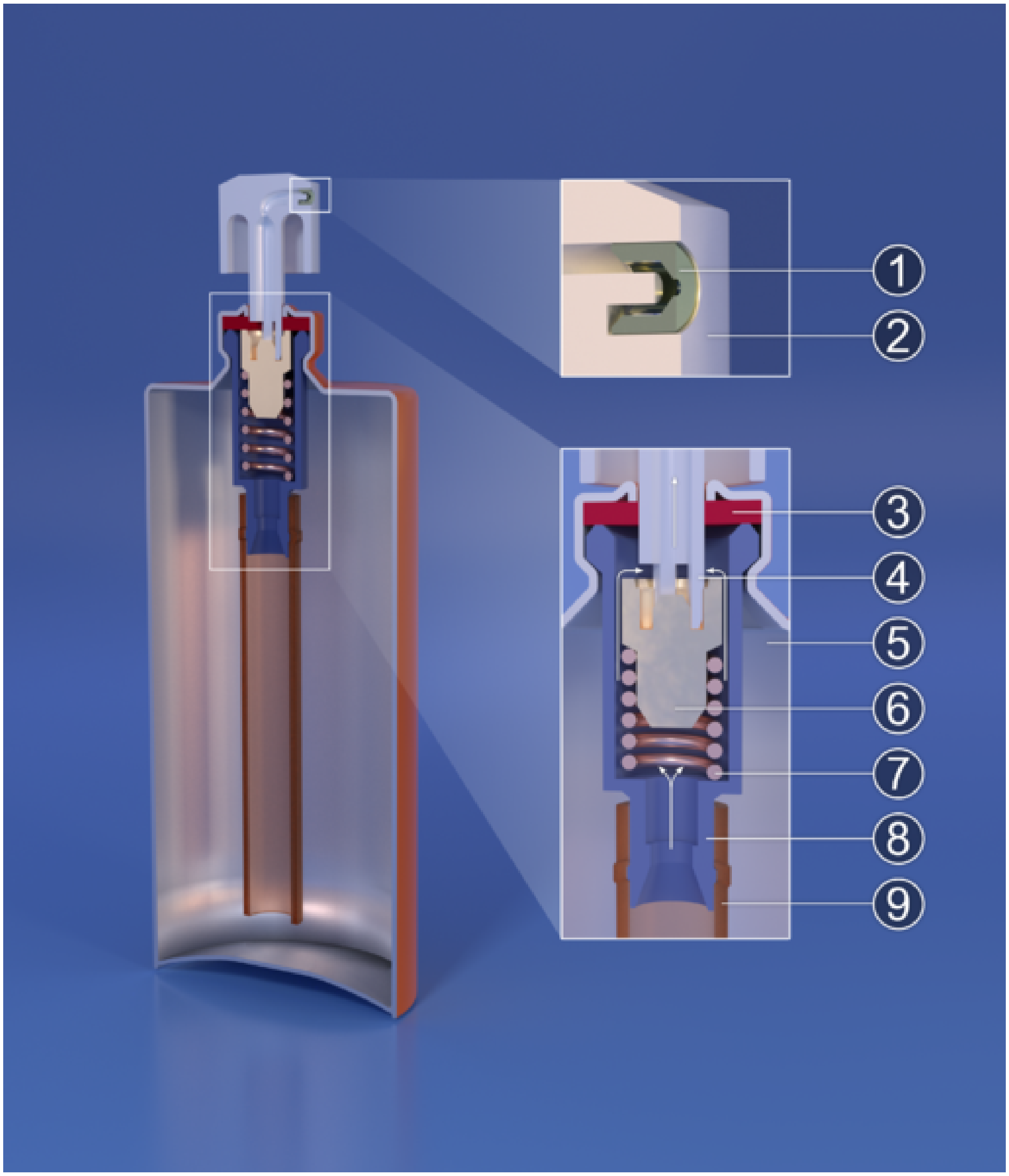
Moments at various sections are calculated per equation (36)

$$M = L_A \cdot \frac{A}{A_{total}} \cdot d_{cp} + \frac{L_{Cyl}}{2} \cdot h \tag{36}$$

where,

$$\begin{aligned} M &= \text{moment at any section (in-lbf)}, \\ A &= \text{area of the section (in}^2\text{)}, \\ A_{total} &= \text{airfoil area (in}^2\text{)}, \\ d_{cp} &= \text{distance to the centroid of the section area (in), and} \\ h &= \text{distance from the airfoil tip to the section (in).} \end{aligned}$$

Area, A, is calculateable from dimensions a, b and h in diagram 37.



$$ \tag{37} $$

Figure 37: Airfoil bending moment dimension diagram.

Simple bending stress σ is per (38):

$$\sigma = \frac{Mc}{I} \tag{38}$$

where,

$$\sigma = \text{bending stress (psi)},$$
$$M = \text{local bending moment (in-lbf)},$$
$$c = \text{distance from the neutral axis to the extreme fiber (in), and}$$
$$I = \text{area moment of inertia (in}^4\text{)}.$$

TO DO: Moments at various sections are calculated by spreadsheet separately using dimensions a, b and h and cross-sectional properties I and c obtained from the 3D CAD model.

2.3.6 Rotating Mass Loads