Autonomic Computer Systems, HS11

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Signal Processing and Transient-State Analysis of Protocols

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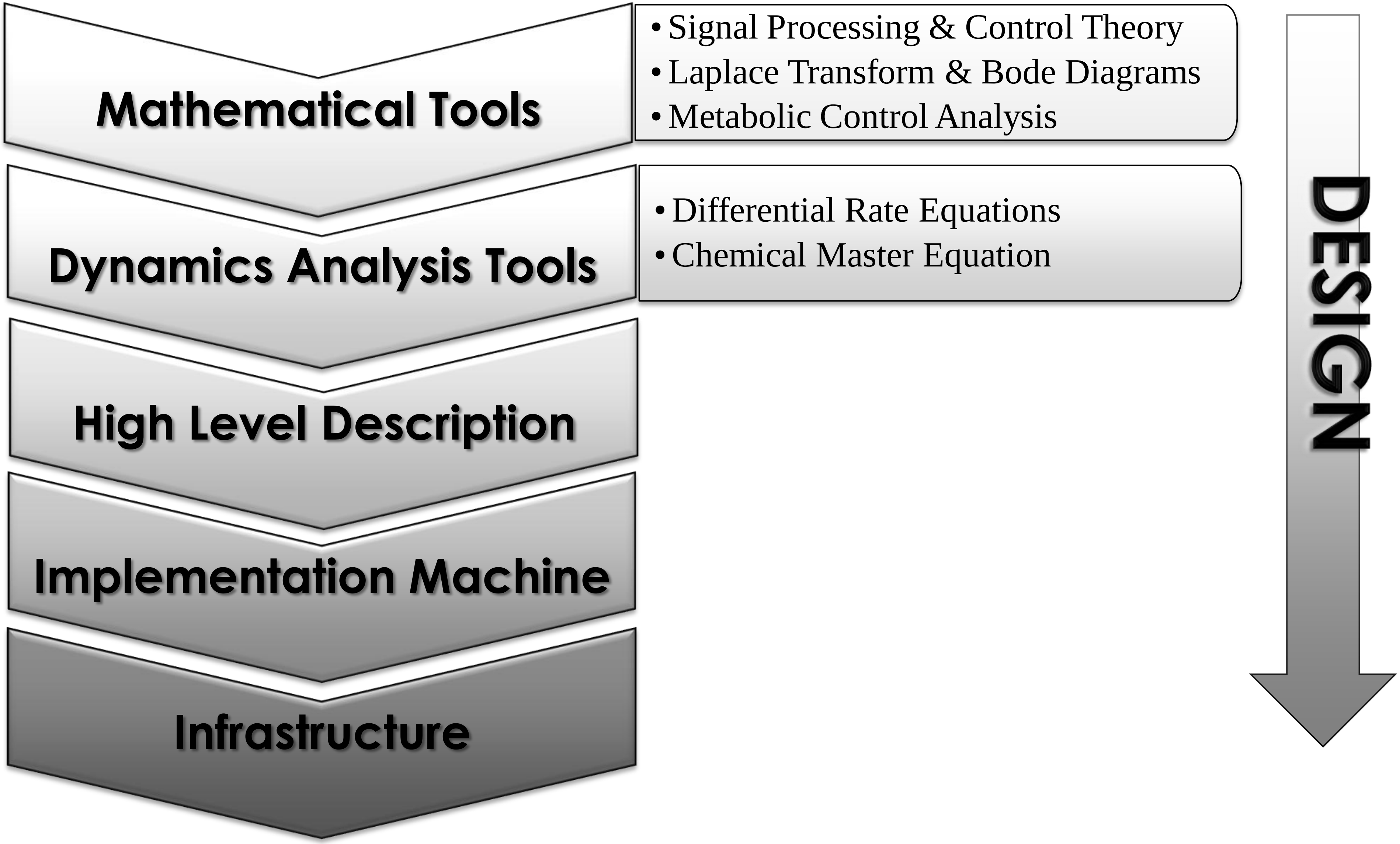
Context


Mathematical Tools
• Signal Processing & Control Theory
• Laplace Transform & Bode Diagrams
• Metabolic Control Analysis
Dynamics Analysis Tools
• Differential Rate Equations
• Chemical Master Equation
High Level Description
Implementation Machine
Infrastructure
DESIGN

Contents

Why Protocols' Dynamics?

-) Design protocols from scratch

-) Understand deeply the protocol's behavior

-) Optimize parameter settings

Challenge

-) Complex and large-scale networks (e.g. the internet)

-) *Complexity* of the detailed analysis at the micro-level

-) *Abstract models* must be manually extracted from source code

-) Approximated results must be close enough to real behaviors

Approach

-) Base on abstract (fluid) models

-) Find similarities and common points with *other established fields*

-) Make use of *well-known tools* of other areas

Signal Processing and Frequency Domain (1)

Why Signal Processing and Frequency-Domain?

-) obtain general results that can be re-used in future

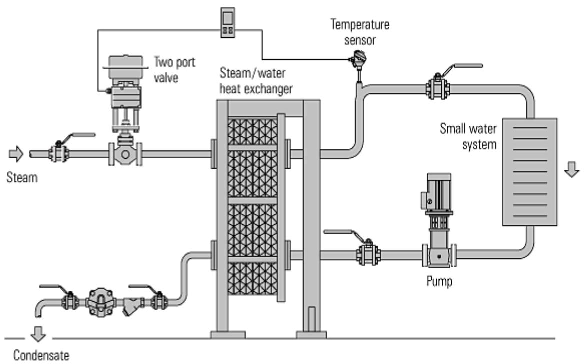
-) composition/decomposition in less complex sub-blocks

-) have a wide and alternative "observation view"

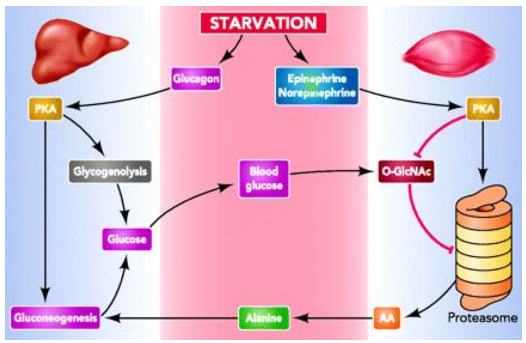
-) avoid complexity of other kinds of analysis

-) reversibility of frequency-analysis into time-analysis

Electronics/Telecomm.



Mechanics



Biology

Signal Processing and Frequency Domain (2)

How can we transpose it to protocols?

Protocols
⇕
Systems

Inputs, parameters and outputs of protocols
⇕
Input and output signals of a system

Protocols' behaviors (dynamic)
⇕
Relationships between input and output signals (TF)

Signal Processing and Frequency Domain (2)

How can we transpose it to protocols?

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⬍

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Protocols' behaviors (dynamic)

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Relationships between input and output signals (TF)

What Kind of Protocols?

-) Protocols that can be described with mathematical rules and relationships

-) A good example: *Chemical Networking Protocols (CNPs)*

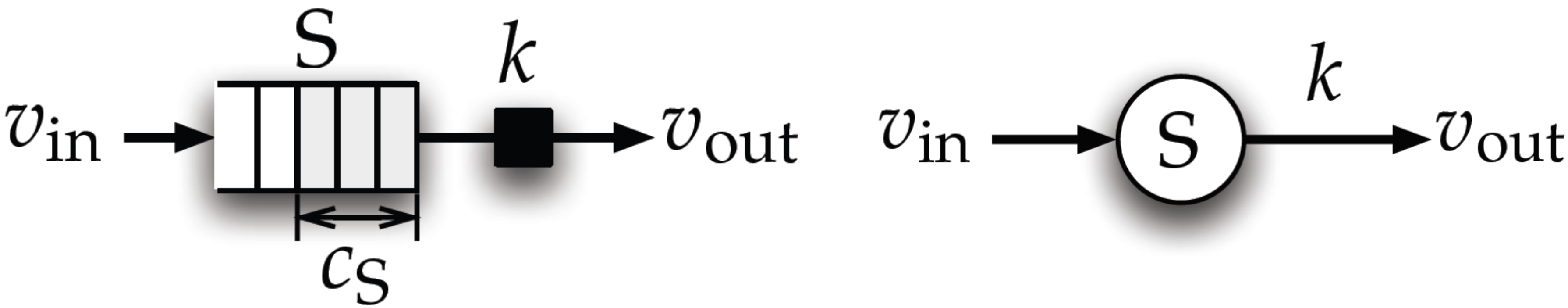
| CNPs offer an *intrinsic description* of their dynamical behavior

| The protocol is based on a related *chemical reaction model*

| A system of differential equations describes the *average behavior* of the protocol in time

Example: a Simple CNP

Packets		*as*	Molecules
Queue	(S)	*as*	Molecules species
Queue length	(c_s)	*as*	Species concentration
Service Rule		*as*	Law of Mass Action



Signal Processing and Frequency Domain

How can we transpose it to protocols?

Protocols

⬍

Systems

Inputs, parameters and outputs of protocols

⬍

Input and output signals of a system

Protocols' behaviors (dynamic)

⬍

Relationships between input and output signals (TF)

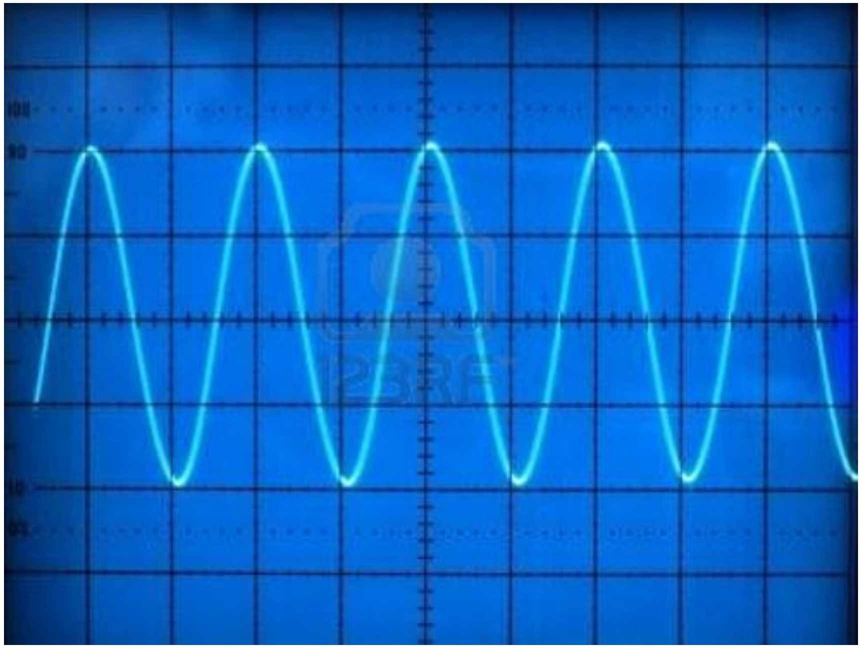
What Are Signals?

"Signals are analog or digital representations of time-varying (or spatial-varying) physical quantities." [~ Wikipedia]

e.g.

Electrocardiograms

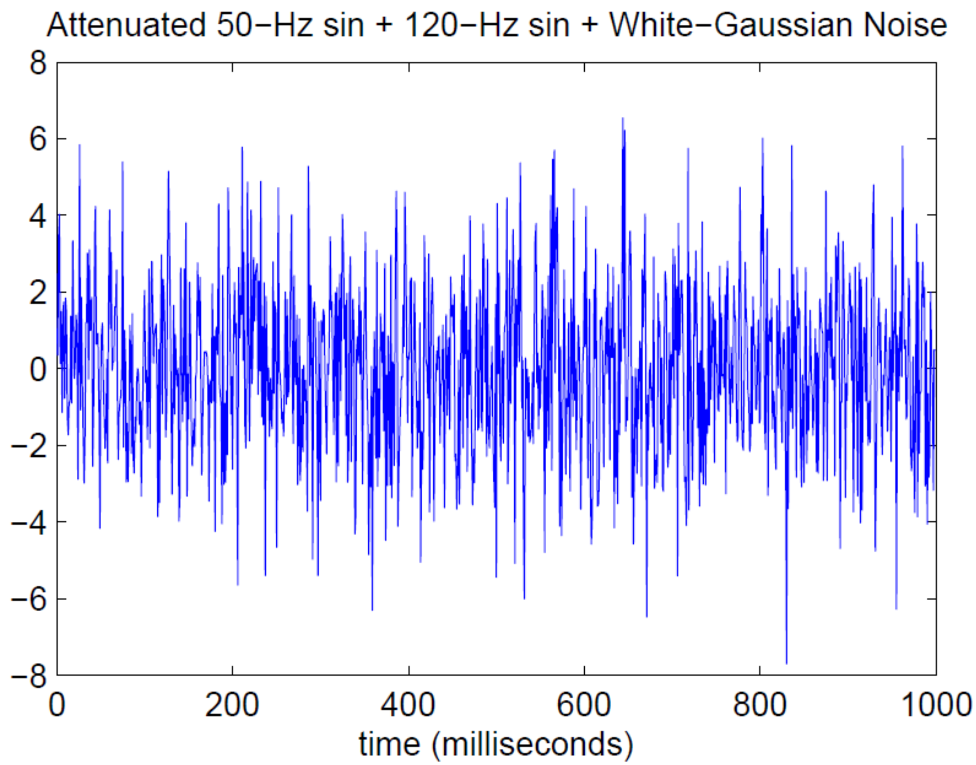
Voltage/current in electrical circuits



Signals and Points of View

```
y = x = 0.7*sin(2*pi*50*t) + sin(2*pi*120*t) + 2*randn(size(t))
```

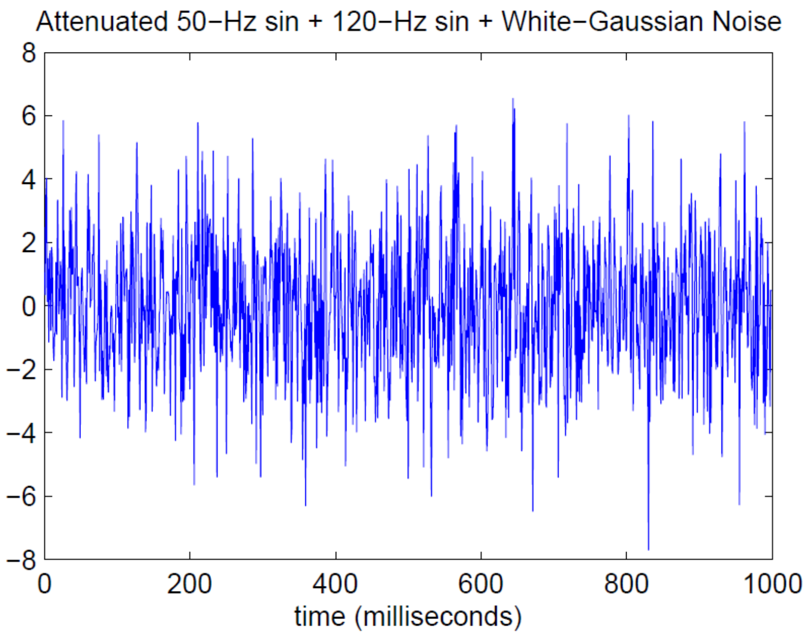
Time Domain



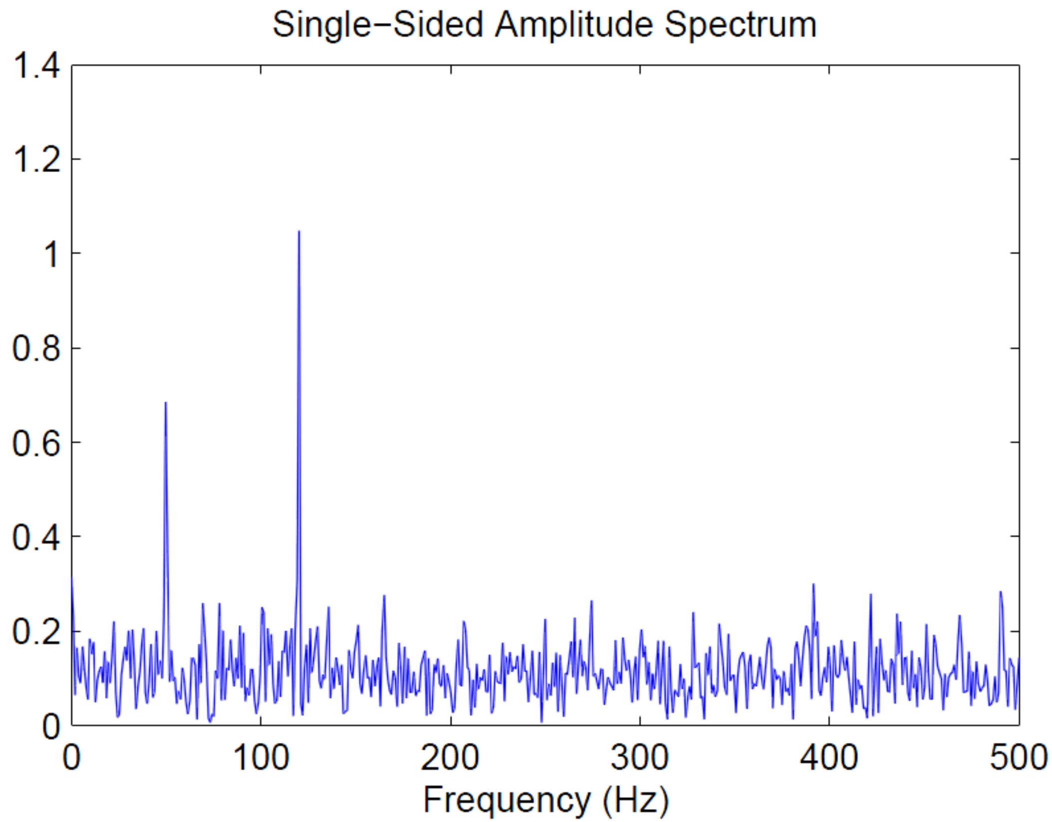
Signals and Points of View

```
y = x = 0.7*sin(2*pi*50*t) + sin(2*pi*120*t) + 2*randn(size(t))
```

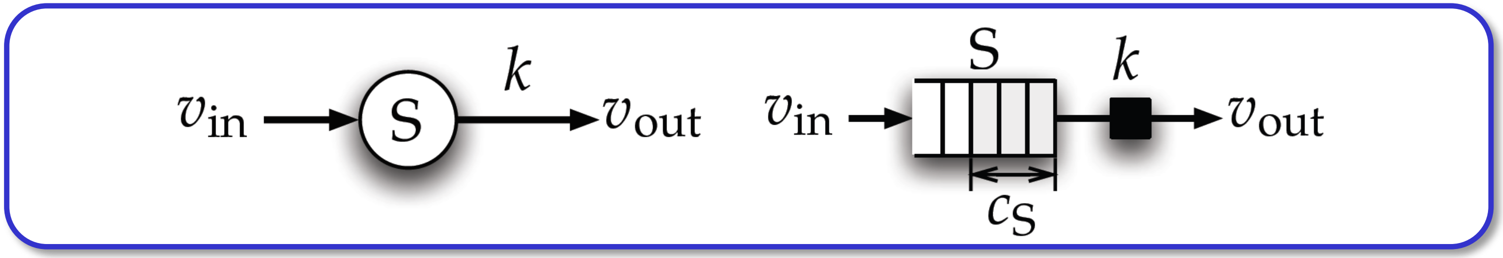
Time Domain

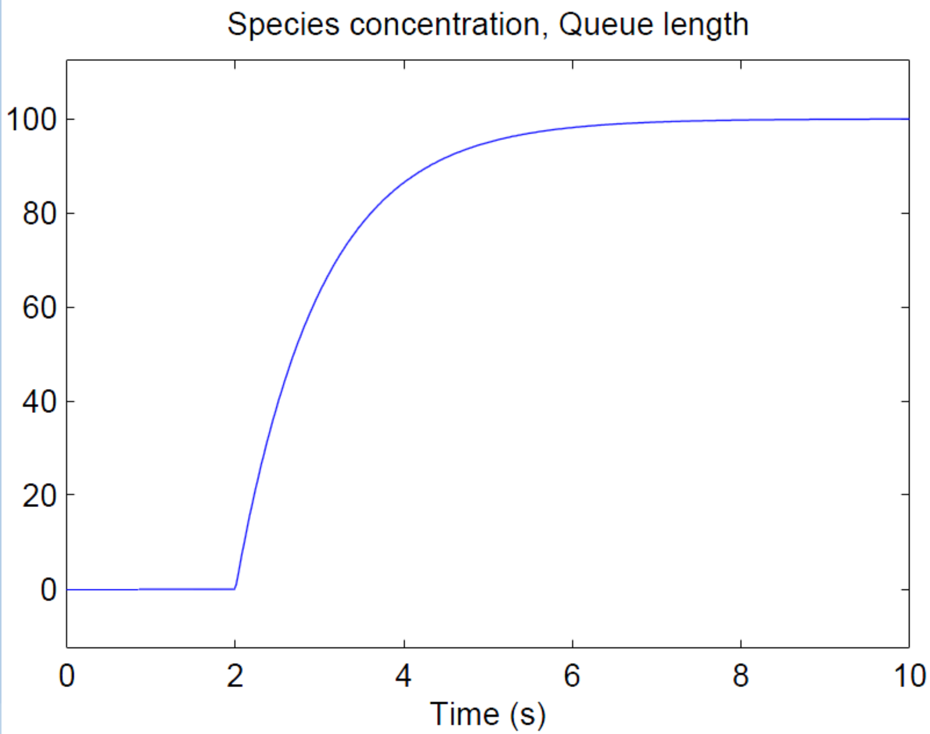


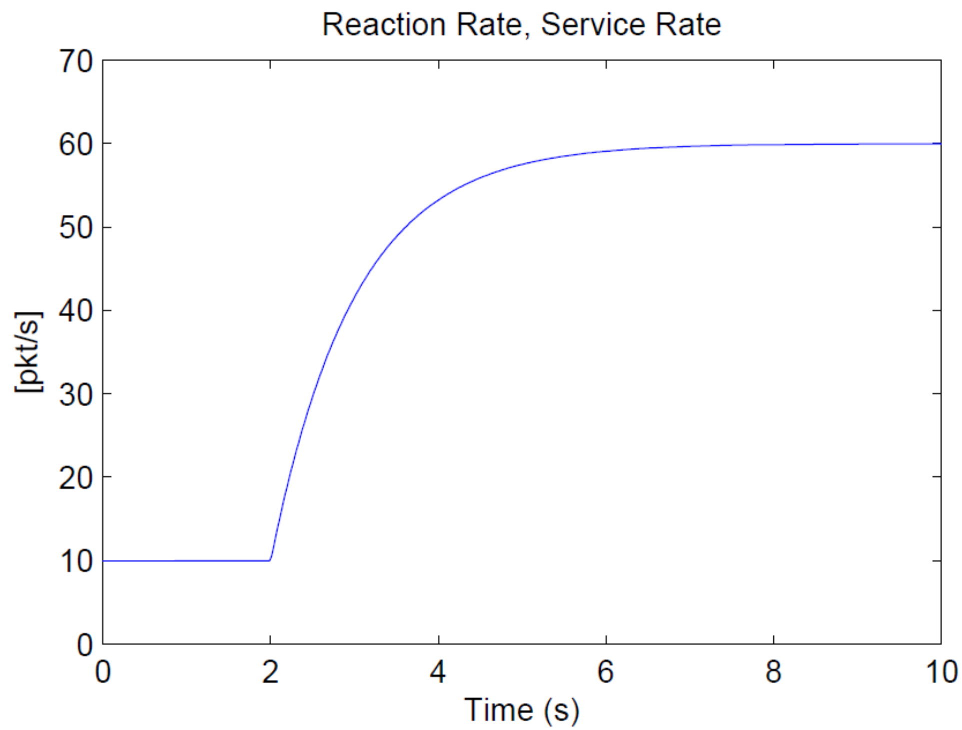
Frequency Domain



Signals in a Simple CNP:







Generally: a quantity which we are interested in!

Signal Processing and Frequency Domain (2)

How can we transpose it to protocols?

Protocols

⬍

Systems

Inputs, parameters and outputs of protocols

⬍

Input and output signals of a system

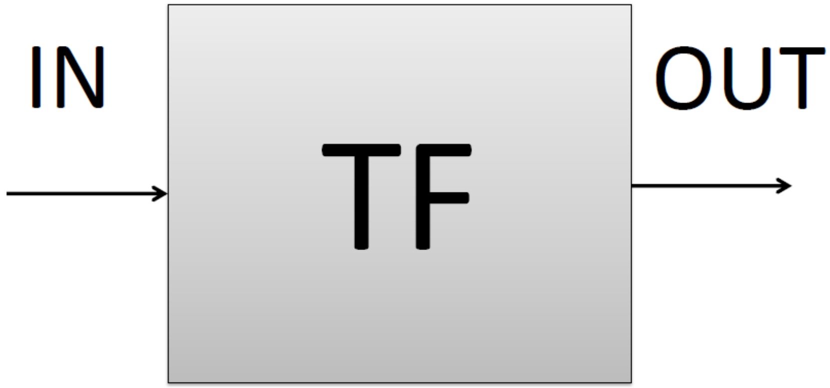
Protocols' behaviors (dynamic)

⬍

Relationships between input and output signals (TF)

What is a Transfer Function (TF)?

"A transfer function is a mathematical representation in terms of frequency, of the relation between the input and output of a Linear Time-Invariant (LTI) system." [~ Wikipedia]



continuous-time signals → Laplace Transform

discrete-time signals → Z-Transform

The Laplace Transform

Good for…

-) Solving *differential* and integral equations

-) Analyzing *Linear Time-Invariant (LTI)* systems

| *Time*-domain: inputs and outputs are functions of time, [s]

| *Frequency*-domain: where the same inputs and outputs are functions of complex angular frequency, [rad/s]

-) Analyzing linear dynamical systems

The Laplace Transform - **Formal Definition**

A linear operator, $\mathcal{L}$, of a function f with a real argument $t \geq 0$ that transforms f to a function F with a complex argument s.

$$F(s) = \mathfrak{L}[f(t)] = \int_0^{\infty} f(t)e^{-st}\ dt \qquad \forall \ \ t \geq 0$$

$$s = \sigma + j\omega$$

!) $f(t)$ must be integrable on $[0,\infty)$

-) Unilateral definition (we deal with causal signals)

-) Essentially bijective for the majority of practical uses

$\Downarrow$

Reversibility of the transformation

The Laplace Transform - **Region of Convergence (ROC)**

If

f is locally integrable on $[0, \infty)$

$\lim_{T \to \infty} \int_0^T f(t) e^{-st}\, dt$ exists

Then

The Laplace transform (*conditionally*) converges

(The *absolute* convergence needs $|f(t)e^{-st}|$)

The set of values for the convergence of *F(s)* is of the form

$Re\{s\} > a \ \ (Re\{s\} \geq a)$

a *is* constant

$a \in Re$

$a \in (-\infty, \infty)$

a depends on the growth behavior of $f(t)$

(The two-sided transform converges absolutely in a strip of the form $a < Re\{s\} < b$ $(a \leq Re\{s\} \leq b)$).

The Laplace Transform – **ROC in Practice**

-) $F(s) = \mathscr{L}[f(t)]$ generally exists only for values that reside in the ROC.

-) ROC relates to the system's **BIBO stability** (Bounded Input Bounded Output)

-) The absolute convergence of the Laplace transform of the impulse response function in the region $Re\{s\} \geq 0$.

Stability In Practice…

1) Calculate the Laplace transform of the impulse response of the system
2) Find the roots of the denominator: poles ($p = \alpha + j\beta$)
3) Look at the *real* part of all poles, if it is always negative, the system is stable ($Re\{p\} = \alpha < 0$?)

The Laplace Transform – **Back to the Time-Domain**

Inverse Laplace Transform:

$$f(t) = \mathfrak{L}^{-1}[F(s)] = \frac{1}{j2\pi} \lim_{T\to\infty} \int_{\sigma-jT}^{\sigma+jT} F(S) \cdot e^{-sT} \quad ds$$

$$Re\{s\} = \sigma$$

(Integration done along the line σ in a way that it resides completely in the convergence region)

(Special case $Re\{s\} = \sigma = 0$ the Inverse Laplace Transform equals the Inverse Fourier Transform)

The Laplace Transform & Company

Fourier Transform

$$F(s) = \mathfrak{L}[f(t)]_{s=j\omega} = \mathfrak{F}[f(t)]$$

(Fourier T. resolves a function into its modes of vibration, the Laplace T. resolves a function into its moments.)

Laplace transform is an extension of the Fourier transform that allows analyses of a broader class of signals and systems (including unstable systems!)

Z-Transform

The unilateral Z-transform is simply the unilateral Laplace T. of an ideally sampled signal.

By a simple substitution

$$F(s) = \mathfrak{L}[f(t)]_{e^{sT}=z} = \sum_{n=0}^{\infty} f(n)e^{-nsT}$$

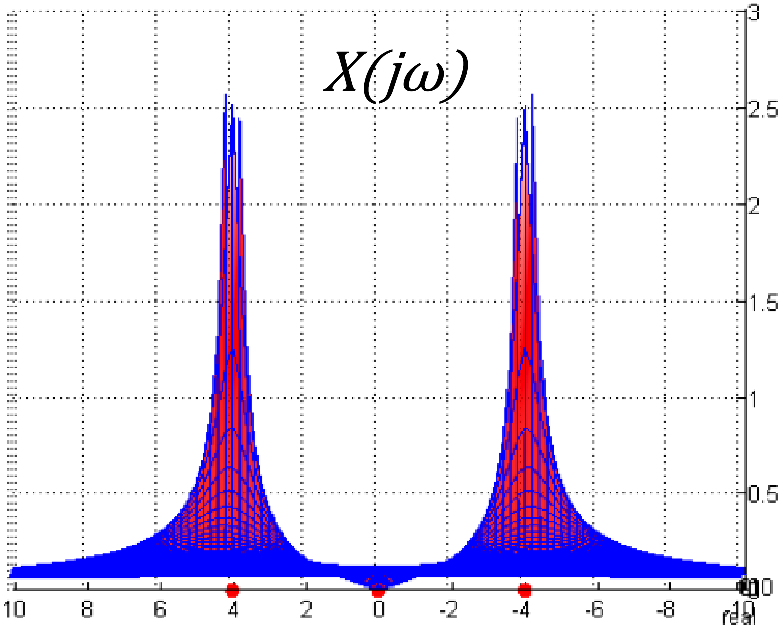
T is the sampling period, [s].

Other: Borel transform, Stieltjes transform, Mellin transform

The Laplace & Fourier Transform

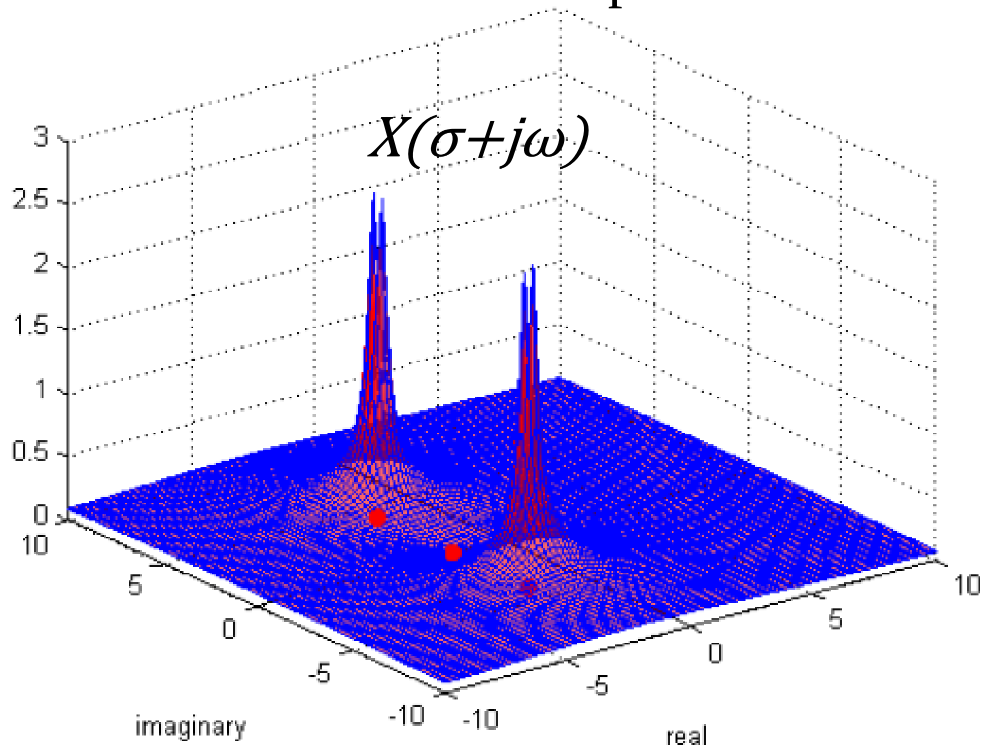
From a 1-D time-domain signal *x(t)* to….

… a 1-D frequency domain complex spectrum



Fourier Transform

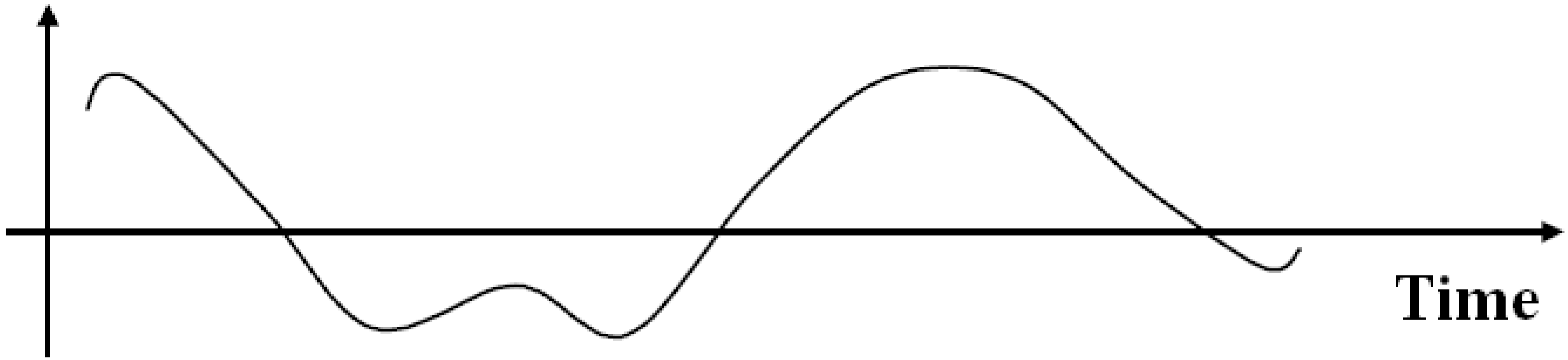
… a 2-D complex function defined over the s-plane



Laplace Transform

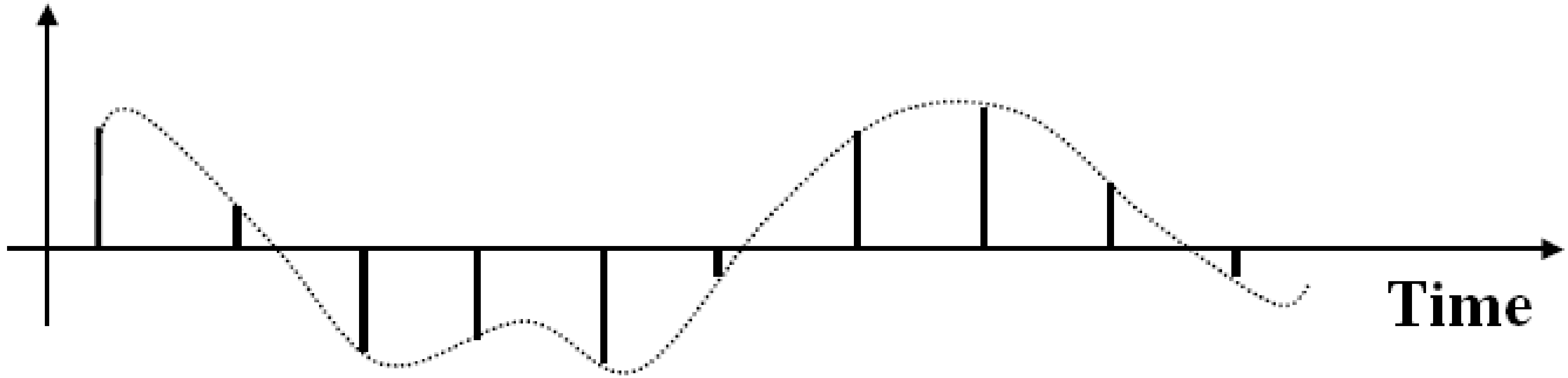
The Laplace & Z-Transform

From a continuous time-domain signal *x(t)* to….



… a 2-D complex function defined over the s-plane.

From a discrete time-domain signal *x(nT)* to….



… a 2-D complex function defined over the Z-plane.

The Laplace Transform in Practice

-) A *linear* transformation from the *time*-domain to the *frequency*-domain

-) An *alternative functional description* of the whole system that often simplifies the behavioral analysis of it.

-) Don't worry: *f*(*t*) - *F*(*s*) can be found directly in tables!

name	Time domain description	Laplace s-domain description	ROC
impulse	$\delta(t)$	1	$\forall\ s$
step	u(t)	$\frac{1}{s}$	$Re\{s\} > 0$
Exponential decay	e^{-at}	$\frac{1}{s+a}$	$Re\{s\} > -a$
…..	…..	……	…..

The Laplace Transform – **Properties (1)**

Many relationships over the originals $f(t)$ correspond to *simpler* relationships over the images $F(s)$.

Differentiation and Integration → multiplication and division

Integral and differential equations → polynomial equations

Differentiation:

$$f'(t) \leftrightarrow sF(s) - f(0)$$

(f twice differentiable and the second derivative of exponential type)

General Differentiation:

$$f^{(n)}(t) \leftrightarrow s^n F(s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(t)$$

(f n-times differentiable and the n^{th} derivative of exponential type)

The Laplace Transform – **Properties (2)**

-) Linearity: $a \cdot f(t) + b \cdot g(t) \leftrightarrow a \cdot F(s) + b \cdot G(s)$

-) Time Scaling $f(at) \leftrightarrow \frac{1}{|a|} \cdot F\left(\frac{s}{a}\right)$

-) Time Shifting (Delay) $f(t-a)u(t-a) \leftrightarrow e^{-as} \cdot F(s)$

!) Multiplication

$$f(t) \cdot g(t) \leftrightarrow \frac{1}{j2\pi} \lim_{T\to\infty} \int_{\sigma-jT}^{\sigma+jT} F(x) \cdot G(s-x) \ dx$$

That has consequences on *non-linear model* analysis!

The Laplace Transform – Two Important Theorems

Initial Value Theorem: $f(0^+) = \lim_{s \to \infty} s \cdot F(s)$

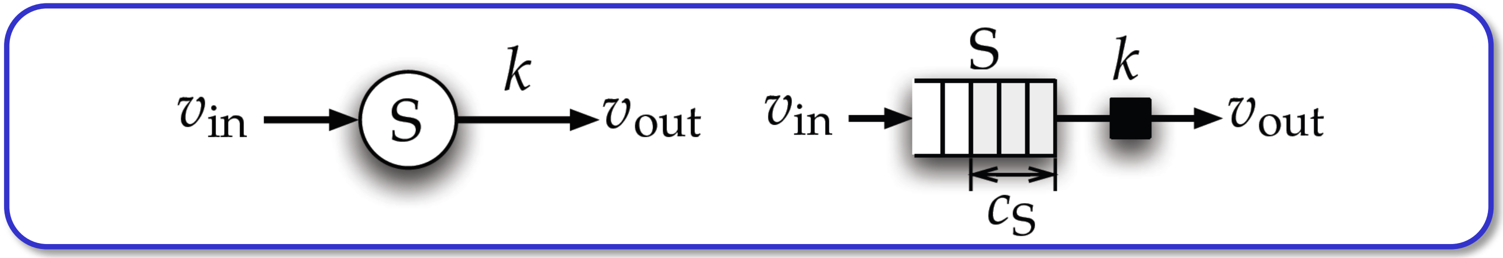
-) continuous-term behavior of the system (steady-state)

Final Value Theorem: $f(\infty) = \lim_{s \to 0} s \cdot F(s)$

-) long-term behavior of the system

!) the system must be stable: all poles of the transfer function must have negative real part.

A simple CNP - Coming Back to the planet Earth!



-) *Differential equation* describing the average behavior of the reaction model (service protocol)

$$\dot{c}_s = v_{in} - k \cdot c_s$$

!) $c_s(t),\ v_{in}(t)$ functions of time

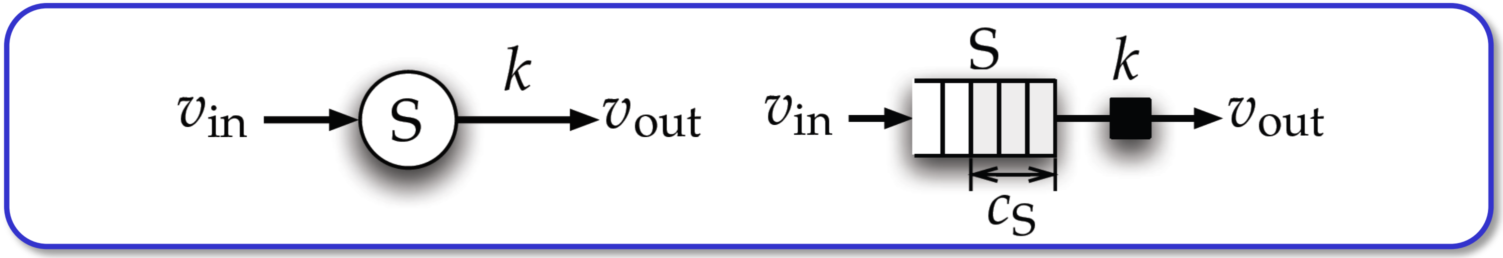
-) Laplace-transforming…

$$s \cdot C_s(s) - c_s(0) = V_{in}(s) - k \cdot C_s(s)$$

$$C_s(s) = \frac{c_s(0)}{s+k} + \frac{V_{in}(s)}{s+k}$$

!) $C_s(s),\ V_{in}(s)$ functions of the complex variable s

A simple CNP (1a)



Assuming ...

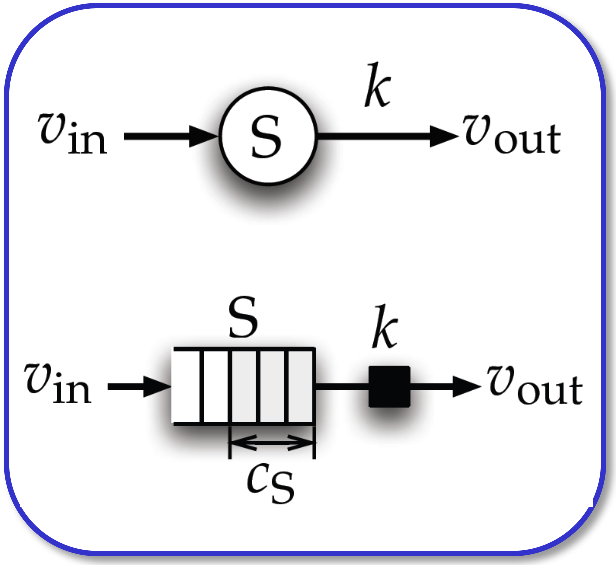
-) an initial amount of packets, $c_s(0)$

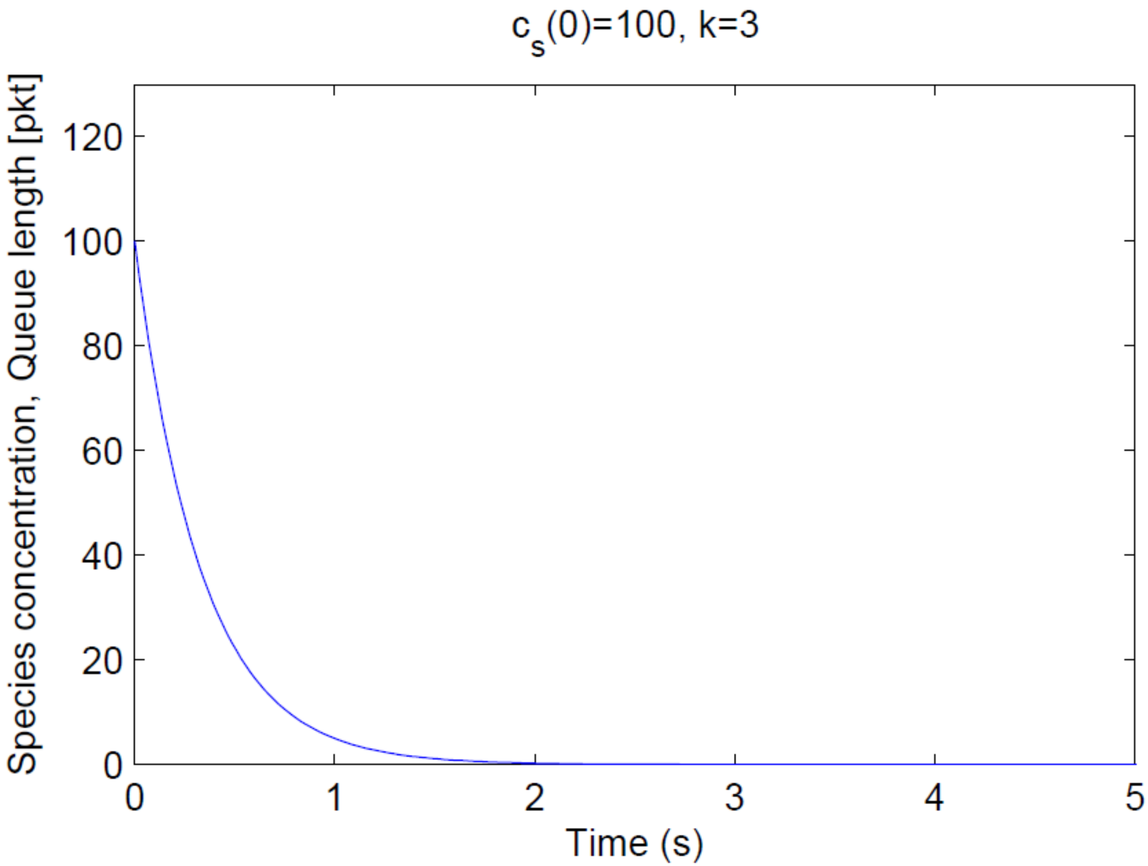
-) no new packets arriving in the queue, $v_{in}(t) = 0$

Anti-transforming $C_s(s)$ → Time-trend of the queue length

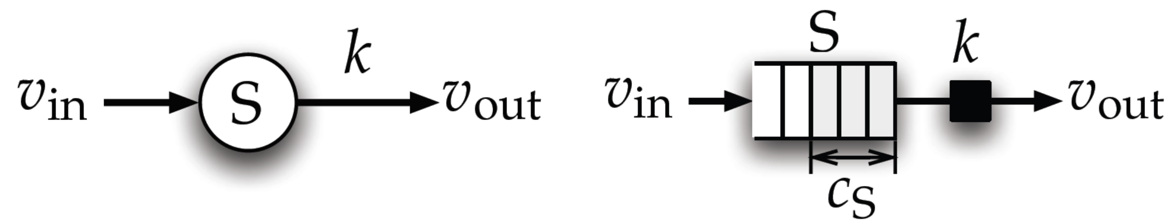
$$c_s(t) = \left(c_s(0) \cdot e^{-kt}\right) \cdot u(t)$$

A simple CNP (1b)





A simple CNP (2a)



Assuming…

-) an empty queue at the beginning, $c_s(0) = 0$

-) a constant generation rate starting from time t=1, $v_{in}(t) = u(t-1)$

-) a service rate parameter, k

What is the time trend of …

-) the queue length, $c_s(t)$?

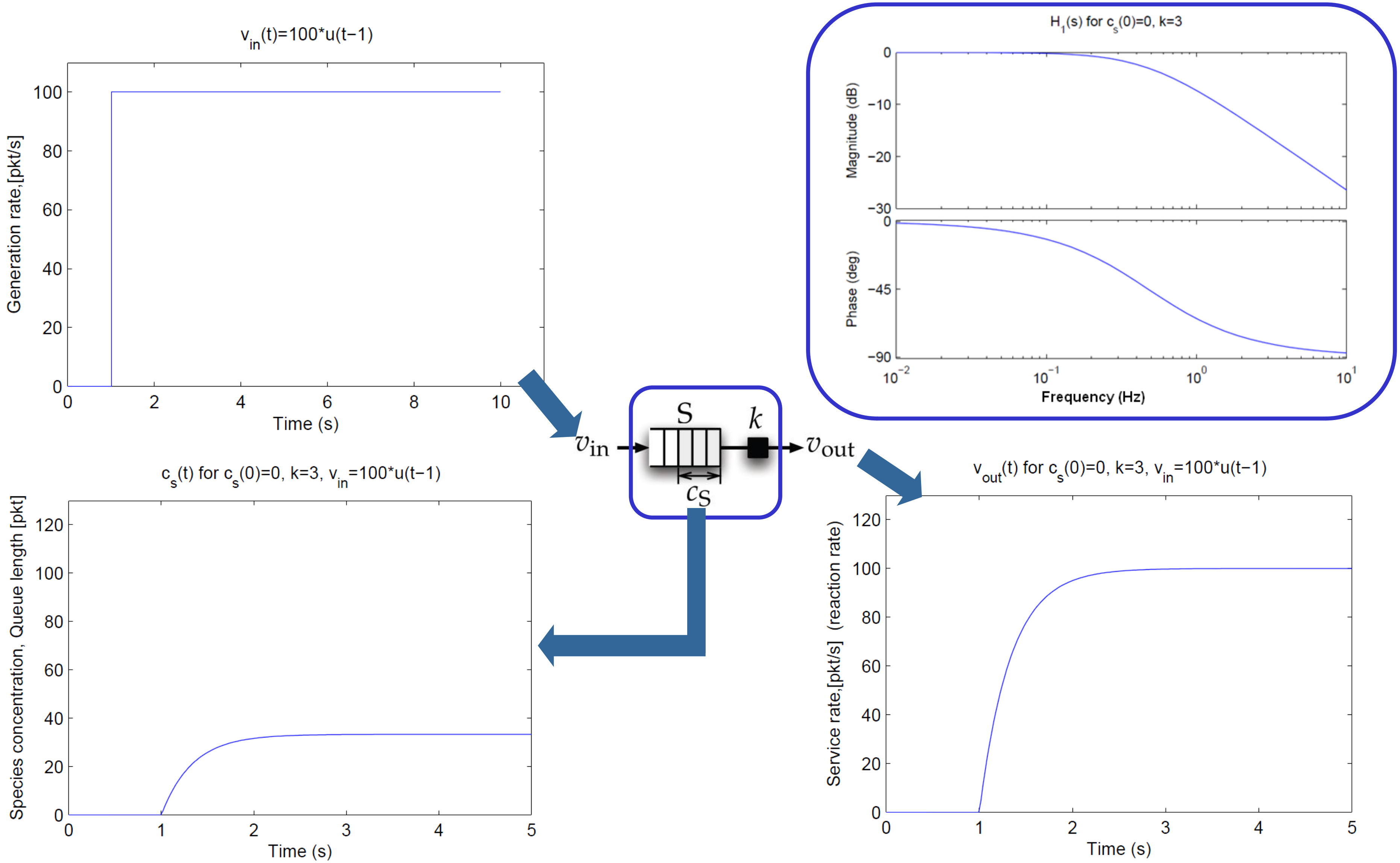
-) the service rate, $kc_s(t)$?

$$TF' = H_1(s) = {V_{out}(s)}/{V_{in}(s)}$$

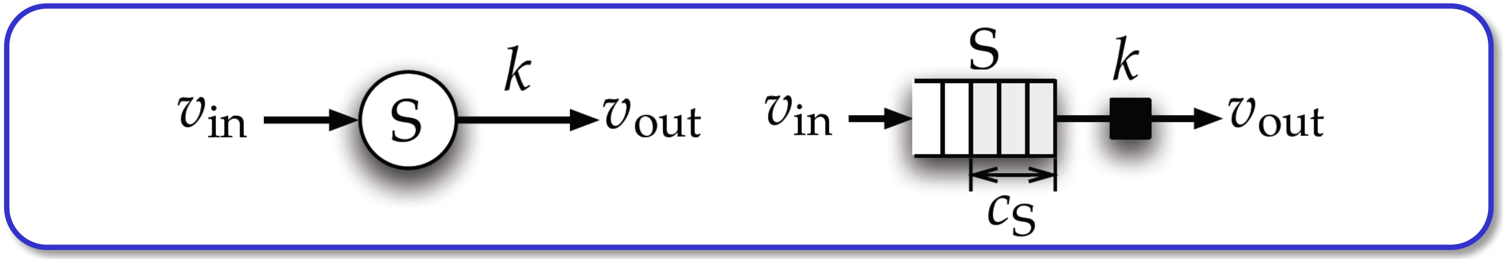
$$\rightarrow C_s(s)\big|_{V_{in}(s)=\frac{1}{s}e^{-s}} \leftrightarrow c_s(t)$$

$$\rightarrow V_{out}(s)\big|_{V_{in}(s)=\frac{1}{s}e^{-s}} \leftrightarrow v_{out}(t)$$

A simple CNP (2b)



A simple CNP (3a)



Assuming…

an empty queue at the beginning

$c_s(0) \neq 0$

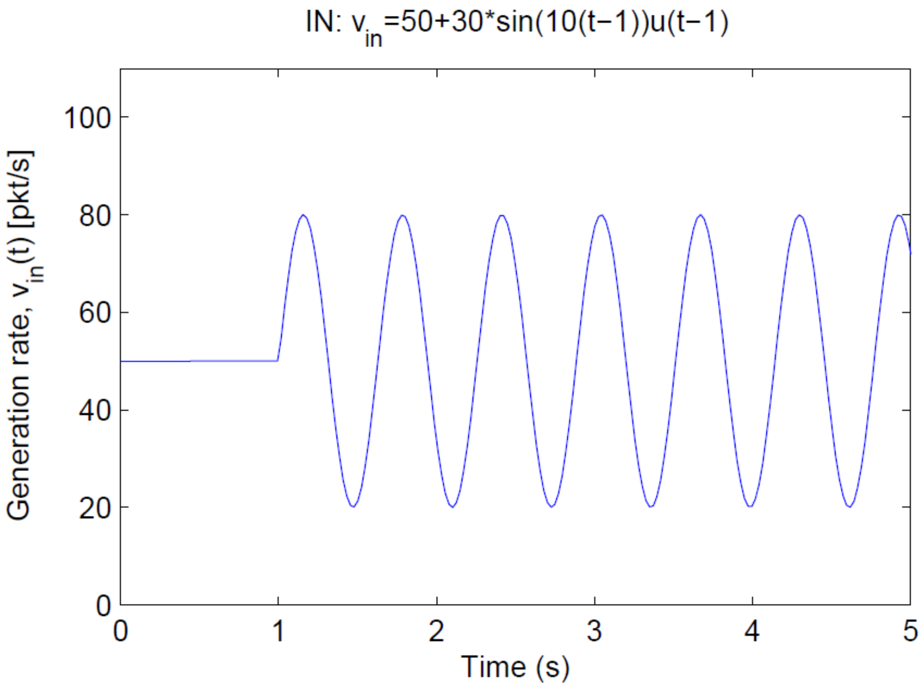
-) a generation rate starting from time t=1, $v_{in}(t) = A\sin(\omega t) \cdot u(t-1)$

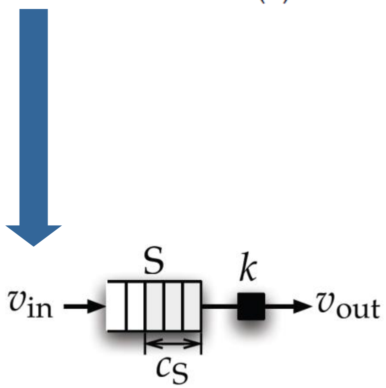
-) a service rate parameter, k

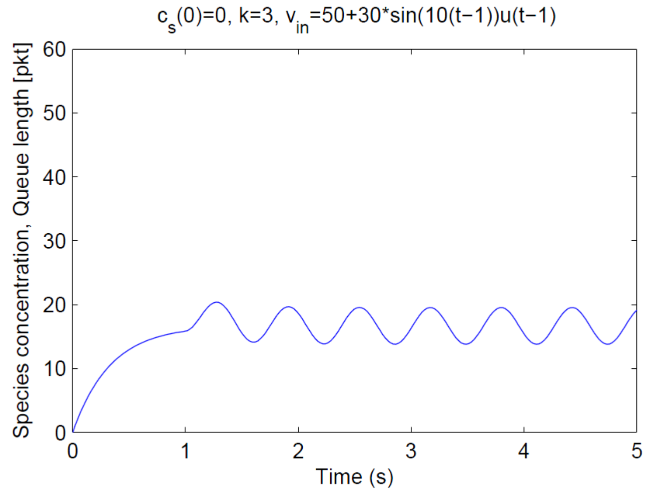
What is the time trend of …

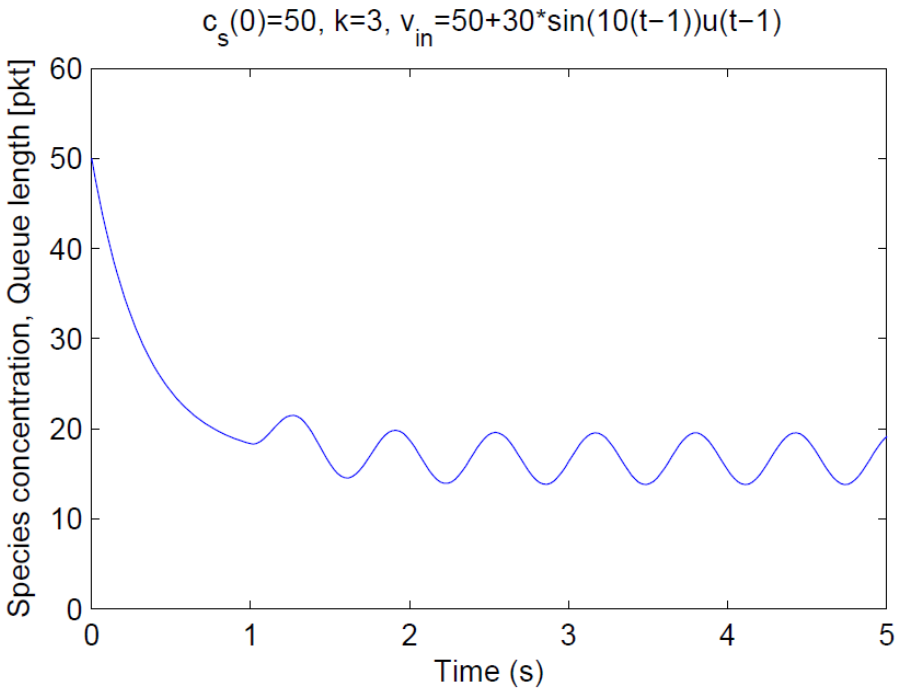
-) the queue length, $c_s(t)$?

A simple CNP (3b)

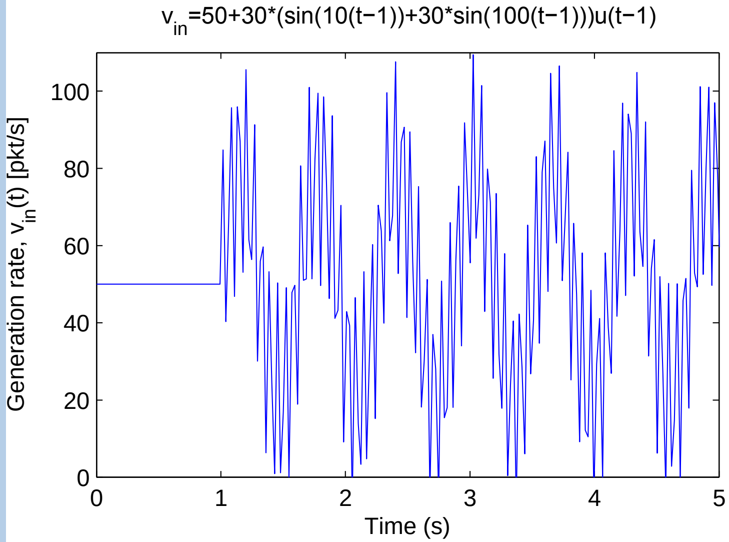


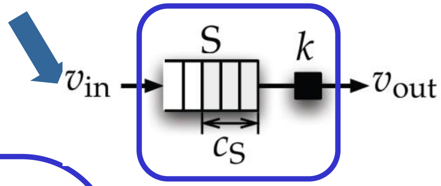


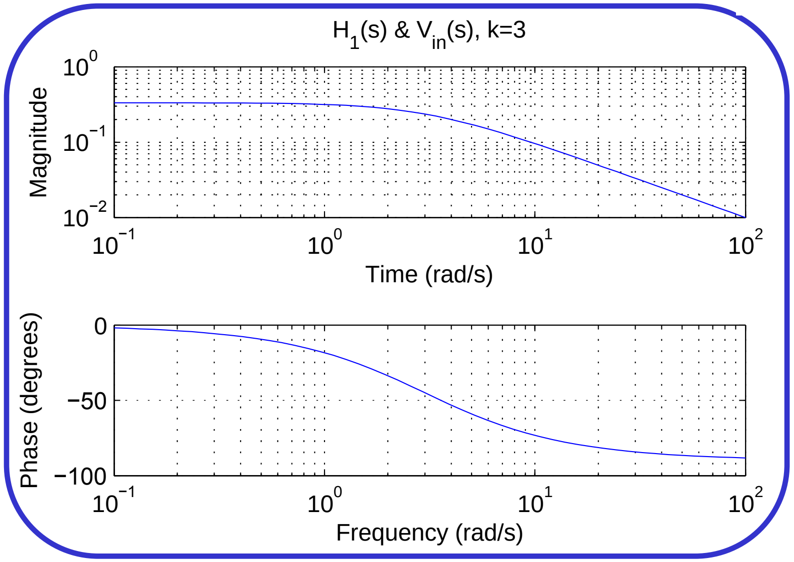


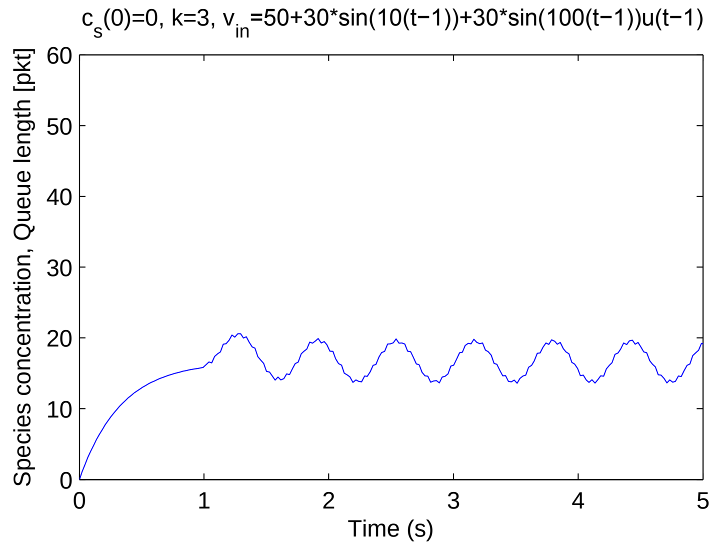


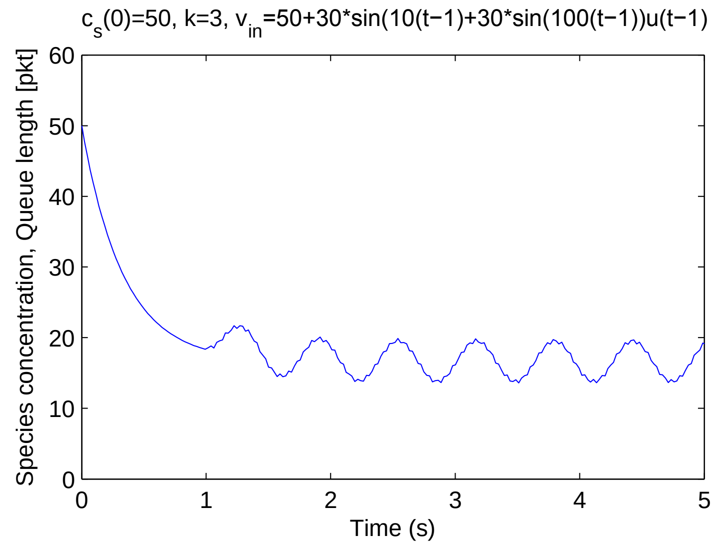
A simple CNP (4)











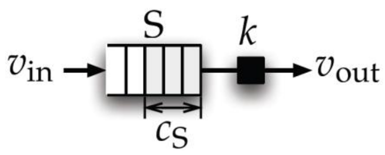
Signal Processing and Frequency Domain

Good for…

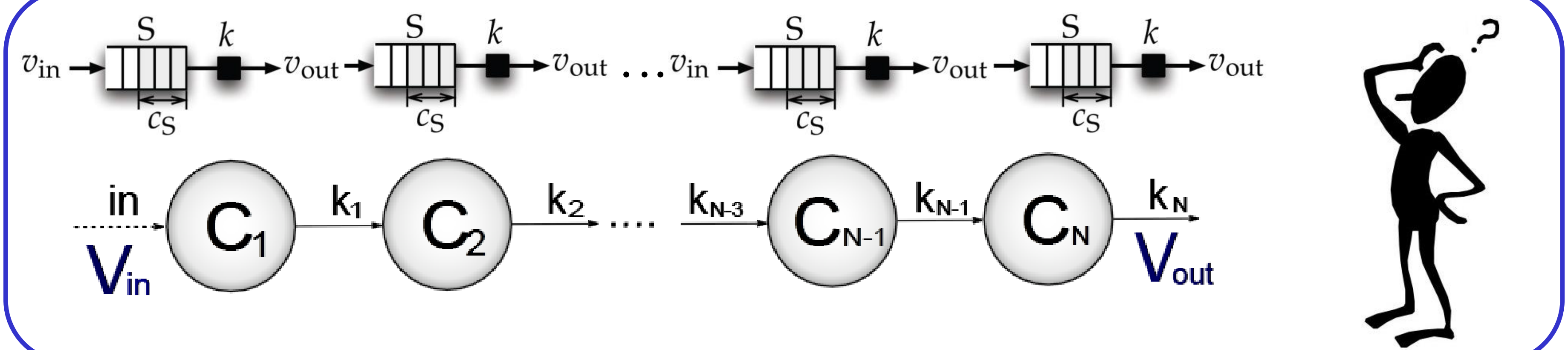
-) …

-) obtaining general results that can be re-used in future

-) …

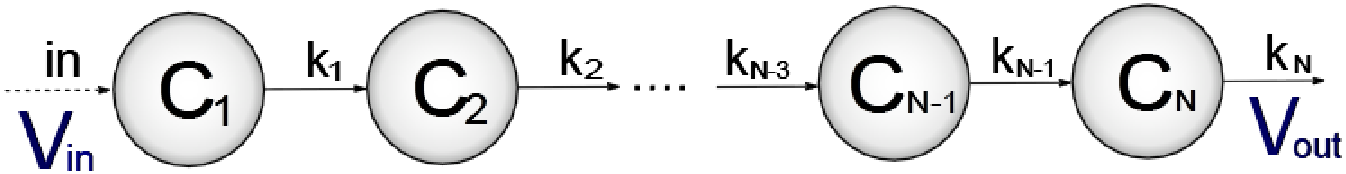


$$TF' = H_1(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{k}{s+k}$$



Module Composition: Series

Series of *N*-Node Networks



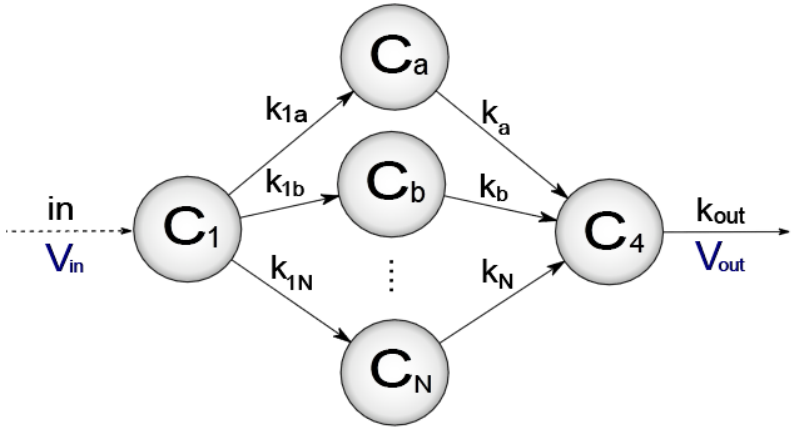
Transfer Function:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = H_1(s) \cdot H_2(s) \ldots H_N(s) = \prod_{n=1}^{N} H_n(s)$$

where $$H_n(s) = \frac{k_n}{s + k_n}$$

Module Composition: **Parallel**

Parallel of *N*-Node Networks

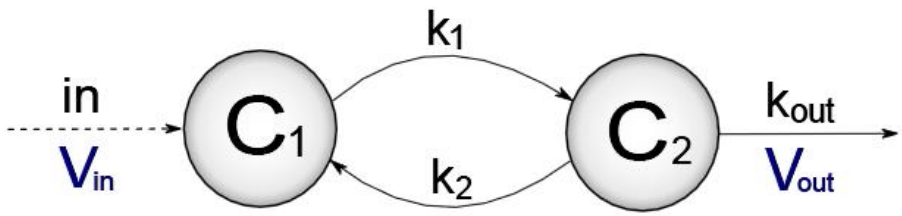


Transfer Function:

$$H(s) = \frac{k_{out}}{s + \sum_{n=1}^{N} k_{1n}} \cdot \left(\sum_{n=1}^{N} \frac{k_{1n} \cdot k_n}{s + k_n} \right) \cdot \frac{1}{s + k_{out}}$$

Module Composition: **Loop**

Loop Networks



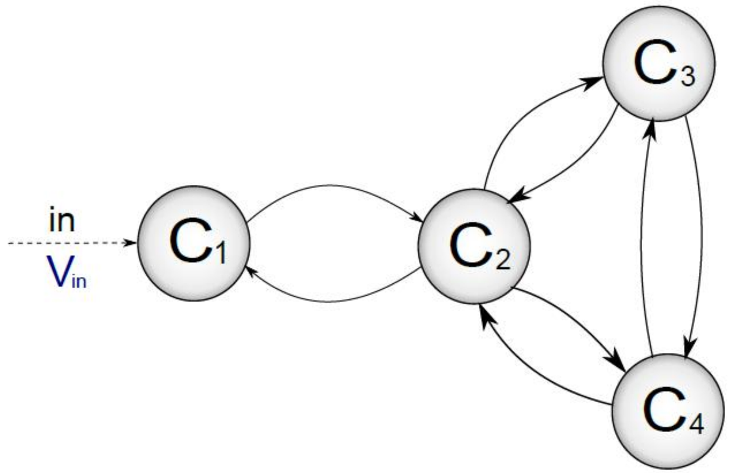
Transfer Function:

$$H(s) = \frac{V_{out}(s)}{v_{in}(s)} = \frac{k_1 k_{out}}{s^2 + s(k_1 + k_2 + k_{out}) + k_{in} k_{out}}$$

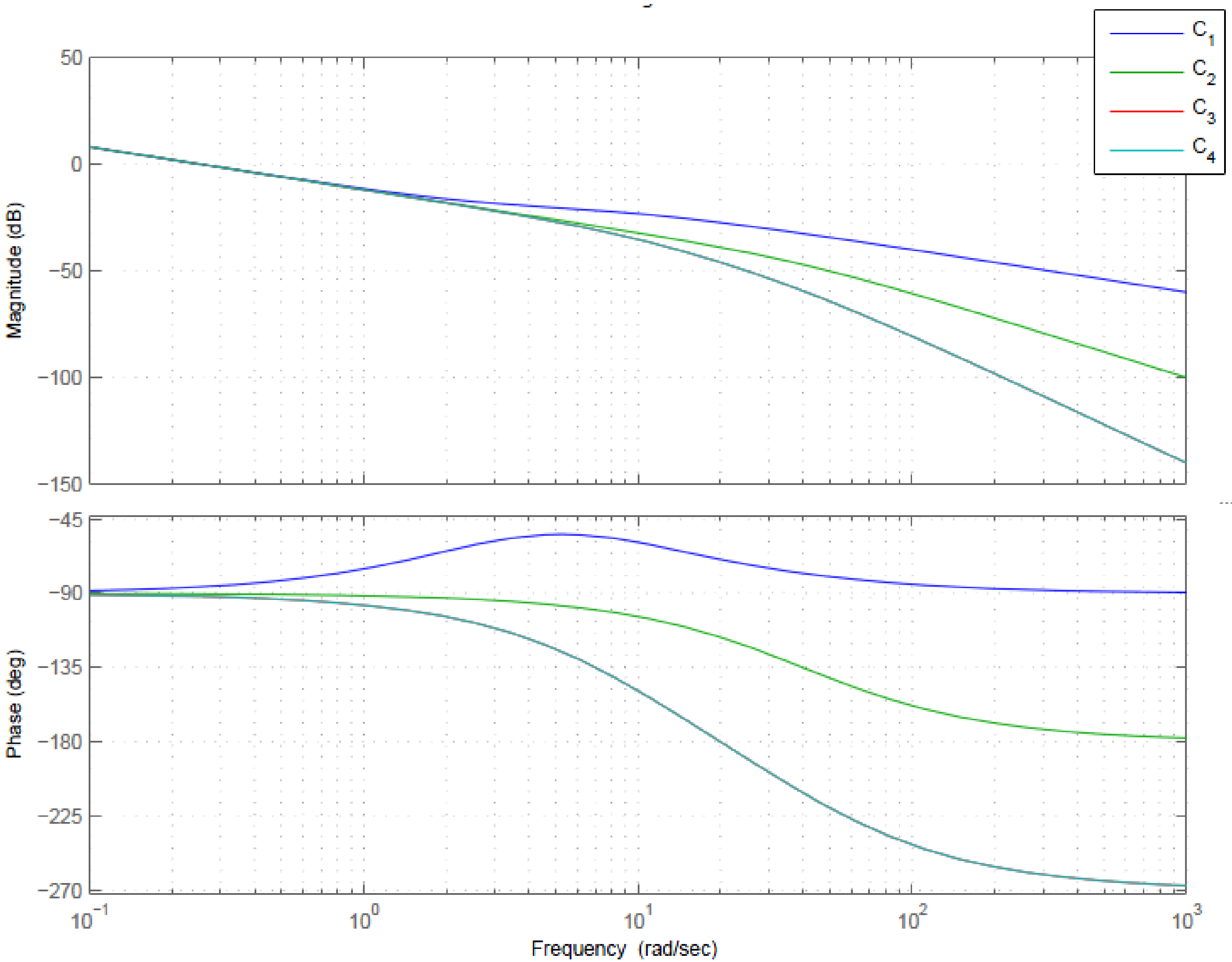
Real CNP analysis

The Disperser Protocol

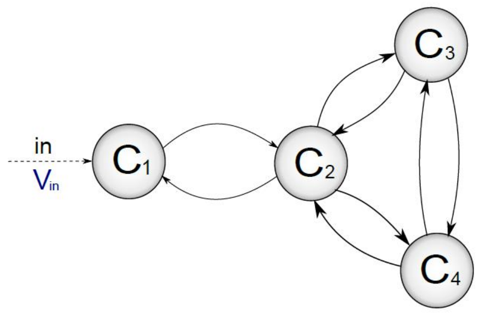
-) Distributed average calculation
-) Comparable to Gossip algorithm
-) Based on linear reaction models



The Disperser Protocol: **Results (1)**

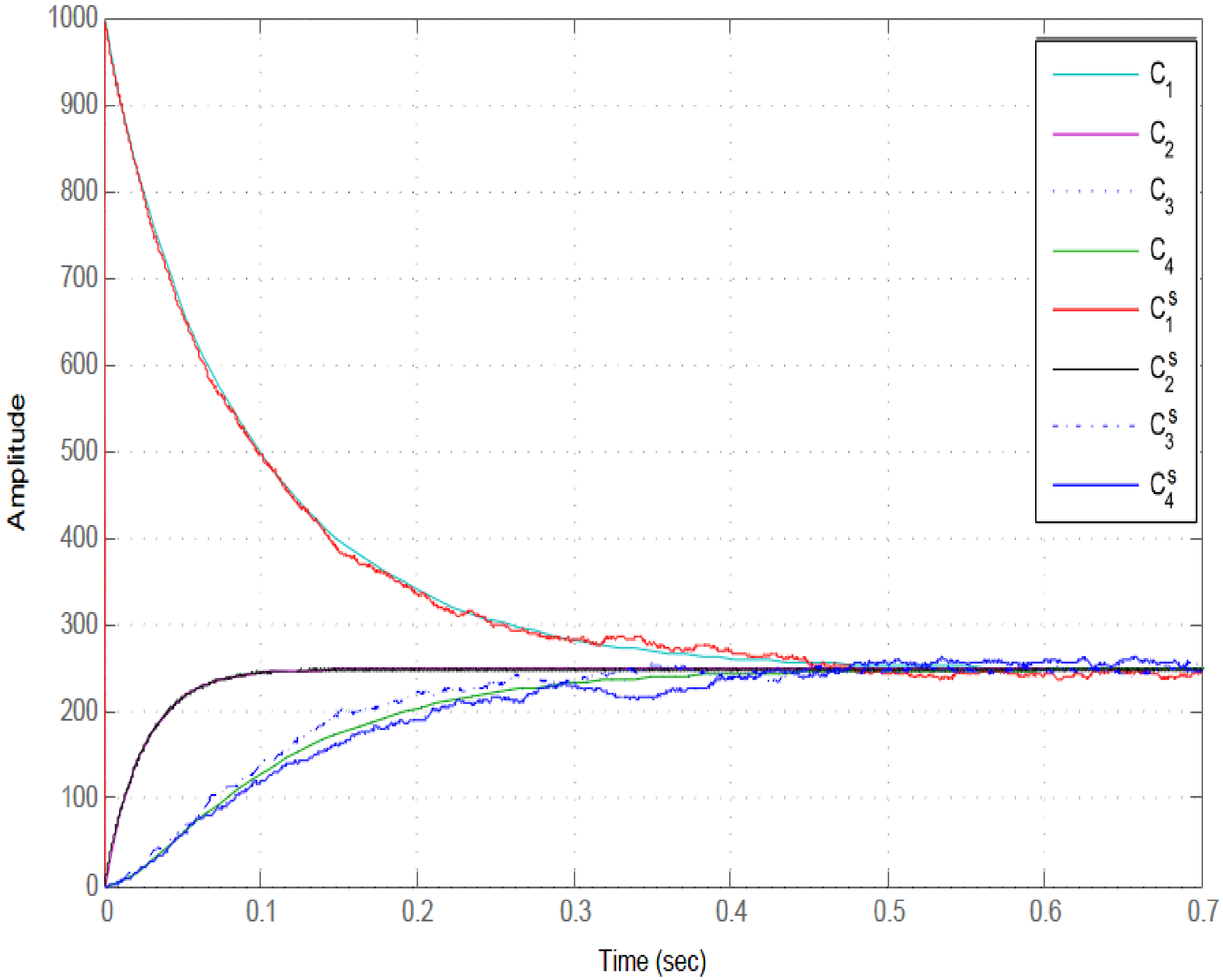


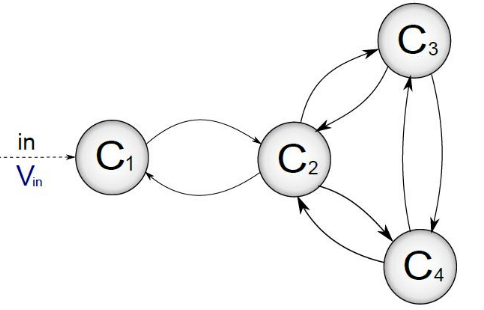
Frequency Transforms



The Disperser Protocol: **Results (2)**

Impulse Responses





CNP: **Why Analysis Deviates from Simulation?**

(i) **Time-continuous discrete-valued signal** — **Time-continuous continuous-valued signal**

High species concentrations → Low deviation

(ii) **Stochastic process** — **Deterministic process**

-) Exact description ↔ Chemical Master Equation (CME) only
The particular behavior of an observed CNP is a realization of a stochastic process
CME solution is a hard task!
Designing may require average behaviors of protocols only
High species concentrations → average ~ realization

-) ODE model describes the average (deterministic) behavior of CNPs.

Randomness from Signal-Processing Point of View (1)

See the original random signal as a (additive) stochastic component plus a deterministic one

Deterministic signal

$$y(t) = x(t) + n(t)$$

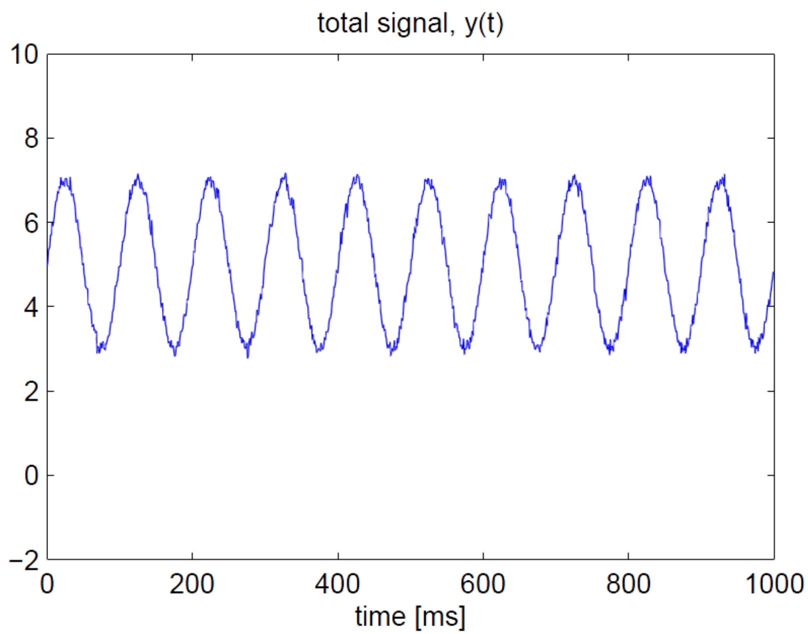
Original signal embeding (i) and (ii) effect

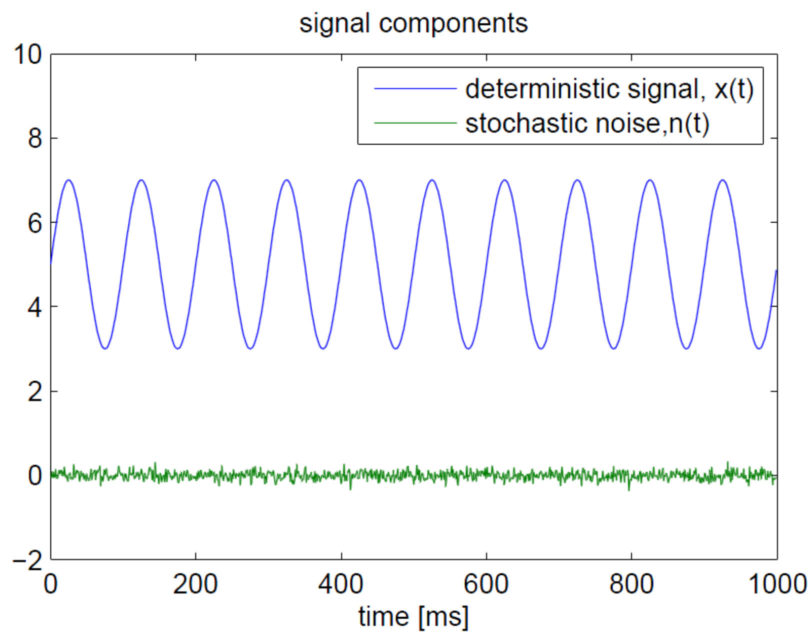
Stochastic component (random noise) (known statistical properties)

Randomness from Signal-Processing Point of View (2)

See the original random signal as a (additive) stochastic component plus a deterministic one

$$y(t) = x(t) + n(t)$$

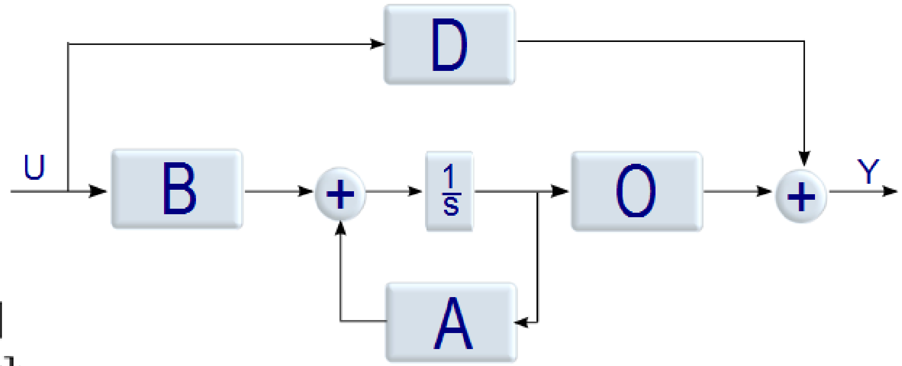




State-Space Description

-) Representation /analysis of multiple-input multiple-output (MIMO) systems

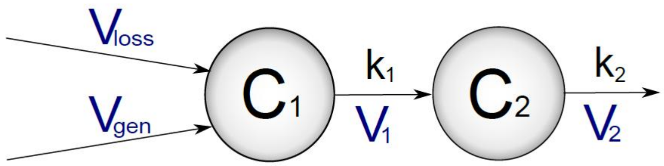
$$\dot{\bar{x}}(t) = \bar{\bar{A}} \cdot \bar{x}(t) + \bar{\bar{B}} \cdot \bar{u}(t)$$
$$\bar{y}(t) = \bar{\bar{O}} \cdot \bar{x}(t) + \bar{\bar{D}} \cdot \bar{u}(t)$$



State Matrix	$\bar{\bar{A}}$	[#states X #states]
Input Matrix	$\bar{\bar{B}}$	[#states X #inputs]
Output Matrix	$\bar{\bar{O}}$	[#outputs X #states]
Direct Transmission Matrix	$\bar{\bar{D}}$	[#outputs X #inputs]
Input Vector	$\bar{u}$	[1 X #inputs]
Output Vector	$\bar{y}$	[1 X #outputs]

$$\bar{\bar{H}}(s) = \bar{\bar{O}} \cdot \left(\bar{\bar{I}}s - \bar{\bar{A}}\right)^{-1} \cdot \bar{\bar{B}} \cdot \bar{\bar{D}} = \begin{array}{cc} \begin{array}{cccc} u_1 & u_2 & \dots & u_c \end{array} & \\ \begin{bmatrix} H_{11}(s) & H_{12}(s) & \cdots & H_{1C}(s) \\ H_{21}(s) & H_{22}(s) & \cdots & H_{2C}(s) \\ \vdots & \vdots & \ddots & \vdots \\ H_{R1}(s) & H_{R2}(s) & \cdots & H_{RC}(s) \end{bmatrix} & \begin{array}{c} y_1 \\ y_2 \\ \vdots \\ y_R \end{array} \end{array}$$

State-Space Representation: **an example**



Differential Equations:

$$\dot{x}_{c_1} = v_{gen} + v_{loss} - k_1 \cdot x_{c_1}$$
$$\dot{x}_{c_2} = k_1 \cdot x_{c_1} - k_2 \cdot x_{c_2}$$

Matlab Code:

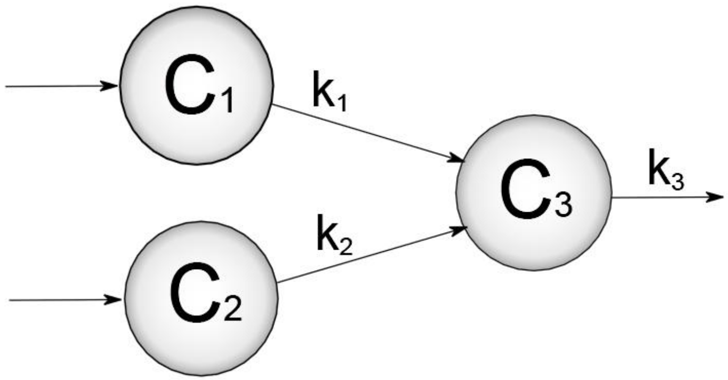
```
x=[Xc1     Xc2 ];
u=[Vgen   Vloss];
y=[k1*Xc1  k2*Xc2  Xc1+Xc2];
A=[-k1 0;k1 -k2]
B=[ 1  1; 0  0 ]
O=[ k1 0; 0  k2; 1  1]
D=[ 0  0; 0  0 ;  0  0]
syms s
H= O * ( s * eye(size(A)) - A )^(-1) *B + D
```

Matlab Result:

$$\overline{\overline{H}}(s) = \begin{bmatrix} \dfrac{k1}{k1 + s}, & \dfrac{k1}{k1 + s} \\[2ex] \dfrac{k1\ k2}{(k1 + s)\ (k2 + s)}, & \dfrac{k1\ k2}{(k1 + s)\ (k2 + s)} \\[2ex] \dfrac{1}{k1 + s} + \dfrac{k1}{(k1 + s)\ (k2 + s)}, & \dfrac{1}{k1 + s} + \dfrac{k1}{(k1 + s)\ (k2 + s)} \end{bmatrix}$$

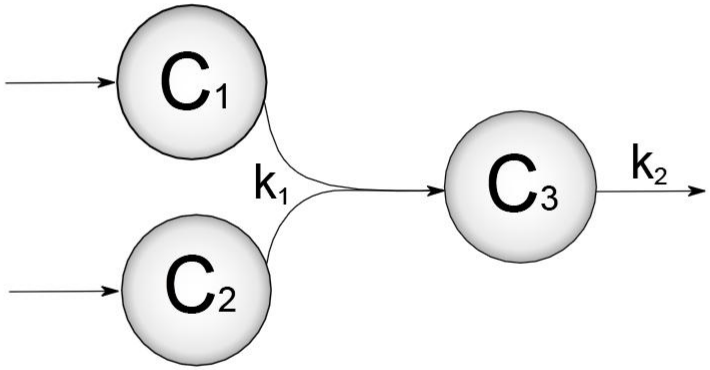
Linear VS Non-linear Reaction Models

Linear Model



$$\dot{c}_3(t) = k_1 c_1 + k_2 c_2 - k_3\, c_3$$

Non-Linear Model



$$\dot{c}_3(t) = k_1 c_1 c_2 - k_2\, c_3$$

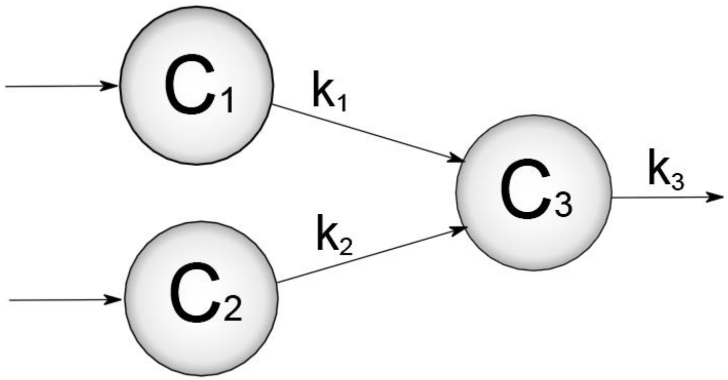
Remind…

Multiplication in time $\leftrightarrow$ Convolution in frequency

$$f(t) \cdot g(t) \quad \leftrightarrow \quad \frac{1}{j2\pi} \lim_{T \to \infty} \int_{\sigma - jT}^{\sigma + jT} F(x) \cdot G(s - x) \quad dx$$

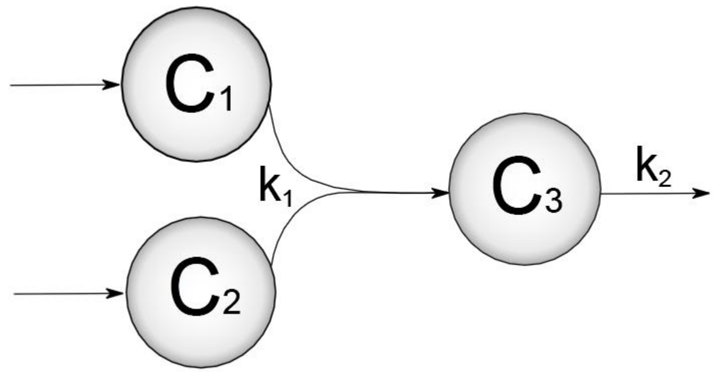
Linear VS Non-linear Reaction Models

Linear Model



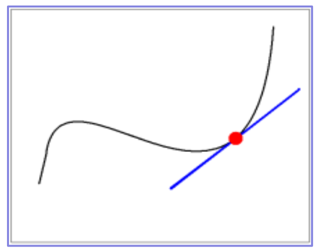
$$\dot{c}_3(t) = k_1 c_1 + k_2 c_2 - k_3\, c_3$$

Non-Linear Model



$$\dot{c}_3(t) = k_1 c_1 c_2 - k_2\, c_3$$

Linearize the system response around a fixed point (the steady state) and give a result that is valid for small perturbations around this point.



Metabolic Control Analysis (MCA)

-) Formal tool for linearizing (from Biology)

-) Nothing else than a linearization of the model behavior around the steady states

-) After the linearization the *state-space description* as well as the *Laplace transform* can be used

-) Already Ingalls[1] hinted the frequency as a useful analysis domain

!) Validity of results only for small perturbation of the input around the steady-states

1] Ingalls, B. «A frequency domain approach to sensitivity analysis of biochemical networks.» Journal of Physical Chemistry B. 2004. 1143-1152, vol.108, no.3.

MCA formalities (1)

State-space description of the linearized system

$$\dot{\bar{x}}(t) = \bar{\bar{A}} \cdot \bar{\gamma}(t) + \bar{\bar{B}} \cdot \bar{p}(t)$$
$$\bar{y}(t) = \bar{\bar{O}} \cdot \gamma(t) + \bar{\bar{D}} \cdot \bar{p}(t)$$

Input quantities

Steady states input

Perturbations of inputs

$$\bar{p}(t) = \bar{u}(t) - \bar{u}_{ss}$$

States

Steady states

Perturbations of states

$$\bar{\gamma}(t) = \bar{x}(t) - \bar{x}_{ss} \quad (\dot{x}(x,u) = 0 \rightarrow x_{ss})$$

MCA formalities (2)

State-space description of the linearized system

$$\dot{\bar{x}}(t) = \bar{\bar{A}} \cdot \bar{y}(t) + \bar{\bar{B}} \cdot \bar{p}(t)$$
$$\bar{y}(t) = \bar{\bar{O}} \cdot \gamma(t) + \bar{\bar{D}} \cdot \bar{p}(t)$$

State Matrix $\longrightarrow$ $\bar{\bar{A}} = \left[\; \bar{\bar{N}} \cdot \dfrac{\partial \bar{a}}{\partial \bar{x}} \; \right]_{(\bar{x}_{ss}, \bar{u}_{ss})}$ $\longleftarrow$ linearized relationship state – state variation

Input Matrix $\longrightarrow$ $\bar{\bar{B}} = \left[\; \bar{\bar{N}} \cdot \dfrac{\partial \bar{a}}{\partial \bar{u}} \; \right]_{(\bar{x}_{ss}, \bar{u}_{ss})}$ $\longleftarrow$ linearized relationship input – state variation

MCA formalities (3)

State-space description of the linearized system

$$\dot{\bar{x}}(t) = \bar{\bar{A}} \cdot \bar{\gamma}(t) + \bar{\bar{B}} \cdot \bar{p}(t)$$
$$\bar{y}(t) = \bar{\bar{O}} \cdot \gamma(t) + \bar{\bar{D}} \cdot \bar{p}(t)$$

Interest in… *States*

Output Matrix

$$\bar{\bar{O}} = \bar{\bar{I}}$$

Direct Transmission Matrix

$$\bar{\bar{D}} = 0$$

Interest in… *Rates*

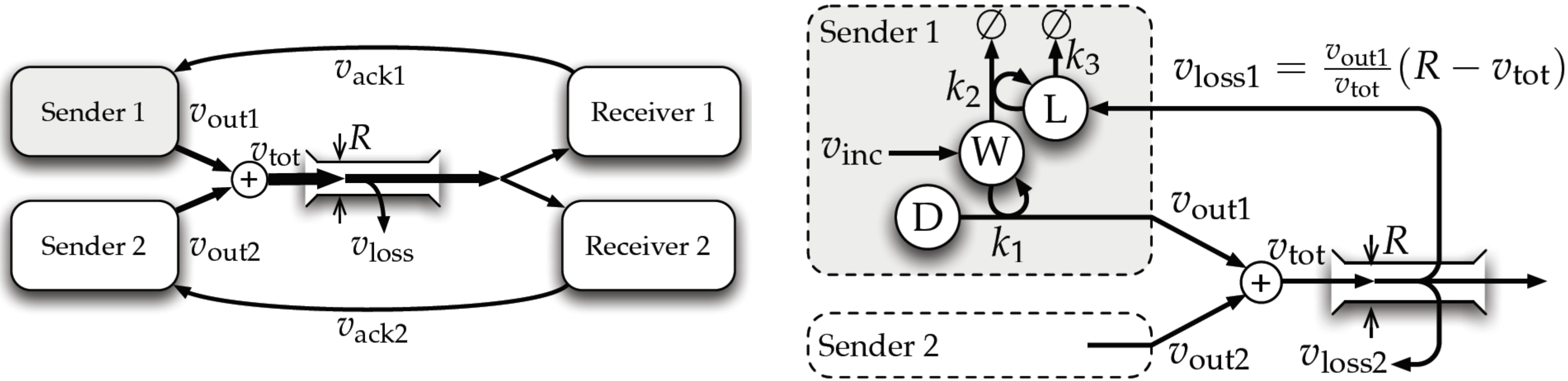
$$\bar{\bar{O}} = \left[\frac{\partial \bar{a}}{\partial \bar{x}} \right]_{(\bar{x}_{ss}, \bar{u}_{ss})}$$

Rate-deviation due to state-perturbation

$$\bar{\bar{D}} = \left[\frac{\partial \bar{a}}{\partial \bar{u}} \right]_{(\bar{x}_{ss}, \bar{u}_{ss})}$$

Rate-deviation due to input-perturbation

Non-linear Model example: **analyzing the C_3A protocol**

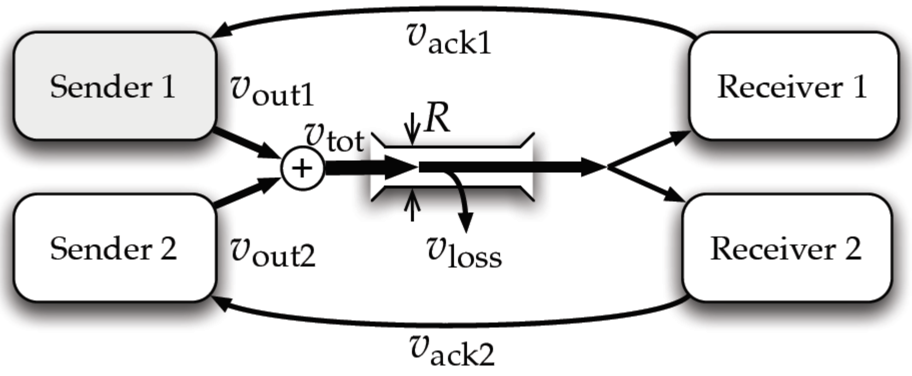


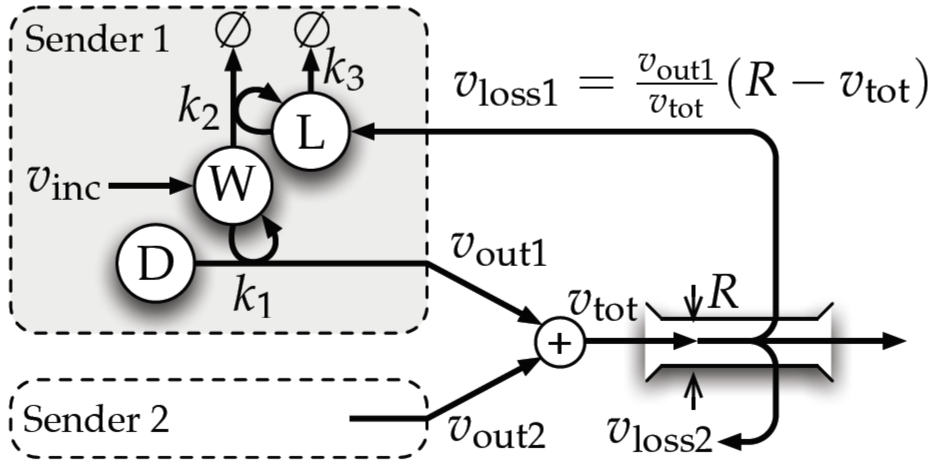
-) How does *sender 1* react to traffic generated by competing users?
(Effect of *n* users aggregate in *sender 2*)

-) What are the important parameters that determine the C_3A behavior?

-) What guidelines should designers follow to optimize their protocol?

MCA of the C_3A Protocol: **Settings**





-) *State system* - species: $\bar{y} = [c_w \quad c_l]$

-) *Input* - perturbation of the competing rate of *sender2*: $\bar{p} = [v_{out2}]$

-) Output - rate of *sender1*, v_{out1}

(!) It is not a proper reaction rate: $\bar{y} = [k_1 c_w]$

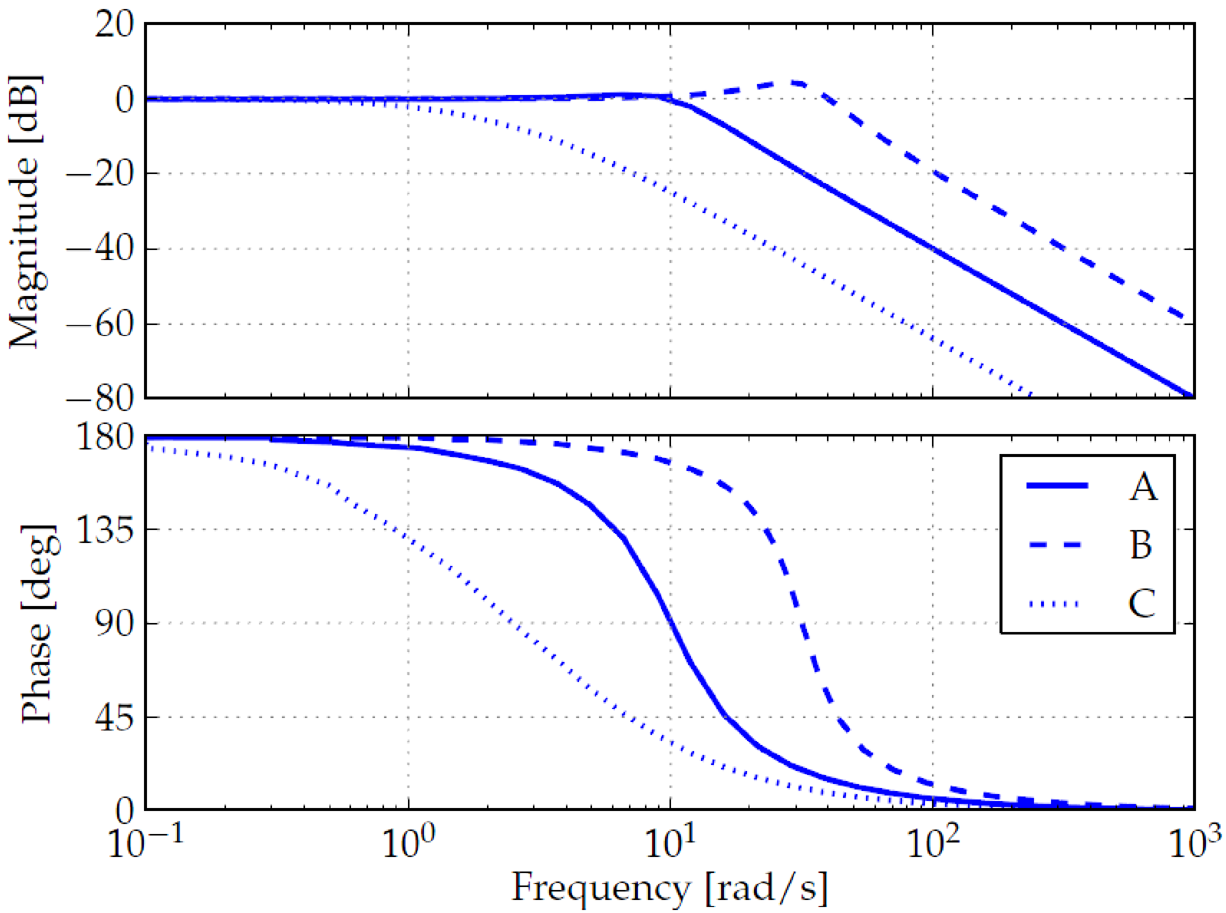
-) *Initial steady states*: *sender2* silent $v_{out2} = 0$

sender1 max.rate $v_{out1} = R$

MCA of the C_3A Protocol: **Results (1a)**

Frequency domain:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{V_{out1}(s)}{V_{out2}(s)} \cong -\frac{k_2 R}{s^2 + k_3 s + k_2 R}, \qquad \left(R \gg \sqrt{\frac{4 k_1 k_3 v_{inc}}{k_2}} \right)$$



MCA of the C_3A Protocol: **Results (1b)**

Frequency domain:

$$H(s)=\frac{V_{out}(s)}{V_{in}(s)}=\frac{V_{out1}(s)}{V_{out2}(s)}\cong-\frac{k_2R}{s^2+k_3s+k_2R},\qquad \left(R\gg\sqrt{\frac{4k_1k_3v_{inc}}{k_2}}\right)$$

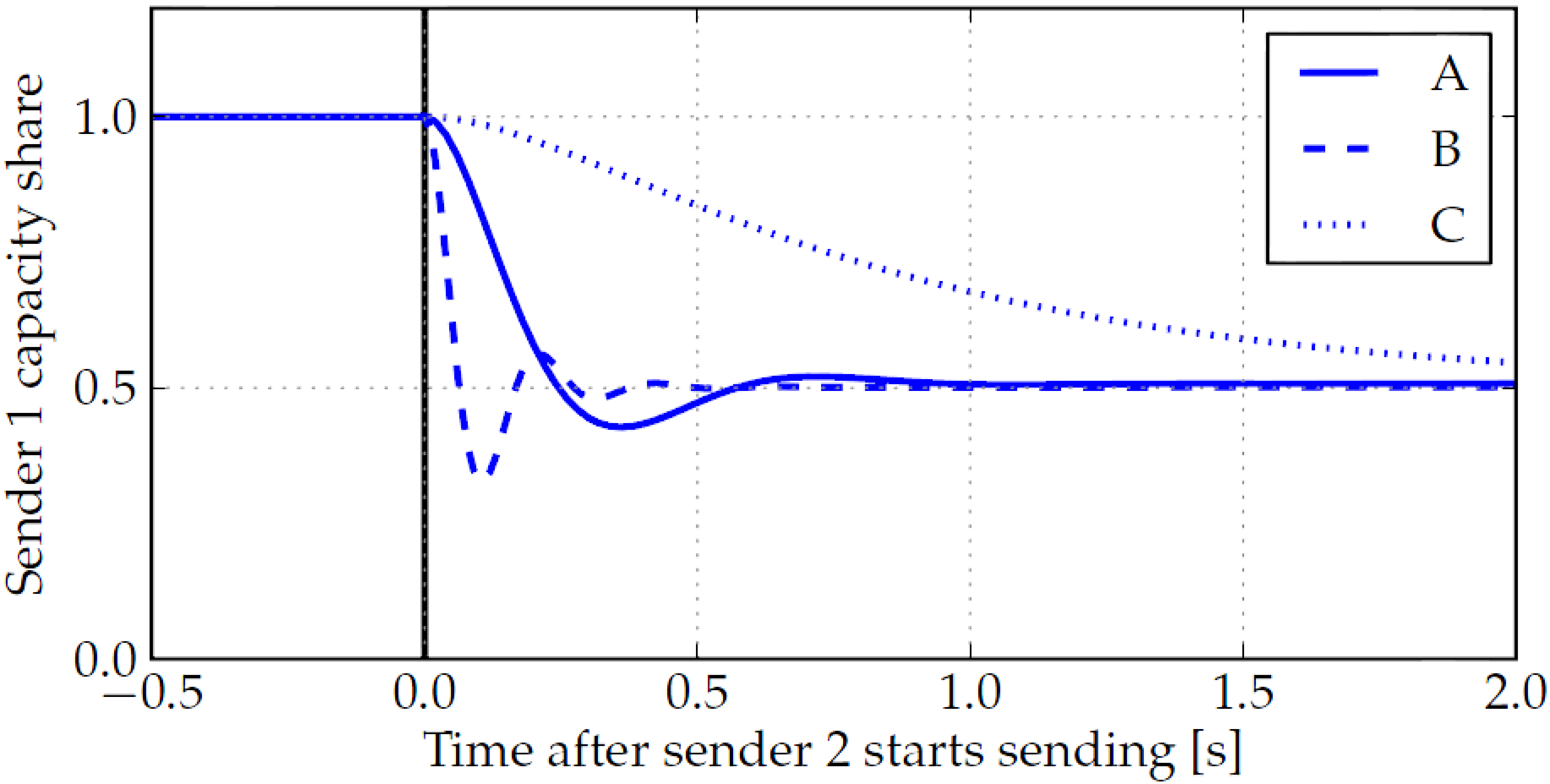
-) 2nd order low-pass filter (2 poles, 0 zeros)

-) nature of poles depends on $k_3, k_2,\ R$ only. → C_3A protocol's behavior

-) real part of poles always negative → Bounded-Input Bounded-Output (BIBO) stability of C_3A protocol

MCA of the C_3A Protocol: **Results (2a)**

Time domain: (step: $v_{out2}(t^-) = 0$pkt/s; $v_{out2}(t^+) = R/2$pkt/s)



MCA of the C_3A Protocol: **Results (2b)**

Time domain: (step: $v_{out2}(t^-) = 0$pkt/s; $v_{out2}(t^+) = R/2$pkt/s)

Under-dampened behavior for $\left(R > k_3{}^2/4k_2\right)$.

The step response is an exponential decaying sinusoid (A and B curve).

Settling time $t_{set} \cong 9.2/k_3$

Oscillation period $t_{osc} = 4\pi/\sqrt{4Rk_2 - k_3{}^2}$

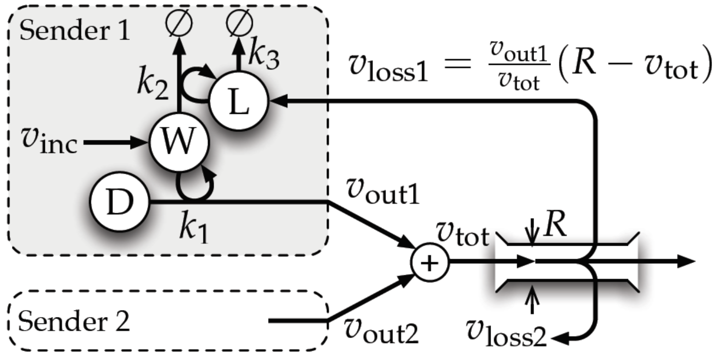
Overshoot [%] $= 100 \cdot e^{e^{-\pi k_3/\sqrt{4k_2R - k_3{}^2}}}$

Over-dampened behavior for $\left(R < k_3{}^2/4k_2\right)$.

The step response is a decaying exponential (C curve).

Settling time $t_{set} \cong 10/\left(k_3 \cdot \sqrt{k_3{}^2 - 4Rk_2}\right)$

MCA of the C_3A Protocol: **Results (3)**



Derived Guidelines for a C_3A Implementation:

-) k_2 large to obtain a quick response

-) k_3 large to obtain a quick response

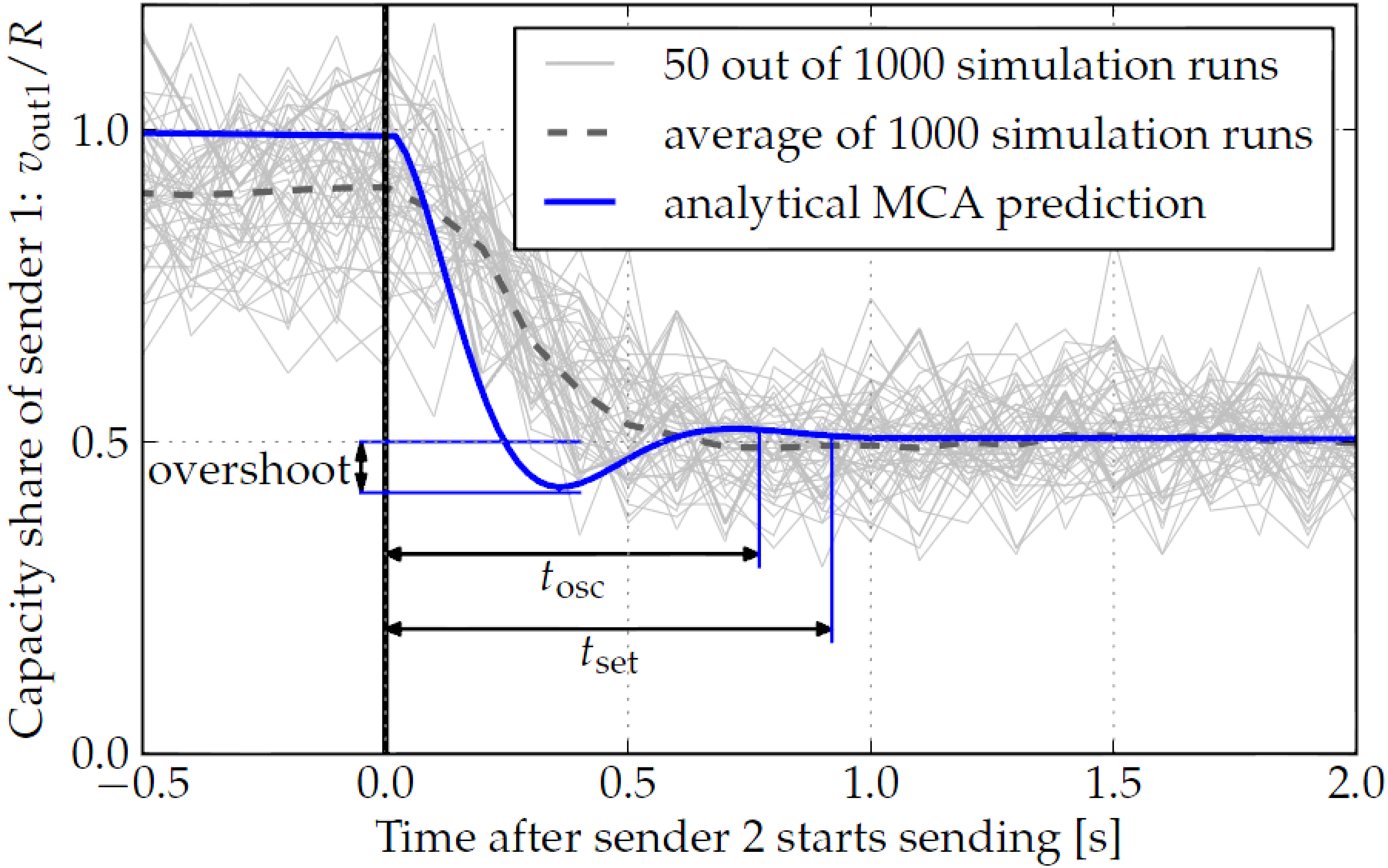
-) k_2 not too small w.r.t. k_3 to contain the overshoot (accurate response)

-) $k_1,\ v_{inc}$ large to quickly probe the channel capacity

-) $k_1,\ v_{inc}$ have marginal effect on C_3A behavior (aggressiveness of packets' transmission and packet-transmission rate increase)

-) Impose $\left(R \gg \sqrt{\frac{4k_1k_3v_{inc}}{k_2}}\right) \rightarrow k_1, k_3, v_{inc}$ small w.r.t. k_2

C_3A Protocol: **Analysis VS Simulation (1)**



C_3A Protocol: **Analysis VS Simulation (2)**

↑) Effect of protocol parameters

↑) Measured times

↓) (Small) deviations between each of the 1000 simulation runs, their mean and what predicted by the analysis.

Reasons

(i) Stochastic-deterministic models

(ii) Continuous-discrete models

(iii) The usefulness of the C_3A requires low loss-rate

→ We must keep low the species concentrations

→ Feedback based on a discrete signal

Lesson's message

✓ A new point of view (for protocols and not only…).

✓ Introduction to the frequency domain.

✓ Basic concepts of control theory.

✓ How control theory can be used in protocol analysis.

✓ Examples for better understanding the previous three lectures.

X Equations are important and must be known.

X Specific results regarding protocols (e.g. Disperser and C_3A).

References

Ingalls, Brian P. *"A frequency domain approach to sensitivity analysis of biochemical networks.*", Journal of Physical Chemistry B. 2004. 1143-1152, vol.108, no.3. http://pubs.acs.org/doi/abs/10.1021/jp036567u

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Monti, Massimo: *A Signal Processing Approach to the Analysis of Chemical Networking Protocols"*MS Thesis, Dep.of Telecommunications Engineering, University of Pisa, and Computer Science Dep., University of Basel, July 2010. http://cn.cs.unibas.ch/pub/doc/2010-msthMonti.pdf

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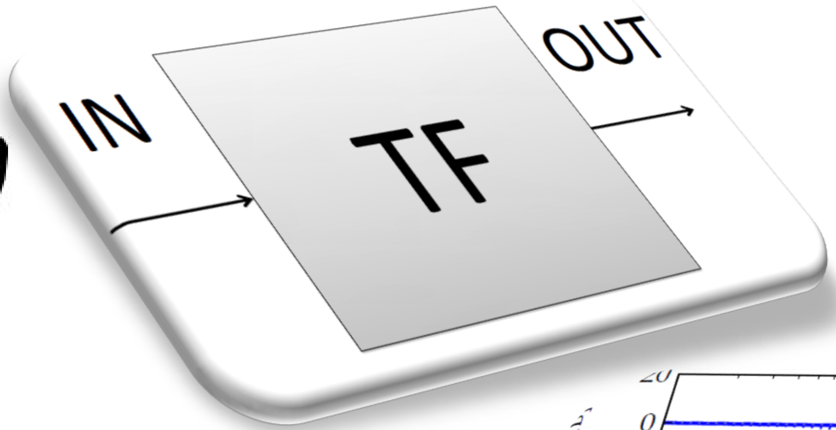
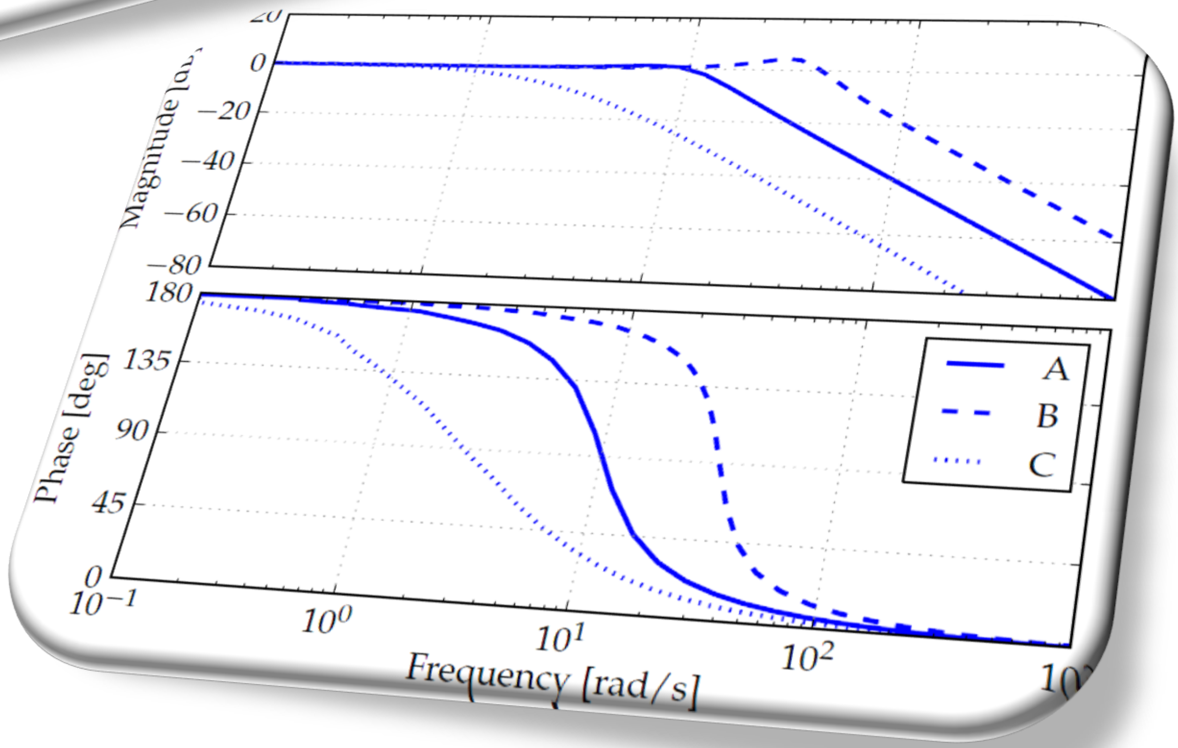
Questions



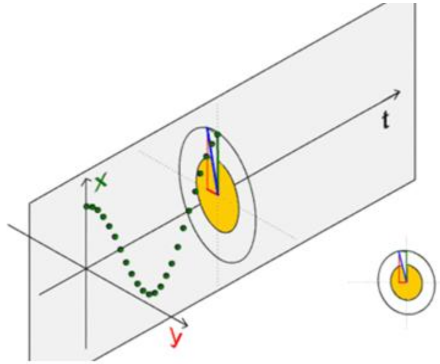


Thank You

More on Transformations…

Laplace, Fourier and Z TRANFORMS

(trying to avoid semantics wars!)



- All based on Fourier series → all complicated functions as a sum of waves (sin & cos)
- Simplify the notation : sin & cos → e^{j} (rotating vector)
- Increase the interval lenght of Fourier series → Fourier transform (integral form)
- Laplace: multiplication by an arbitrary decaying exponential, e^{-ct} (c big enough to ensure the convergence)
- In the discrete-time domain → z transform (continuous signal multiplied by a time-shifted impulse)