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ABSTRACT

The first objective of this paper is to study the existence of greenness in green bonds. For this objective, we propose a new model of price correlations between green bonds and energy commodities. The second objective is to examine the performance of green bonds over conventional bonds. We propose a new model of the expected return, the risk, and the performance ratio of green bond premiums defined by the log price differences between green and conventional bonds so as to address the second objective. Empirical studies using the data of green and conventional bond indices and crude oil prices show that the Bloomberg Barclays MSCI and the S&P green bond indices tend to have positive correlations with and increase in line with both WTI and Brent crude oil prices while the Solactive green bond index tends to have negative correlations with and decrease in line with both WTI and Brent crude oil prices. From the empirical evidence of the positive relationship between energy and environmental value, it is suggested that the greenness is incorporated in the Bloomberg Barclays MSCI and the S&P green bond. In contrast taking it into account that the conventional S&P bond index has negative correlations with WTI and Brent crude oil prices which are the same as the results of the Solactive green bond index, the Solactive green bond index may not fully represent the characteristics of green bonds in the sense of environmental value. We also demonstrate that the expected returns of green bond premiums are positive while decreasing and that the risks of green bond premiums are slightly decreasing but almost flat over time in the recent years, resulting in positive but decreasing information ratios. It implies that green bond investment performance is superior to conventional bond investment performance but the superiority is decaying over time.

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1. Introduction

Green bonds are fixed income investments that fund environmentally friendly projects. Bonds for investments in renewable energy and energy-saving buildings are typical examples. Green bond issuance has recently been increasing as observed in the year 2019 when a new global record of green bond issuance USD 257.7 billion was achieved (Climate Bonds Initiative, 2019). However do green bonds properly include environmental value? It has empirically been shown that environmental assets such as emission credits and clean energy indices have positive correlations with energy values such as fossil fuels (see e.g., Kanamura (2016), Gupta (2017)). If a green bond also has value as an environmental asset, it is expected to show a positive correlation with the value of fossil fuels. In this study, based on the relationship

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with fossil fuels, we seek a solution to the research question whether green bonds have environmental value. Furthermore, even if we can observe the environmental value associated with energy value, the important question is whether the greenness leads to the investment performance more than the passive investment. To address the question, we need to employ the performance evaluation model of green bond investments driven by energy value.

As the first school of the literature on green bonds, we have research to examine the characteristics of green bonds including the performance of green bonds. Pham (2016) is the first research to analyse the volatility behaviour of the green bond market by using data on daily closing prices of the S&P green bond indices between April 2010 and April 2015. Hachenberg and Schiereck (2018) find that rating classes AA-BBB of green bonds trade marginally tighter for the respective period compared to non-green bonds of the same issuers. Furthermore, they find that financial and corporate green bonds trade tighter than their comparable non-green bonds, and government-related green bonds on the other hand trade marginally wider. Karpf and Mandel (2018) show that, although returns on conventional bonds are on average higher than green bonds, the differences can largely be explained by the fundamental properties of the bonds. Flammer (2018) shows that green bonds yield i) positive announcement returns, ii) improvements

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Table 1Basic Statistics of Green Bond Indices (GBs), S&P Bond Index (SPB), and Crude Oil Prices: The data covers from November 3, 2014 to December 31, 2018, which is obtained from the Bloomberg.

	MSCI GB	SP GB	SLA GB	SPB	WTI	Brent
Mean	99.538	130.724	117.318	106.184	52.574	56.842
Std. Dev.	3.517	4.744	2.739	4.177	10.414	12.010
Skewness	0.222	0.133	-1.153	-0.340	0.294	0.262
Kurtosis	1.969	1.896	4.501	1.577	2.702	2.530
Observations	1086	1086	1086	1086	1086	1086

in long-term value and operating performance, iii) improvements in environmental performance, iv) increases in green innovations, and v) an increase in ownership by long-term and green investors. Nanayakkara and Colombage (2019) find that green bonds are traded at a premium of 63 basis points, compared with a comparable corporate bond issue. Zerbib (2019) demonstrates that the yield of a green bond is lower than that of a conventional bond by estimating the yield differential between a green bond and a counterfactual conventional bond from July 2013 to December 2017. By using a simple framework that incorporates assets with nonpecuniary utility, Baker et al. (2018) find that green municipal bonds are issued at a premium to otherwise similar ordinary bonds. Broadstock and Cheng (2019) find that the correlations between green and black, i.e., conventional, bonds are time varying and they are sensitive to: changes in financial market volatility; economic policy uncertainty; daily economic activity; oil prices and; uniquely constructed measures of positive and negative news-based sentiment towards green bonds. Baulkaran (2019) demonstrates that the cumulative abnormal returns due to the green bond issuance announcement are positive and significant, implying that shareholders view this form of financing as value-enhancing and that funds from green bonds issuance are used to undertake profitable green projects or as a means of risk mitigation. Hyun et al. (2020) find that the comparison of liquidity adjusted yield premiums of green bonds versus synthetic conventional bonds indicates that, on average, there is no robust and significant yield premium or discount on green bonds. As the second school, we have research to investigate the relationship between green bonds and the other markets. Gormus et al. (2018) demonstrate that energy markets are found to impact the entire high-yield bond market, which may include greenness, from both price and volatility perspectives. Reboredo (2018) provides the structure of dependence between green bond and financial markets, offering the practical implications for investors in terms of i) the diversification benefits of green bonds in investor portfolios and ii) how green bond prices could be impacted by price oscillations in the financial markets. In particular, Reboredo (2018) shows that green bonds prices weakly comove with energy commodity markets. These researches are interesting in the sense that they examine the characteristics of green bonds including the performance of green bonds and investigate the relationship between green bonds and the other markets. But they do not study the existence of greenness in green bonds and the investment performance by using the existing relationship of environmental asset and energy price correlation with the energy price as a new angle as long as we know.

This paper analyses the correlations between green bond prices as environmental value and crude oil prices as fossil fuel value in order to clarify the existence of greenness in green bonds and the investment performance. First, we propose a price model based on the supply and demand of both assets and its correlation model, using a new framework called "a structural type model" incorporating economic backgrounds. We also propose a model of the expected return, the risk, and the performance ratio of green bond premiums defined by the log price differences between green and conventional bonds. Based on the price correlation model, we empirically analyse the correlations between the two assets based on the data, and we deepen the understandings of the structure of the relationship between environmental value and fossil fuel value. On the other hand, as a reduced type model that does not include an economic background but focuses on the goodness of fit to data, using the dynamic conditional correlation model of Engle (2002) which is an existing econometric model, we empirically analyse the correlations between fossil fuel value and environmental value as in energy prices and green bond indices, respectively. Comparing the results from both models, we verify the robustness of the proposed structural type model. More importantly, we demonstrate that the expected returns of green bond premiums are positive while decreasing and that the risks of green bond premiums are slightly decreasing but almost flat over time in the recent years, resulting in positive but decreasing information ratios. It implies that green bond investment performance is superior to conventional bond investment performance but the superiority is decaying over time. It also implies positive expected returns of green bond premiums defined in this paper may lead to negative yield differences between green and conventional bonds taking account of bond price-yield relationship.

This paper is organised as follows. Section 2 proposes a new correlation model between green bond and crude oil prices and a new model of the expected return, the risk, and the performance ratio of green bond premiums by using the supply and demand relationship. Section 3 conducts empirical studies regarding the existence of greenness in green bonds and the investment performance by using the Bloomberg

Table 2Correlation Model Parameter Estimation between Bloomberg Barclays MSCI Green Bond Index (MSCI GB) and WTI Crude Oil Price: Note all estimates are statistically significant judging from the corresponding standard errors.

Parameters	a_1	$\sigma_{\!\scriptscriptstyle D}$	c_1	a_{2G}	$\sigma_{\!\scriptscriptstyle VG}$	c_{2G}	$ ho_G$	p_G	q_G
Estimates Std Errors Loglikelihood AIC SIC	-0.178 0.001 7131 -14245 -14205	2.636E+03 4.950E-01	1.358E+04 7.444E-01	0.650 0.000	7.567E+03 2.846E-03	6.972E+03 2.674E-01	-0.078 0.000	6.572E-01 4.191E-02	-1.908E-02 7.810E-04

Table 3Correlation Model Parameter Estimation between S&P Green Bond Index (SP GB) and WTI Crude Oil Price: Note all estimates except ρ_G are statistically significant judging from the corresponding standard errors.

Parameters	a_1	$\sigma_{\!\scriptscriptstyle D}$	c_1	a_{2G}	σ_{VG}	c_{2G}	$ ho_G$	p_G	q_G
Estimates Std Errors Loglikelihood AIC SIC	-0.168 0.001 7242 -14468 -14428	1.032E+02 4.060E-01	5.114E+02 7.192E-01	0.431 0.003	7.514E+03 2.832E-03	1.877E+04 1.011E+00	-2.763E-04 1.148E-02	2.184E+01 6.012E-02	-4.972E-01 1.148E-03

Table 4Correlation Model Parameter Estimation between Solactive Green Bond Index (SLA GB) and WTI Crude Oil Price: Note all estimates except ρ_G are statistically significant judging from the corresponding standard errors.

Parameters	a_1	$\sigma_{\!\scriptscriptstyle D}$	c_1	a_{2G}	σ_{VG}	c_{2G}	$ ho_G$	p_G	q_G
Estimates Std Errors Loglikelihood AIC SIC	-0.166 0.001 7417 -14817 -14777	9.532E+01 4.322E-01	4.677E+02 7.117E-01	0.480 0.003	7.513E+03 2.792E-03	1.835E+04 1.027E+00	-2.694E-04 1.311E-02	-1.381E+01 6.062E-02	5.446E-01 1.147E-03

Barclays MSCI, the S&P, and the Solactive green bond indices and WTI and Brent crude oil prices. Section 4 concludes.

2. The model

2.1. A model of price correlations between green bonds and energy assets

As an application of a structural type model, we propose a price model based on the supply and demand of green bonds that includes the structure in which the supply of environmental assets is influenced by the demand amount of fossil fuels and the correlation model with supply-demand based fossil fuel prices. In the case of green bonds, with reference to empirical evidence of a downward sloping demand curve of treasury notes in Kamara (1994) (also see Tanner (1975)) and an inelastic supply curve for bond yields in Grande et al. (2013), we propose a model that the equilibrium green bond price is determined by the intersection of the inelastic supply curve with the downward sloping demand curve.

We model crude oil prices and green bond prices using the supply and demand relationship. We assume that a crude oil price P_t is given by the inverse Box-Cox supply curve function in Eq. (1) and the volume D_t process corresponding to demand in Eq. (3) under short-term price inelasticity of demand. We also assume that a green bond index B_t^i is given by the inverse Box-Cox demand curve in Eq. (2) and the volume V_t^i process corresponding to supply in Eq. (4) (e.g., Kanamura (2013), Kanamura (2015)) under short-term green bond price inelasticity of supply. Kanamura (2015) suggests that V_t^i is the same direction with trading volume. Here we assume that crude oil prices affect the volume processes of the green bond index in order to consider the impacts of crude oil prices on green bond values referring to the impact of crude oil demand shock to aggregate U.S. bond index real returns for 8 months shown in Kang et al. (2014). We have the equilibrium prices of crude oil

 P_t and a green bond B_t^i , respectively as follows.

$$P_{t} = \left(1 + a_{1} \frac{D_{t}}{c_{1}}\right) \frac{1}{a_{1}},\tag{1}$$

$$B_t^i = \left(1 + a_{2i} \frac{\overline{V}_t^i - V_t^i}{c_{2i}}\right)^{\frac{1}{a_{2i}}},\tag{2}$$

$$dD_t = \mu_D dt + \sigma_D dw_t, \tag{3}$$

$$dV_t^i = \alpha_i(P_t)dD_t + \sigma_{Vi}dz_t^i. \tag{4}$$

Note that $E_t[dw_t dz_t^i] = \rho_i dt$ and i = G and C represent green and conventional bonds, respectively. Here we assume that the average maturity of green bond indices is constant because of the rebalance of contributed green bonds included in the index.

By using Ito's Lemma, we have

$$\frac{dP_t}{P_t} = \mu_p dt + \sigma_P dw_t, \tag{5}$$

$$\sigma_P = \frac{P_t^{-a_1}}{c_1} \sigma_D,\tag{6}$$

$$\mu_P = \frac{\mu_D}{\sigma_D} \sigma_P + \frac{1 - a_1}{2} \sigma_P^2,\tag{7}$$

$$\frac{dB_t^i}{B_+^i} = \mu_{Bi}dt + \sigma_{Bi}du_t^i, \tag{8}$$

 Table 5

 Correlation Model Parameter Estimation between S&P Bond Index (SPB) and WTI Crude Oil Price: Note all estimates are statistically significant judging from the corresponding standard errors.

Parameters	a_1	$\sigma_{\!\scriptscriptstyle D}$	c_1	a_{2C}	σ_{VC}	c_{2C}	$ ho_{ m c}$	p_C	q_C
Estimates Std Errors Loglikelihood AIC SIC	-0.448 0.000 7786 -15554 -15509	3.853E+01 5.431E-08	5.541E+02 5.315E-08	4.939 0.000	6.301E+02 6.972E-08	1.778E-06 2.733E-09	1.052E-01 9.389E-09	4.238E+00 1.152E-05	3.499E-03 2.423E-07

Table 6Correlation Model Parameter Estimation between Bloomberg Barclays MSCI Green Bond Index (MSCI GB) and Brent Crude Oil Price: Note all estimates are statistically significant judging from the corresponding standard errors.

Parameters	a_1	$\sigma_{\!\scriptscriptstyle D}$	c_1	a_{2G}	σ_{VG}	c_{2G}	$ ho_G$	p_G	q_G
Estimates Std Errors Loglikelihood AIC SIC	0.308 0.001 7189 -14362 -14323	4.131E+03 4.185E-01	3.367E+03 7.290E-01	0.599 0.000	7.532E+03 2.808E-03	8.772E+03 2.525E-01	-0.114 0.000	3.858E-01 3.798E-02	-1.137E-02 6.698E-04

Table 7Correlation Model Parameter Estimation between S&P Green Bond Index (SP GB) and Brent Crude Oil Price: Note all estimates except ρ_G are statistically significant judging from the corresponding standard errors.

Parameters	a_1	$\sigma_{\!\scriptscriptstyle D}$	c_1	a_{2G}	σ_{VG}	c_{2G}	ρ_G	p_G	q_G
Estimates Std Errors Loglikelihood AIC SIC	0.311 0.001 7303 -14589 -14549	1.221E+02 4.354E-01	9.838E+01 6.801E-01	0.432 0.003	7.515E+03 2.793E-03	1.867E+04 1.010E+00	0.000 0.013	1.977E+01 5.972E-02	-3.972E-01 1.130E-03

$$\sigma_{Bi} = \frac{\left(B_t^i\right)^{-a_{2i}}}{c_{2i}} \overline{\sigma}_{Bi},\tag{9}$$

$$\overline{\sigma}_{Bi} = \sqrt{\alpha_i (P_t)^2 \sigma_D^2 + \sigma_{Vi}^2 - 2\rho_i \alpha_i (P_t) \sigma_D \sigma_{Vi}}, \tag{10}$$

$$\mu_{Bi} = -\alpha_i(P_t) \frac{\mu_D}{\overline{\sigma}_{Bi}} \sigma_{Bi} + \frac{1 - a_{2i}}{2} \sigma_{Bi}^2, \tag{11}$$

$$du_t^i = \frac{1}{\overline{\sigma_{Pi}}} \left(-\alpha_i(P_t)\sigma_D dw_t + \sigma_{Vi} dz_t^i \right), \tag{12}$$

$$\rho_{PB^{i}} = \frac{1}{dt} \operatorname{corr} \left(\frac{dB_{t}^{i}}{B_{t}^{i}}, \frac{dP_{t}}{P_{t}} \right)$$

$$=\frac{-\alpha_{i}P_{t}\sigma_{D}+\rho_{i}\sigma_{Vi}}{\bar{\sigma}_{Bi}}.$$
(13)

Note that

$$\frac{\partial \rho_{PB^i}}{\partial P} = -\frac{\sigma_D \sigma_{Vi}^2}{\overline{\sigma}_{Bi}^3} (1 - \rho_i^2) \frac{\partial \alpha_i}{\partial P}. \tag{14}$$

If $\frac{\partial \alpha_i}{\partial P}$ <0 is shown, $\rho_{PB'}$ is an increasing function of crude oil prices. We obtained a model of price *cor*relations between green bonds and energy commodities, referred to as "CR model."

Judging from Eqs. (6) and (9) the volatilities in crude oil prices and green bond indices are driven by crude oil prices and green bond indices, respectively. If the inverse Box-Cox function parameter " a_1 " is positive, the volatilities in crude oil prices decrease in line with crude oil prices, which is referred to as the "leverage effect" often observed in financial markets. In opposite if the inverse Box-Cox function parameter " a_1 " is negative, the volatilities in crude oil prices increase in line with

crude oil prices, which is referred to as the "inverse leverage effect" often observed in energy markets. The same characteristics hold for green and conventional bonds through " a_{2i} ."

2.2. A green bond premium model

Although the price difference between a green bond and a regular vanilla bond may seem to be non-existent at first glance, it is considered that the difference gradually appears in time series when expanding for green value. If a model capable of observing or predicting such a difference can be constructed, it is highly likely that this research will be widely used for practical purposes in the future. Thus, we define a green bond premium by the log price difference of green and conventional bonds:

$$GBP_t = \log B_t^C - \log B_t^C \tag{15}$$

where ^G and ^C represent green and conventional bonds, respectively. Because bond prices decrease in line with the yields, the green bond premiums defined correspond to negative yield differences between green and conventional bonds as in Zerbib (2019).

Then we examine the expected return and volatility of green bond premiums.

Proposition 1.

$$\frac{1}{dt}E_t[dGBP_t] = \mu_{BG} - \mu_{BC} \tag{16}$$

$$\sqrt{\frac{1}{dt}V_{t}[dGBP_{t}]} = \sqrt{\sigma_{BG}^{2} + \sigma_{BC}^{2} - 2\rho_{GC}\sigma_{BG}\sigma_{BC}}$$
(17)

Note that ρ_{GC} is the correlation of price returns between green and conventional bonds.

Table 8Correlation Model Parameter Estimation between Solactive Green Bond Index (SLA GB) and Brent Crude Oil Price: Note all estimates except ρ_G are statistically significant judging from the corresponding standard errors.

Parameters	a_1	$\sigma_{\!\scriptscriptstyle D}$	<i>c</i> ₁	a_{2G}	σ_{VG}	c_{2G}	ρ_G	p_G	q_G
Estimates	0.316	1.317E+02	1.039E+02	0.480	7.514E+03	1.832E+04	0.000	-6.172E+00	2.675E-01
Std Errors	0.001	4.964E-01	6.609E-01	0.003	2.756E-03	1.008E+00	0.014	6.010E-02	1.129E-03
Loglikelihood	7474								
AIC	-14933								
SIC	-14893								

Table 9Correlation Model Parameter Estimation between S&P Bond Index (SPB) and Brent Crude Oil Price: Note all estimates are statistically significant judging from the corresponding standard errors. While *q_C* is negative, the correlations are negative, implying that the both relationship is negative while there exists the comovement of the correlations with Brent crude oil prices.

Parameters	a_1	$\sigma_{\!\scriptscriptstyle D}$	c_1	a_{2C}	$\sigma_{ m VC}$	c _{2C}	$\rho_{\mathcal{C}}$	p_C	q_C
Estimates Std Errors Loglikelihood AIC SIC	0.266 0.000 7872 -15727 -15682	7.269E+02 4.289E-08	6.997E+02 9.453E-06	5.492 0.000	7.522E+03 3.598E-07	1.515E-06 2.135E-09	0.352 0.000	7.197E+00 1.711E-07	-3.647E-02 7.615E-06

Table 10
DCC Model Parameter Estimation between Bloomberg Barclays MSCI Green Bond Index (MSCI GB) and WTI Crude Oil Price: Note the parameters except $ω_2$ and $θ_1$ are statistically significant.

Parameters	ω_1	α_1	β_1	ω_2	α_2	β_2	θ_1	θ_2
Estimates	2.062E-07	0.029	0.953	6.450E-06	0.057	0.933	0.041	0.881
Std Errors	2.671E-08	0.011	0.010	4.458E-06	0.020	0.025	0.022	0.096
Loglikelihood	7.223E + 03							
AIC	-1.443E+04							
SIC	-1.439E+04							

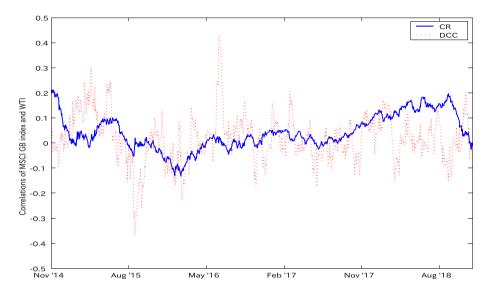


Fig. 1. Comparison of CR and DCC on Correlations between Bloomberg Barclays MSCI Green Bond Index (MSCI GB) and WTI Crude Oil Price: Note the figure suggests that both models demonstrate time varying and positive correlations on average between MSCI GB and WTI crude oil price returns.

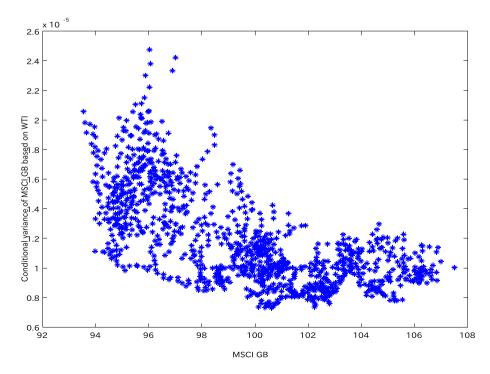


Fig. 2. Price and Volatility of Bloomberg Barclays MSCI Green Bond Index (MSCI GB) from DCC model: Note the figure represents negative price-volatility relationship, which is consistent with $a_{2G} > 0$ from CR model.

Table 11DCC Model Parameter Estimation between S&P Green Bond Index (SP GB) and WTI Crude Oil Price: Note the parameters except ω_1 , ω_2 , and θ_1 are statistically significant.

Parameters	ω_1	α_1	β_1	ω_2	α_2	β_2	θ_1	θ_2
Estimates Std Errors Loglikelihood AIC SIC	5.091E-11 1.251E-08 7.360E+03 -1.471E+04 -1.467E+04	1.660E-02 8.023E-04	9.829E-01 1.262E-03	6.450E-06 4.458E-06	0.057 0.020	0.933 0.025	0.039 0.022	0.877 0.099

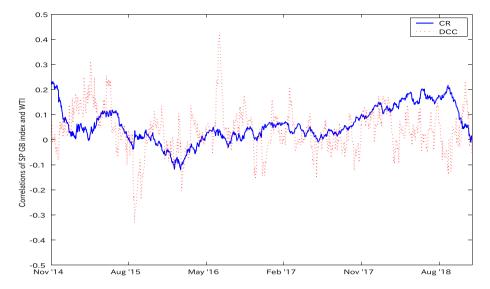


Fig. 3. Comparison of CR and DCC on Correlations between S&P Green Bond Index (SP GB) and WTI Crude Oil Price: Note the figure suggests that both models demonstrate time varying and positive correlations on average between SP GB and WTI crude oil price returns.

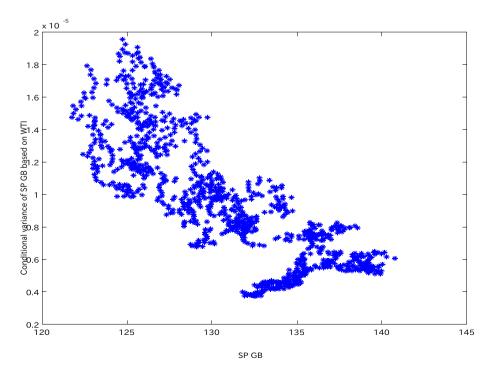


Fig. 4. Price and Volatility of S&P Green Bond Index (SP GB) from DCC model: Note the figure represents negative price-volatility relationship, which is consistent with $a_{2G} > 0$ from CR model.

Table 12 DCC Model Parameter Estimation between Solactive Green Bond Index (SLA GB) and WTI Crude Oil Price: Note the parameters except ω_2 are statistically significant.

Parameters	ω_1	α_1	β_1	ω_2	α_2	β_2	θ_1	θ_2
Estimates Std Errors	1.218E-07 1.116E-08	0.037 0.017	0.945 0.018	6.450E-06 4.458E-06	0.057 0.020	0.933 0.025	0.032 0.010	0.927 0.026
Loglikelihood	7.545E+03	0.017	0.016	4,4362-00	0.020	0.023	0.010	0.020
AIC SIC	-1.507E+04 -1.503E+04							

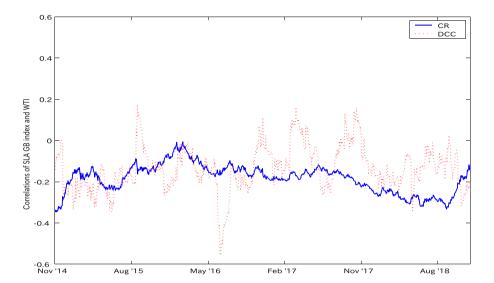


Fig. 5. Comparison of CR and DCC on Correlations between Solactive Green Bond Index (SLA GB) and WTI Crude Oil Price: Note the figure suggests that both models demonstrate time varying and almost negative correlations between SLA GB and WTI crude oil price returns.

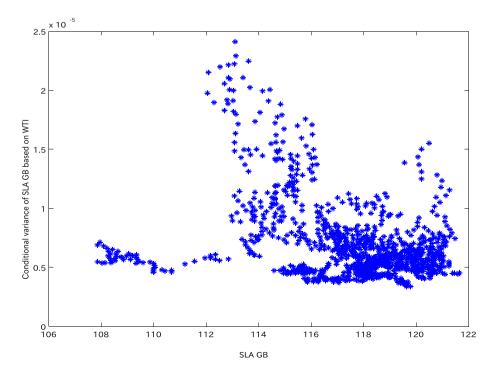


Fig. 6. Price and Volatility of Solactive Green Bond Index (SLA GB) from DCC model: Note the figure represents negative price-volatility relationship, which is consistent with $a_{2G} > 0$ from CR model.

Table 13 DCC Model Parameter Estimation between S&P Bond Index (SPB) and WTI Crude Oil Price: Note the parameters except ω_2 and θ_1 are statistically significant.

Parameters	ω_1	α_1	β_1	ω_2	α_2	β_2	θ_1	θ_2
Estimates Std Errors	3.219E-08 3.285E-09	0.024 0.009	0.969 0.009	6.450E-06 4.458E-06	0.057 0.020	0.933 0.025	0.007 0.005	0.988 0.009
Loglikelihood AIC SIC	7.695E+03 -1.537E+04 -1.533E+04							

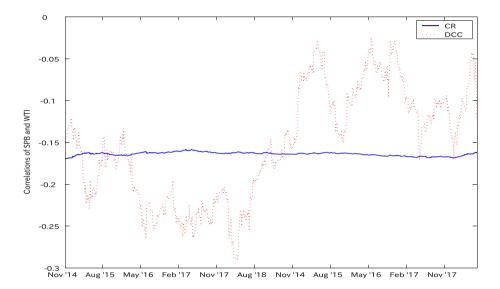


Fig. 7. Comparison of CR and DCC on Correlations between S&P Bond Index (SPB) and WTI Crude Oil Price: Note the figure suggests that both models demonstrate time varying and almost negative correlations between SPB and WTI crude oil price returns.

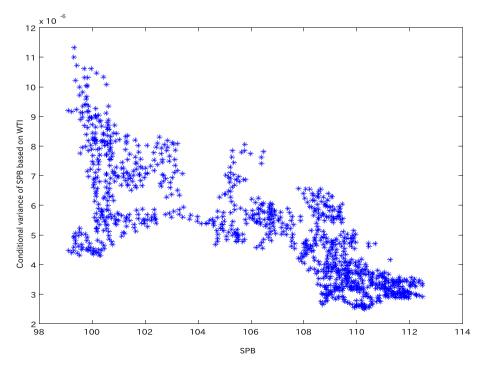


Fig. 8. Price and Volatility of S&P Bond Index (SPB) from DCC model: Note the figure represents negative price-volatility relationship, which is consistent with $a_{2C} > 0$ from CR model.

Table 14DCC Model Parameter Estimation between Bloomberg Barclays MSCI Green Bond Index (MSCI GB) and Brent Crude Oil Price: Note the parameters except ω_2 are statistically significant.

Parameters	ω_1	α_1	β_1	ω_2	α_2	β_2	θ_1	θ_2
Estimates	2.062E-07	0.029	0.953	7.492E-06	0.065	0.922	0.052	0.817
Std Errors	2.671E-08	0.011	0.010	4.563E-06	0.019	0.024	0.022	0.110
Loglikelihood	7.269E+03							
AIC	-1.452E+04							
SIC	-1.448E+04							

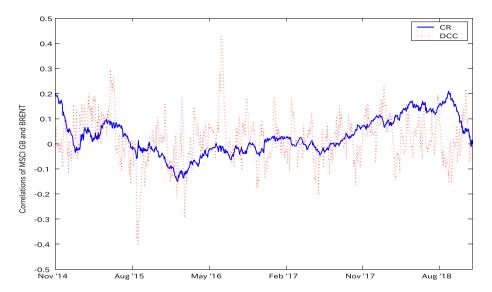


Fig. 9. Comparison of CR and DCC on Correlations between Bloomberg Barclays MSCI Green Bond Index (MSCI GB) and Brent Crude Oil Price: Note the figure suggests that both models demonstrate time varying and positive correlations on average between MSCI GB and Brent crude oil price returns.

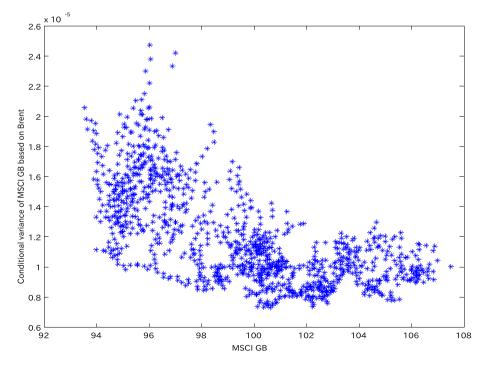


Fig. 10. Price and Volatility of Bloomberg Barclays MSCI Green Bond Index (MSCI GB) from DCC model: Note the figure represents negative price-volatility relationship, which is consistent with $a_{2G} > 0$ from CR model.

Table 15DCC Model Parameter Estimation between S&P Green Bond Index (SP GB) and Brent Crude Oil Price: Note the parameters except ω_1 and ω_2 are statistically significant.

Parameters	ω_1	$lpha_1$	β_1	ω_2	$lpha_2$	β_2	θ_1	θ_2
Estimates Std Errors Loglikelihood AIC SIC	5.091E-11 1.251E-08 7.407E+03 -1.480E+04 -1.476E+04	1.660E-02 8.023E-04	9.829E-01 1.262E-03	7.492E-06 4.563E-06	0.065 0.019	0.922 0.024	0.054 0.021	0.807 0.092

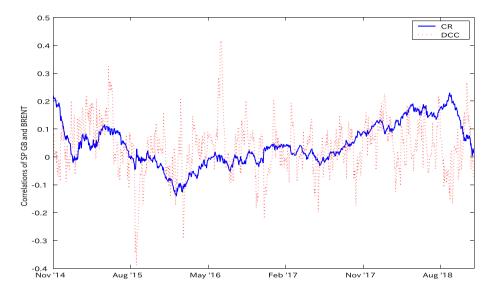


Fig. 11. Comparison of CR and DCC on Correlations between S&P Green Bond Index (SP GB) and Brent Crude Oil Price: Note the figure suggests that both models demonstrate time varying and positive correlations on average between SP GB and Brent crude oil price returns.

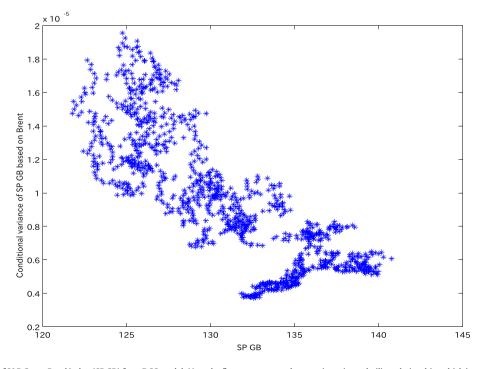


Fig. 12. Price and Volatility of S&P Green Bond Index (SP GB) from DCC model: Note the figure represents the negative price-volatility relationship, which is consistent with $a_{2G} > 0$ from CR model.

Table 16DCC Model Parameter Estimation between Solactive Green Bond Index (SLA GB) and Brent Crude Oil Price: Note the parameters except ω_2 are statistically significant.

Parameters	ω_1	$lpha_1$	β_1	ω_2	$lpha_2$	β_2	θ_1	θ_2
Estimates Std Errors Loglikelihood AIC SIC	1.218E-07 1.116E-08 7.589E+03 -1.516E+04 -1.512E+04	0.037 0.017	0.945 0.018	7.492E-06 4.563E-06	0.065 0.019	0.922 0.024	0.041 0.019	0.869 0.088

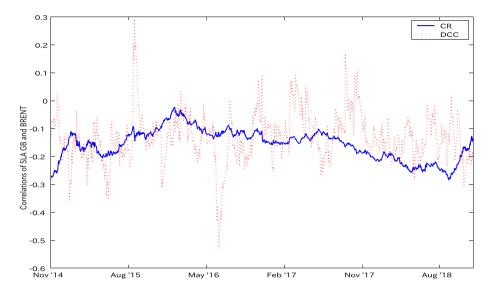


Fig. 13. Comparison of CR and DCC on Correlations between Solactive Green Bond Index (SLA GB) and Brent Crude Oil Price: Note the figure suggests that both models demonstrate time varying and almost negative correlations between SLA GB and Brent crude oil price returns.

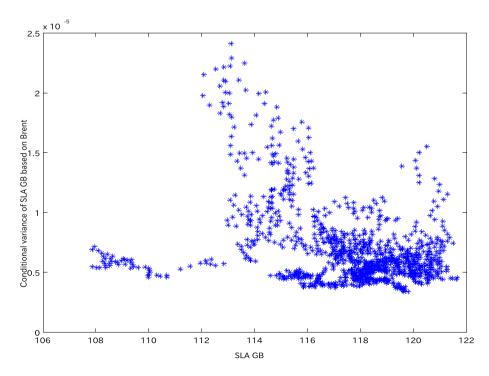


Fig. 14. Price and Volatility of Solactive Green Bond Index (SLA GB) from DCC model: Note the figure represents negative price-volatility relationship, which is consistent with $a_{2G} > 0$ from CR model.

Table 17 DCC Model Parameter Estimation between S&P Bond Index (SPB) and Brent Crude Oil Price: Note the parameters except ω_2 and θ_1 are statistically significant.

Parameters	ω_1	$lpha_1$	β_1	ω_2	$lpha_2$	β_2	θ_1	θ_2
Estimates	3.219E-08	0.024	0.969	7.492E-06	0.065	0.922	0.020	0.922
Std Errors	3.285E-09	0.009	0.009	4.563E-06	0.019	0.024	0.014	0.032
Loglikelihood	7.743E + 03							
AIC	-1.547E+04							
SIC	-1.543E+04							

Proof. By using Ito's lemma to Eq. (15), we derive the proposition.

We also define the information ratio of green bond premiums as follows:

$$IR_{GBP} = \frac{\mu_{GBP}}{\sigma_{GRP}} \tag{18}$$

where $\mu_{GBP} = \frac{1}{dt} E_t[dGBP_t]$ and $\sigma_{GBP} = \sqrt{\frac{1}{dt} V_t[dGBP_t]}$. We call the expected return, volatility, and information ratio of green bond premiums as "a green bond premium model."

3. Empirical studies

3.1. Data

We use the Bloomberg Barclays MSCI, the S&P, and the Solactive green bond index (MSCI GB, SP GB, and SLA GB, respectively) and WTI and Brent crude oil prices. We also use the S&P bond index (SPB) as the benchmark of conventional bonds. The data covers from November 3, 2014 to December 31, 2018, which is obtained from the Bloomberg. Note that we normalise SPB using the average of MSCI GB with the lowest average green bond index as an example, i.e., SPB is 99.538 on November 3, 2014, in order to conduct the equal footing between green and conventional bonds. The basic statistics are reported in Table 1. The skewness of MSCI GB, SP GB, WTI, and Brent are positive, i.e., the distributions are right-skewed while the skewness of SLA GB and SPB are negative, i.e., the distributions are left-skewed.

3.2. The model parameter estimation

We estimate the parameters of the correlation model, i.e., CR model. between green bond indices and crude oil prices proposed in this paper by using the maximum likelihood estimation. Then we need to specify $\alpha_i(P_t)$ and assume that $\alpha_i(P_t)$ is a linear function of crude oil price (P_t) for simplicity:

$$\alpha_i(P_t) = p_i + q_i P_t. \tag{19}$$

Note that the lower letter i = G. C represent green and conventional bonds, respectively. In order to estimate the model parameters, we discretise the model:

$$\Delta \log P_{t} = \left[\frac{\mu_{D}}{\sigma_{D}} \sigma_{P} - \frac{a_{1}}{2} \sigma_{P}^{2} \right] \Delta t + \sigma_{P} \varepsilon_{t}, \tag{20}$$

$$\sigma_P = \frac{P_t^{-a_1}}{c_1} \sigma_D,\tag{21}$$

$$\Delta \log B_t^i = \left[-\alpha_i(P_t) \frac{\mu_D}{\overline{\alpha}_{Bi}} \sigma_{Bi} - \frac{a_{2i}}{2} \sigma_{Bi}^2 \right] \Delta t + \sigma_{Bi} \eta_t^i, \tag{22}$$

$$\sigma_{Bi} = \frac{\left(B_t^i\right)^{-a_{2i}}}{c_{2i}} \overline{\sigma}_{Bi},\tag{23}$$

$$\overline{\sigma}_{Bi} = \sqrt{\alpha_i (P_t)^2 \sigma_D^2 + \sigma_{Vi}^2 - 2\rho_i \alpha_i (P_t) \sigma_D \sigma_{Vi}}, \tag{24}$$

$$\overline{\sigma}_{Bi} = \sqrt{\alpha_{i}(P_{t})^{2}\sigma_{D}^{2} + \sigma_{Vi}^{2} - 2\rho_{i}\alpha_{i}(P_{t})\sigma_{D}\sigma_{Vi}},$$

$$\begin{pmatrix} Var(\varepsilon_{t}) & Cov(\varepsilon_{t}, \eta_{t}^{i}) \\ Cov(\varepsilon_{t}, \eta_{t}^{i}) & Var(\eta_{t}^{i}) \end{pmatrix} = \begin{pmatrix} \Delta t & \rho_{pg^{i}}\Delta t \\ \rho_{pg^{i}}\Delta t & \Delta t \end{pmatrix},$$
(25)

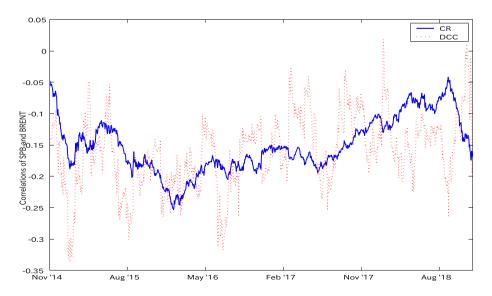


Fig. 15. Comparison of CR and DCC on Correlations between S&P Bond Index (SPB) and Brent Crude Oil Price: Note the figure suggests that both models demonstrate time varying and almost negative correlations between SPB and Brent crude oil price returns.

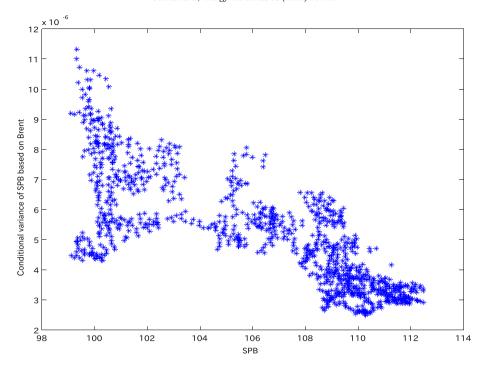


Fig. 16. Price and Volatility of S&P Bond Index (SPB) from DCC model: Note the figure represents negative price-volatility relationship, which is consistent with $a_{2C} > 0$ from CR model.

$$\rho_{PB^{i}} = \frac{-\alpha_{i}(P_{t})\sigma_{D} + \rho_{i}\sigma_{Vi}}{\overline{\sigma}_{Bi}}.$$
 (26)

Here we take $\Delta t=\frac{1}{252}$ and assume $\mu_D=0$ for simplicity. Note that $\varepsilon_t \sim N(0,\Delta t)$ and $\eta_t^i \sim N(0,\Delta t)$.

The parameter estimation results of CR model for MSCI GB and WTI are shown in Table 2. All estimates are statistically significant judging from the corresponding standard errors. In particular, q_G is negative,

implying that the correlations between MSCI GB and WTI increase in line with WTI from Eqs. (14) and (19). a_{2G} is obtained as a positive value, implying that the volatility of MSCI GB decreases in line with MSCI GB. The results using SP GB and WTI are the same as the results using MSCI GB and WTI as in Table 3.

The parameter estimation results of CR model for SLA GB and WTI are shown in Table 4. All estimates except ρ_G are statistically significant judging from the corresponding standard errors. In particular, q_G is positive, implying that the correlations between SLA GB and WTI decrease in line with WTI from Eqs. (14) and (19) unlike MSCI GB and SP GB results. a_{2G} is obtained as a positive value, implying that the volatility of

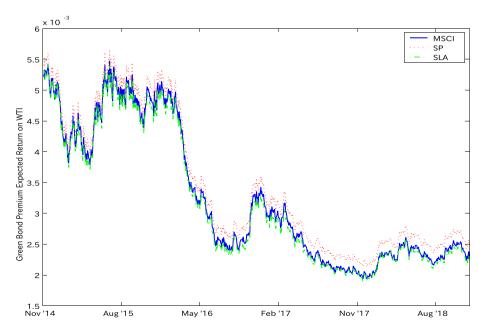


Fig. 17. Green Bond Premium Expected Return for Bloomberg Barclays MSCI (MSCI), S&P (SP), and Solactive (SLA) Green Bonds on WTI: Note the figure suggests that green bond premiums are positive while decreasing over time.

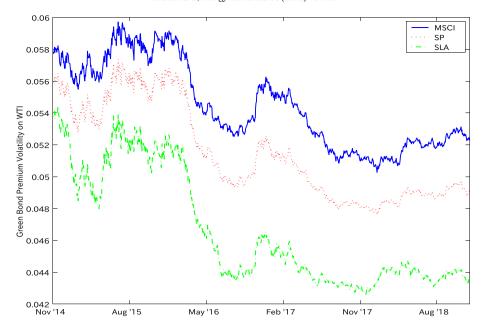


Fig. 18. Green Bond Premium Volatility for Bloomberg Barclays MSCI (MSCI), S&P (SP), and Solactive (SLA) Green Bonds on WTI: Note the figure demonstrates that the risks of green bond premiums are slightly decreasing but almost flat over time in the recent years.

SLA GB decreases in line with SLA GB. The parameter estimation results of CR model for SPB and WTI (See Table 5.) are different from the results of MSCI GB and SP GB, but the same as those of SLA GB. This may be consistent with the results of Kang et al. (2014) in that a positive oil market-specific demand shock is associated with significant decreases in aggregate bond index real returns for 8 months following the shock.

Now we use Brent crude oil in replace of WTI crude oil. The results using MSCI GB, SP GB, and SLA GB to Brent crude oil are the same as the results using MSCI GB, SP GB and SLA GB to WTI crude oil as shown in Tables 6, 7, and 8, respectively.

The parameter estimation results of CR model for SPB and Brent are shown in Table 9. All estimates are statistically significant judging from the corresponding standard errors. In particular, q_C is negative, implying that the correlations between SPB and Brent increase in line with Brent from Eqs. (14) and (19). While q_C is negative for SPB and Brent crude oil price returns, the correlations are negative shown in Fig. 15 in the next subsection, implying that the both relationship is negative while there exists the comovement of the correlations with Brent crude oil prices. a_{2C} is obtained as a positive value, implying that the volatility of SPB decreases in line with SPB.

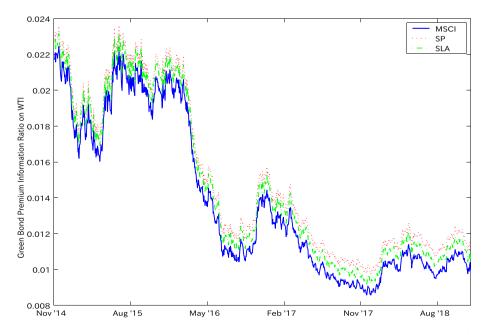


Fig. 19. Green Bond Premium Information Ratio for Bloomberg Barclays MSCI (MSCI), S&P (SP), and Solactive (SLA) Green Bonds on WTI: Note the figure demonstrates positive but decreasing information ratios.

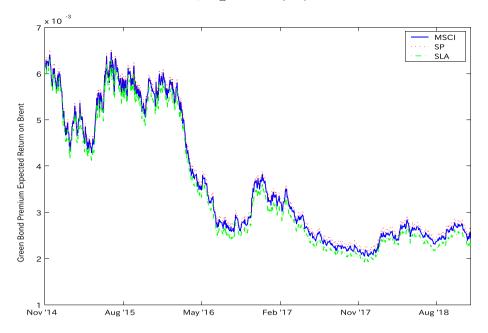


Fig. 20. Green Bond Premium Expected Return for Bloomberg Barclays MSCI (MSCI), S&P (SP), and Solactive (SLA) Green Bonds on Brent: Note the figure suggests that green bond premiums are positive while decreasing over time.

SLA GB is negatively affected by energy prices while MSCI GB and SP GB are positively affected by energy prices. As the result, MSCI GB and SP GB possess the characteristics of environmental assets while SLA GB does not have those from the perspective of the relationship between energy and environmental value.

3.3. Comparisons with DCC model

We apply the dynamic conditional correlation (DCC) model of Engle (2002) to the green bond indices including MSCI GB, SP GB, and SLA GB and energy prices including WTI and Brent crude oil prices so as to examine the robustness of CR model we propose. We also investigate the relation of SPB with WTI and Brent crude oil prices as the

comparison between green and conventional bonds. The details of DCC model are given in Appendix A.

The estimation results for MSCI GB and WTI crude oil prices are reported in Table 10. The parameters except ω_2 and θ_1 are statistically significant. The statistical significance of α_1 , β_1 , α_2 and β_2 suggests the GARCH effects in MSCI GB and WTI crude oil price returns. The correlations between MSCI GB and WTI crude oil price returns are time varying because of the significance of θ_2 . Fig. 1 shows the comparisons of the correlations between MSCI GB and WTI crude oil price returns from CR model with the correlations from DCC model. It suggests that both models demonstrate time varying and positive correlations on average between MSCI GB and WTI crude oil price returns. Fig. 2 represents the negative price-volatility relationship obtained from DCC model, which is consistent with $a_{2G} > 0$ obtained from CR model. Because CR

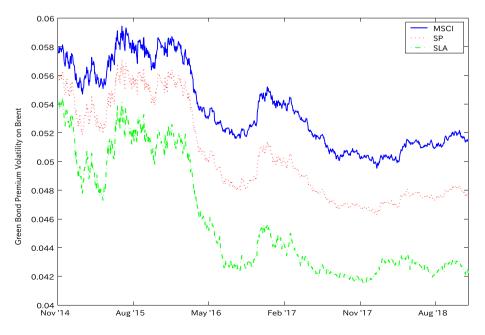


Fig. 21. Green Bond Premium Volatility for Bloomberg Barclays MSCI (MSCI), S&P (SP), and Solactive (SLA) Green Bonds on Brent: Note the figure demonstrates that the risks of green bond premiums are slightly decreasing but almost flat over time in the recent years.

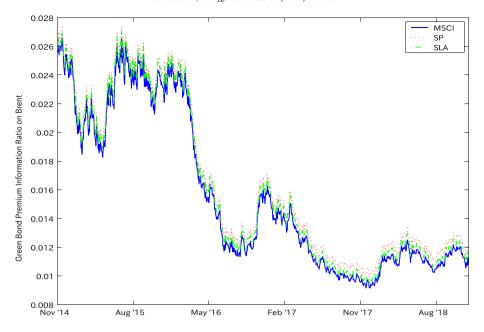


Fig. 22. Green Bond Premium Information Ratio for Bloomberg Barclays MSCI (MSCI), S&P (SP), and Solactive (SLA) Green Bonds on Brent: Note the figure demonstrates positive but decreasing information ratios. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

model demonstrates the same level of the correlations to DCC model and the same negative price-volatility relationship, we can safely say that our proposed CR model works well as the correlation model between MSCI GB and WTI crude oil price returns. The results for SP GB and WTI crude oil prices are the same as the results for MSCI GB and WTI crude oil prices as in Table 11 and Figs. 3 and 4.

The estimation results for SLA GB and WTI crude oil prices are reported in Table 12. The parameters except ω_2 are statistically significant. The statistical significance of α_1 , β_1 , α_2 and β_2 suggests the GARCH effects in SLA GB and WTI crude oil price returns. The correlations between SLA GB and WTI crude oil price returns are time varying because of the significance of θ_1 and θ_2 . Fig. 5 shows the comparisons of the correlations between SLA GB and WTI crude oil price returns from CR model with the correlations from DCC model. It suggests that both models demonstrate time varying and almost negative correlations between SLA GB and WTI crude oil price returns. The result is completely different from the results of MSCI GB and SP GB in Figs. 1 and 3 from the point of plus or minus signs of the correlations. Fig. 6 represents the negative price-volatility relationship obtained from DCC model, which is consistent with $a_{2G} > 0$ obtained from CR model. Because CR model demonstrates the same level of the correlations to DCC model and the same negative price-volatility relationship, we can safely say that our proposed CR model works well as the correlation model between SLA GB and WTI crude oil price returns. From the point of plus or minus signs of the correlations, the estimation results for the SPB and WTI crude oil price (Table 13 and Fig. 7) are completely different from the results of MSCI GB and SP GB in Figs. 1 and 3 while the same as the result of SLA GB in Fig. 5. Fig. 8 represents the negative price-volatility relationship obtained from DCC model, which is consistent with $a_{2C} > 0$ obtained from CR model. Because CR model demonstrates the same level of the correlations to DCC model and the same negative price-volatility relationship, we can safely say that our proposed CR model works well as the correlation model between SPB and WTI crude oil price returns.

By comparison, we employ Brent crude oil in replace of WTI crude oil. We do not have any difference between Brent and WTI crude oil in the correlations between the bond indices and crude oil prices for MSCI GB, SP GB, SLA GB, and SPB as in Table 14 and Figs. 9 and 10, Table 15 and Figs. 11 and 12, Table 16 and Figs. 13 and 14, and Table 17 and Figs. 15 and 16, respectively.

Taking it into account that the conventional SPB has negative correlations with WTI and Brent crude oil which are the same as the results of SLA GB, SLA GB may not fully represent the characteristics of green bonds in the sense of environmental value. The differences of SLA GB from MSCI GB and SP GB are highlighted again from the correlation comparisons between CR model and DCC model. Now we investigate the reason of the differences from the point of qualitative aspects of the green bond indices. SP GB introduces market value weights. MSCI GB employs multi-currency market value weights. On the other hand, SLA GB calculates the value weighted with the maximum 5% cap per bond. The difference of SLA GB from MSCI GB and SP GB may come from the fact that SLA GB does not track the green bond market properly due to the upper limit of green bond weighting. It implies that the weighting of the green bond index needs to be carefully considered in order to properly represent the green bond market.

3.4. Green bond premiums and investment performance

We calculate the expected returns, volatilities and information ratios of green bond premiums by the log price differences between green and conventional bonds. Note that we employ the whole data sample correlations between green and conventional bond index returns for ρ_{GC} in Eq. (17). The results of the expected returns, volatilities and information ratios of green bond premiums based on WTI crude oil are reported in Figs. 17, 18, and 19, respectively. Fig. 17 suggests that the expected returns of green bond premiums are positive while decreasing over time. In particular, SP GB has highest expected returns among them from Fig. 17. From Fig. 18, we demonstrate that the risks of green bond premiums are slightly decreasing but almost flat over time in the recent years. In particular, SLA GB demonstrates the lowest green bond premium volatility from Fig. 18. Fig. 19 demonstrates positive but decreasing information ratios. SP GB gives the highest performance in Fig. 19 from the perspective of risk and return. It can be seen from Figs. 17 and 18 that SLA GB has lower risks and lower returns than other green bonds. This may be the cause of the similarities to the conventional SPB.

The results of the expected returns, volatilities and information ratios of green bond premiums based on Brent crude oil are reported in Figs. 20, 21, and 22, respectively. Fig. 20 suggests that the expected returns of green bond premiums are positive while decreasing over

time. In particular, SP GB has highest expected returns among them from Fig. 20. Fig. 21 demonstrates that the risks of green bond premiums are slightly decreasing but almost flat over time in the recent years. SLA GB demonstrates the lowest green bond premium volatility from Fig. 21. Fig. 22 demonstrates positive but decreasing information ratios. SP GB gives the highest performance in Fig. 22 from the perspective of risk and return. It can be seen from Figs. 20 and 21 that SLA GB has lower risks and lower returns than other green bonds. This may be the cause of the similarities to the conventional SPB. These results are the same as the results using WTI crude oil.

We demonstrated that the expected returns of green bond premiums are positive while decreasing and that the risks of green bond premiums are slightly decreasing but almost flat over time in the recent years, resulting in positive but decreasing information ratios. It also implies positive expected returns of green bond premiums defined in this paper may lead to negative yield differences between green and conventional bonds taking account of bond price-yield relationship. This is consistent with the results of Zerbib (2019).

4. Conclusions

This paper studied the existence of greenness in green bonds and the investment performance by using the relationship between green bonds and energy commodities as a new angle. We proposed a new model of price correlations between green bonds and energy commodities. Based on the correlation model, we also proposed a new model of the expected return, the risk, and the performance ratio of green bond premiums defined by the log price differences between green and conventional bonds. Empirical studies using the data of green and conventional bond indices and crude oil prices showed that the Bloomberg Barclays MSCI and the S&P green bond indices tend to have positive correlations with and increase in line with both WTI and Brent crude oil prices while the Solactive green bond index tends to have negative correlations with and decrease in line with both WTI and Brent crude oil prices. From the empirical evidence of the positive relationship between energy and environmental value, it was suggested that the greenness is incorporated in the Bloomberg Barclays MSCI and the S&P green bond. In contrast, taking it into account that the conventional S&P bond index has negative correlations with WTI and Brent crude oil which are the same as the results of the Solactive green bond index, the Solactive green bond index may not fully represent the characteristics of green bonds in the sense of environmental value. Because the Bloomberg Barclays MSCI and the S&P green bond indices employ market value weights while the Solactive green bond index calculates the value weighted with the maximum 5% cap per bond, the difference of the Solactive from the Bloomberg Barclays MSCI and the S&P green bond index may come from the fact that the Solactive green bond index does not track the green bond market properly due to the upper limit of green bond weighting. It implies that the weighting of the green bond index needs to be carefully considered in order to properly represent the green bond market. Furthermore because the price correlation model we propose demonstrated the same levels of the correlations and the same negative price-volatility relationship to the existing dynamic conditional correlation model, we can safely say that our price correlation model works well as the correlation model between green bond and crude oil price returns. More importantly, we demonstrated that the expected returns of green bond premiums are positive while decreasing and that the risks of green bond premiums are slightly decreasing but almost flat over time in the recent years. These results of the risks and the returns make information ratios positive but decreasing. It implies that green bond investment performance is superior to conventional bond investment performance but the superiority is decaying over time. It also implies positive expected returns of green bond premiums defined in this paper may lead to negative yield differences between green and conventional bonds taking account of bond price-yield relationship as in Zerbib (2019).

Because this study analyses the data up to 2018, the analysis relies on the past trends of energy and environmental value. However, the possibility of decoupling of energy assets and environmental assets, such as green bonds, cannot be denied in the future. We think that this research can be used as a means to capture the decoupling and leave this research as a future research topic.

CRediT authorship contribution statement

Takashi Kanamura: Conceptualization, Methodology, Software, Formal analysis, Writing - original draft, Writing - review & editing.

Appendix A. DCC model

We model the log return of prices y_t using the Engle's DCC model as follows

$$y_t = \varepsilon_t \sim N(0, H_t),$$
 (A1)

$$\varepsilon_t = D_t \eta_t,$$
 (A2)

$$D_{t} = \operatorname{diag}\left[h_{1,t}^{\frac{1}{2}}h_{2,t}^{\frac{1}{2}}\right], \tag{A3}$$

where $y_t = (y_{1,t}, y_{2,t})'$, $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$, and $\eta_t = (\eta_{1,t}, \eta_{2,t})'$. For i = 1, 2. we have

$$h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}, \tag{A4}$$

$$H_t = E[\varepsilon_t \varepsilon_t' | F_{t-1}] = D_t R_t D_t, \tag{A5}$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}, (A6)$$

$$Q_t = (1 - \theta_1 - \theta_2)Q + \theta_1 \eta_{t-1} \eta'_{t-1} + \theta_2 Q_{t-1}, \tag{A7}$$

where Q_t^* is the diagonal component of the square root of the diagonal elements of Q_t^1 and F_{t-1} is the filtration at time t-1. Eq. (A4) represents the GARCH(1,1) effect for each price return, which may generally be observed in energy and financial markets. The conditional correlations are calculated using Eq. (A6) where Eq. (A7) represents time varying conditional covariance. The scale parameters θ_1 and θ_2 represent the effects of previous standardized shock and conditional correlation persistence, respectively. If either of the estimations of θ_1 or θ_2 in Eq. (A7) is statistically significant, the correlation structure of the pairs demonstrates time varying. The estimation is conducted as two steps: First, conditional volatilities are estimated using univariate GARCH(1,1) model. Second, the parameters of the conditional variance are estimated using the standardized residuals obtained from the first step. Here the loglikelihood function (L) for the bivariate model is given by

$$L = -\frac{1}{2} \sum_{t=1}^{T} \left(2 \log 2\pi + 2 \log |D_t| + \log |R_t| + \eta_t' R_t^{-1} \eta_t \right). \tag{A8}$$

After the estimation of parameters using the QMLE, the time-varying conditional correlations are empirically calculated using the errors (η_t) obtained from each GARCH(1,1) model.

Define
$$Q_t = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}$$
. Then, $Q_t^* = \begin{pmatrix} \sqrt{q_{11}} & 0 \\ 0 & \sqrt{q_{22}} \end{pmatrix}$

 2 θ_2 represents the persistence of the conditional covariance matrix. Because the standardized shock η_t is used for the calculation, θ_2 is approximately considered as the conditional correlation persistence.

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