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# Lecture 4: Dense Neural Networks

Date: 15/05/2020

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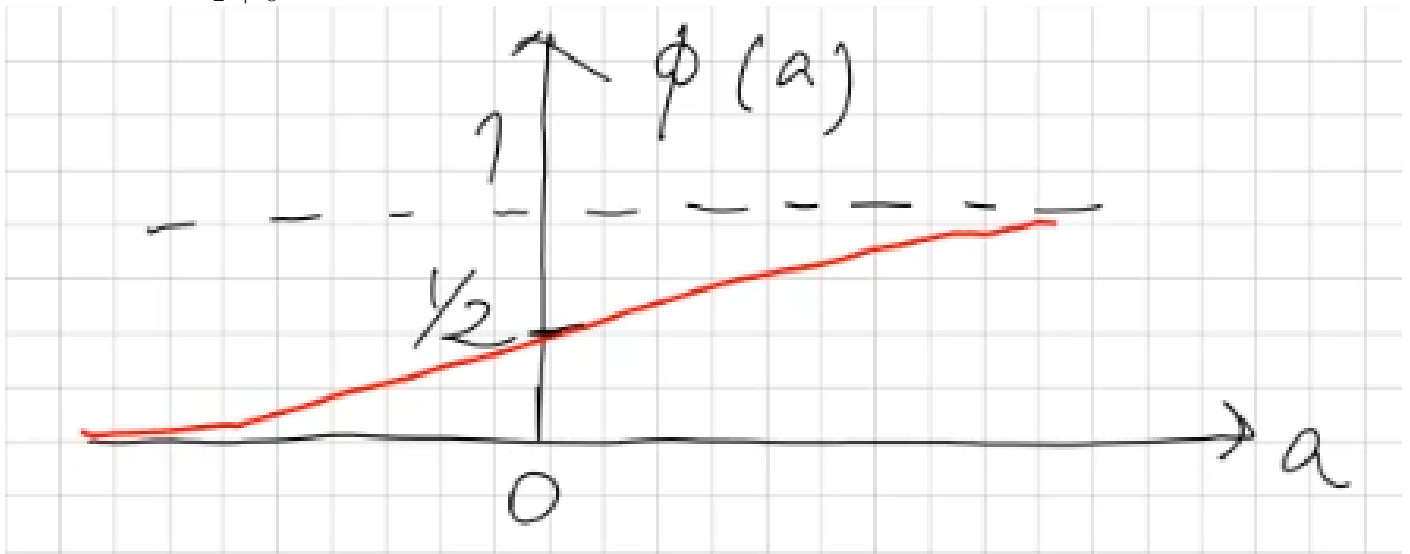
## 1.1. Activation function

Mild requirements on  $\phi()$ :

- nonlinear in general  $\rightarrow$  fundamental
- smooth, differentiable  $\rightarrow$  for training
- simple calculation  $\rightarrow$  low complexity
- Slides 4-6; 4-7 activation function types

### 1.1.1. Sigmoid activation function

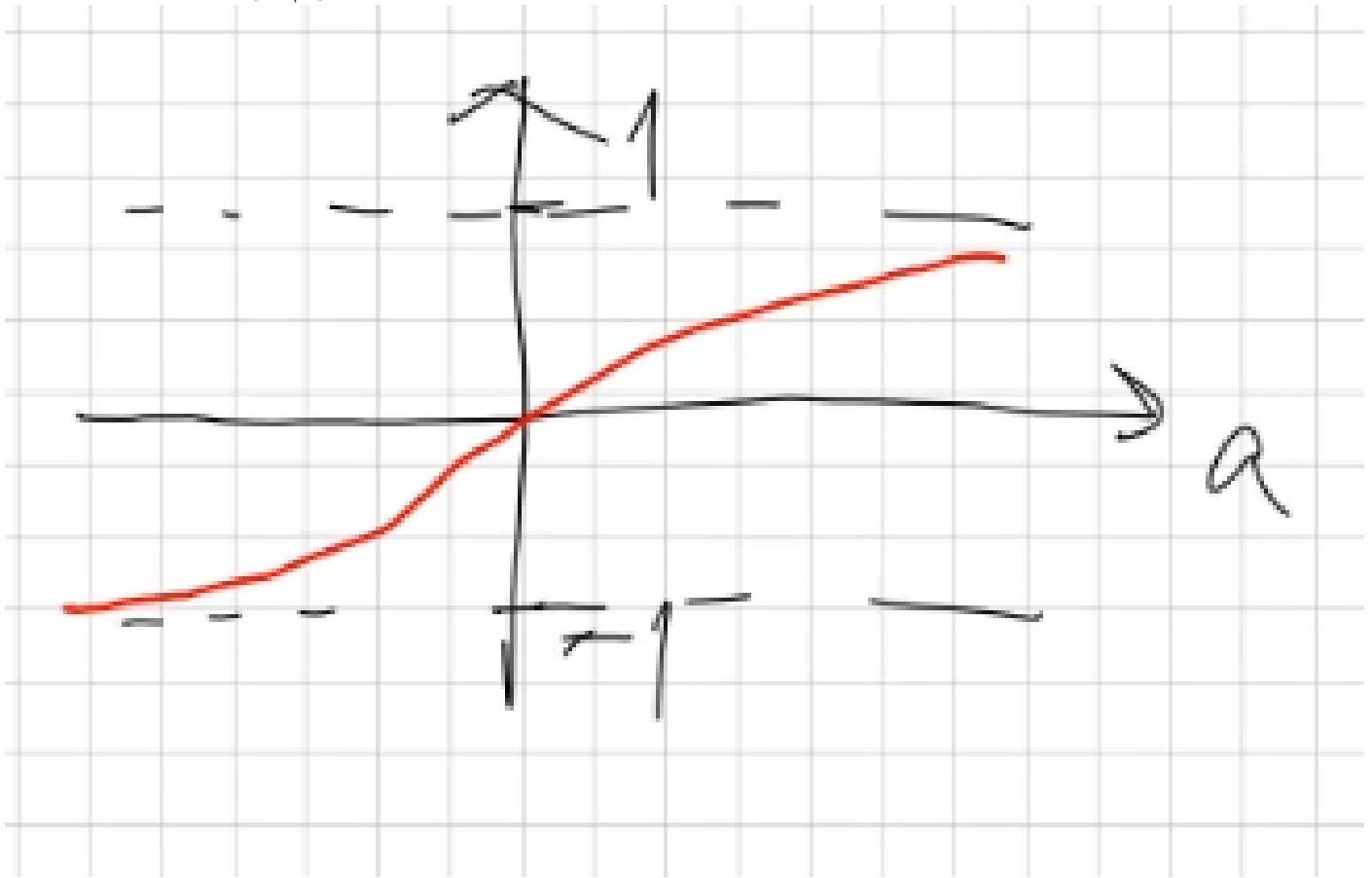
$$\phi(a) = \sigma(a) = \frac{1}{1 + e^{-a}}$$



- $0 < \phi < 1 = \hat{\text{prob.}}$
  - symmetry:  $\phi(-a) = 1 - \phi(a)$
  - derivative:  $\frac{d\phi(a)}{da} = \dots = \frac{e^{-a}}{(1 + e^{-a})^2} = \phi(a) \cdot \phi(-a) = \phi(a)(1 - \phi(a)), \in (0, 1)$
- easy calculative
- widely used in conventional NN (shallow)

### 1.1.2. hyperbolic tangent activation function

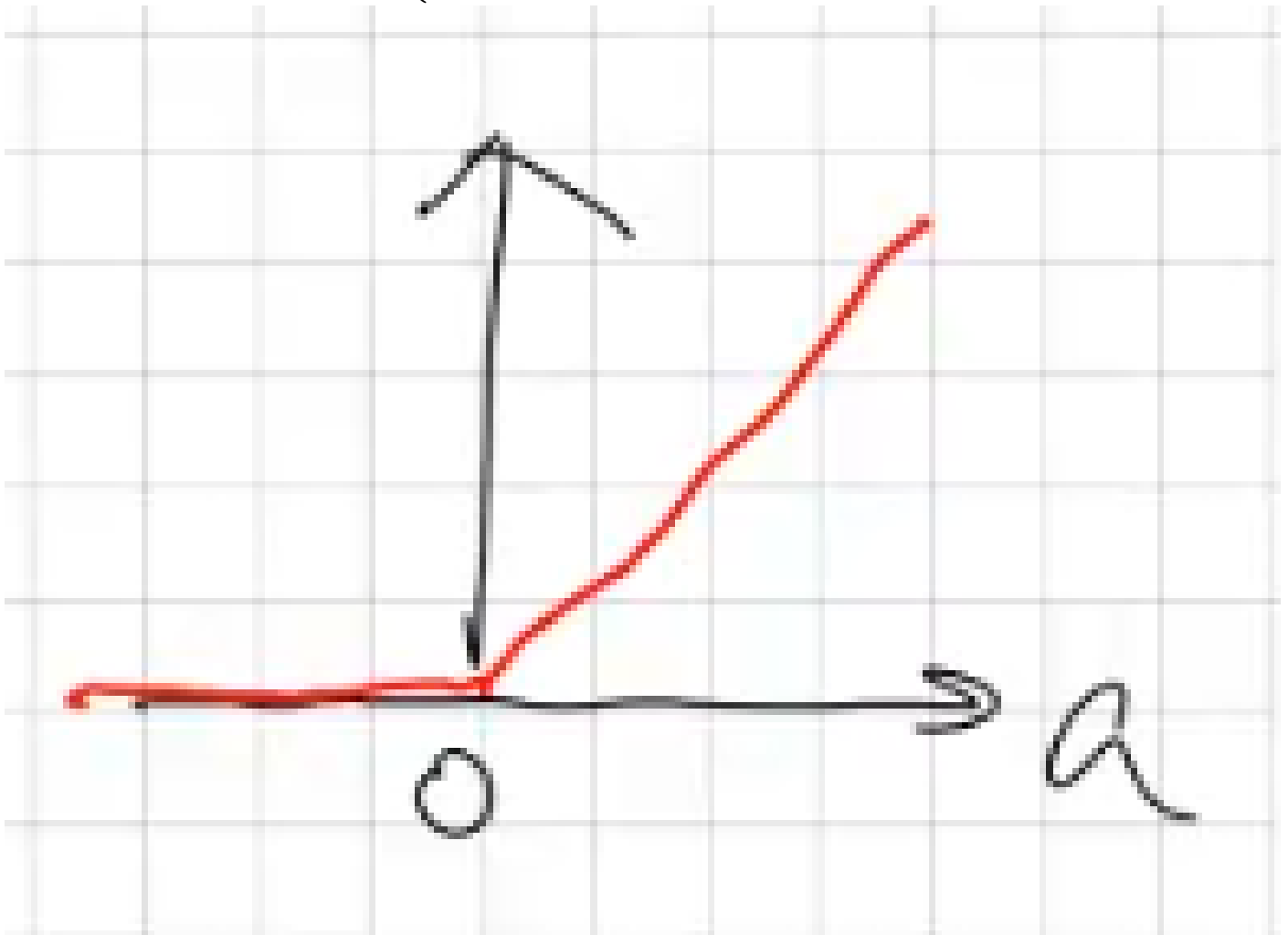
$$\phi(a) = \tanh(a) = \frac{e^a - e^{-a}}{e^a + e^{-a}}$$



like sigmoid but another output range

### 1.1.3. rectifier linear unit(ReLU)

$$\phi(a) = \text{ReLU}(a) = \max(a, 0) = \begin{cases} a & a \geq 0 \\ 0 & a < 0 \end{cases}$$



•≡ diode

simple calculation

$$\bullet \frac{d\phi}{da} = \begin{cases} 1 & a > 0 \\ 0 & a < 0 \end{cases} = u(a), u(0) = 0 \text{ typically used}$$

•most popular in DNN

•Details on 4-7

### 1.1.4. Softmax activation function(classification problem)

$$\phi(\underline{a} : \underline{a} = [a_i] \in \mathbb{R}^c \rightarrow \mathbb{R}^c$$

$$\phi(\underline{a}) = \text{softmax}(\underline{a}) = \begin{bmatrix} \phi_1(\underline{a}) \\ \vdots \\ \phi_c(\underline{a}) \end{bmatrix}$$

$$\phi(\underline{a}) = \frac{a^{a_i}}{\sum_{j=1}^c e^{a_j}}, \in (0, 1), \sum_{i=1}^c \phi_i(\underline{a}) = 1$$

•maps  $\underline{a} \in \mathbb{R}^c$  to a categorical PMF with  $c$  classes

• $a_i$  large  $\rightarrow \phi_i(\underline{a})$  close to 1

• $a_i$  small  $\rightarrow \phi_i(\underline{a})$  close to 0

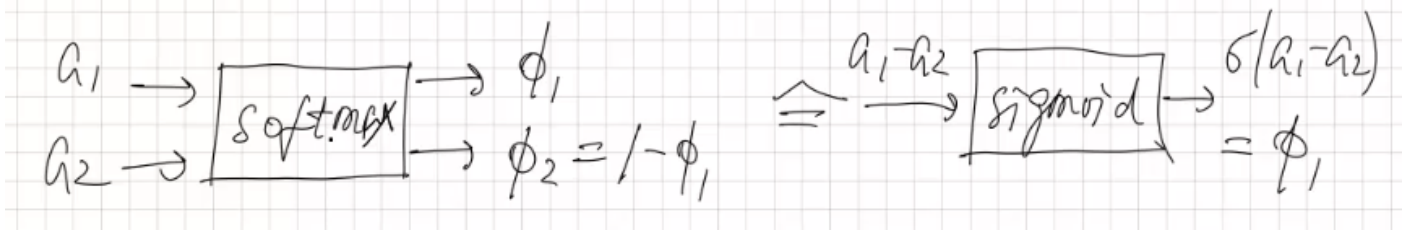
•used in the output layer for classification problems

## 1.2. Special case $c=2$ , binary classification problem

$$\phi_1(\underline{a}) = \frac{e^{a_1}}{e^{a_1} + e^{a_2}} = \frac{1}{1 + e^{-(a_1 - a_2)}} = \sigma(a_1 - a_2)$$

$$\phi_2(\underline{a}) = \frac{e^{a_2}}{e^{a_1} + e^{a_2}} = 1 - \phi_1(\underline{a}) = \sigma(a_2 - a_1)$$

i.e. softmax



one sigmoid output is sufficient for binary classification instead of 2 output softmax!

**Derivative of softmax:**

$$\frac{\partial \phi_i(\underline{a})}{\partial a_j} = \dots = \begin{cases} \phi_i(\underline{a})(1 - \phi_i(\underline{a})) & i = j \\ -\phi_i(\underline{a})\phi_j(\underline{a}) & i \neq j \end{cases}$$