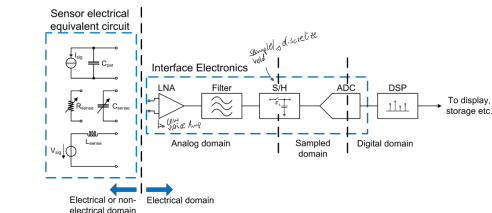


# Sensor Principles

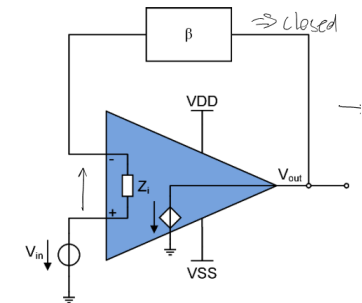
## A generic sensor interface



**Decibel:**  $\frac{V_{dB}}{V_0} = 20 \log_{10}(V/V_0)$ ,  
 $\frac{P_{dB}}{P_0} = 10 \log_{10}(P/P_0)$ ,  
 $\log_{10}(2) \approx 0,3$ ,  $\log_{10}(10) = 1$

## OPAMP Basics

### Opamps in feedback



Gain:  $T(\omega) = \beta \cdot A_0(\omega)$   
DC gain sets the acc. of the closed loop amplifier. Phase Margin:  $PM = 180^\circ + \angle(T(\omega_1)) | T(\omega_1) | = 1$   
Gain margin:  $GM = 20 \cdot \log_{10}(\frac{1}{|T(\omega_2)|}) | \angle(T(\omega_2)) = -180^\circ$

### Generic Transfer function

$$V_x = \beta \cdot V_{out} = \beta \cdot A(\omega) \cdot (V_{in} - V_x) = \beta \cdot A(\omega) \cdot (V_{in} - \beta \cdot V_{out})$$

$$A_{CL} = \frac{V_{out}}{V_{in}} = \frac{A(\omega)}{1 + \beta \cdot A(\omega)} \quad A(\omega) \gg 1 \Rightarrow \frac{1}{\beta}$$

### Acc and gain

$$\epsilon = \frac{A_{CL,nom} - A_{CL}}{A_{CL,nom}}$$

$$A_{OL}(Error, A_{CL}) = A_{CL} \cdot (\frac{1}{Error} - 1)$$

### Negative Feedback and linear operation

$$A(\omega) \gg 1 \Rightarrow \text{virtual short at the input } \Delta V \approx 0$$

$$Z_i \rightarrow \infty \Rightarrow i_{oa} \approx 0$$

Voltage Drive  $\rightarrow$  negative feedback

Current drive  $\rightarrow$  positive feedback

A non-dominant pole located at a frequency lower than the unity gain frequency / GBW causes ringing in the time domain and peaking in the frequency domain.

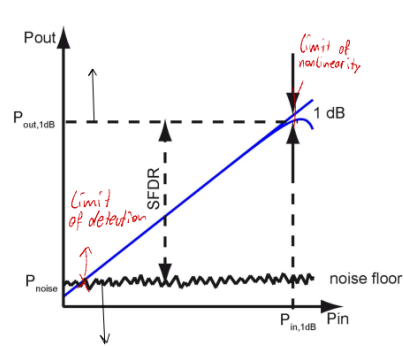
## Errors and Noise

**Limit of detection (LOD):** minimum measurable input amplitude ( $SNR \approx 0$ )

**Dynamic range (DR):** ratio of max and min amplitude within inaccuracy levels.

Lower limit: Noise floor

Upper Limit: Distortion



### Error types:

**Deterministic:** source loading, offset, gain error  $\rightarrow$

Removed by calibration

**Random:** thermal noise, 1/f noise  $\rightarrow$  Mitigated by circuit design to compensate

### Quantification:

$$\text{Absolute: } \Delta x = |\hat{x} - x_0|$$

$$\text{Relative: } \left| \frac{\Delta x}{x_0} \right| = \left| \frac{\hat{x} - x_0}{\hat{x}} \right|$$

Max inaccuracy:  $\Delta x_{max} | x \in [\hat{x} - \Delta x_{max}, \hat{x} + \Delta x_{max}]$

### Error Propagation

$$y = f(x_1, x_2, \dots, x_N)$$

Deterministic fluctuations of  $x_i \rightarrow$  total error:

$$\Delta y \approx \sum_{i=1}^N \frac{\partial f}{\partial x_i} \cdot \Delta x_i$$

Partial derivative  $\frac{\partial f}{\partial x_i}$  is called **sensitivity**

Additive errors are best specified absolute and multiplicative errors are best specified relative

### Interference:

Unwanted coupling of external signal

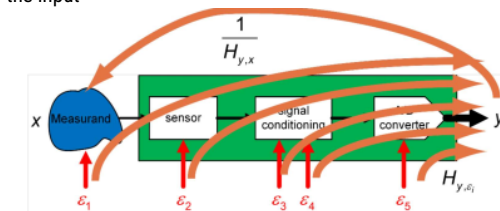
Noise: random fluctuations from setup  $\rightarrow$  can be modeled as error sources

## Combining Error sources

### Output referred noise

Effect of an error-source on the output

**Input referred noise** Equivalent effect of the error-source on the input



Linearize system

$$y_t = f(x_0) + \left. \frac{df}{dx} \right|_{x_0} \cdot (x - x_0)$$

**Taylor Series:**  $T_n(x, a) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x - a)^k$

Apply superposition principle

Find TF from ES to output for all sources

$$y : H_{y,ES} \Rightarrow y_{out,\epsilon_2} = H_{y,\epsilon_2} \cdot \epsilon_2$$

### Compute sum:

$$\text{Deterministic Error: } y_{out, tot} = \sum_{i=1}^N y_{out, i}$$

$$\text{Random Error: } y_{out, tot} = \sqrt{\sum_{i=1}^N y_{out, i}^2}$$

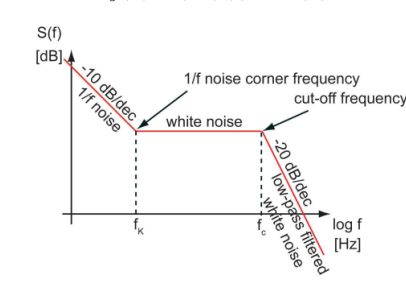
### Lin. System noise:

Refer result back to input wit

$$H_{y,x} : x_{\epsilon_2} = \frac{y_{out,\epsilon_2}}{H_{y,x}} = H_{y,\epsilon_2} \cdot \frac{\epsilon_2}{H_{y,x}}$$

## Wiener-Khintchine-Theorem:

$$PSD = S_y(f) = |H(f)|^2 \cdot S_x(f)$$



## Random Process

**WSS:** Wide sense stationary: Mean is independent of time t and the ACF only depends on  $\tau = t_2 - t_1$

**ACF:** Power spectrum of white noise can be extracted from ACF with fourier transform

**Noise Power:** Total Noise Power is the ACF  $R(0)$

$$S_x(f) = F \{ R_x(\tau) \} = \int_{-\infty}^{\infty} R_x(\tau) \cdot e^{-j2\pi f \tau} d\tau$$
$$R_x(\tau) = F^{-1} \{ S_x(f) \} = \int_{-\infty}^{\infty} S_x(f) \cdot e^{+j2\pi f \tau} df$$

$$\text{Noise power: } P_x = \int_{-\infty}^{\infty} S_x(f) \cdot df = R_x(0)$$

Noise voltage:

$$V_{n,rms}^2 = \int_{-\infty}^{\infty} S_{v_n}(f) \cdot df = \int_0^{\infty} S_{v_n}^+(f) \cdot df$$

Noise current:

$$I_{n,rms}^2 = \int_{-\infty}^{\infty} S_{i_n}(f) \cdot df = \int_0^{\infty} S_{i_n}^+(f) \cdot df$$

## Noise Types

**Thermal noise:** excitation of charge carriers (white)  $\rightarrow$  mitigate lower temp. lower resistance

**Shot noise:** carriers randomly crossing the barrier, dependent on DC bias and white

**Flicker Noise:** due to traps in semiconduct. 1/f spectral density. MOS trans at low freq.

**Thermalnoise Theorem:** Every closed system at temp. T has average. Energy of  $kT/2 \rightarrow S_u(f) = 2kTR$  (double-sided) math  $S_u(f) = 4kTR$  (single-sided) physics

## Langevin Approach kt/C noise

$$PSD \text{ noise voltage of } V_n = S_{v_n}(f) = 4kT \cdot \Re \{ Z(j2\pi f) \}$$

$$\rightarrow \overline{V_n^2} = kT \left[ \frac{1}{C_0} - \frac{1}{C_0} \right] = \frac{kT}{C}$$

PSD Noise current:

$$S_{I_n} = \frac{4kT}{\Re \{ Z(j2\pi f) \}} \quad \text{MOS IRN}$$

$$S_{\Delta V_{nG-tot}^2} = 4kT \cdot R_{nG-tot}, \quad R_{nG-tot} = \frac{\rho}{W \cdot L \cdot f} + \frac{\gamma_n D}{G_m}$$

with GateExcess noise factor

$$\gamma_{nD} = (n = 1.3) \cdot [0.5WI; 2/3SI]$$

**Bipolar Trans IRN:**

$$S_{\Delta V_{nR}^2} = 4kT \cdot R_B$$

## Noise Analysis

small-signal-equivalent is valid.

$$\text{Total IRN: } S_{n, out}(f) = \sum_{k=1}^N |H_k(f)|^2 \cdot S_{n_k}(f)$$

N uncorrelated NS.

$H_k(f)$  TF from NS ton output. IRN:

$$S_{V_{neq,IRN}}(f) = \frac{S_{V_{out}}(f)}{|A(f)|^2} \text{ with } A(f) \text{ is TF}$$

## Sensor types

Information domain  $\rightarrow$  Electrical domain

Transduction: Converting a signal from the energy domain into another.

Sensors and actuators are transducers

## Sources of error

Noise, sensitivity to unintended quantities, Noise, EMI

Tandem transducers: Multiple steps to target domain. cross-sensitivity, sensitivity to undesired quantity

## Sensor classification:

### Active Sensors:

Require external source of excitation

### Passive / self-generating sensors:

Generate their own electrical output signal

Draws all required energy from the measurand (source loading) E: Potentiometer for angle measurements.

### Modulating sensors:

Measure desired quantity by modulating

Additional source with modulated energy. Also adds error. E:

Non-contact displacement measurement (rotating disk)

### Analog vs. Digital:

Analog: time and value continuous

Digital: Discrete outputs

### Deflection mode sensors:

Response to an output is a deviation from the equilibrium position

### Null mode sensors:

Sensor or instruments exert an influence the measured system opposing the effect of the measurand. Ideally the result is a 0 measurement, typically achieved by feedback. The opposing influence is then the sensor output. Slower than deflection, but more accurate.

### Resistive sensors - strain gauges

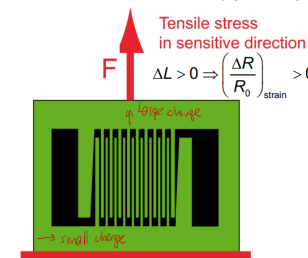
Change in geometry under mechanical stress produces associated resistance change

Volume.  $= \text{const.} \rightarrow$

$$\frac{\partial V}{V_0} = \frac{L_0}{V_0} \cdot \partial A + \frac{A_0}{V_0} \cdot \partial L \wedge \frac{\partial V}{V_0} = 0 \Rightarrow \frac{\partial A}{A_0} = -\frac{\partial L}{L_0}$$
$$\rightarrow \frac{\partial R}{R_0} = \frac{\partial \rho}{\rho_0} + 2 \frac{\partial L}{L_0}$$
$$\rightarrow \frac{\partial R}{R_0} = \alpha \cdot \frac{\partial L}{L_0} + 2 \frac{\partial L}{L_0} = (\alpha + 2) \cdot \frac{\partial L}{L_0} \triangleq k$$

k: gauge factor

$$\alpha \text{ proportionality factor } \frac{\partial \rho}{\rho_0} \propto \frac{\partial L}{L_0}$$



## Readout resistive sensors

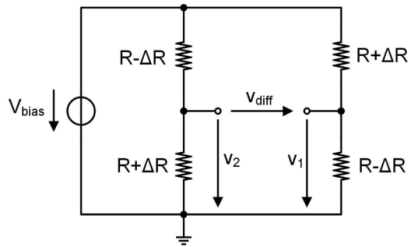
Use a half bridge resistive divider

top element sensing resistor

$$\frac{v_{out}}{V_{bias}} = \frac{R}{2R + \Delta R} \Leftrightarrow \frac{\Delta R}{R} = \frac{V_{bias}}{v_{out}} - 2$$

Full bridge to remove offset, 2 sensing elements more sensitive,

remove nonlinearity by implementing differential measurements  
Best use 4point full bridge, no nonlinearity



$V_{diff} = v_2 - v_1 = \frac{\Delta R}{R} \cdot V_{bias}$   
Sensitivity  $S = 45mV/V$  excitation for 1V input  
Accuracy  $A = \frac{v_{diff}(\frac{\Delta R}{R}) - V_{diff,lin}(\frac{\Delta R}{R})}{V_{diff}(\frac{\Delta R}{R})}$   
Deviation from the ideal bridge

## BridgeParameters

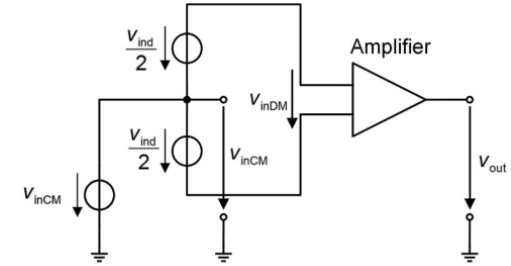
**Bridge resistance:** Unloaded  $R$  across the signal terminals  
**Offset error** Outputvoltage at 0 input  
**Drift:** Outputchange conditioned on environmental condition

## Differential/ Commonmode signals

**Recall:**  
 $V_1 = \frac{1}{2} \cdot \left(1 - \frac{\Delta R}{R}\right) \cdot V_{bias}$ ,  $v_2 = \frac{1}{2} \cdot \left(1 + \frac{\Delta R}{R}\right) \cdot V_{bias}$

**Differential signal:**  $V_{diff} = V_2 - V_1 = \frac{\Delta R}{R} \cdot V_{bias}$

**Common-mode signal:**  $V_{CM} = \frac{v_2 + v_1}{2} = \frac{V_{bias}}{2}$



**DiffGain:**  $V_{out} = A_{DM} \cdot V_{inDM}$

**CMMGain:**  $V_{out} = A_{CM} \cdot v_{inCM}$

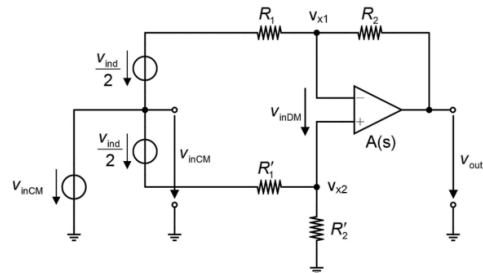
Ideally  $A_{cm}$  is 0 or  $A_{cm} \ll A_{dm}$

Commonmode rejection ratio(CMRR):

$$CMRR \triangleq \left| \frac{A_{DM}}{A_{CM}} \right| = \left| \frac{dv_{out}/dv_{inDM}}{dv_{out}/dv_{inCM}} \right|$$

Often in dB

**Opampbase difference amplifier:**



1st:  $A(s) = \infty$ ,  $R_2 = R_2' = \alpha \cdot R_1 = \alpha R_1'$

$A_{DM} = -\alpha$ ,  $A_{CM} = 0$ ,  $CMRR = \infty$   
2nd assume  $A(s) = \infty$ ,  $R_2 = \alpha \cdot R_1$ ,  
 $R_2' = (\alpha + \Delta\alpha) \cdot R_1'$ ,  $R_1' = R_1 + \Delta R$

$$A_{DM} = -\left(\alpha + \frac{\Delta\alpha}{2 \cdot (1 + \alpha + \Delta\alpha)}\right)$$

$$A_{CM} = \frac{\Delta\alpha}{1 + \alpha + \Delta\alpha}$$

$$CMRR = -\frac{1}{2} + \frac{\alpha \cdot (1 + \alpha + \Delta\alpha)}{\Delta\alpha}$$

$$3d: A(s) = A_{DC} / \left(1 + s \cdot \frac{A_{DC}}{GBW}\right)$$

$$A_{DM} = -\left(\alpha + \frac{\Delta\alpha}{2 \cdot (1 + \alpha + \Delta\alpha)}\right) \cdot \frac{1}{1 + \frac{1}{A_{DC}} + \frac{1}{A_{DC}} + (1 + \alpha) \cdot \frac{s}{GBW}}$$

$$A_{CM} = \frac{\Delta\alpha}{1 + \alpha + \Delta\alpha} \cdot \frac{1}{1 + \frac{1}{A_{DC}} + \frac{1}{A_{DC}} + (1 + \alpha) \cdot \frac{s}{GBW}}$$

$$CMRR = -\frac{1}{2} + \frac{\alpha \cdot (1 + \alpha + \Delta\alpha)}{\Delta\alpha}$$

## Magnetic Field Sensors

$V_{hall} = G \cdot \frac{\mu \cdot \rho}{h} \cdot l \cdot B = G \cdot \frac{1}{e \cdot n \cdot h} \cdot l \cdot B = S_l \cdot l \cdot B$   
geometry factor

**Error sources:**

- Thermal noise

-1/f noise

-Noise of conditioning electronics(minor)

-Offset Noise power:  $V_{noise} \sqrt{4kT \cdot R \cdot \Delta f}$

Signal power:  $V_{hall} S_l \cdot I \cdot B$

$$\text{Resolution: } B_{min} \hat{=} \frac{V_{noise}}{V_{hall}/B} = \frac{\sqrt{4kT \cdot R \cdot \Delta f}}{S_l \cdot I}$$

With  $S_l = \frac{\mu \cdot \rho}{h \cdot e \cdot n} \cdot G$

At  $B = 0$  Hall plate can be modeled as wheatstone bridge,

non-uniform stress causes intrinsic offset(5 to 30mT)

Cancel intrinsic offset by coupling 90° turned sensors,

current and sense ports are swapped

Idea: Hall voltage is in phase and gradient caused offsets are than 180° out of phase

**Spinning current method:**

Sense and current ports are switched every clock cycle

Idea: A DC component equal to the offset and a frequency component proportional to the hall voltage is produced

(Fourier Expansion of a square wave)

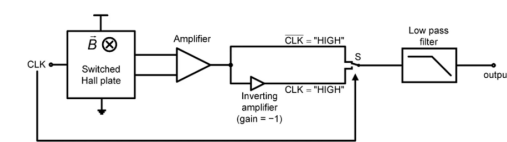
By modulating the information to the clock frequency noise and offset are greatly mitigated.

Multiplying by 1 and -1 to demodulate back to low frequency.

Low pass stage to remove signal around the 2nd harmonic

This allow to see the base noise floor of the sensor not 1/f noise.

In the conventional implementation 2 bias currents are used in the low power configuration the bias form the plate is reused for the amplifier.



## Temperature Sensors

Seebeck effect: Convert temp. gradient to electromotive force  $F(T) \triangleq \int_{T_{stand.}}^T (S_+(T') - S_-(T')) dT' \rightarrow$

$E_{emf} = F(T_{sense} - T_{ref})$  **Temperature stable**

**references:**

$$V_{REF}(T) = V_{CTAT}(T) + K \cdot V_{PTAT}(T)$$

PTAT: Proportional to absolute temperature

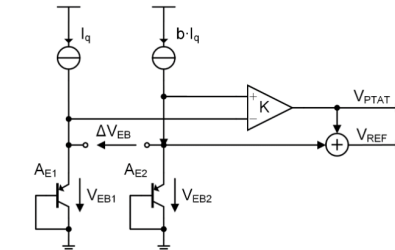
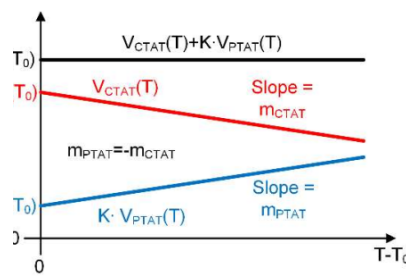
CTAT: Complementary to absolute temperature

Scale one slope and add the function for a temp. constant reference

This can be built with a diode or a BJT with a shorted base and collector

Butt current is also temperature dependent so use diff pair.

**Bandgap reference:**



$$V_{PTAT} = \Delta V_{EB} = V_{EB1} - V_{EB2} \approx$$

$$\frac{kT}{q} \cdot \ln \left[ \frac{I_{q1} \cdot I_{S1} \cdot A_{E2}}{I_{S2} \cdot A_{E1} \cdot I_{q2}} \right] = \frac{kT}{q} \cdot \ln \left[ \frac{A_{E2}}{A_{E1}} \right]$$

Scale currents or diodes to make equal bias

$$V_{EB} = \underbrace{E_g/q}_{V_G} - \underbrace{\frac{kT}{q}}_{U_T} \cdot \ln \left( \frac{I'_S}{I_D} \right) =$$

$$V_G - U_T \cdot \ln \left( \frac{K_1 \cdot T^\gamma}{I_D} \right)$$

$$V_G \approx V_{G0K} - a \cdot T$$

$$V_G = V_{EB2} = V_{CTAT} \text{ With } I'_S > I_D \text{ but also}$$

temperature dependent  $\rightarrow$  curved CTAT.

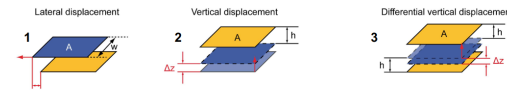
**Combine CTAT and PTAT: Series:**

$$V_{REF} = I_{PTAT} \cdot R_2 + V_{CTAT} = \frac{R_2}{R_1} \cdot V_{PTAT} + V_{CTAT}$$

Parallel:  $V_{REF} = (I_{PTAT} + I_{CTAT}) \cdot R_3 =$

$$\frac{R_3}{R_1} \cdot V_{PTAT} + \frac{R_3}{R_2} \cdot V_{CTAT}$$

## Capa. Sensor Readout



$$1 \text{ linear: } \Delta C = C - C_0 = \epsilon \cdot \frac{W}{h} \cdot \Delta x$$

$$2 \text{ non-linear: } \Delta C = \frac{\epsilon \cdot A}{h} \cdot \frac{\Delta z/h}{1 + \Delta z/h} \approx \frac{\epsilon \cdot A}{h^2} \cdot \Delta z$$

$$3 \text{ reduced non-linear with diff readout:}$$

$$\Delta C = \epsilon \cdot \frac{A}{h} \cdot \left( \frac{2 \cdot \Delta z/h}{1 - [\Delta z/h]^2} \right) = \frac{2 \cdot \epsilon \cdot A}{h^2} \cdot \Delta z$$

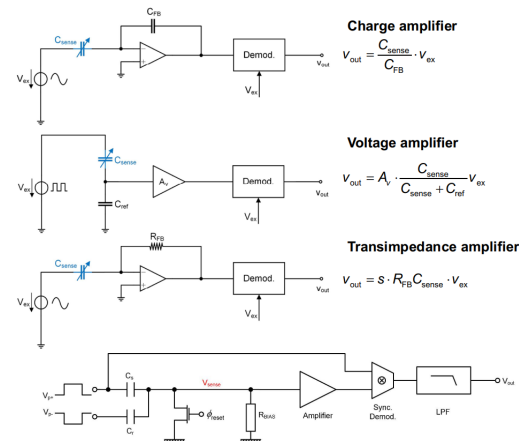
**OpenLoop readout:**

$$\text{ChargeAmp: } V_{out} = \frac{C_{sense}}{C_{FB}} \cdot V_{ex}$$

$$\text{ChargeAmpNonideal: } \frac{V_{out}}{V_{in}} = - \frac{j\omega \cdot R_{FB} C_{in}}{\left(1 + \frac{j\omega}{GBW}\right) \cdot \left(1 + \frac{j\omega}{\omega_{FB}}\right)}$$

$$\text{VoltAmp: } v_{out} = A_v \cdot \frac{C_{sense}}{C_{sense} + C_{ref}} \cdot V_{ex}$$

$$\text{TransimpAmp: } v_{out} = s \cdot R_{FB} C_{sense} \cdot v_{ex}$$

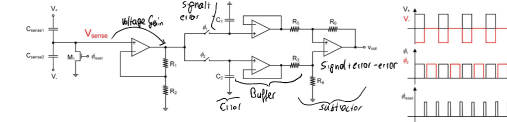


$V_{Sense}$  not grounded sensitive to leakage currents

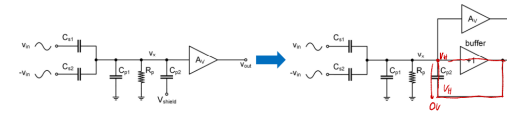
Bias resistor provides well defined DC-point.  $R_{Bias}$  needs to be large because of the highpass forming, corner frequency needs to be low enough compared to excitation freq.

Periodic reset can also be used where  $V_{p+} = V_{p-} = 0$

Correlated double sampling can also be used:



## Bootstrapping: VoltAmp



$$C_{s1} = C_{s0} + \Delta C / 2, C_{s2} = C_{s0} - \Delta C / 2, v_{out}$$

becomes sensitive to parasitic capacitance

Can be bootstrapped out with voltage buffer driving  $V_{shield}$

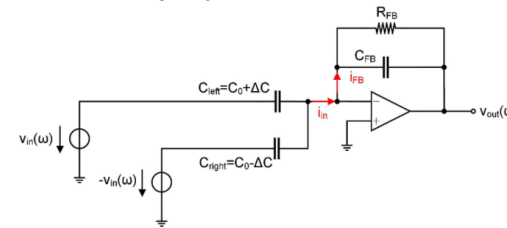
$$v_{out} = \frac{\Delta C}{2C_{s0} + C_{p1} + (C_{p2})} \cdot A_v \cdot v_{in}$$

Gain of output is reduced

$C_{p2}$  gets removed by  $V_{shield}$

Can also use TIA, the virtual ground helps with the parasitic capacitance, this also works in the charge amplifier

**DiffReadoutChargeAmp:**



**Req  $V_{ex}$ :**

Error will propagate to output directly

Accuracy more important for small  $\Delta C$

Mismatch or drift will introduce errors

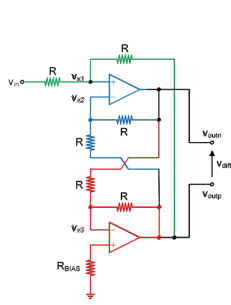
Sinewave can be created with center-tapped transformer or active balun

Rectangular witch switched excitation schemes

**DiffReadoutChargeAmp:**

- Assuming ideal opamps ( $A_{op} \mapsto \infty$ )  
 $\Rightarrow V_{x2} = V_{x1}, V_{x3} = 0$

- The part of the circuit highlighted in red is a simple inverting amplifier with input voltage  $v_{x1}$  and output voltage  $v_{outp}$
- The part of the circuit highlighted in blue is a simple non-inverting amplifier with input voltage  $v_{x1}$  and with its reference potential pulled to  $v_{outp}$
- The part of the circuit highlighted in green is a voltage divider between  $v_{in}$  and  $v_{outp}$



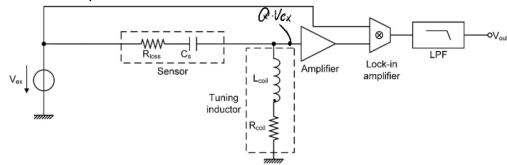
$$V_{outp} = V_{in} = -V_{outn}$$

### Differential Capa sensing:

$$\Delta v_{x, amp} \approx -\frac{\Delta x}{x_0} \cdot V_{ex}$$

### Resonant readout:

$$\omega_{LC} = \frac{1}{\sqrt{L_{coil} \cdot C_s}}$$



$$\text{DoubleDiffReadout: } \Delta V_{out} = -\Delta V_{ex} \cdot \left( \frac{\Delta C_s}{C_i} \cdot \left[ 1 - \frac{C_s}{C_s + C_i + C_p} \right] - \frac{\Delta C_i + \Delta C_p}{C_i} \cdot \frac{C_s}{C_s + C_i + C_p} \right)$$

gain error                      offset error

## Differential Measurements

- Inherent rejection to common-mode interference and noise
- Wider Signal swing for a given supply voltage
- Minimum effect of even order distortion including DC offset
- Can lead to improved sensor linearity

But respond to some degree to common mode signal

### Ideal DiffPair:

$$\text{CMRR} \triangleq 20 \cdot \log \left( \left| \frac{A_{DM}}{A_{CM-to-DM}} \right| \right) =$$

$$20 \cdot \log \left( \frac{2G_{mav} \cdot r_{out}}{\Delta R_L / R_{Lav} + \Delta G_m / G_{mav}} \right)$$

$$\text{PSRR} \triangleq 20 \cdot \log \left( \left| \frac{A_{DM}}{A_{VDD}} \right| \right) = 20 \cdot \log \left( \left| \frac{A_{DM}}{\Delta V_{out} / \Delta V_{DD}} \right| \right)$$

Process variation causes mismatch between resistors and current source is not ideal

Input referred offset is the voltage needed to get zero volts differential output

$$\text{IRO: } \Delta V_G = V_{OS} = -\frac{I_{D_{av}}}{G_{mav}} \cdot \left( \frac{\Delta R_L}{R_{Lav}} + \frac{\Delta \beta}{\beta_{av}} \right) - \Delta V_{T0}$$

OffsetVoltage: 10mV MOS; 1mV BJT, 120  $\mu$ , trimmed BJT

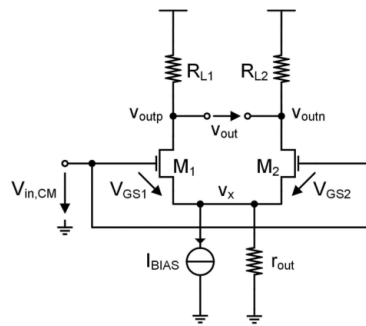
$$\text{CMRR} \cdot V_{OS} \approx G_{mav} \cdot r_{out} \cdot V_{OV}, V_{OV} = \frac{I_{D_{av}}}{G_{mav}}$$

### PSRR:

$$\text{PSRR}_{VDD} \cong \frac{G_{mav} R_{Lav}}{g_{outM1,2} \cdot \left( \frac{g_{outL} \cdot \Delta R_L - 2 \cdot R_{Lav} \cdot \Delta G_m}{G_{mav}} \right)}$$

$$\text{PSRR}_{VSS} \cong$$

$$G_{mav} \cdot R_{Lav} / g_{out} \cdot \left( \Delta R_L + R_{Lav} \cdot \frac{\Delta G_m}{G_{mav}} \right)$$



## Modulation

Synonyms: Coherent detection, synchronous demodulation, lock-in amplification, chopping

All modulation techniques, square wave is called chopping

Leads to better low freq. specification, smaller 1/f, bigger CMRR and PSRR

**Trimming:** Measuring static error offset and gain and adjusting the value of a component to reduce the error to 0  
 Low complexity, no bandwidth limit, but reqs. measure equipment

Also reqs. memory element

### Dyn. offset cancel:

usually no measure eq. but more complex circuits, reduce bandwidth

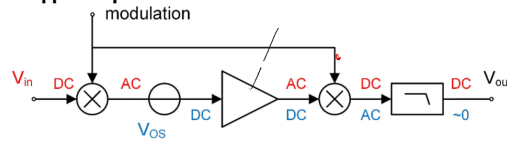
### AutoZeroing:

periodically measure offset and subtract from input(time domain)

### Chopping:

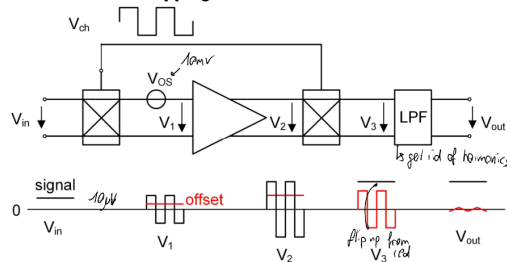
Modulate signal above 1/f noise(freq. domain)

### Chopper Amps



implemented with polarity reversing switch

### Time-Domain-Chopping:



Complete suppression of 1/f noise if  $f_{chop} > 1/f$  corner freq., but up-modulated offset must be filtered out. loss of bandwidth and residual chopper ripple. **Charge Injection**

Injected charge splits half-half

$$\text{Charge: } Q = W \cdot L \cdot C_{ox} \cdot [V_{\varphi} - V_{in} - V_{T0} (\sqrt{V_{in}})]$$

Linear w.r.t to  $W \cdot L$  non-linear w.r.t.  $V_{in}$   $V_{out} = V_{in} (1 + \frac{W \cdot L \cdot C_{ox}}{2CL}) - \frac{W \cdot L \cdot C_{ox}}{2CL} \cdot [V_{\varphi} - V_{T0} (\sqrt{V_{in}})]$

$$\text{gain error} \quad \text{offset \& distortion}$$

### Clock feedthrough

Overlap capacitance of trans.:  $C_{OV}$

$$\Delta V_{out} = \frac{C_{ov}}{C_{ov} + C_L} \cdot V_{\varphi}$$

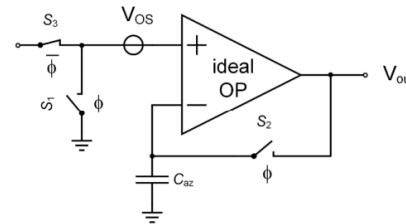
Asymmetric clock duty cycle causes demodulation signal to have DC component which feeds through

### Bandwidth gain acc

Limited amplifier Bandwidth causes output signal to not be perfectly square, therefore less gain

### AutoZeroing, LF Noise Reduce

Sampling unwanted signal during  $\Phi_1$ , storing, and subtracting during  $\Phi_2$ , input is disconnected during  $\Phi_1$



Error stored on  $C_{AZ}$  will slowly leak away,  $C_{AZ}$  as large as possible

### Mitigate Charge Inject.

-Use min size switch

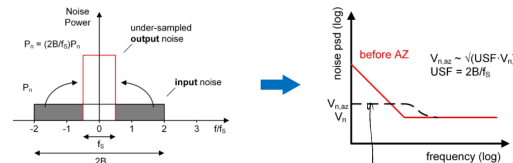
- **Diff Sampling:** Const offset and nonlin. is reduced

- **Comp Switch:** NMOS and PMOS in parallel cancel opposite charge packets, can only occur for one  $V_{in}$ , clock feedthrough can't be cancelled perfectly because of different overlap capacitance.

- **Dummy Switch:** Add a dummy switch of half size to such up injection of M1, but equal charging splitting rarely holds, but clock feedthrough is also mitigated

- **BottomplateSampling:** Disconnect  $C_{AZ}$ 's bottomplate from ground slightly before M1,  $C_{az}$  bottom plate is then floating when M1 is opened and no charge can be injected.

Requires additional clock.



No ripples, like chopping, offset of a few  $\mu V$  can be reached, main problems switching spikes, leakage currents and finite gain

### Correlated Double Sampling (CDS):

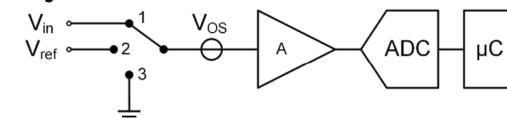
Special case of AZ

$$\text{Phase 1 (calib): } V_1 = V(t_1) = A \cdot (0 + V_{OS})$$

$$\text{Phase 2 (measure): } V_2 = V(t_2) = A \cdot (V_{in} + V_{OS})$$

$$\Rightarrow (V_2 - V_1) = A \cdot V_{in}$$

### 3 signal method:



Find:  $V_{in}$ ,  $A$ ,  $V_{OS}$

$$\text{Phase 1: } V_1 = A \cdot (V_{in} + V_{OS})$$

$$\text{Phase 2: } V_2 = A \cdot (V_{ref} + V_{OS})$$

$$\text{Phase 3: } V_3 = A \cdot V_{OS}$$

Acc. is limited by ADC resolution

### DEM:

Switch nominally identical components with a clock

Acc. is limited by mismatch of switch resistance

Significantly reduces average error

$$2 \text{ Resistors in parallel: } \frac{R_1 = R + \Delta R}{R_2 = R - \Delta R} \rightarrow$$

$$\text{Gain}_{av} = \frac{(1 + \frac{R + \Delta R}{R - \Delta R}) + (1 + \frac{R - \Delta R}{R + \Delta R})}{2} = 2$$

LPF needed like chopping, can be easily combined

Reduces bandwidth and need more components

## Coherent Detection

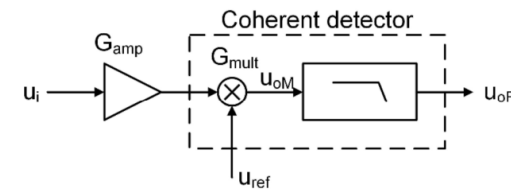
synonyms: coherent detection, synchronous demodulation, lock-in amplification, chopping

Like chopping but sinewave instead of rect. Good for:

- low-bandwidth quasi static signals with high noise
- if high dynamic range is req.

Mems, Infrared, magnetic sensors, strain gauges.

**Behaves like bandpass but isn't one**



### Amplitude Synchron. detection:

$$u_{oM} = G_{amp} \cdot G_{mult} \cdot [\hat{u}_i \cdot \sin(\omega_i t) \cdot \hat{u}_r \sin(\omega_r t)]$$

$$u_{oM} = G_{amp} \cdot G_{mult} \cdot \frac{\hat{u}_i \hat{u}_r}{2} \cdot [1 - \cos(2\omega_i t)]$$

$$\xrightarrow{LP} u_{oF} = G_{LPF} \cdot G_{amp} \cdot G_{mult} \cdot \frac{\hat{u}_i \hat{u}_r}{2} \text{ for } \omega_r = \omega_i$$

### Phase:

$$u_{oM} = G_{amp} \cdot G_{mult} \cdot [\hat{u}_i \cdot \sin(\omega_i t + \varphi) \cdot \hat{u}_r \cdot \sin(\omega_r t)]$$

$$u_{oF} = G_{LPF} \cdot G_{amp} \cdot G_{mult} \cdot \frac{\hat{u}_i \hat{u}_r}{2} \cdot \cos(\varphi)$$

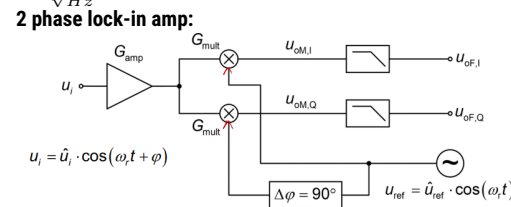
Phase sensitive coherent detector

### SNR:

LOD for  $10nV/\sqrt{Hz}$  white noise and  $f_{LPF} = 1Hz$

$$\rightarrow \frac{10nV}{\sqrt{Hz}} \cdot \sqrt{1Hz} = 10nV$$

### 2 phase lock-in amp:



$$A_0 =$$

$$u_{oF,I} = G_{amp} \cdot G_{mult} \cdot G_{LPF} \cdot \frac{\hat{u}_i \cdot \hat{u}_r}{2} \cdot \cos(\varphi)$$

$$u_{oF,Q} = G_{amp} \cdot G_{mult} \cdot G_{LPF} \cdot \frac{\hat{u}_i \cdot \hat{u}_r}{2} \cdot \sin(\varphi)$$

$$\hat{u}_i = \frac{1}{G_{amp} \cdot G_{mult} \cdot G_{LPF}} \cdot \sqrt{u_{of,I}^2 + u_{of,Q}^2}$$

$$\varphi = \tan^{-1} \left( \frac{u_{of,Q}}{u_{of,I}} \right)$$

$$\varphi = \tan^{-1} \left( \frac{u_{of,Q}}{u_{of,I}} \right)$$

$$\varphi = \tan^{-1} \left( \frac{u_{of,Q}}{u_{of,I}} \right)$$

$$\varphi = \tan^{-1} \left( \frac{u_{of,Q}}{u_{of,I}} \right)$$

$$\varphi = \tan^{-1} \left( \frac{u_{of,Q}}{u_{of,I}} \right)$$

$$\varphi = \tan^{-1} \left( \frac{u_{of,Q}}{u_{of,I}} \right)$$

$$\varphi = \tan^{-1} \left( \frac{u_{of,Q}}{u_{of,I}} \right)$$

$$\varphi = \tan^{-1} \left( \frac{u_{of,Q}}{u_{of,I}} \right)$$

$$\varphi = \tan^{-1} \left( \frac{u_{of,Q}}{u_{of,I}} \right)$$

$$\varphi = \tan^{-1} \left( \frac{u_{of,Q}}{u_{of,I}} \right)$$

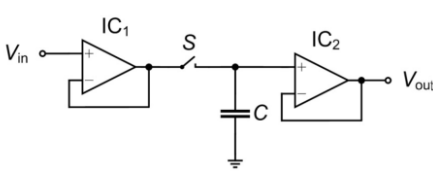
## DACs

Prerequisite for sampling is the Nyquist theorem to be able to perfectly reconstruct a band-limited signal.

$$f_s \geq f_N = 2 \cdot f_b$$

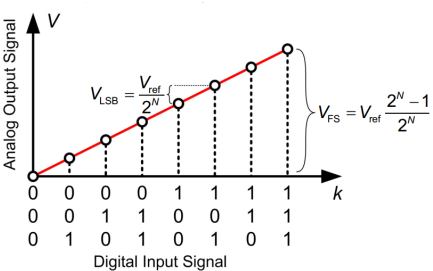
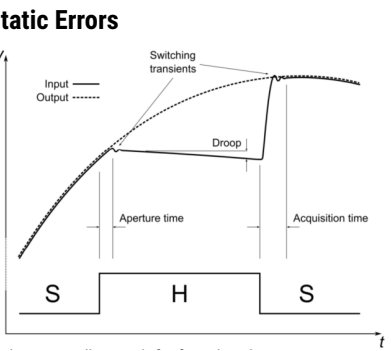
**Sample and hold:**





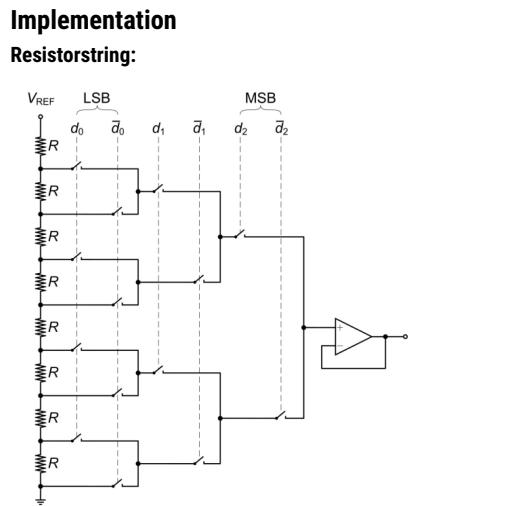
Sample: S is closed  $V_{out} = V_{in}$   
 Hold: S is open C holds  $V_{out}$   
 $IC_1$  with small  $Z_1$  for fast charge of C  
 $IC_2$  with  $Z_{in}$  for slow discharge of C  
 S with small  $R_{on}$

**Characteristics:**  
 Aperture time: time for switch to open  
 Droop: discharge of capacitor  
 Acquisition time: time to switch and charge capacitor  
 Switching transients: voltage buffer ringing  
**Design C:**  
 Large for small droop, small for fast charge  
 Charge depends on  $R_{on}$  and  $Z_{out}$  or  $I_{out,max}$  of  $IC_1$   
 Large enough for sufficient small  $\frac{kT}{C}$

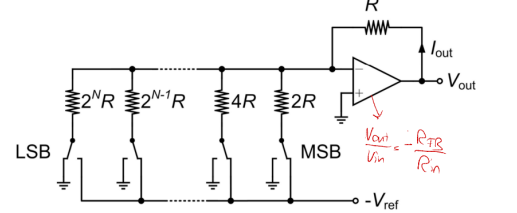


**Gain Error:**  $V_{gain,error} = V_{MSB,avg} - V_{MSB,ideal}$   
**Offset:**  $V_{OS} = V(000)$   
 Does not affect linearity, easy to compensate  
**Diff nonlin (DNL):**  
 Measure of nonuniformity, quantifies for each of the k binary input combinations the deviation of each step from the ideal stepsize of one LSB  
 $DNL(k) = \frac{V(k) - V(k-1) - V_{LSB}}{V_{LSB}} = \frac{\Delta V(k)}{V_{LSB}}$  If DNL smaller than -1 the output is smaller than the previous and it's non-monotonic  
 Monotonicity is critical for feedback: turns negative feedback into positive feedback  
 Occurs most often when switching the MSB  
**Intefral nonlin (INL):** Deviation of the output val from the

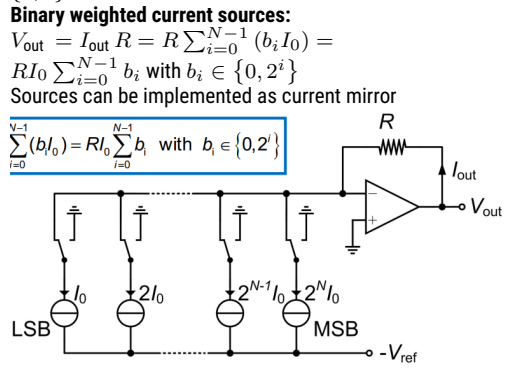
ideal val:  
 $INL(k) = \frac{V(k) - V_{ideal}(k)}{V_{LSB}}$   
**Dynamic Errors**  
**Jitter:** Max jitter ( $\Delta t$ ) for error below one lsb:  
 $\Delta t < \frac{V_{LSB}}{\pi f_{MAX} V_{FS}} \approx \frac{1}{2^N \pi f_{MAX}}$   
**Glitches:**  
 Turn on and turn off time not precisely synchronized. In the moment of switching one bit to another the value can briefly be both or none of the bits.



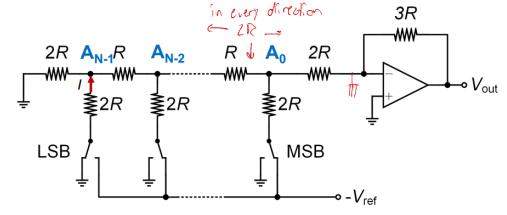
Simple voltage divider, inherently monotonic, amount of resistors proportional  $2^N$ , Area  $A \propto W_R \cdot L_R \cdot 2^N$   
**Binary weighted resistive voltage divider**  
 Inverting summing amplifier



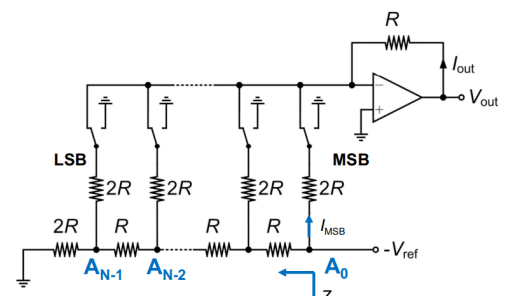
For each bit  $i$ :  $V_{out,i} = d_i \cdot \frac{R}{2^{i+1}R} = d_i \cdot \frac{1}{2^{i+1}}$ ,  $i = 0, \dots, N-1$ ,  $d_i \in \{0, 1\}$   
 $V_{out} = \sum_{i=0}^{N-1} V_{out,i} = V_{ref} \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}}$ ,  $d_i \in \{0, 1\}$



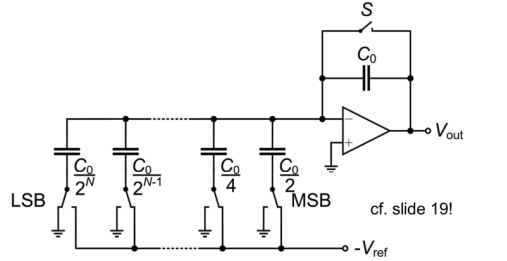
**Binary DAC:** few number of components, large ratios of resistors, currents and switches, monotonicity not guaranteed, not linear, bas acc, need precise matching, prone to glitches, large portion switches.  
**Thermometer wighted:**  
 Current steering DAC, reduced glitches, monotonic,  $2^N$  sources, binary to thermometer decoder needed



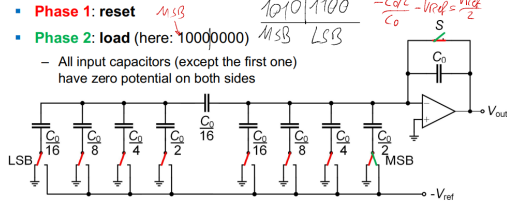
Branch current:  $I = -\frac{V_{ref}}{2R + (2R \parallel 2R)} = -\frac{V_{ref}}{3R}$   
 $V_{out,i} = -d_i \frac{I}{2^{i+1}} \cdot 3R = V_{ref} \cdot d_i \cdot \frac{1}{2^{i+1}}$   
 $V_{out} = \sum_{i=0}^{N-1} V_{out,i} = V_{ref} \cdot \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}}$



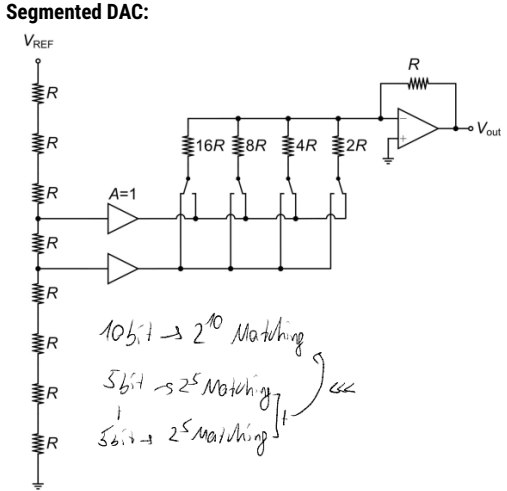
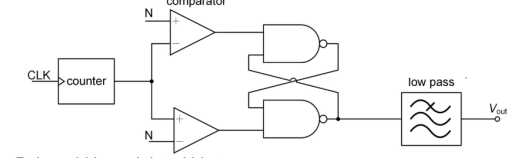
**Inverse R2R-Ladder:**  
 $V_{out,i} = -d_i \cdot \frac{V_{ref} / (2R)}{2^i} \cdot R = V_{ref} \cdot d_i \cdot \frac{1}{2^{i+1}}$   
 $V_{out} = \sum_{i=0}^{N-1} V_{out,i} = V_{ref} \cdot \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}}$   
**Bin weighted Cap. volt divider:**  
 $V_{out,i} = d_i \cdot \frac{C_0 / 2^{i+1}}{C_0} \cdot V_{ref}$ ,  $d_i \in \{0, 1\}$ ,  $i = 0, \dots, N-1$   
 $V_{out} = \sum_{i=0}^{N-1} V_{out,i} = V_{ref} \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}}$  where  $d_i \in \{0, 1\}$  Capas are weighted in the denominator, but otherwise topology same as



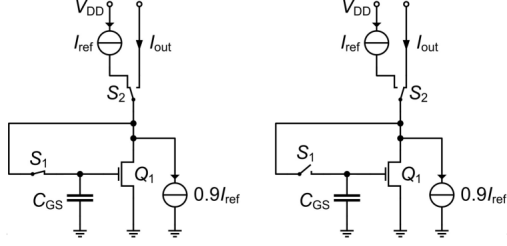
**Realization with less capas:**



**Switch between lsb and msb group reset and load**  
 $V_{out} = \alpha_{right} \cdot V_{ref}$   
 $\alpha_{right} = \sum_{i=1}^{N_{right}} d_{i,right}$  with  $d_{i,right} \in \{0, 2^{-i}\}$   
 $V_{out} = \frac{\alpha_{left}}{16} V_{ref}$   
 $\alpha_{left} = \sum_{i=1}^{16} d_{i,left}$  with  $d_{i,left} \in \{0, 2^{-i}\}$   
**PWM DAC:**



Course DAC(MSBs) feeds fine DAC (LSBs), matching needs to be instead  $2^{10}$  for 10 bit only  $2 \cdot 2^5$



Each source supposed to have  $I_{ref}$ , Assume one only has  $0.9I_{ref}$   
 Calib phase:  $V_{GS1}$  settles so that  $Q_1$  draws  $0.1I_{ref}$   
 Operat. Phase:  $Q_1$  calibs. so that  $I_{d1} = I_{ref}$