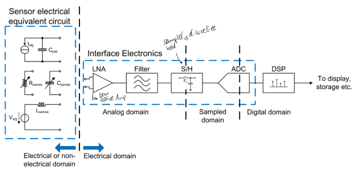
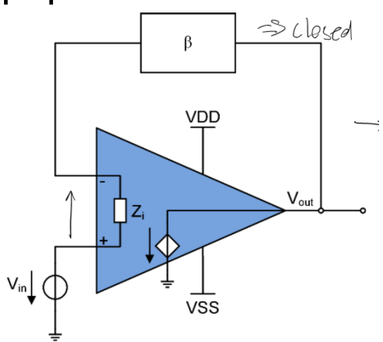
Sensor Principles

A generic sensor interface



OPAMP Basics

Opamps in feedback



Gain : $T(\omega) = \beta \cdot A_0(\omega)$

Phase Margin:

$PM = 180° + \angle(T(\omega_1)|_{|T(\omega_1)|=1}$ Gain margin:$GM =$

$20 \cdot log_{10}(\frac{1}{T(\omega_2)})|_{\angle(T(\omega_2)=-180°}$ **Generic Transfer function**

$V_x = \beta \cdot V_{\text{out}} = \beta \cdot A(\omega) \cdot (V_{\text{in}} - V_x) =$
$\beta \cdot A(\omega) \cdot (V_{\text{in}} - \beta \cdot V_{\text{out}})$

$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{A(\omega)}{1+\beta \cdot A(\omega)} \overset{A(\omega)]\gg 1}{\approx} \frac{1}{\beta}$ **Negative Feedback and linear operation**

$A(\omega) \gg 1 \Rightarrow$ virtual short at the input

$\Delta V \approx 0$

$Z_i \rightarrow \infty \Rightarrow i_{oa} \approx 0$

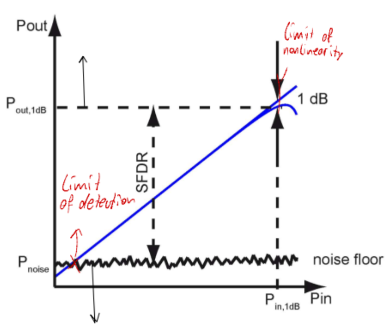
Voltage Drive $\rightarrow$ negative feedback

Current drive $\rightarrow$ positive feedback

Errors and Noise

Limit of detection(LOD): minimum measurable input amplitude ($SNR \approx 0$)

Dynamic range()DR: ratio of max and min amplitude within inaccuracy levels.



Errortypes:

Deterministic: source loading, offset, gain error

Random: thermal noise, 1/f noise

Quantification:

Absolute : $\Delta x = |\hat{x} - x_0|$

Relative: $\left|\frac{\Delta x}{x_0}\right| = \left|\frac{\hat{x}-x_0}{\hat{x}}\right|$

Max inaccuracy:

$\Delta x_{max} | x \in [\hat{x} - \Delta x_{\max}, \hat{x} + \Delta x_{\max}]$

Error Propagation

$y = f(x_1, x_2, \ldots, x_N)$

Deterministic fluctuations of $x_i \rightarrow$ total error:

$\Delta y \approx \sum_{i=1}^{N} \frac{\partial f}{\partial x_i} \cdot \Delta x_i$

Partial derivative $\frac{\delta f}{\delta x_i}$ is called **sensitivity**

Additive errors are best specified absolute and multiplicative errors are best specified relative

Interference:

Unwanted coupling of external signal

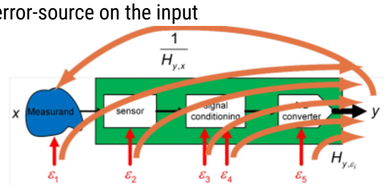
Noise: random fluctuations from setup $\rightarrow$ can be modeled as error sources

Combining Error sources

Output referred noise

Effect of an error-source on the output

Input referred noise Equivalent effect of the error-source on the input



Find TF from ES to output

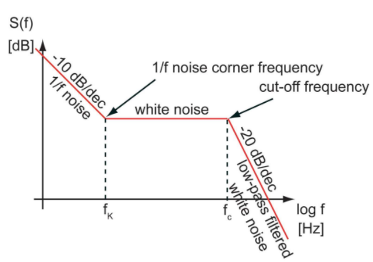
$y : H_{y,ES} \Rightarrow y_{\text{out},\epsilon_2} = H_{y,\epsilon_2} \cdot \epsilon_2$

Refer result back to input wit

$H_{y,x} : x_{\epsilon_2} = \frac{y_{\text{out},\epsilon_2}}{H_{y,x}} = H_{y,\epsilon_2} \cdot \frac{\epsilon_2}{H_{y,x}}$

Lin. System noise:

$PSD = S_y(f) = |H(f)|^2 \cdot S_x(f)$



Noise Types

Thermal noise: excitation of charge carriers(white)

Shot noise: carriers randomly crossing the barrier, dependent on DC bias and white

Flicker Noise:, due to traps in semiconduct. 1/f spectral density. MOS trans at low freq.

Thermalnoise Theorem: Every closed system at temp. T has average. Energy of $kT/2 \rightarrow$

$S_u(f) = 2kTR($ double-sided $)$ math

$S_u(f) = 4kTR($ single-sided $)$ physics

Langevin Approach kt/C noise

PSD noise voltage of V_n

$= S_{v_n}(f) = 4kT \cdot \Re\{Z(j2\pi f)$

$\rightarrow \overline{V_n^2} = kT\left[\frac{1}{C_\infty} - \frac{1}{C_0}\right] = \frac{kT}{C}$

MOS IRN

$S_{\Delta V^2_{\text{nG-tot}}} = 4kT \cdot R_{\text{nG-tot}}, \quad R_{\text{nG-tot}} =$
$\frac{\rho}{W \cdot L \cdot f} + \frac{\gamma_{nD}}{G_m}$ with GateExcess Noise factor
$:\gamma_{nD} = (n = 1.3) \cdot [0.5WI; 2/3SI]$

Bipolar Trans IRN:

$S_{\Delta V^2_{nR}} = 4kT \cdot R_B$

Noise Aanalysis

small-signal-equivalent is valid.

Total ORN:

$S_{n,\text{ out}}(f) = \sum_{k=1}^{N} |H_k(f)|^2 \cdot S_{nk}(f)$

N uncorrelated NS.

$H_k(f)$ TF from NS ton output. IRN:

$S_{V_{\text{neq,IRN}}}(f) = \frac{S_{V_{\text{nout}}}(f)}{|A(f)|^2}$ with $A(f)$ is TF

Sensor types

Information domain $\rightarrow$ Electrical domain

Transduction: Converting a signal from the energy domain into another.

Sensors and actuators are transducers

Sources of eerror

Noise, sensitivity to unintended quantities, Noise, EMI

$$\begin{pmatrix} E_{mag} \\ E_{mech} \\ E_{therm} \\ E_{opt} \\ E_{chem} \\ E_{elec} \end{pmatrix} = \begin{pmatrix} \alpha_{mag,mag} & \alpha_{mag,mech} & \alpha_{mag,therm} & \alpha_{mag,opt} & \alpha_{mag,chem} & \alpha_{mag,elec} \\ \alpha_{mech,mag} & \alpha_{mech,mech} & \alpha_{mech,therm} & \alpha_{mech,opt} & \alpha_{mech,chem} & \alpha_{mech,elec} \\ \alpha_{therm,mag} & \alpha_{therm,mech} & \alpha_{therm,therm} & \alpha_{therm,opt} & \alpha_{therm,chem} & \alpha_{therm,elec} \\ \alpha_{opt,mag} & \alpha_{opt,mech} & \alpha_{opt,therm} & \alpha_{opt,opt} & \alpha_{opt,chem} & \alpha_{opt,elec} \\ \alpha_{chem,mag} & \alpha_{chem,mech} & \alpha_{chem,therm} & \alpha_{chem,opt} & \alpha_{chem,chem} & \alpha_{chem,elec} \\ \alpha_{elec,mag} & \alpha_{elec,mech} & \alpha_{elec,therm} & \alpha_{elec,opt} & \alpha_{elec,chem} & \alpha_{elec,elec} \end{pmatrix} \cdot \begin{pmatrix} E_{mag} \\ E_{mech} \\ E_{therm} \\ E_{opt} \\ E_{chem} \\ E_{elec} \end{pmatrix} + \begin{pmatrix} E_{mag,os} \\ E_{mech,os} \\ E_{therm,os} \\ E_{opt,os} \\ E_{chem,os} \\ E_{elec,os} \end{pmatrix}$$

Tandem transducers: Multiple steps to target domain.

cross-sensitivity, sensitivity to undesired quantity

Sensor classification:

Active Sensors:

Require external source of excitation

Passive / self-generating sensors:

Generate their own electrical output signal

Draws all required energy from the measurand(source loading) E: Potentiometer for angle measurements. **Modulating sensors:** Measure desired quantity by modulating

Additional source with modulated energy. Also adds error. E: Non0contact displacement measrement (rotating disk)

Analog vs. Digital:

Analog: time and value continuous

Digital: Discrete outputs

Deflection mode sensors:

Response to an output is a deviation from the equilibrium position

Null mode sensors:

Sensor or instruments exert an influence the measured system opposing the effect of the measurand. Ideally the result is a 0 measurement, typically achieved by feedback. The opposing influence is then the sensor output. Slower than deflection, but more accurate.

Resistive sensors - strain gauges

Change in geometry under mechanical stress produces associated resistance change

Volum. == const. $\rightarrow$

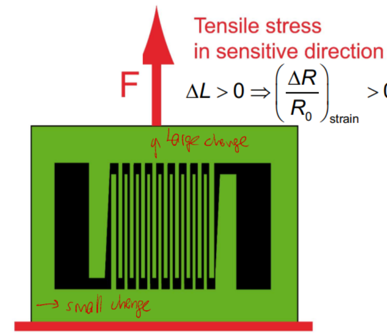
$\frac{\partial V}{V_0} = \frac{L_0}{V_0} \cdot \partial A + \frac{A_0}{V_0} \cdot \partial L \wedge \frac{\partial V}{V_0} = 0 \Rightarrow$

$\frac{\partial A}{A_0} = -\frac{\partial L}{L_0} \rightarrow \frac{\partial R}{R_0} = \frac{\partial \rho}{\rho_0} + 2\frac{\partial L}{L_0}$

$\rightarrow \frac{\partial R}{R_0} = \alpha \cdot \frac{\partial L}{L_0} + 2\frac{\partial L}{L_0} = \underbrace{(\alpha + 2)}_{\triangleq k} \cdot \frac{\partial L}{L_0}$

k: gauge factor

α proportionality factor $\frac{\partial \rho}{\rho_0} \propto \frac{\partial L}{L_0}$



Readout resistive sensors

Use a half bridge resistive divider

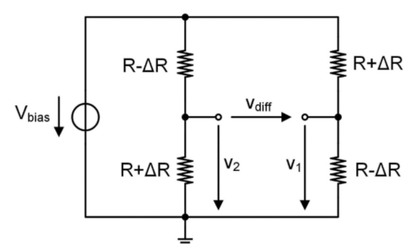
top element sensing resistor

$\frac{v_{\text{out}}}{V_{\text{bias}}} = \frac{R}{2R+\Delta R} \Leftrightarrow \frac{\Delta R}{R} = \frac{V_{\text{bias}}}{v_{\text{out}}} - 2$

Full bridge to remove offset, 2 sensing elements more sensitive,

remove nonlinearity by implementing differential measurements

Best use 4point full bridge



$V_{\text{diff}} = v_2 - v_1 = \frac{\Delta R}{R} \cdot V_{\text{bias}}$

Sensitivity $S = 45mV/V$ excitation for 1V input

Accuracy $A = \frac{v_{\text{diff}}\left(\frac{\Delta R}{R}\right) - V_{\text{diff, lin}}\left(\frac{\Delta R}{R}\right)}{V_{\text{diff}}\left(\frac{\Delta R}{R}\right)}$

Deviation from the ideal bridge

BridgeParameters

Bridge resistance: Unloaded R across the signal terminals

Offset error Outputvoltage at 0 input

Drift: Outputchange conditioned on environmental condition

Differential/ Commonmode signals

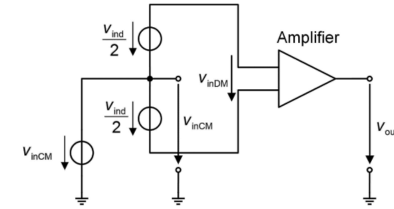
Recall: $V_1 = \frac{1}{2} \cdot \left(1 - \frac{\Delta R}{R}\right) \cdot V_{\text{bias}}, \quad v_2 =$
$\frac{1}{2} \cdot \left(1 + \frac{\Delta R}{R}\right) \cdot V_{\text{bias}}$

Differential signal:

$V_{\text{diff}} = V_2 - V_1 = \frac{\Delta R}{R} \cdot V_{\text{bias}}$

Common-mode signal:

$V_{\text{CM}} = \frac{v_2+v_1}{2} = \frac{V_{\text{bias}}}{2}$



DiffGain: $V_{\text{out}} = A_{\text{DM}} \cdot V_{\text{inDM}}$

CMMGain: $V_{\text{out}} = A_{\text{CM}} \cdot v_{\text{inCM}}$

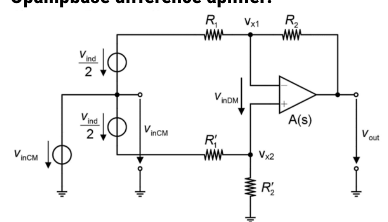
Ideally A_{cm} is 0 or $A_{cm} \ll A_{dm}$

Commonmode rejection ratio(CMRR):

$\text{CMRR} \triangleq \left|\frac{A_{\text{DM}}}{A_{\text{CM}}}\right| = \left|\frac{\mathrm{d}v_{\text{out}}/\mathrm{d}v_{\text{inDM}}}{\mathrm{d}v_{\text{out}}/\mathrm{d}v_{\text{inCM}}}\right|$

Often in **dB**

Opampbase difference aplifier:



1st: $A(s) = \infty, R_2 = R_2' = \alpha \cdot R_1 = \alpha R_1'$

$A_{DM} = -\alpha, A_{CM} = 0, CMRR = \infty$

2^{nd} assume $A(s) = \infty, R_2 = \alpha \cdot R_1,$
$R_2' = (\alpha + \Delta\alpha) \cdot R_1', R_1' = R_1 + \Delta R$

$A_{\text{DM}} = -\left(\alpha + \frac{\Delta\alpha}{2\cdot(1+\alpha+\Delta\alpha)}\right)$

$A_{\text{CM}} = \frac{\Delta\alpha}{1+\alpha+\Delta\alpha}$

$\text{CMRR} = -\frac{1}{2} + \frac{\alpha\cdot(1+\alpha+\Delta\alpha)}{\Delta\alpha}$

3d: $A(s) = A_{\text{DC}} / \left(1 + s \cdot \frac{A_{\text{DC}}}{\text{GBW}}\right)$

$A_{\text{DM}} = -\left(\alpha + \frac{\Delta\alpha}{2\cdot(1+\alpha+\Delta\alpha)}\right) \cdot$
$\frac{1}{1+\frac{1}{A_{\text{DC}}}+\frac{\alpha}{A_{\text{DC}}}+(1+\alpha)\cdot\frac{s}{\text{GBW}}}$

$A_{\text{CM}} =$
$\frac{\Delta\alpha}{1+\alpha+\Delta\alpha} \cdot \frac{1}{1+\frac{1}{A_{\text{DC}}}+\frac{\alpha}{A_{\text{DC}}}+(1+\alpha)\cdot\frac{s}{\text{GBW}}}$

$\text{CMRR} = -\frac{1}{2} + \frac{\alpha\cdot(1+\alpha+\Delta\alpha)}{\Delta\alpha}$

Magnetic Field Sensors

$V_{\text{hall}} = G \cdot \frac{\mu\cdot\rho}{h} \cdot l \cdot B = G \cdot \frac{1}{e\cdot n\cdot h} \cdot l \cdot B =$
$S_I \cdot l \cdot B$ G: geometry factor **Noise sources:**

- Thermal noise

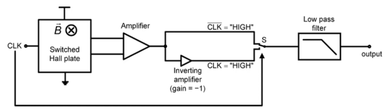
-1/f noise

-Noise of conditioning electronics

Noise power: $V_{noise}\sqrt{4kT \cdot R \cdot \Delta f}$

Signal power: $V_{hall} S_I \cdot I \cdot B$

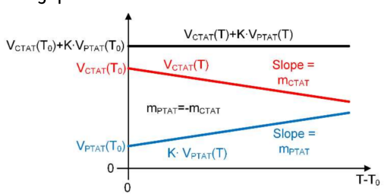
Resolution: $B_{\min} \hat{=} \frac{V_{\text{noise}}}{V_{\text{hall}}/B} = \frac{\sqrt{4kT\cdot R\cdot \Delta f}}{S_I\cdot I}$
With $S_I = \frac{\mu\cdot\rho}{h\cdot e\cdot n}\cdot G$
At $B = 0$ Hall plate can be modeled as wheatstone bridge, non-uniform stress causes intrinsic offset(5 to 30mT)
Cancel intrinsic offset by coupling ° turned sensors, current and sense ports are swapped
Idea: Hall voltage is in phase and gradient caused offsets are than $180°$ out of phase
Spinning current method:
Sense and current ports are switched every clock cycle
Idea: A DC component equal to the offset and a frequency component proportional to the hall voltage is produced **(Fourier Expansion of a square wave)**
By modulating the information to the clock frequency noise and offset are greatly mitigated.



Multiplying by 1 and -1 to demodulate back to low frequency.
Low pass stage to remove signal around the 2nd harmonic
This allow to see the base noise floor of the sensor not 1/f noise.
In the conventional implementation 2 bias currents are used in the low power configuration the bias form the plate is reused for the amplifier.

Temperature Sensors

Seebeck effect: Convert temp. gradient to electromotive force
$F(T) \hat{=} \int_{T_{stand.}}^{T} (S_+(T') - S_-(T'))dT'$
$\to E_{emf} = F(T_{sense} - F(T_{ref})$
Temperature stable references:
$V_{\text{REF}}(T) = V_{\text{CTAT}}(T) + K\cdot V_{\text{PTAT}}(T)$
Bandgap reference:



PTAT: Proportional to absolute temperature
CTAT: Complementary to absolute temperature
Scale one slope and add the function for a temp. constant reference
This can be built with a diode or a BJT with a shorted base and collector
Butt current is also temperature dependent so use diff pair.
$V_{\text{PTAT}} = \Delta V_{\text{EB}} = V_{\text{EB1}} - V_{\text{EB2}} \approx \frac{kT}{q}\cdot\ln\left[\frac{I_q\cdot I_S'\cdot A_{E2}}{I_S'\cdot A_{E1}\cdot I_q}\right] = \frac{kT}{q}\cdot\ln\left[\frac{A_{E2}}{A_{E1}}\right]$

Scale currents or diodes to make equal bias
$V_{\text{EB}} = \underbrace{E_g/q}_{V_G} - \underbrace{\frac{kT}{q}}_{U_T}\cdot\ln\left(\frac{I_s'}{I_D}\right) =$
$V_G - U_T\cdot\ln\left(\frac{K_1\cdot T^{\gamma}}{I_D}\right)$
$V_G \approx V_{G0K} - a\cdot T$
With $I_S' > I_D$ but also temperature dependent $\to$ curved CTAT.
Combine CTAT and PTAT: Series:
$V_{\text{REF}} = I_{\text{PTAT}}\cdot R_2 + V_{\text{CTAT}} = \frac{R_2}{R_1}\cdot V_{\text{PTAT}} + V_{\text{CTAT}}$
Parallel:
$V_{\text{REF}} = (I_{\text{PTAT}} + I_{\text{CTAT}})\cdot R_3 = \frac{R_3}{R_1}\cdot V_{\text{PTAT}} + \frac{R_3}{R_2}\cdot V_{\text{CTAT}}$