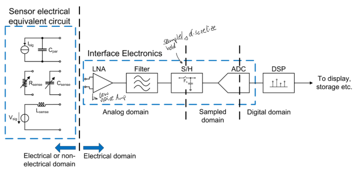
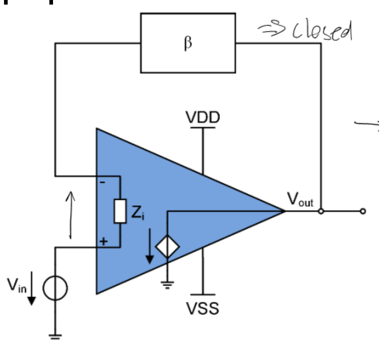
Sensor Principles

A generic sensor interface



OPAMP Basics

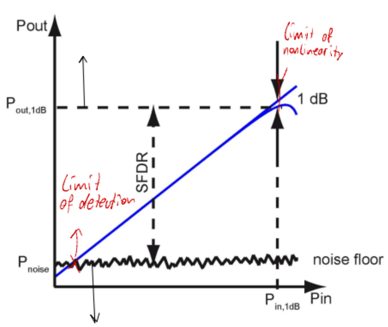
Opamps in feedback



Gain : $T(\omega) = \beta \cdot A_0(\omega)$
Phase Margin:
$PM = 180° + \angle(T(\omega_1))|_{|T(\omega_1)|=1}$ Gain margin:$GM = 20 \cdot log_{10}(\frac{1}{T(\omega_2)})|_{\angle(T(\omega_2))=-180°}$ **Generic Transfer function**
$V_x = \beta \cdot V_{out} = \beta \cdot A(\omega) \cdot (V_{in} - V_x) = \beta \cdot A(\omega) \cdot (V_{in} - \beta \cdot V_{out})$
$\frac{V_{out}}{V_{in}} = \frac{A(\omega)}{1+\beta \cdot A(\omega)} \overset{A(\omega)]\gg 1}{\approx} \frac{1}{\beta}$ **Negative Feedback and linear operation**
$A(\omega) \gg 1 \Rightarrow$ virtual short at the input
$\Delta V \approx 0$
$Z_i \to \infty \Rightarrow i_{oa} \approx 0$
Voltage Drive $\to$ negative feedback
Current drive $\to$ positive feedback

Errors and Noise

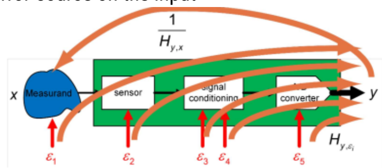
Limit of detection(LOD): minimum measurable input amplitude ($SNR \approx 0$)
Dynamic range()DR: ratio of max and min amplitude within inaccuracy levels.



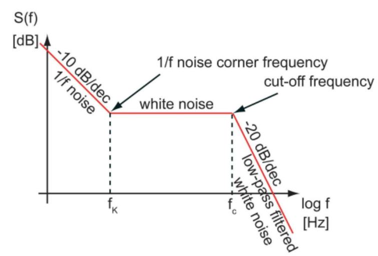
Errortypes:
Deterministic: source loading, offset, gain error
Random: thermal noise, 1/f noise
Quantification:
Absolute : $\Delta x = |\hat{x} - x_0|$
Relative: $\left|\frac{\Delta x}{x_0}\right| = \left|\frac{\hat{x}-x_0}{\hat{x}}\right|$
Max inaccuracy:
$\Delta x_{max} | x \in [\hat{x} - \Delta x_{\max}, \hat{x} + \Delta x_{\max}]$
Error Propagation
$y = f(x_1, x_2, \dots, x_N)$
Deterministic fluctuations of $x_i \to$ total error:
$\Delta y \approx \sum_{i=1}^{N} \frac{\partial f}{\partial x_i} \cdot \Delta x_i$
Partial derivative $\frac{\delta f}{\delta x_i}$ is called **sensitivity**
Additive errors are best specified absolute and multiplicative errors are best specified relative
Interference:
Unwanted coupling of external signal
Noise: random fluctuations from setup $\to$ can be modeled as error sources

Combining Error sources

Output referred noise
Effect of an error-source on the output
Input referred noise Equivalent effect of the error-source on the input



Find TF from ES to output
$y : H_{y,ES} \Rightarrow y_{out,\epsilon_2} = H_{y,\epsilon_2} \cdot \epsilon_2$
Refer result back to input wit
$H_{y,x} : x_{\epsilon_2} = \frac{y_{out,\epsilon_2}}{H_{y,x}} = H_{y,\epsilon_2} \cdot \frac{\epsilon_2}{H_{y,x}}$
Lin. System noise:
$PSD = S_y(f) = |H(f)|^2 \cdot S_x(f)$



Noise Types

Thermal noise: excitation of charge carriers(white)
Shot noise: carriers randomly crossing the barrier, dependent on DC bias and white
Flicker Noise:, due to traps in semiconduct. 1/f spectral density. MOS trans at low freq.
Thermalnoise Theorem: Every closed system at temp. T has average. Energy of $kT/2 \to$
$S_u(f) = 2kTR($ double-sided $)$ math
$S_u(f) = 4kTR($ single-sided $)$ physics

Langevin Approach kt/C noise

PSD noise voltage of V_n
$= S_{v_n}(f) = 4kT \cdot \Re\{Z(j2\pi f)\}$
$\to \overline{V_n^2} = kT\left[\frac{1}{C_\infty} - \frac{1}{C_0}\right] = \frac{kT}{C}$

MOS IRN

$S_{\Delta V^2_{nG\text{-tot}}} = 4kT \cdot R_{nG\text{-tot}}$, $R_{nG\text{-tot}} = \frac{\rho}{W \cdot L \cdot f} + \frac{\gamma_{nD}}{G_m}$ with GateExcess Noise factor $:\gamma_{nD} = (n = 1.3) \cdot [0.5WI; 2/3SI]$
Bipolar Trans IRN:
$S_{\Delta V^2_{nR}} = 4kT \cdot R_B$

Noise Aanalysis

small-signal-equivalent is valid.
Total ORN:
$S_{n,\text{ out}}(f) = \sum_{k=1}^{N} |H_k(f)|^2 \cdot S_{nk}(f)$
N uncorrelated NS.
$H_k(f)$ TF from NS ton output. IRN:
$S_{V_{\text{neq,IRN}}}(f) = \frac{S_{V_{\text{nout}}}(f)}{|A(f)|^2}$ with $A(f)$ is TF

Sensor types

Information domain $\to$ Electrical domain
Transduction: Converting a signal from the energy domain into another.
Sensors and actuators are transducers

Sources of eerror

Noise, sensitivity to unintended quantities, Noise, EMI

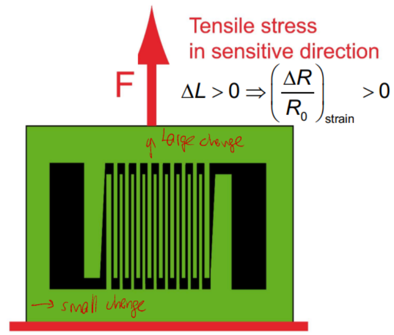
$$\begin{pmatrix} E_{mag} \\ E_{mech} \\ E_{therm} \\ E_{opt} \\ E_{chem} \\ E_{elec} \end{pmatrix} = \begin{pmatrix} \alpha_{mag,mag} & \alpha_{mag,mech} & \alpha_{mag,therm} & \alpha_{mag,opt} & \alpha_{mag,chem} & \alpha_{mag,elec} \\ \alpha_{mech,mag} & \alpha_{mech,mech} & \alpha_{mech,therm} & \alpha_{mech,opt} & \alpha_{mech,chem} & \alpha_{mech,elec} \\ \alpha_{therm,mag} & \alpha_{therm,mech} & \alpha_{therm,therm} & \alpha_{therm,opt} & \alpha_{therm,chem} & \alpha_{therm,elec} \\ \alpha_{opt,mag} & \alpha_{opt,mech} & \alpha_{opt,therm} & \alpha_{opt,opt} & \alpha_{opt,chem} & \alpha_{opt,elec} \\ \alpha_{chem,mag} & \alpha_{chem,mech} & \alpha_{chem,therm} & \alpha_{chem,opt} & \alpha_{chem,chem} & \alpha_{chem,elec} \\ \alpha_{elec,mag} & \alpha_{elec,mech} & \alpha_{elec,therm} & \alpha_{elec,opt} & \alpha_{elec,chem} & \alpha_{elec,elec} \end{pmatrix} \cdot \begin{pmatrix} E_{mag} \\ E_{mech} \\ E_{therm} \\ E_{opt} \\ E_{chem} \\ E_{elec} \end{pmatrix} + \begin{pmatrix} E_{mag,os} \\ E_{mech,os} \\ E_{therm,os} \\ E_{opt,os} \\ E_{chem,os} \\ E_{elec,os} \end{pmatrix}$$

Tandem transducers: Multiple steps to target domain.
cross-sensitivity, sensitivity to undesired quantity

Sensor classification:

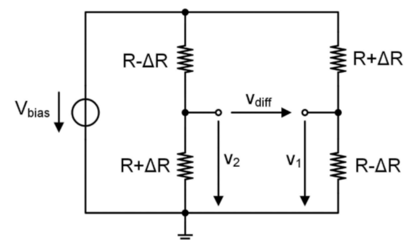
Active Sensors:
Require external source of excitation
Passive / self-generating sensors:
Generate their own electrical output signal
Draws all required energy from the measurand(source loading) E: Potentiometer for angle measurements. **Modulating sensors:** Measure desired quantity by modulating
Additional source with modulated energy. Also adds error. E: Non0contact displacement measrement (rotating disk)
Analog vs. Digital:
Analog: time and value continuous
Digital: Discrete outputs
Deflection mode sensors:
Response to an output is a deviation from the equilibrium position
Null mode sensors:
Sensor or instruments exert an influence the measured system opposing the effect of the measurand. Ideally the result is a 0 measurement, typically achieved by feedback. The opposing influence is then the sensor output. Slower than deflection, but more accurate.
Resistive sensors - strain gauges
Change in geometry under mechanical stress produces associated resistance change
Volum. == const. $\to$
$\frac{\partial V}{V_0} = \frac{L_0}{V_0} \cdot \partial A + \frac{A_0}{V_0} \cdot \partial L \wedge \frac{\partial V}{V_0} = 0 \Rightarrow$
$\frac{\partial A}{A_0} = -\frac{\partial L}{L_0} \to \frac{\partial R}{R_0} = \frac{\partial \rho}{\rho_0} + 2\frac{\partial L}{L_0}$
$\to \frac{\partial R}{R_0} = \alpha \cdot \frac{\partial L}{L_0} + 2\frac{\partial L}{L_0} = \underbrace{(\alpha+2)}_{\triangleq k} \cdot \frac{\partial L}{L_0}$

k: gauge factor
α proportionality factor $\frac{\partial \rho}{\rho_0} \propto \frac{\partial L}{L_0}$



Readout resistive sensors

Use a half bridge resistive divider
top element sensing resistor
$\frac{v_{out}}{V_{bias}} = \frac{R}{2R+\Delta R} \Leftrightarrow \frac{\Delta R}{R} = \frac{V_{bias}}{v_{out}} - 2$
Full bridge to remove offset, 2 sensing elements more sensitive,
remove nonlinearity by implementing differential measurements
Best use 4point full bridge



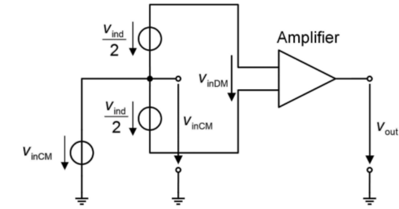
$V_{diff} = v_2 - v_1 = \frac{\Delta R}{R} \cdot V_{bias}$
Sensitivity $S = 45mV/V$ excitation for 1V input
Accuracy $A = \frac{v_{diff}(\frac{\Delta R}{R}) - V_{diff,\text{ lin}}(\frac{\Delta R}{R})}{V_{diff}(\frac{\Delta R}{R})}$
Deviation from the ideal bridge

BridgeParameters

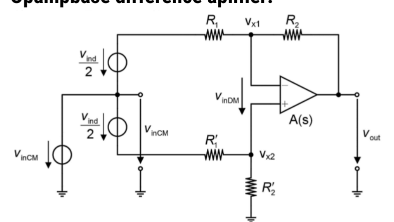
Bridge resistance: Unloaded R across the signal terminals
Offset error Outputvoltage at 0 input
Drift: Outputchange conditioned on environmental condition

Differential/ Commonmode signals

Recall: $V_1 = \frac{1}{2} \cdot \left(1 - \frac{\Delta R}{R}\right) \cdot V_{bias}$, $v_2 = \frac{1}{2} \cdot \left(1 + \frac{\Delta R}{R}\right) \cdot V_{bias}$
Differential signal:
$V_{diff} = V_2 - V_1 = \frac{\Delta R}{R} \cdot V_{bias}$
Common-mode signal:
$V_{CM} = \frac{v_2 + v_1}{2} = \frac{V_{bias}}{2}$



DiffGain: $V_{out} = A_{DM} \cdot V_{inDM}$
CMMGain: $V_{out} = A_{CM} \cdot v_{inCM}$
Ideally A_{cm} is 0 or $A_{cm} \ll A_{dm}$
Commonmode rejection ratio(CMRR):
$\text{CMRR} \triangleq \left|\frac{A_{DM}}{A_{CM}}\right| = \left|\frac{dv_{out}/dv_{inDM}}{dv_{out}/dv_{inCM}}\right|$
Often in **dB**
Opampbase difference aplifier:

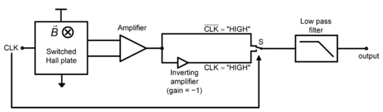


1st: $A(s) = \infty, R_2 = R_2' = \alpha \cdot R_1 = \alpha R_1'$
$A_{DM} = -\alpha, A_{CM} = 0, CMRR = \infty$
2[nd] assume $A(s) = \infty, R_2 = \alpha \cdot R_1$,
$R_2' = (\alpha + \Delta\alpha) \cdot R_1', R_1' = R_1 + \Delta R$
$A_{DM} = -\left(\alpha + \frac{\Delta\alpha}{2\cdot(1+\alpha+\Delta\alpha)}\right)$
$A_{CM} = \frac{\Delta\alpha}{1+\alpha+\Delta\alpha}$
$\text{CMRR} = -\frac{1}{2} + \frac{\alpha\cdot(1+\alpha+\Delta\alpha)}{\Delta\alpha}$
3d: $A(s) = A_{DC}/\left(1 + s \cdot \frac{A_{DC}}{\text{GBW}}\right)$
$A_{DM} = -\left(\alpha + \frac{\Delta\alpha}{2\cdot(1+\alpha+\Delta\alpha)}\right) \cdot \frac{1}{1+\frac{1}{A_{DC}}+\frac{\alpha}{A_{DC}}+(1+\alpha)\cdot\frac{s}{\text{GBW}}}$
$A_{CM} = \frac{\Delta\alpha}{1+\alpha+\Delta\alpha} \cdot \frac{1}{1+\frac{1}{A_{DC}}+\frac{\alpha}{A_{DC}}+(1+\alpha)\cdot\frac{s}{\text{GBW}}}$
$\text{CMRR} = -\frac{1}{2} + \frac{\alpha\cdot(1+\alpha+\Delta\alpha)}{\Delta\alpha}$

Magnetic Field Sensors

$V_{hall} = G \cdot \frac{\mu\cdot\rho}{h} \cdot l \cdot B = G \cdot \frac{1}{e\cdot n\cdot h} \cdot l \cdot B = S_l \cdot l \cdot B$ G: geometry factor **Noise sources:**
- Thermal noise
-1/f noise
-Noise of conditioning electronics
Noise power: $V_{noise}\sqrt{4kT \cdot R \cdot \Delta f}$
Signal power: $V_{hall} S_I \cdot I \cdot B$

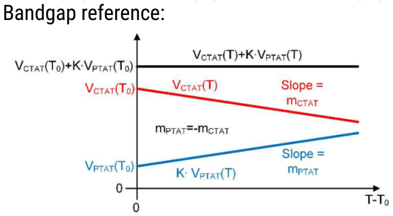
Resolution: $B_{\min}\hat{\wedge}\frac{V_{\text{noise}}}{V_{\text{hall}}/B} = \frac{\sqrt{4kT\cdot R\cdot \Delta f}}{S_I\cdot I}$
With $S_I = \frac{\mu\cdot\rho}{h\cdot e\cdot n}\cdot G$
At $B=0$ Hall plate can be modeled as wheatstone bridge, non-uniform stress causes intrinsic offset(5 to 30mT)
Cancel intrinsic offset by coupling ° turned sensors, current and sense ports are swapped
Idea: Hall voltage is in phase and gradient caused offsets are than $180°$ out of phase
Spinning current method:
Sense and current ports are switched every clock cycle
Idea: A DC component equal to the offset and a frequency component proportional to the hall voltage is produced **(Fourier Expansion of a square wave)**
By modulating the information to the clock frequency noise and offset are greatly mitigated.



Multiplying by 1 and -1 to demodulate back to low frequency.
Low pass stage to remove signal around the 2nd harmonic
This allow to see the base noise floor of the sensor not 1/f noise.
In the conventional implementation 2 bias currents are used in the low power configuration the bias form the plate is reused for the amplifier.

Temperature Sensors

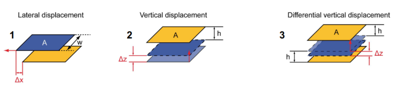
Seebeck effect: Convert temp. gradient to electromotive force
$F(T) \hat{=} \int_{T_{stand.}}^{T} (S_+(T') - S_-(T'))dT'$
$\rightarrow E_{emf} = F(T_{sense} - F(T_{ref})$
Temperature stable references:
$V_{\text{REF}}(T) = V_{\text{CTAT}}(T) + K\cdot V_{\text{PTAT}}(T)$
Bandgap reference:



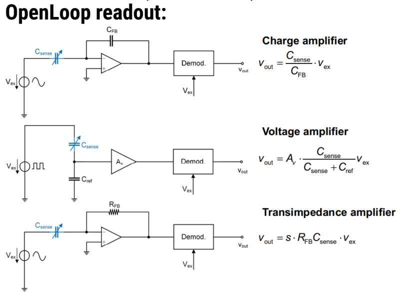
PTAT: Proportional to absolute temperature
CTAT: Complementary to absolute temperature
Scale one slope and add the function for a temp. constant reference
This can be built with a diode or a BJT with a shorted base and collector
Butt current is also temperature dependent so use diff pair.
$V_{\text{PTAT}} = \Delta V_{\text{EB}} = V_{\text{EB1}} - V_{\text{EB2}} \approx \frac{kT}{q}\cdot\ln\left[\frac{I_q\cdot I_S'\cdot A_{E2}}{I_S'\cdot A_{E1}\cdot I_q}\right] = \frac{kT}{q}\cdot\ln\left[\frac{A_{E2}}{A_{E1}}\right]$

Scale currents or diodes to make equal bias
$V_{\text{EB}} = \underbrace{E_g/q}_{V_G} - \underbrace{\frac{kT}{q}}_{U_T}\cdot\ln\left(\frac{I_s'}{I_D}\right) =$
$V_G - U_T\cdot\ln\left(\frac{K_1\cdot T^\gamma}{I_D}\right)$
$V_G \approx V_{G0K} - a\cdot T$
With $I_S' > I_D$ but also temperature dependent $\rightarrow$ curved CTAT.
Combine CTAT and PTAT: Series:
$V_{\text{REF}} = I_{\text{PTAT}}\cdot R_2 + V_{\text{CTAT}} = \frac{R_2}{R_1}\cdot V_{\text{PTAT}} + V_{\text{CTAT}}$
Parallel:
$V_{\text{REF}} = (I_{\text{PTAT}} + I_{\text{CTAT}})\cdot R_3 = \frac{R_3}{R_1}\cdot V_{\text{PTAT}} + \frac{R_3}{R_2}\cdot V_{\text{CTAT}}$

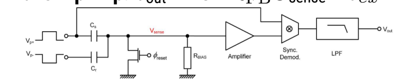
Capa. Sensor Readout



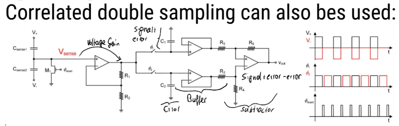
1: $\Delta C = C - C_0 = \varepsilon\cdot\frac{W}{h}\cdot\Delta x$
2: $\Delta C = \frac{\varepsilon\cdot A}{h}\cdot\frac{\Delta z/h}{1+\Delta zh} \approx \frac{\varepsilon\cdot A}{h^2}\cdot\Delta z$
3: $\Delta C = \varepsilon\cdot\frac{A}{h}\cdot\left(\frac{2\cdot\Delta z/h}{1-[\Delta z/h]^2}\right) = \frac{2\cdot\varepsilon\cdot A}{h^2}\cdot\Delta z$
OpenLoop readout:



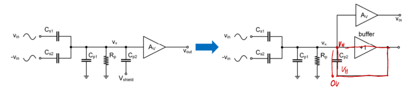
ChargeAmp: $V_{\text{out}} = \frac{C_{\text{sense}}}{C_{\text{FB}}}\cdot V_{\text{ex}}$ **ChargeAmp-Nonideal:** $\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{j\omega\cdot R_{\text{FB}}C_{\text{in}}}{\left(1+\frac{j\omega}{\text{GBW}}\right)\cdot\left(1+\frac{j\omega}{\omega_{\text{FB}}}\right)}$
VoltAmp: $v_{\text{out}} = A_v\cdot\frac{C_{\text{sense}}}{C_{\text{sense}}+C_{\text{ref}}}V_{\text{ex}}$
TransImpAmp: $v_{\text{out}} = s\cdot R_{\text{FB}}C_{\text{sense}}\cdot v_{ex}$



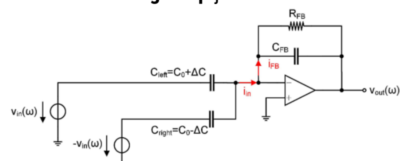
V_{Sense} not grounded sensitive to leakage currents
Bias resistor provides well defined DC-point.
R_{Bias} needs to be large because of the highpass forming, corner frequency needs to be low enough compared to excitation freq.
Periodic reset can also be used where $V_{p+} = V_{p-} = 0$
Correlated double sampling can also bes used:



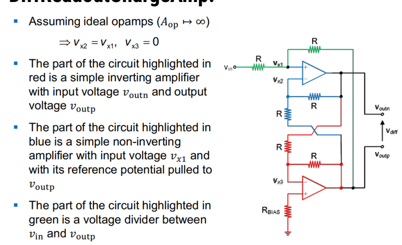
Bootstrapping: VoltAmp



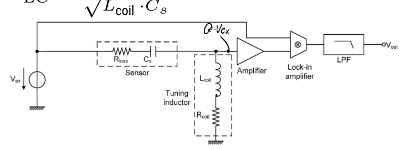
$C_{s1} = C_{s0} + \Delta C/2 C_{s2} = C_{s0} - \Delta C/2$, v_{out} becomes sensitive to parasitic capacitance
Can be bootstrapped out with voltage buffer driving V_{shield}
$v_{\text{out}} = \frac{\Delta C}{2C_{s0}+C_{p1}+(C_{p2})}\cdot A_v\cdot v_{\text{in}}\; C_{p2}$ gets removed by V_{shied}
Can also use TIA, the virtual ground helps with the parasitic capacitance, this also works in the charge amplifier
DiffReadoutChargeAmp:j



Req V_{ex}:
Error will propagate to output directly
Accuracy more important for small ΔC
Mismatch or drift will introduce errors
Sinewave can be created with center-tapped transformer or active balun
Rectangular witch switched excitation schemes
DiffReadoutChargeAmp:



$V_{\text{outp}} = V_{\text{in}} = -V_{\text{outn}}$
Differential Capa sensing:
$\Delta v_{x,\,\text{amp}} \approx -\frac{\Delta x}{x_0}\cdot V_{\text{ex}}$
Resonant readout:
$\omega_{\text{LC}} = \frac{1}{\sqrt{L_{\text{coil}}\cdot C_s}}$

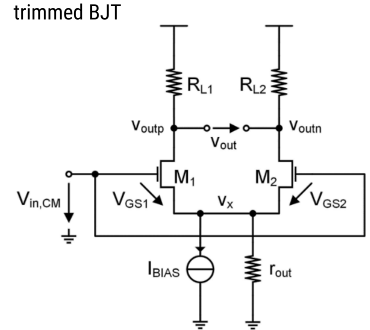


DoubleDiffReadout:
$\Delta V_{\text{out}} = -\Delta V_{ex}\cdot\left(\frac{\Delta C_s}{C_i}\cdot\underbrace{\left[1-\frac{C_s}{C_s+C_i+C_p}\right]}_{\text{gain error}} - \underbrace{\frac{\Delta C_i+\Delta C_p}{C_i}\cdot\frac{C_s}{C_s+C_i+C_p}}_{\text{offset error}}\right)$

Differential Measurements

- Inherent rejection to common-mode interference ans noise
- Wider Signal swing for a given supply voltage
- Minimum effect of even order distortion including DC offset
- Can lead to improved sensor linearity

But respond to some degree to common mode signal
Ideal DiffPair:
CMMR: $\triangleq 20\cdot\log\left(\left|\frac{A_{\text{DM}}}{A_{\text{CM-to-DM}}}\right|\right) = 20\cdot\log\left(\frac{2G_{\text{mav}}\cdot r_{\text{out}}}{\Delta R_L/R_{\text{Lav}}+\Delta G_{\text{m}}/G_{\text{mav}}}\right]$
PSRR: $\triangleq 20\cdot\log\left(\left|\frac{A_{\text{DM}}}{A_{\text{VDD}}}\right|\right) = 20\cdot\log\left(\left|\frac{A_{\text{DM}}}{\Delta V_{\text{out}}/\Delta\text{VDD}}\right|\right)$ Process variation causes mismatch between resistors and current source is not ideal
Input referred offset is the voltage needed to get zero volts differential output
IRO: $\Delta V_G = V_{\text{OS}} = -\frac{I_{\text{Dav}}}{G_{\text{mav}}}\cdot\left(\frac{\Delta R_{\text{L}}}{R_{\text{Lav}}}+\frac{\Delta\beta}{\beta_{\text{av}}}\right) - \Delta V_{\text{TO}}$
OffsetVoltage: 10mV **MOS** ; 1mV **BJT**, 120 μ, trimmed BJT

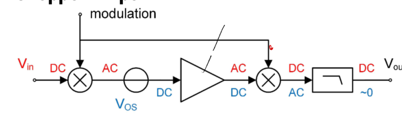


$CMRR\cdot V_{os} \approx G_{mav}\cdot r_{out}\cdot V_{OV}$,
$V_{OV} = \frac{I_{DAV}}{G_{mav}}$ **PSRR:**
$\text{PSRR}_{\text{VDD}} \cong G_{\text{mav}}R_{\text{Lav}}/\left(g_{\text{outM1,2}}\cdot\left(\frac{g_{\text{out}}}{G_{\text{mav}}}\cdot\Delta R_{\text{L}} - 2\cdot R_{\text{L}}\frac{\Delta G_{\text{m}}}{G_{\text{mav}}}\right)\right)$
$\text{PSRR}_{\text{VSS}} \cong G_{\text{mav}}\cdot R_{\text{Lav}}/g_{\text{out}}\cdot\left(\Delta R_{\text{L}} + R_{\text{Lav}}\cdot\frac{\Delta G_{\text{m}}}{G_{\text{mav}}}\right)$

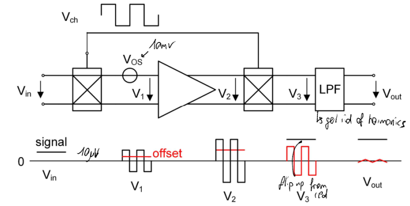
Modulation

Stnontms: Coherent detectrion, synchronous demodulation, lock-in amplification, chopping
All modulation techniques, square wave is called chopping
Leads to better low freq. specification, smaller 1/f, bigger CMRR and PSRR
Trimming: Measuring static error offset and gain and adjusting the value of a component to reduce the error to 0
Low complexity, no bandwith limit, but reqs. measure equipment
Also reqs. memory element
Dyn. offset cancel:
usually no measure eq. but more complex circuits, reduce bandwidth
AutoZeroing:
periodically measure offset and substract from input(time domain)
Chopping:
Modulate signal above 1/f noise(freq. domain)

Chopper Amps



implemented with polarity reversing switch
Time-Domain-Chopping:



Complete suppression of 1/f noise if $f_{chop} > 1/f$ corner freq., but up-modulated offset must be filtered out. loss of bandwidth and residual chopper ripple. **Charge Injection** Injected charge splits half-half
Charge:
$Q = W\cdot L\cdot C_{\text{ox}}\cdot[V_\varphi - V_{\text{in}} - V_{\text{TO}}\,(\sqrt{V_{\text{in}}})]$
Linear w.r.t to $W\cdot L$ non-linear w.r.t. V_{in}
$V_{\text{out}} = V_{\text{in}}\,\underbrace{(1+\frac{W\cdot L\cdot C_{ox}}{2C_L})}_{\text{gain error}} - \underbrace{\frac{W\cdot L\cdot C_{ox}}{2C_L}\cdot\left[V_\varphi - V_{T0}\left(\sqrt{V_{\text{in}}}\right)\right]}_{\text{offset \& dostortion}}$
Clock feedthrough
Overlap capacitance of trans.: C_{OV}
$\Delta V_{\text{out}} = \frac{C_{\text{ov}}}{C_{\text{ov}}+C_L}\cdot V_\varphi$
Asymmetric clock duty cycle causes demodulation signal to have DC component which feeds through
Bandwidth,gain/acc
Limited applifier Bandwidth causes output signal to not be perfectly square, therefore less gain
AutoZeroing