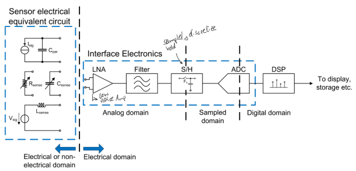
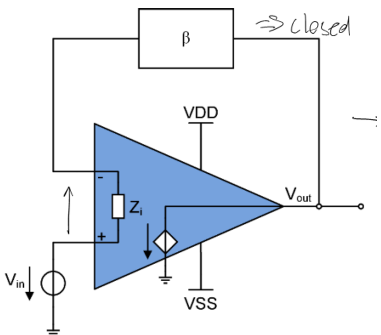
Sensor Principles

A generic sensor interface



OPAMP Basics

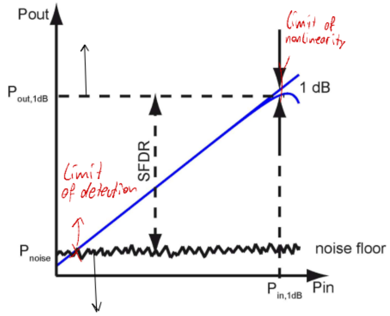
Opamps in feedback



Gain : $T(\omega)=\beta\cdot A_0(\omega)$
DC gain sets the acc. of the closed loop amplifier. Phase Margin:
$PM=180^{\circ}+\angle(T(\omega_1)|_{|T(\omega_1)|=1}$
Gain margin:$GM=$
$20\cdot log_{10}(\frac{1}{T(\omega_2)})|_{\angle(T(\omega_2)=-180^{\circ}}$
Generic Transfer function
$V_x=\beta\cdot V_{\text{out}}=\beta\cdot A(\omega)\cdot(V_{\text{in}}-V_x)=$
$\beta\cdot A(\omega)\cdot(V_{\text{in}}-\beta\cdot V_{\text{out}})$
$A_{CL}=\frac{V_{\text{out}}}{V_{\text{in}}}=\frac{A(\omega)}{1+\beta\cdot A(\omega)}\overset{A(\omega)\gg 1}{\approx}\frac{1}{\beta}$
Acc and gain
$A_{OL}(Error,A_CL)=A_{CL}\cdot(\frac{1}{Error}-1)$
Negative Feedback and linear operation
$A(\omega)\gg 1\Rightarrow$ virtual short at the input
$\Delta V\approx 0$
$Z_i\rightarrow\infty\Rightarrow i_{oa}\approx 0$
Voltage Drive $\rightarrow$ negative feedback
Current drive $\rightarrow$ positive feedback
A non-dominant pole located at a frequency lower than the unity gain frequency / GBW causes ringing in the time domain and peaking in the frequency domain.

Errors and Noise

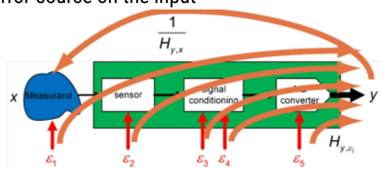
Limit of detection(LOD): minimum measurable input amplitude ($SNR\approx 0$)
Dynamic range(DR): ratio of max and min amplitude within inaccuracy levels.
Lower limit: Noise floor
Upper Limit: Distortion



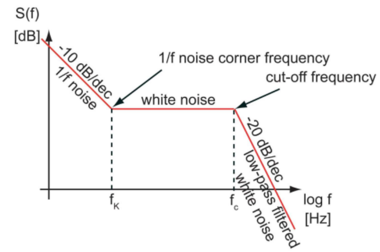
Errortypes:
Deterministic: source loading, offset, gain error $\rightarrow$ Removed by calibration
Random: thermal noise, 1/f noise $\rightarrow$ Mitigated by circuit design to compensate
Quantification:
Absolute : $\Delta x=|\hat{x}-x_0|$
Relative: $\left|\frac{\Delta x}{x_0}\right|=\left|\frac{\hat{x}-x_0}{\hat{x}}\right|$
Max inaccuracy:
$\Delta x_{max}|x\in[\hat{x}-\Delta x_{\max},\hat{x}+\Delta x_{\max}]$
Error Propagation
$y=f(x_1,x_2,\ldots,x_N)$
Deterministic fluctuations of $x_i\rightarrow$ total error:
$\Delta y\approx\sum_{i=1}^{N}\frac{\partial f}{\partial x_i}\cdot\Delta x_i$
Partial derivative $\frac{\delta f}{\delta x_i}$ is called **sensitivity**
Additive errors are best specified absolute and multiplicative errors are best specified relative
Interference:
Unwanted coupling of external signal
Noise: random fluctuations from setup $\rightarrow$ can be modeled as error sources

Combining Error sources

Output referred noise
Effect of an error-source on the output
Input referred noise Equivalent effect of the error-source on the input



Linearize system
Apply superposition principle
Find TF from ES to output for all sources
$y:H_{y,ES}\Rightarrow y_{\text{out},\epsilon_2}=H_{y,\epsilon_2}\cdot\epsilon_2$
Compute sum:
Deterministic Error: $y_{\text{out, tot}}=\sum_{i=1}^{N}y_{\text{out},i}$
Random Error: $y_{\text{out,tot}}=\sqrt{\sum_{i=1}^{N}y_{\text{out},i}^2}$
Lin. System noise:
Refer result back to input wit
$H_{y,x}:x_{\epsilon_2}=\frac{y_{\text{out},\epsilon_2}}{H_{y,x}}=H_{y,\epsilon_2}\cdot\frac{\epsilon_2}{H_{y,x}}$
Wiener-Khintchine-Theorem:
$PSD=S_y(f)=|H(f)|^2\cdot S_x(f)$



Random Process

WSS: Wide sense stationary: Mean is independent of time t and the ACF only depends on $\tau=t_2-t_1$
ACF: Power spectrum of white noise can be extracted from ACF with fourier transform
Noise Power: Total Noise Power is the ACF $R(0)$
$S_x(f)=F\{R_x(\tau)\}=\int_{-\infty}^{\infty}R_x(\tau)\cdot e^{-j2\pi f\tau}d\tau$
$R_x(\tau)=F^{-1}\{S_x(f)\}=\int_{-\infty}^{\infty}S_x(f)\cdot e^{+j2\pi f\tau}df$
Noise power:
$P_x=\int_{-\infty}^{\infty}S_x(f)\cdot\mathrm{d}f=R_x(0)$
Noise voltage: $V_{n,ms}^2=$
$\int_{-\infty}^{\infty}S_{v_n}(f)\cdot df=\int_0^{\infty}S_{v_n}^{+}(f)\cdot df$
Noise current: $I_{n,\text{rms}}^2=$
$\int_{-\infty}^{\infty}S_{i_n}(f)\cdot\mathrm{d}f=\int_0^{\infty}S_{i_n}^{+}(f)\cdot\mathrm{d}f$

Noise Types

Thermal noise: excitation of charge carriers(white) $\rightarrow$ mitigate lower temp. lower resistance
Shot noise: carriers randomly crossing the barrier, dependent on DC bias and white
Flicker Noise:, due to traps in semiconduct. 1/f spectral density. MOS trans at low freq.
Thermalnoise Theorem: Every closed system at temp. T has average. Energy of $kT/2\rightarrow$
$S_u(f)=2kTR($ double-sided $)$ math
$S_u(f)=4kTR($ single-sided $)$ physics

Langevin Approach kt/C noise

PSD noise voltage of V_n
$=S_{v_n}(f)=4kT\cdot\Re\{Z(j2\pi f)$
$\rightarrow\overline{V_n^2}=kT\left[\frac{1}{C_\infty}-\frac{1}{C_0}\right]=\frac{kT}{C}$
PSD Noise current:
$S_{I_n}=\frac{4kT}{\Re\{Z(j2\pi f)}$ **MOS IRN**
$S_{\Delta V_{\text{nG-tot}}^2}=4kT\cdot R_{\text{nG-tot}}$, $R_{\text{nG-tot}}=$
$\frac{\rho}{W\cdot L\cdot f}+\frac{\gamma_{nD}}{G_m}$ with GateExcess Noise factor
$:\gamma_{nD}=(n=1.3)\cdot[0.5WI;2/3SI]$
Bipolar Trans IRN:
$S_{\Delta V_{nR}^2}=4kT\cdot R_B$

Noise Aanalysis

small-signal-equivalent is valid.
Total IRN:
$S_{n,\text{ out}}(f)=\sum_{k=1}^{N}|H_k(f)|^2\cdot S_{nk}(f)$
N uncorrelated NS.

$H_k(f)$ TF from NS ton output. IRN:
$S_{V_{\text{neq,IRN}}}(f)=\frac{S_{V_{\text{nout}}}(f)}{|A(f)|^2}$ with $A(f)$ is TF

Sensor types

Information domain $\rightarrow$ Electrical domain
Transduction: Converting a signal from the energy domain into another.
Sensors and actuators are transducers

Sources of error

Noise, sensitivity to unintended quantities, Noise, EMI

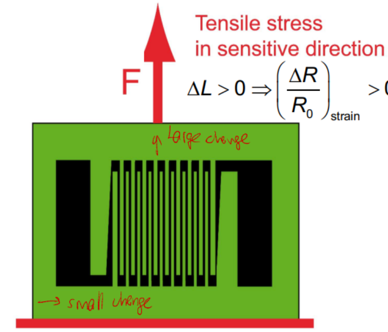
$$\begin{pmatrix}E_{\text{mag}}\\E_{\text{mech}}\\E_{\text{therm}}\\E_{\text{opt}}\\E_{\text{chem}}\\E_{\text{elec}}\end{pmatrix}=\begin{pmatrix}\alpha_{\text{mag,mag}}&\alpha_{\text{mag,mech}}&\alpha_{\text{mag,therm}}&\alpha_{\text{mag,opt}}&\alpha_{\text{mag,chem}}&\alpha_{\text{mag,elec}}\\\alpha_{\text{mech,mag}}&\alpha_{\text{mech,mech}}&\alpha_{\text{mech,therm}}&\alpha_{\text{mech,opt}}&\alpha_{\text{mech,chem}}&\alpha_{\text{mech,elec}}\\\alpha_{\text{therm,mag}}&\alpha_{\text{therm,mech}}&\alpha_{\text{therm,therm}}&\alpha_{\text{therm,opt}}&\alpha_{\text{therm,chem}}&\alpha_{\text{therm,elec}}\\\alpha_{\text{opt,mag}}&\alpha_{\text{opt,mech}}&\alpha_{\text{opt,therm}}&\alpha_{\text{opt,opt}}&\alpha_{\text{opt,chem}}&\alpha_{\text{opt,elec}}\\\alpha_{\text{chem,mag}}&\alpha_{\text{chem,mech}}&\alpha_{\text{chem,therm}}&\alpha_{\text{chem,opt}}&\alpha_{\text{chem,chem}}&\alpha_{\text{chem,elec}}\\\alpha_{\text{elec,mag}}&\alpha_{\text{elec,mech}}&\alpha_{\text{elec,therm}}&\alpha_{\text{elec,opt}}&\alpha_{\text{elec,chem}}&\alpha_{\text{elec,elec}}\end{pmatrix}\cdot\begin{pmatrix}E_{\text{mag}}\\E_{\text{mech}}\\E_{\text{therm}}\\E_{\text{opt}}\\E_{\text{chem}}\\E_{\text{elec}}\end{pmatrix}+\begin{pmatrix}E_{\text{mag,os}}\\E_{\text{mech,os}}\\E_{\text{therm,os}}\\E_{\text{opt,os}}\\E_{\text{chem,os}}\\E_{\text{elec,os}}\end{pmatrix}$$

Tandem transducers: Multiple steps to target domain.
cross-sensitivity, sensitivity to undesired quantity

Sensor classification:

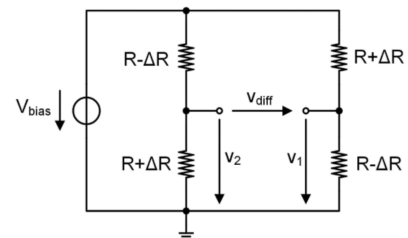
Active Sensors:
Require external source of excitation
Passive / self-generating sensors:
Generate their own electrical output signal
Draws all required energy from the measurand(source loading) E: Potentiometer for angle measurements. **Modulating sensors:**
Measure desired quantity by modulating
Additional source with modulated energy. Also adds error. E: Non-contact displacement measurement (rotating disk)
Analog vs. Digital:
Analog: time and value continuous
Digital: Discrete outputs
Deflection mode sensors:
Response to an output is a deviation from the equilibrium position
Null mode sensors:
Sensor or instruments exert an influence the measured system opposing the effect of the measurand. Ideally the result is a 0 measurement, typically achieved by feedback. The opposing influence is then the sensor output. Slower than deflection, but more accurate.
Resistive sensors - strain gauges
Change in geometry under mechanical stress produces associated resistance change
Volum. == const. $\rightarrow$
$\frac{\partial V}{V_0}=\frac{L_0}{V_0}\cdot\partial A+\frac{A_0}{V_0}\cdot\partial L\wedge\frac{\partial V}{V_0}=0\Rightarrow$
$\frac{\partial A}{A_0}=-\frac{\partial L}{L_0}\rightarrow\frac{\partial R}{R_0}=\frac{\partial\rho}{\rho_0}+2\frac{\partial L}{L_0}$
$\rightarrow\frac{\partial R}{R_0}=\alpha\cdot\frac{\partial L}{L_0}+2\frac{\partial L}{L_0}=\underbrace{(\alpha+2)}_{\triangleq k}\cdot\frac{\partial L}{L_0}$

k: gauge factor
α proportionality factor $\frac{\partial\rho}{\rho_0}\propto\frac{\partial L}{L_0}$



Readout resistive sensors

Use a half bridge resistive divider
top element sensing resistor
$\frac{v_{out}}{V_{\text{bias}}}=\frac{R}{2R+\Delta R}\Leftrightarrow\frac{\Delta R}{R}=\frac{V_{\text{bias}}}{v_{\text{out}}}-2$
Full bridge to remove offset, 2 sensing elements more sensitive,
remove nonlinearity by implementing differential measurements
Best use 4point full bridge, no nonlinearity



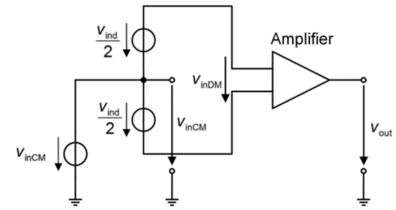
$V_{\text{diff}}=v_2-v_1=\frac{\Delta R}{R}\cdot V_{\text{bias}}$
Sensitivity $S=45mV/V$ excitation for 1V input
Accuracy $A=\frac{v_{\text{diff}}(\frac{\Delta R}{R})-V_{\text{diff, lin}}(\frac{\Delta R}{R})}{V_{\text{diff}}(\frac{\Delta R}{R})}$
Deviation from the ideal bridge

BridgeParameters

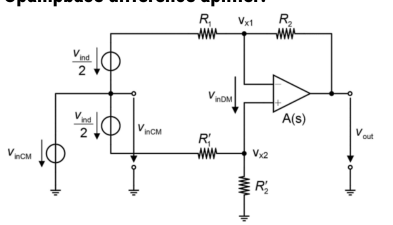
Bridge resistance: Unloaded R across the signal terminals
Offset error Outputvoltage at 0 input
Drift: Outputchange conditioned on environmental condition

Differential/ Commonmode signals

Recall: $V_1=\frac{1}{2}\cdot\left(1-\frac{\Delta R}{R}\right)\cdot V_{\text{bias}}$, $v_2=\frac{1}{2}\cdot\left(1+\frac{\Delta R}{R}\right)\cdot V_{\text{bias}}$
Differential signal:
$V_{\text{diff}}=V_2-V_1=\frac{\Delta R}{R}\cdot V_{\text{bias}}$
Common-mode signal:
$V_{\text{CM}}=\frac{v_2+v_1}{2}=\frac{V_{\text{bias}}}{2}$



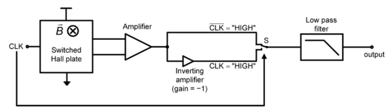
DiffGain: $V_{out} = A_{DM} \cdot V_{inDM}$
CMMGain: $V_{out} = A_{CM} \cdot v_{inCM}$
Ideally A_{cm} is 0 or $A_{cm} \ll A_{dm}$
Commonmode rejection ratio(CMRR):
$\mathrm{CMRR} \triangleq \left|\frac{A_{DM}}{A_{CM}}\right| = \left|\frac{dv_{out}/dv_{inDM}}{dv_{out}/dv_{inCM}}\right|$
Often in **dB**
Opampbase difference aplifier:



1st: $A(s) = \infty, R_2 = R_2' = \alpha \cdot R_1 = \alpha R_1'$
$A_{DM} = -\alpha, A_{CM} = 0, CMRR = \infty$
2nd assume $A(s) = \infty, R_2 = \alpha \cdot R_1$,
$R_2' = (\alpha + \Delta\alpha) \cdot R_1', R_1' = R_1 + \Delta R$
$A_{DM} = -\left(\alpha + \frac{\Delta\alpha}{2\cdot(1+\alpha+\Delta\alpha)}\right)$
$A_{CM} = \frac{\Delta\alpha}{1+\alpha+\Delta\alpha}$
$\mathrm{CMRR} = -\frac{1}{2} + \frac{\alpha\cdot(1+\alpha+\Delta\alpha)}{\Delta\alpha}$
3d: $A(s) = A_{DC}/\left(1 + s \cdot \frac{A_{DC}}{GBW}\right)$
$A_{DM} = -\left(\alpha + \frac{\Delta\alpha}{2\cdot(1+\alpha+\Delta\alpha)}\right) \cdot \frac{1}{1+\frac{1}{A_{DC}}+\frac{\alpha}{A_{DC}}+(1+\alpha)\cdot\frac{s}{GBW}}$
$A_{CM} = \frac{\Delta\alpha}{1+\alpha+\Delta\alpha} \cdot \frac{1}{1+\frac{1}{A_{DC}}+\frac{\alpha}{A_{DC}}+(1+\alpha)\cdot\frac{s}{GBW}}$
$\mathrm{CMRR} = -\frac{1}{2} + \frac{\alpha\cdot(1+\alpha+\Delta\alpha)}{\Delta\alpha}$

Magnetic Field Sensors

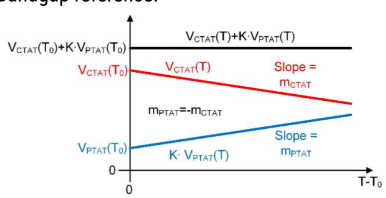
$V_{hall} = G \cdot \frac{\mu\cdot\rho}{h} \cdot l \cdot B = G \cdot \frac{1}{e\cdot n\cdot h} \cdot l \cdot B = S_l \cdot l \cdot B$ G: geometry factor
Error sources:
- Thermal noise
-1/f noise
-Noise of conditioning electronics(minor)
-Offset Noise power: $V_{noise}\sqrt{4kT \cdot R \cdot \Delta f}$
Signal power: $V_{hall} S_I \cdot I \cdot B$
Resolution: $B_{min} \hat{\wedge} \frac{V_{noise}}{V_{hall}/B} = \frac{\sqrt{4kT\cdot R\cdot\Delta f}}{S_l \cdot I}$
With $S_I = \frac{\mu\cdot\rho}{h\cdot e\cdot n} \cdot G$
At $B = 0$ Hall plate can be modeled as wheatstone bridge, non-uniform stress causes intrinsic offset(5 to 30mT)
Cancel intrinsic offset by coupling $90°$ turned sensors, current and sense ports are swapped
Idea: Hall voltage is in phase and gradient caused offsets are than $180°$ out of phase
Spinning current method:
Sense and current ports are switched every clock cycle
Idea: A DC component equal to the offset and a frequency component proportional to the hall voltage is produced **(Fourier Expansion of a square wave)**
By modulating the information to the clock frequency noise and offset are greatly mitigated.



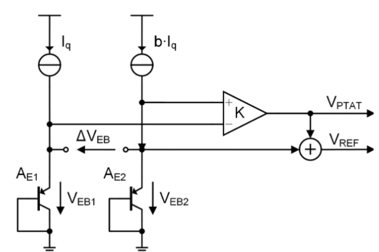
Multiplying by 1 and -1 to demodulate back to low frequency.
Low pass stage to remove signal around the 2nd harmonic
This allow to see the base noise floor of the sensor not 1/f noise.
In the conventional implementation 2 bias currents are used in the low power configuration the bias form the plate is reused for the amplifier.

Temperature Sensors

Seebeck effect: Convert temp. gradient to electromotive force
$F(T) \hat{=} \int_{T_{stand.}}^{T} (S_+(T') - S_-(T'))dT'$
$\rightarrow E_{emf} = F(T_{sense} - F(T_{ref})$
Temperature stable references:
$V_{REF}(T) = V_{CTAT}(T) + K \cdot V_{PTAT}(T)$
Bandgap reference:

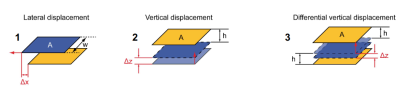


PTAT: Proportional to absolute temperature
CTAT: Complementary to absolute temperature
Scale one slope and add the function for a temp. constant reference
This can be built with a diode or a BJT with a shorted base and collector
Butt current is also temperature dependent so use diff pair.

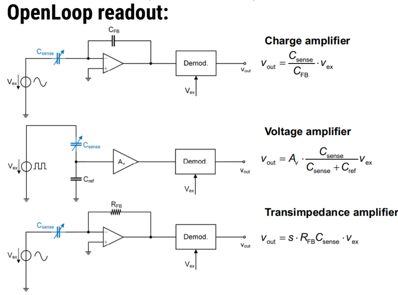


$V_{PTAT} = \Delta V_{EB} = V_{EB1} - V_{EB2} \approx \frac{kT}{q} \cdot \ln\left[\frac{I_q \cdot I_S' \cdot A_{E2}}{I_S' \cdot A_{E1} \cdot I_q}\right] = \frac{kT}{q} \cdot \ln\left[\frac{A_{E2}}{A_{E1}}\right]$
Scale currents or diodes to make equal bias
$V_{EB} = \underbrace{E_g/q}_{V_G} - \underbrace{\frac{kT}{q}}_{U_T} \cdot \ln\left(\frac{I_s'}{I_D}\right) =$
$V_G - U_T \cdot \ln\left(\frac{K_1 \cdot T^\gamma}{I_D}\right)$
$V_G \approx V_{G0K} - a \cdot T$
$V_G = V_{EB2} = V_{CTAT}$ With $I_S' > I_D$ but also temperature dependent $\rightarrow$ curved CTAT.
Combine CTAT and PTAT: Series:
$V_{REF} = I_{PTAT} \cdot R_2 + V_{CTAT} = \frac{R_2}{R_1} \cdot V_{PTAT} + V_{CTAT}$
Parallel:
$V_{REF} = (I_{PTAT} + I_{CTAT}) \cdot R_3 = \frac{R_3}{R_1} \cdot V_{PTAT} + \frac{R_3}{R_2} \cdot V_{CTAT}$

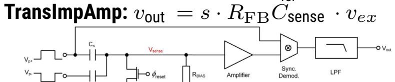
Capa. Sensor Readout



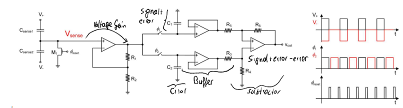
1 linear: $\Delta C = C - C_0 = \varepsilon \cdot \frac{W}{h} \cdot \Delta x$
2 non-linear:
$\Delta C = \frac{\varepsilon\cdot A}{h} \cdot \frac{\Delta z/h}{1+\Delta z h} \approx \frac{\varepsilon\cdot A}{h^2} \cdot \Delta z$
3 reduced non-linear with diff readout:
$\Delta C = \varepsilon \cdot \frac{A}{h} \cdot \left(\frac{2\cdot\Delta z/h}{1-[\Delta z/h]^2}\right) = \frac{2\cdot\varepsilon\cdot A}{h^2} \cdot \Delta z$
OpenLoop readout:



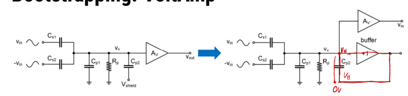
ChargeAmp: $V_{out} = \frac{C_{sense}}{C_{FB}} \cdot V_{ex}$ **ChargeAmp-Nonideal:** $\frac{V_{out}}{V_{in}} = -\frac{j\omega \cdot R_{FB} C_{in}}{\left(1+\frac{j\omega}{GBW}\right)\cdot\left(1+\frac{j\omega}{\omega_{FB}}\right)}$
VoltAmp: $v_{out} = A_v \cdot \frac{C_{sense}}{C_{sense}+C_{ref}} V_{ex}$
TransImpAmp: $v_{out} = s \cdot R_{FB} C_{sense} \cdot v_{ex}$



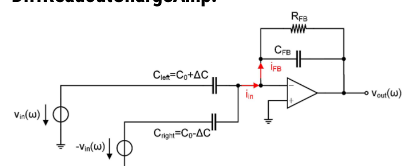
V_{Sense} not grounded sensitive to leakage currents
Bias resistor provides well defined DC-point.
R_{Bias} needs to be large because of the highpass forming, corner frequency needs to be low enough compared to excitation freq.
Periodic reset can also be used where $V_{p+} = V_{p-} = 0$
Correlated double sampling can also bes used:



Bootstrapping: VoltAmp

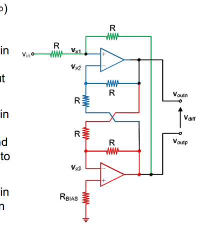


$C_{s1} = C_{s0} + \Delta C/2 C_{s2} = C_{s0} - \Delta C/2$, v_{out} becomes sensitive to parasitic capacitance
Can be bootstrapped out with voltage buffer driving V_{shield}
$v_{out} = \frac{\Delta C}{2C_{s0}+C_{p1}+(C_{p2})} \cdot A_v \cdot v_{in}$
Gain of output is reduced
C_{p2} gets removed by V_{shied}
Can also use TIA, the virtual ground helps with the parasitic capacitance, this also works in the charge amplifier
DiffReadoutChargeAmp:

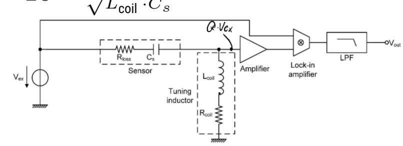


Req V_{ex}**:**
Error will propagate to output directly
Accuracy more important for small ΔC
Mismatch or drift will introduce errors
Sinewave can be created with center-tapped transformer or active balun
Rectangular witch switched excitation schemes
DiffReadoutChargeAmp:

- Assuming ideal opamps ($A_{op} \mapsto \infty$)
  $\Rightarrow v_{x2} = v_{x1}, \ v_{x3} = 0$
- The part of the circuit highlighted in red is a simple inverting amplifier with input voltage v_{outn} and output voltage v_{outp}
- The part of the circuit highlighted in blue is a simple non-inverting amplifier with input voltage v_{x1} and with its reference potential pulled to v_{outp}
- The part of the circuit highlighted in green is a voltage divider between v_{in} and v_{outp}



$V_{outp} = V_{in} = -V_{outn}$
Differential Capa sensing:
$\Delta v_{x,\,amp} \approx -\frac{\Delta x}{x_0} \cdot V_{ex}$
Resonant readout:
$\omega_{LC} = \frac{1}{\sqrt{L_{coil} \cdot C_s}}$

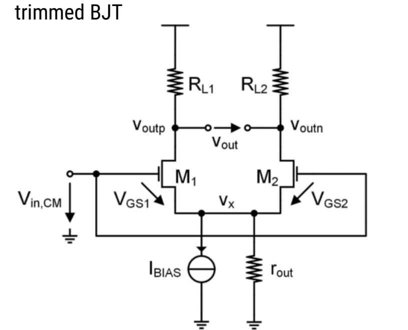


DoubleDiffReadout:
$\Delta V_{out} = -\Delta V_{ex} \cdot \left(\frac{\Delta C_s}{C_i}\right.$ ·

$\underbrace{\left[1 - \frac{C_s}{C_s+C_i+C_p}\right]}_{\text{gain error}} - \underbrace{\frac{\Delta C_i + \Delta C_p}{C_i} \cdot \frac{C_s +}{}}_{\text{offset error}}$

Differential Measurements

- Inherent rejection to common-mode interference ans noise
- Wider Signal swing for a given supply voltage
- Minimum effect of even order distortion including DC offset
- Can lead to improved sensor linearity
But respond to some degree to common mode signal
Ideal DiffPair:
CMMR: $\triangleq 20 \cdot \log\left(\left|\frac{A_{DM}}{A_{CM-to-DM}}\right|\right) =$
$20 \cdot \log\left(\frac{2G_{mav}\cdot r_{out}}{\Delta R_L/R_{Lav}+\Delta G_m/G_{mav}}\right]$
PSRR: $\triangleq 20 \cdot \log\left(\left|\frac{A_{DM}}{A_{vDD}}\right|\right) =$
$20 \cdot \log\left(\left|\frac{A_{DM}}{\Delta V_{out}/\Delta VDD}\right|\right)$ Process variation causes mismatch between resistors and current source is not ideal
Input referred offset is the voltage needed to get zero volts differential output
IRO: $\Delta V_G = V_{OS} = -\frac{I_{Dav}}{G_{mav}} \cdot \left(\frac{\Delta R_L}{R_{Lav}} + \frac{\Delta\beta}{\beta_{av}}\right) - \Delta V_{TO}$
OffsetVoltage: 10mV **MOS** ; 1mV **BJT**, 120 μ, trimmed BJT

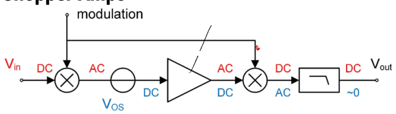


$CMRR \cdot V_{os} \approx G_{mav} \cdot r_{out} \cdot V_{OV}$, $V_{OV} = \frac{I_{DAV}}{G_{mav}}$ **PSRR:**
$\mathrm{PSRR}_{vDD} \cong G_{mav} R_{Lav} / \left(g_{outM1,2} \cdot \left(\frac{g_{out}}{G_{mav}} \cdot \Delta R_L - 2\right.\right.$
$\mathrm{PSRR}_{vss} \cong G_{mav} \cdot R_{Lav} / g_{out} \cdot \left(\Delta R_L + R_{Lav} \cdot \frac{\Delta G_m}{G_{mav}}\right)$

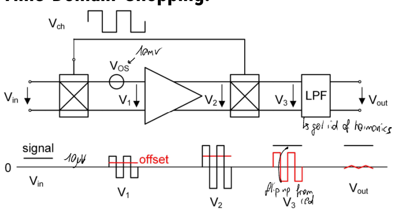
Modulation

Synonyms: Coherent detection, synchronous demodulation, lock-in amplification, chopping
All modulation techniques, square wave is called chopping
Leads to better low freq. specification, smaller 1/f, bigger CMRR and PSRR
Trimming: Measuring static error offset and gain and adjusting the value of a component to reduce the error to 0
Low complexity, no bandwith limit, but reqs. measure equipment
Also reqs. memory element

Dyn. offset cancel:
usually no measure eq. but more complex circuits, reduce bandwidth
AutoZeroing:
periodically measure offset and substract from input(time domain)
Chopping:
Modulate signal above 1/f noise(freq. domain)
Chopper Amps



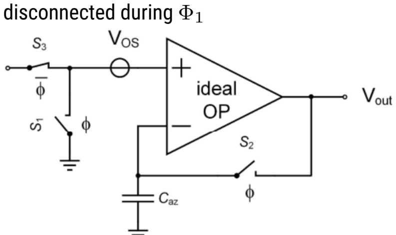
implemented with polarity reversing switch
Time-Domain-Chopping:



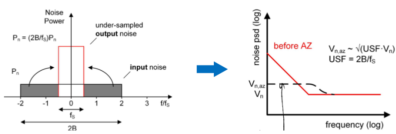
Complete suppression of 1/f noise if $f_{chop} > 1/f$ corner freq., but up-modulated offset must be filtered out. loss of bandwidth and residual chopper ripple. **Charge Injection**
Injected charge splits half-half
Charge:
$Q = W \cdot L \cdot C_{\text{ox}} \cdot [V_\varphi - V_{\text{in}} - V_{\text{T0}}(\sqrt{V_{\text{in}}})]$
Linear w.r.t to $W \cdot L$ non-linear w.r.t. V_{in}

$$V_{\text{out}} = V_{\text{in}}\Big(1 + \underbrace{\frac{W \cdot L \cdot C_{ox}}{2C_L}}_{\text{gain error}}\Big) - \underbrace{\frac{W \cdot L \cdot C_{ox}}{2C_L} \cdot \left[V_\varphi - V_{T0}\left(\sqrt{V_{\text{in}}}\right)\right]}_{\text{offset \& dostortion}}$$

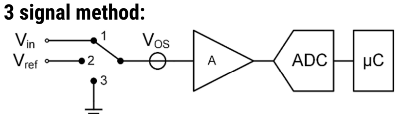
Clock feedthrough
Overlap capacitance of trans.: C_{OV}
$\Delta V_{\text{out}} = \frac{C_{\text{ov}}}{C_{\text{ov}} + C_L} \cdot V_\varphi$
Asymmetric clock duty cycle causes demodulation signal to have DC component which feeds through
Bandwidth gain acc
Limited applifier Bandwidth causes output signal to not be perfectly square, therefore less gain
AutoZeroing, LF Noise Reduce
Sampling unwanted signal during Φ_1, storing, and subtracting during Φ_2, input is disconnected during Φ_1



Error stored on C_{AZ} will slowly leak away, C_{AZ} as large as possible
Mitigate Charge Inject.
-Use min size switch
- **Diff Sampling:** Const offset and nonlin. is reduced
-**Comp Switch:** NMOS and PMOS in parallel cancel opposite charge packets, can only occur for one V_{in}, clock feedtrough can't be cancelled perfet because of different overlap capacitance.
-**Dummy Switch:** Add a dummy switch of half size to such up injection of M1, but equal charging splitting rarely holds, but clock feedtrough is also mitigated
-**BottomplateSampling:** Disconnect C_{AZ}'s bottomplate from ground slightly before M1, C_{az} bottom plate is then floating when M1 is opened and no charge can be injected. Requires additional clock.



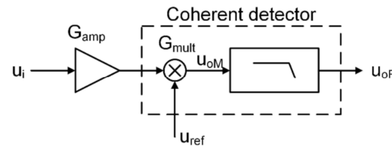
No ripples, like chopping, offset of a few μV can be reached, main problems switching spikes, leakage currents and finite gain
Correlated Double Sampling (CDS):
Special case of AZ
Phase 1 (calib):
$V_1 = V(t_1) = A \cdot (0 + VOS)$
Phase 2 (measure):
$V_2 = V(t_2) = A \cdot (V_{in} + V_{OS})$
$\Rightarrow (V_2 - V_1) = A \cdot V_{in}$
3 signal method:



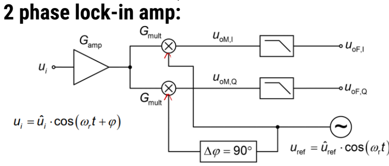
Find: V_{in}, A, V_{OS}
Phase 1: $V_1 = A \cdot (V_{\text{in}} + V_{\text{OS}})$
Phase 2: $V_2 = A \cdot (V_{\text{ref}} + V_{\text{OS}})$
Phase 1: $V_3 = A \cdot V_{\text{OS}}$
Acc. is limited by ADC resolution
DEM:
Switch nomically identical components with a clock
Acc. is limited by mismatch of switch resistance
Significantly reduces average error
2 Resistors in parallel: $\begin{matrix} R_1 = R + \Delta R \\ R_2 = R - \Delta R \end{matrix} \rightarrow$
$Gain_{av} = \frac{\left(1 + \frac{R+\Delta R}{R-\Delta R}\right) + \left(1 + \frac{R-\Delta R}{R+\Delta R}\right)}{2} = 2$
LPF needed like chopping, can be easily combined
Reduces bandwidth and need more components

Coheren Detection

synonyms: coherent detection, synchronous demodulation, lock-in amplification, chopping
Like chopping but sinewave instead of rect.
Good for:
- low-bandwidth quasi static signals with high noise
- if high dynamic range is req.
Mems, Infrared, magnetic sensors, strain gauges.
Behaves like bandpass but isn't one



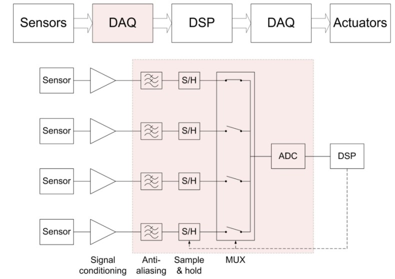
Amplitude Synchron. detection:
$u_{oM} = G_{\text{Amp}} \cdot G_{\text{mult}} \cdot [\hat{u}_i \cdot \sin(\omega_i t) \cdot \hat{u}_r \sin(\omega_r t)]$
$u_{oM} = G_{\text{Amp}} \cdot G_{\text{mult}} \cdot \frac{\hat{u}_i \hat{u}_r}{2} \cdot [1 - \cos(2\omega_i t)]$
$\overset{LP}{\Rightarrow} u_{oF} = G_{\text{LPF}} \cdot G_{\text{Amp}} \cdot G_{\text{mult}} \cdot \frac{\hat{u}_i \hat{u}_r}{2}$
for $\omega_r = \omega_i$
Phase:
$u_{oM} = G_{\text{Amp}} \cdot G_{\text{mult}} \cdot [\hat{u}_i \cdot \sin(\omega_i t + \varphi) \cdot \hat{u}_r \cdot \sin(\omega_r t)]$
$u_{oF} = G_{\text{LPF}} \cdot G_{\text{Amp}} \cdot G_{\text{mult}} \cdot \frac{\hat{u}_i \hat{u}_r}{2} \cdot \cos(\varphi)$
Phase sensitive coherent detector
SNR:
LOD for $10nV/\sqrt{Hz}$ white noise and $f_{LPF} = 1Hz \rightarrow \frac{10nV}{\sqrt{Hz}} \cdot \sqrt{1Hz} = 10nV$
2 phase lock-in amp:



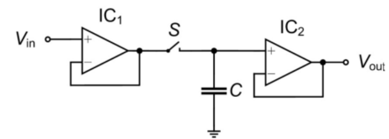
$A_0 =$
$u_{\text{oF},I} = G_{\text{amp}} \cdot G_{\text{mult}} \cdot G_{\text{LPF}} \cdot \frac{\hat{u}_i \cdot \hat{u}_r}{2} \cdot \cos(\varphi)$
$u_{\text{oF},Q} = G_{\text{amp}} \cdot G_{\text{mult}} \cdot G_{\text{LPF}} \cdot \frac{\hat{u}_i \cdot \hat{u}_r}{2} \cdot \sin(\varphi)$
$\Delta\varphi =$
$\hat{u}_i = \frac{1}{G_{\text{amp}} \cdot G_{\text{mult}} \cdot G_{\text{LPF}}} \cdot \sqrt{u_{\text{of},I}^2 + u_{\text{oF},Q}^2}$
$\varphi = \tan^{-1}\left(\frac{u_{\text{of},Q}}{u_{\text{oF},I}}\right)$

DACs

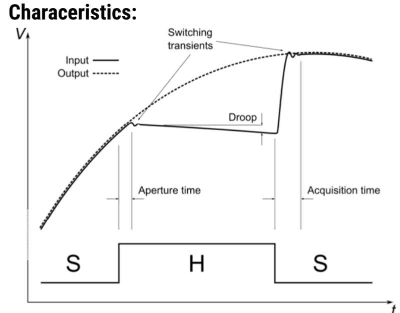
Prerequisite for sampling is the Nyquist theorem to be able to perfectly reconstruct a band-limited signal. $f_s \geq f_N = 2 \cdot f_b$



Sample and hold:

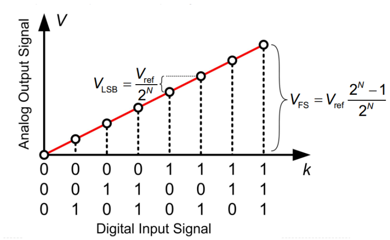


Sample: S is closed $V_{out} = V_{in}$
Hold: S is open C holds V_{out}
IC_1 with small Z_1 for fast charge of C
IC_2 with Z_{in} for slow discharge of C
S with small R_{on}
Characteristics:



Aperture time: time for switch to open
Droop: discharge of capacitor
Acquisition time: time to switch and charge capacitor
Switching transients: voltage buffer ringing
Design C:
Large for small droop, small for fast charge
Charge depends on R_{on} and Z_{out} or $I_{out,max}$ of IC_1
Large enough for sufficient small $\frac{kT}{C}$

Static Errors



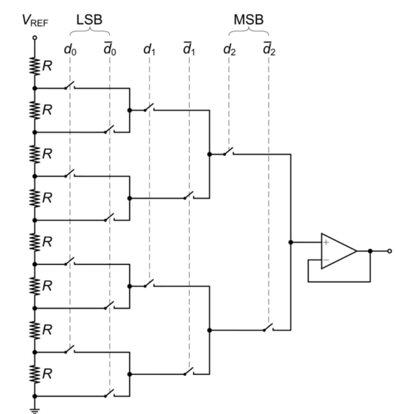
Gain Error: $V_{\text{gain,error}} = V_{\text{MSB avg}} - V_{\text{MSB ideal}}$
Offset: $V_{\text{os}} = V(000)$
Does not affect linearity, easy to compensate
Diff nonlin (DNL):
Measure of nonuniformity, quantifies for each of the k binary input combinations the deviation of each step from the ideal stepsize of one LSB
$\text{DNL}(k) = \frac{V(k) - V(k-1) - V_{\text{LSB}}}{V_{\text{LSB}}} = \frac{\Delta V(k)}{V_{\text{LSB}}}$ If DNL smaller than -1 the output is smaller than the previous and it's non-monotonic
Monotonicity is critical for feedback: turns negative feedback into positive feedback
Occurs most often when switching the MSB
Intefral nonlin (INL): Deviation of the output val from the ideal val:
$\text{INL}(k) = \frac{V(k) - V_{\text{ideal}}(k)}{V_{\text{LSB}}}$

Dynamic Errors

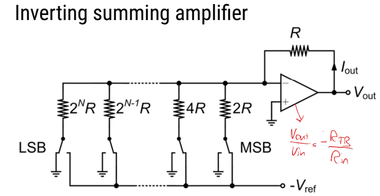
Jitter: Max jitter (Δt) for error below one lsb:
$\Delta t < \frac{V_{\text{LSB}}}{\pi f_{\text{MAX}} V_{\text{FS}}} \approx \frac{1}{2^N \pi f_{\text{MAX}}}$
Glitches:
Turn on and turn off time not precisely synchronized. In the moment of switching one bit to another the value can briefly be both or none of the bits.

Implementation

Resistorstring:



Simple voltage divider, inherently monotonic, amount of resistors proportional 2^N, Area $A \propto W_R \cdot L_R \cdot 2^N$
Binary weighted resisitve voltage divider
Inverting summing amplifier



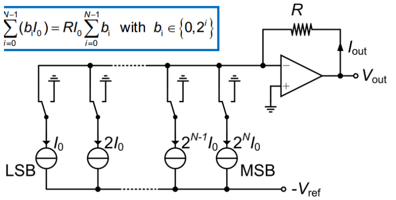
For each bit i: $V_{\text{out},i} = d_i \cdot \frac{R}{2^{i+1}R} = d_i \cdot \frac{1}{2^{i+1}}, \quad i = 0,..,N-1, d_i \in \{0,1\}$

$V_{out} = \sum_{i=0}^{N-1} V_{out,i} = V_{ref} \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}}, \quad d_i \in \{0,1\}$

Binary weighted current sources:

$V_{out} = I_{out} R = R \sum_{i=0}^{N-1} (b_i I_0) = R I_0 \sum_{i=0}^{N-1} b_i$ with $b_i \in \{0, 2^i\}$

Sources can be implemented as current mirror



Binary DACS: few number of components, large ratios of resistors, currents and switches, monotonicity not guaranteed, not linear, bas acc, need precise matching, prone to glitches, large portion switches.

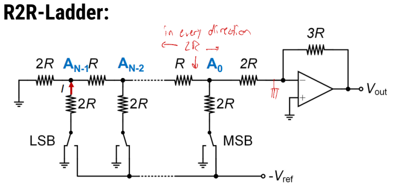
Thermometer wighted:

Current steering DAC, reduced glitches, monotonic, 2^N sources, binary to thermometer decoder needed

D	b_1	b_2	t_2	t_1	t_0
0	0	0	0	0	0
1	0	1	0	0	1
2	1	0	0	1	1
3	1	1	1	1	1

Same topology as binary weighted but sources are not weighted just I_0

R2R-Ladder:



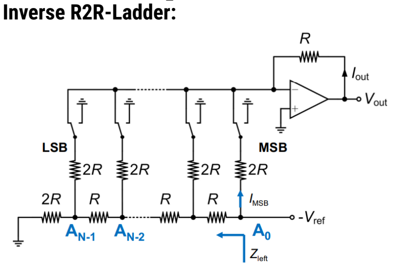
Branch current:

$I = -\frac{V_{ref}}{2R+(2R\|2R)} = -\frac{V_{ref}}{3R}$

$V_{out,i} = -d_i \frac{I}{2^{i+1}} \cdot 3R = V_{ref} \cdot d_i \cdot \frac{1}{2^{i+1}}$

$V_{out} = \sum_{i=0}^{N-1} V_{out,i} = V_{ref} \cdot \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}}$

Inverse R2R-Ladder:



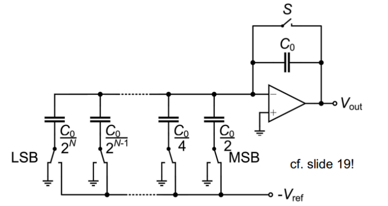
$V_{out,i} = -d_i \cdot \frac{V_{ref}/(2R)}{2^i} \cdot R = V_{ref} \cdot d_i \cdot \frac{1}{2^{i+1}}$

$V_{out} = \sum_{i=0}^{N-1} V_{out,i} = V_{ref} \cdot \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}}$

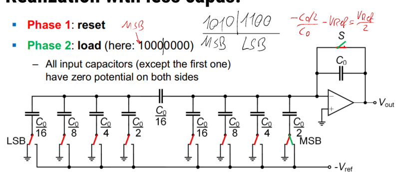
Bin weighted Cap. volt divider:

$V_{out,i} = d_i \cdot \frac{C_0/2^{i+1}}{C_0} \cdot V_{ref}, \quad d_i \in \{0,1\}, i = 0.., N-1$

$V_{out} = \sum_{i=0}^{N-1} V_{out,i} = V_{ref} \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}} V_{out,i}$ where $d_i \in \{0,1\}$ Capas are weighted in the denominator, but otherwise topology same as



Realization with less capas:

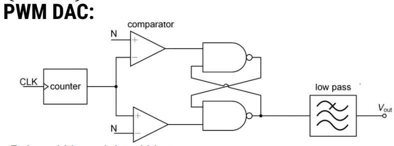


Switch between lsb and msb group reset and load $V_{out} = \alpha_{right} \cdot V_{ref}$ $\alpha_{right} = \sum_{i=1}^{N_{right}} d_{i,right}$ with $d_{i,right} \in \{0, 2^{-i}\}$

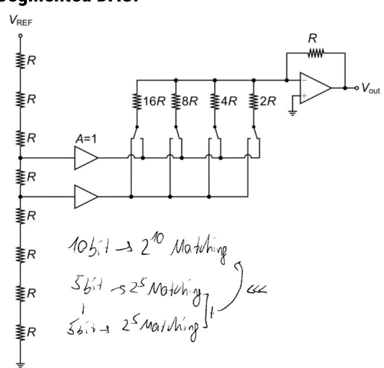
$V_{out} = \frac{\alpha_{left} V_{ref}}{16}$

$\alpha_{left} = \sum_{i=1}^{N} d_{i,left}$ with $d_{i,left} \in \{0, 2^{-i}\}$

PWM DAC:

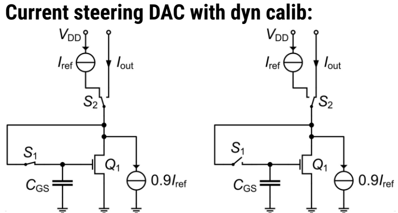


Segmented DAC:



Course DAC(MSBs) feeds fine DAC (LSBs), matching needs to be instead $2^1 0$ for 10 bit only $2 \cdot 2^5$

Current steering DAC with dyn calib:



Each source supposed to have I_{ref}, Assume one only has $0.9 I_{Ref}$

Calib phase: V_{GS1} settles so that Q_1 draws $0.1 I_{Ref}$

Operat. Phase: Q_1 calibs. so that $I_{d1} = I_{ref}$