

Resolution: $B_{\min} \hat{\Delta} \frac{V_{\text{noise}}}{V_{\text{hall}}/B} = \frac{\sqrt{4kT \cdot R \cdot \Delta f}}{S_I \cdot I}$

With $S_I = \frac{\mu \cdot \rho}{h \cdot e \cdot n} \cdot G$

At $B = 0$ Hall plate can be modeled as wheatstone bridge, non-uniform stress causes intrinsic offset(5 to 30mT)

Cancel intrinsic offset by coupling ° turned sensors, current and sense ports are swapped

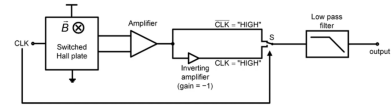
Idea: Hall voltage is in phase and gradient caused offsets are than 180° out of phase

Spinning current method:

Sense and current ports are switched every clock cycle

Idea: A DC component equal to the offset and a frequency component proportional to the hall voltage is produced (**Fourier Expansion of a square wave**)

By modulating the information to the clock frequency noise and offset are greatly mitigated.



Multiplying by 1 and -1 to demodulate back to low frequency.

Low pass stage to remove signal around the 2nd harmonic

This allow to see the base noise floor of the sensor not 1/f noise.

In the conventional implementation 2 bias currents are used in the low power configuration the bias form the plate is reused for the amplifier.

Temperature Sensors

Seebeck effect: Convert temp. gradient to electromotive force

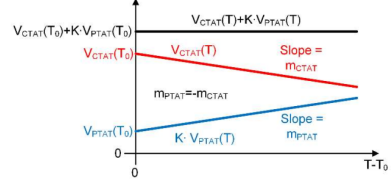
$F(T) \hat{=} \int_{T_{\text{stand}}}^T (S_+(T') - S_-(T')) dT'$

$\rightarrow E_{\text{emf}} = F(T_{\text{sense}} - F(T_{\text{ref}}))$

Temperature stable references:

$V_{\text{REF}}(T) = V_{\text{CTAT}}(T) + K \cdot V_{\text{PTAT}}(T)$

Bandgap reference:



PTAT: Proportional to absolute temperature

CTAT: Complementary to absolute temperature

Scale one slope and add the function for a temp. constant reference

This can be built with a diode or a BJT with a shorted base and collector

Butt current is also temperature dependent so use diff pair.

$$V_{\text{PTAT}} = \Delta V_{\text{EB}} = V_{\text{EB1}} - V_{\text{EB2}} \approx \frac{kT}{q} \cdot \ln \left[\frac{I_{q1} \cdot I_{S \cdot AE2}}{I_{S \cdot AE1} \cdot I_{q2}} \right] = \frac{kT}{q} \cdot \ln \left[\frac{A_{E2}}{A_{E1}} \right]$$

Scale currents or diodes to make equal bias

$$V_{\text{EB}} = \underbrace{E_g/q}_{V_G} \cdot \underbrace{\ln \left(\frac{I'_S}{I_D} \right)}_q = V_G - U_T \cdot \ln \left(\frac{K_1 \cdot T^\gamma}{I_D} \right)$$

$$V_G \approx V_{G0K} - a \cdot T$$

With $I'_S > I_D$ but also temperature dependent \rightarrow curved CTAT.

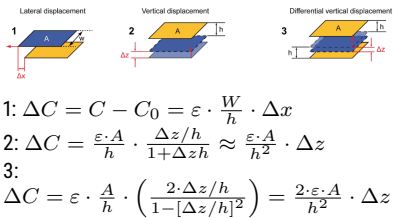
Combine CTAT and PTAT: Series:

$$V_{\text{REF}} = I_{\text{PTAT}} \cdot R_2 + V_{\text{CTAT}} = \frac{R_2}{R_1} \cdot V_{\text{PTAT}} + V_{\text{CTAT}}$$

Parallel:

$$V_{\text{REF}} = (I_{\text{PTAT}} + I_{\text{CTAT}}) \cdot R_3 = \frac{R_3}{R_1} \cdot V_{\text{PTAT}} + \frac{R_3}{R_2} \cdot V_{\text{CTAT}}$$

Capa. Sensor Readout

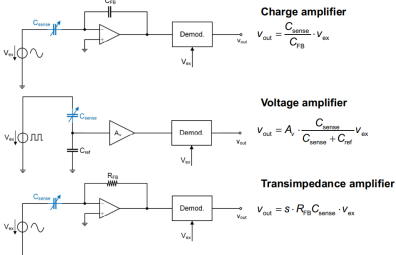


1: $\Delta C = C - C_0 = \epsilon \cdot \frac{W}{h} \cdot \Delta z$

2: $\Delta C = \frac{\epsilon \cdot A}{h} \cdot \frac{\Delta z/h}{1 + \Delta z/h} \approx \frac{\epsilon \cdot A}{h^2} \cdot \Delta z$

3: $\Delta C = \epsilon \cdot \frac{A}{h} \cdot \left(\frac{2 \cdot \Delta z/h}{1 - [\Delta z/h]^2} \right) = \frac{2 \cdot \epsilon \cdot A}{h^2} \cdot \Delta z$

OpenLoop readout:



ChargeAmp: $V_{\text{out}} = \frac{C_{\text{sense}}}{C_{\text{FB}}} \cdot V_{\text{ex}}$

Nonideal: $\frac{V_{\text{out}}}{V_{\text{in}}} = - \frac{j\omega \cdot R_{\text{FB}} \cdot C_{\text{in}}}{(1 + \text{GBW}) \cdot (1 + \frac{j\omega}{\omega_{\text{FB}}})}$

VoltAmp: $v_{\text{out}} = A_v \cdot \frac{C_{\text{sense}}}{C_{\text{sense}} + C_{\text{ref}}} V_{\text{ex}}$

TransImpAmp: $v_{\text{out}} = s \cdot R_{\text{FB}} C_{\text{sense}} \cdot v_{\text{ex}}$

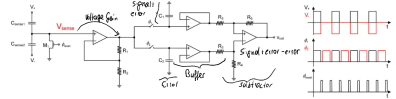
V_{Sense} not grounded sensitive to leakage currents

Bias resistor provides well defined DC-point.

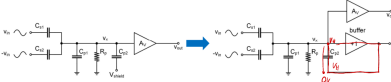
R_{Bias} needs to be large because of the highpass forming, corner frequency needs to be low enough compared to excitation freq.

Periodic reset can also be used where $V_{p+} = V_{p-} = 0$

Correlated double sampling can also be used:



Bootstrapping: VoltAmp



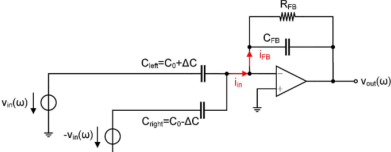
$C_{s1} = C_{s0} + \Delta C/2$ $C_{s2} = C_{s0} - \Delta C/2$, v_{out} becomes sensitive to parasitic capacitance

Can be bootstrapped out with voltage buffer driving V_{shield}

$v_{\text{out}} = \frac{\Delta C}{2C_{s0} + C_{p1} + (C_{p2})} \cdot A_v \cdot v_{\text{in}}$ C_{p2} gets removed by V_{shield}

Can also use TIA, the virtual ground helps with the parasitic capacitance, this also works in the charge amplifier

DiffReadoutChargeAmp:



Req V_{ex} :

Error will propagate to output directly

Accuracy more important for small ΔC

Mismatch or drift will introduce errors

Sinewave can be created with center-tapped transformer or active balun

Rectangular witch switched excitation schemes

DiffReadoutChargeAmp:

- Assuming ideal opamps ($A_{\text{op}} \rightarrow \infty$) $\Rightarrow V_{\text{GS}} = V_{\text{in}}$, $V_{\text{AS}} = 0$
- The part of the circuit highlighted in red is a simple inverting amplifier with input voltage V_{outin} and output voltage V_{outp}
- The part of the circuit highlighted in blue is a simple non-inverting amplifier with input voltage V_{in} and with its reference potential pulled to V_{outp}
- The part of the circuit highlighted in green is a voltage divider between V_{in} and V_{outp}

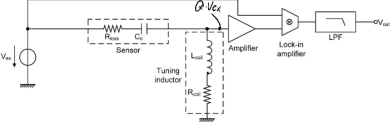
$V_{\text{outp}} = V_{\text{in}} = -V_{\text{outn}}$

Differential Capa sensing:

$\Delta v_{\text{x, amp}} \approx - \frac{\Delta x}{x_0} \cdot V_{\text{ex}}$

Resonant readout:

$\omega_{\text{LC}} = \frac{1}{\sqrt{L_{\text{coil}} \cdot C_s}}$



DoubleDiffReadout:

$\Delta V_{\text{out}} = -\Delta V_{\text{ex}} \cdot \left(\frac{\Delta C_s}{C_i} \cdot \left[1 - \frac{C_s}{C_s + C_i + C_p} \right] - \frac{\Delta C_i + \Delta C_p}{C_i} \cdot C_s + \frac{\Delta C_s}{C_i} \cdot C_s \right)$

gain error

offset error

Differential Measurements

- Inherent rejection to common-mode interference ans noise
- Wider Signal swing for a given supply voltage

- Minimum effect of even order distortion including DC offset

- Can lead to improved sensor linearity

But respond to some degree to common mode signal

Ideal DiffPair:

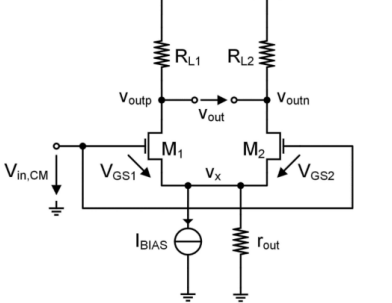
$\text{CMRR} \hat{=} 20 \cdot \log \left(\left| \frac{A_{\text{DM}}}{A_{\text{CM-to-DM}}} \right| \right) = 20 \cdot \log \left(\frac{2G_{\text{mav}} \cdot r_{\text{out}}}{\Delta R_L / R_{\text{Lav}} + \Delta G_{\text{m}} / G_{\text{mav}}} \right)$

$\text{PSRR} \hat{=} 20 \cdot \log \left(\left| \frac{A_{\text{DM}}}{A_{\text{VDD}}} \right| \right) = 20 \cdot \log \left(\left| \frac{A_{\text{DM}}}{\Delta v_{\text{out}} / \Delta V_{\text{DD}}} \right| \right)$ Process variation causes mismatch between resistors and current source is not ideal

Input referred offset is the voltage needed to get zero volts differential output

IRO: $\Delta V_G = V_{\text{OS}} = - \frac{I_{\text{DAV}}}{G_{\text{mav}}} \cdot \left(\frac{\Delta R_L}{R_{\text{Lav}}} + \frac{\Delta \beta}{\beta_{\text{AV}}} \right) - \Delta V_{\text{TO}}$

OffsetVoltage: 10mV **MOS**; 1mV **BJT**, 120 μ , trimmed BJT



$\text{CMRR} \cdot V_{\text{OS}} \approx G_{\text{mav}} \cdot r_{\text{out}} \cdot V_{\text{OV}}$

$V_{\text{OV}} = \frac{I_{\text{DAV}}}{G_{\text{mav}}}$ **PSRR:**

$\text{PSRR}_{V_{\text{DD}}} \hat{=} G_{\text{mav}} R_{\text{Lav}} / \left(g_{\text{outM1,2}} \cdot \left(\frac{g_{\text{out}}}{G_{\text{mav}}} \cdot \Delta R_L - 2 \cdot \frac{\Delta G_{\text{m}}}{G_{\text{mav}}} \right) \right)$

$\text{PSRR}_{V_{\text{SS}}} \hat{=} G_{\text{mav}} \cdot R_{\text{Lav}} / g_{\text{out}} \cdot \left(\Delta R_L + R_{\text{Lav}} \cdot \frac{\Delta G_{\text{m}}}{G_{\text{mav}}} \right)$

Modulation

Stonmts: Coherent detectrion, synchronous demodulation, lock-in amplification, chopping

All modulation techniques, square wave is called chopping

Leads to better low freq. specification, smaller 1/f, bigger CMRR and PSRR

Trimming: Measuring static error offset and gain and adjusting the value of a component to reduce the error to 0

Low complexity, no bandwidth limit, but reqs. measure equipment

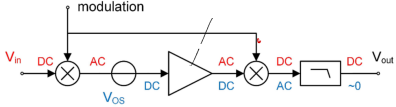
Also reqs. memory element

Dyn. offset cancel: usually no measure eq. but more complex circuits, reduce bandwidth

AutoZeroing: periodically measure offset and substract from input(time domain)

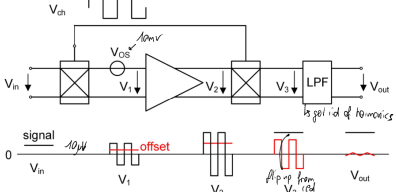
Chopping: Modulate signal above 1/f noise(freq. domain)

Chopper Amps



implemented with polarity reversing switch

Time-Domain-Chopping:



Complete suppression of 1/f noise if $f_{\text{chop}} > 1/f$ corner freq., but up-modulated offset must be filtered out. loss of bandwidth and residual chopper ripple. **Charge Injection**

Injected charge splits half-half

Charge:

$Q = W \cdot L \cdot C_{\text{ox}} \cdot [V_{\phi} - V_{\text{in}} - V_{\text{TO}} (\sqrt{V_{\text{in}}})]$

Linear w.r.t $W \cdot L$ non-linear w.r.t V_{in}

$V_{\text{out}} = V_{\text{in}} (1 + \underbrace{\frac{W \cdot L \cdot C_{\text{ox}}}{2C_L}}_{\text{gain error}}) - \underbrace{\frac{W \cdot L \cdot C_{\text{ox}}}{2C_L} \cdot [V_{\phi} - V_{\text{TO}} (\sqrt{V_{\text{in}}})]}_{\text{offset \& distortion}}$

Clock feedthrough

Overlap capacitance of trans.: C_{OV}

$\Delta V_{\text{out}} = \frac{C_{\text{OV}}}{C_{\text{ov}} + C_L} \cdot V_{\phi}$

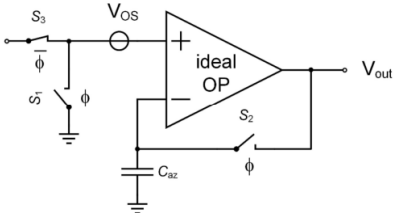
Asymmetric clock duty cycle causes demodulation signal to have DC component which feeds through

Bandwidth gain/acc

Limited applifier Bandwidth causes output signal to not be perfectly square, therefore less gain

AutoZeroing, LF Noise Reduce

Sampling unwanted signal during Φ_1 , storing, and subtracting during Φ_2 , input is disconnected during Φ_1



Error stored on C_{AZ} will slowly leak away, C_{AZ} as large as possible

Mitigate Charge Inject.

-Use min size switch

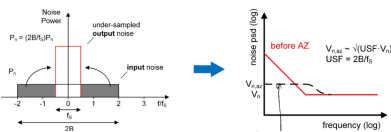
- **Diff Sampling:** Const offset and nonlin. is reduced

- **Comp Switch:** NMOS and PMOS in parallel cancel opposite charge packets, can only occur for one V_{in} , clock feedthrough can't be

cancelled perfer because of different overlap capacitance.

-Dummy Switch: Add a dummy switch of half size to such up injection of M1, but equal charging splitting rarely holds, but clock feedtrough is also mitigated

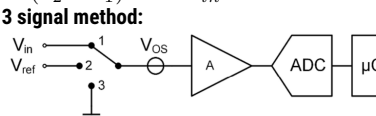
-BottomplateSampling: Disconnect C_{AZ} 's bottomplate from ground slightly before M1, C_{az} bottom plate is then floating when M1 is opened and no charge can be injected. Requires additional clock.



No ripples, like chopping, offset of a few μV can be reached, main problems switching spikes, leakage currents and finite gain

Correlated Double Sampling (CDS):

Special case of AZ
Phase 1 (calib):
 $V_1 = V(t_1) = A \cdot (0 + VOS)$
Phase 2 (measure):
 $V_2 = V(t_2) = A \cdot (Vin + VOS)$
 $\Rightarrow (V_2 - V_1) = A \cdot Vin$



Find: V_{in}, A, VOS
Phase 1: $V_1 = A \cdot (Vin + VOS)$
Phase 2: $V_2 = A \cdot (Vref + VOS)$
Phase 1: $V_3 = A \cdot VOS$

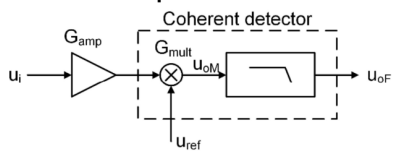
Acc. is limited by ADC resolution
DEM:
Switch nomically identical components with a clock
Acc. is limited by mismatch of switch resistance
Significantly reduces average error

2 Resistors in parallel: $R_1 = R + \Delta R$
 $R_2 = R - \Delta R$
 \rightarrow
 $Gain_{av} = \frac{(1 + \frac{R + \Delta R}{R - \Delta R}) + (1 + \frac{R - \Delta R}{R + \Delta R})}{2} = 2$
LPF needed like chopping, can be easily combined
Reduces bandwidth and need more components

Coheren Detection

synonyms: coherent detection, synchronous demodulation, lock-in amplification, chopping
Like chopping but sinewave instead of rect.
Good for:
- low-bandwidth quasi static signals with high noise
- if high dynamic range is req.
Mems, Infrared, magnetic sensors, strain gauges.

Behaves like bandpass but isn't one

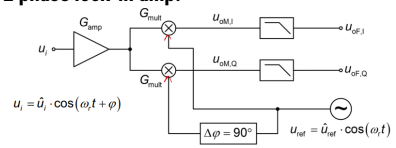


Amplitude Synchron. detection:

$$u_{oM} = G_{amp} \cdot G_{mult} \cdot [\hat{u}_i \cdot \sin(\omega_i t) \cdot \hat{u}_r \sin(\omega_r t)]$$
$$u_{oM} = G_{amp} \cdot G_{mult} \cdot \frac{\hat{u}_i \hat{u}_r}{2} \cdot [1 - \cos(2\omega_i t)]$$
$$\xrightarrow{LP} u_{oF} = G_{LPF} \cdot G_{amp} \cdot G_{mult} \cdot \frac{\hat{u}_i \hat{u}_r}{2}$$

for $\omega_r = \omega_i$ **Phase:** $u_{oM} = G_{amp} \cdot G_{mult} \cdot [\hat{u}_i \cdot \sin(\omega_i t + \varphi) \cdot \hat{u}_r \cdot \sin(\omega_r t)]$

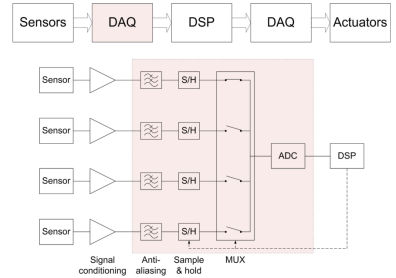
$u_{oF} = G_{LPF} \cdot G_{amp} \cdot G_{mult} \cdot \frac{\hat{u}_i \hat{u}_r}{2} \cdot \cos(\varphi)$
Phase sensitive coherent detector
SNR: LOD for $10nV/\sqrt{Hz}$ white noise and $f_{LPF} = 1Hz \rightarrow \frac{10nV}{\sqrt{Hz}} \cdot \sqrt{1Hz} = 10nV$



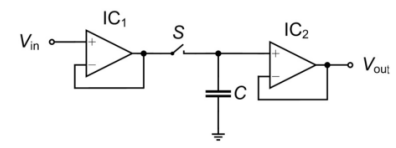
$$A_0 = u_{oF,I} = G_{amp} \cdot G_{mult} \cdot G_{LPF} \cdot \frac{\hat{u}_i \cdot \hat{u}_r}{2} \cdot \cos(\varphi)$$
$$u_{oF,Q} = G_{amp} \cdot G_{mult} \cdot G_{LPF} \cdot \frac{\hat{u}_i \cdot \hat{u}_r}{2} \cdot \sin(\varphi)$$
$$\Delta\varphi = \frac{1}{G_{amp} \cdot G_{mult} \cdot G_{LPF}} \cdot \sqrt{u_{oF,I}^2 + u_{oF,Q}^2}$$
$$\hat{u}_i = \frac{1}{G_{amp} \cdot G_{mult} \cdot G_{LPF}} \cdot \sqrt{u_{oF,I}^2 + u_{oF,Q}^2}$$
$$\varphi = \tan^{-1} \left(\frac{u_{oF,Q}}{u_{oF,I}} \right)$$

DACs

Prerequisite for sampling is the Nyquist theorem to be able to perfectly reconstruct a band-limited signal. $f_s \geq f_N = 2 \cdot f_b$



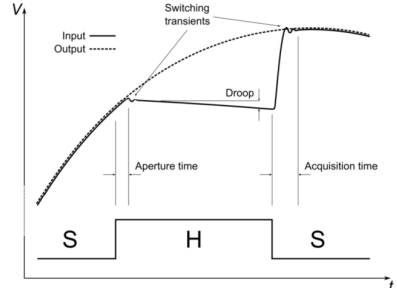
Sample and hold:



Sample: S is closed $V_{out} = V_{in}$
Hold: S is open C holds V_{out}

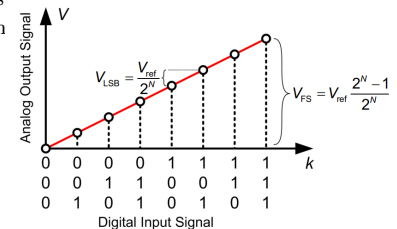
IC_1 with small Z_1 for fast charge of C
 IC_2 with Z_{in} for slow discharge of C
S with small R_{on}

Characteristics:



Aperture time: time for switch to open
Droop: discharge of capacitor
Acquisition time: time to switch and charge capacitor
Switching transients: voltage buffer ringing
Design C:
Large for small droop, small for fast charge
Charge depends on R_{on} and Z_{out} or $I_{out,max}$ of IC_1
Large enough for sufficient small $\frac{kT}{C}$

Static Errors



Gain Error: $V_{gain,error} = V_{MSB,avg} - V_{MSB,ideal}$
Offset: $V_{OS} = V(000)$
Does not affect linearity, easy to compensate
Diff nonlin (DNL):
Measure of nonuniformity, quantifies for each of the k binary input combinations the deviation of each step from the ideal stepsize of one LSB
 $DNL(k) = \frac{V(k) - V(k-1) - V_{LSB}}{V_{LSB}} = \frac{\Delta V(k)}{V_{LSB}}$
If DNL smaller than -1 the output is smaller than the previous and it's non-monotonic
Monotonicity is critical for feedback: turns negative feedback into positive feedback
Occurs most often when switching the MSB
Integral nonlin (INL): Deviation of the output val from the ideal val:
 $INL(k) = \frac{V(k) - V_{ideal}(k)}{V_{LSB}}$

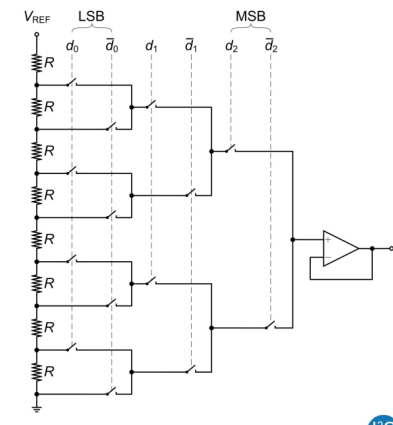
Dynamic Errors

Jitter: Max jitter (Δt) for error below one lsb:
 $\Delta t < \frac{V_{LSB}}{\pi f_{MAX} V_{FS}} \approx \frac{1}{2^N \pi f_{MAX}}$
Glitches:

Turn on and turn off time not precisely synchronized. In the moment of switching one bit to another the value can briefly be both or none of the bits.

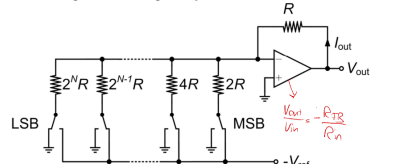
Implementation

Resistorstring:

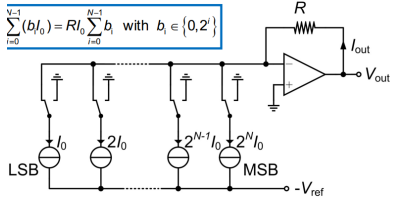


Simple voltage divider, inherently monotonic, amount of resistors proportional 2^N , Area $A \propto W_R \cdot L_R \cdot 2^N$

Binary weighted resistive voltage divider
Inverting summing amplifier



For each bit i : $V_{out,i} = d_i \cdot \frac{V_{ref}}{2^{i+1} R} = d_i \cdot \frac{1}{2^{i+1}} \cdot \frac{V_{ref}}{R}$
 $V_{out} = \sum_{i=0}^{N-1} V_{out,i} = \frac{V_{ref}}{R} \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}}$
 $V_{ref} \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}}, d_i \in \{0, 1\}$
Binary weighted current sources:
 $V_{out} = I_{out} R = R \sum_{i=0}^{N-1} (b_i I_0) = R I_0 \sum_{i=0}^{N-1} b_i$ with $b_i \in \{0, 2^i\}$
Sources can be implemented as current mirror



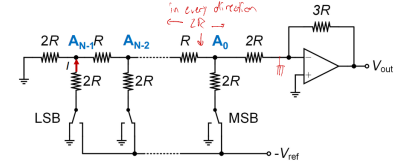
Binary DACs: few number of components, large ratios of resistors, currents and switches, monotonicity not guaranteed, not linear, bas acc, need precise matching, prone to glitches, large portion switches.
Thermometer wighted:
Current steering DAC, reduced glitches, monotonic, 2^N sources, binary to thermometer decoder needed

D	b1	b2	t2	t1	t0
0	0	0	0	0	0
1	0	1	0	0	1
2	1	0	0	1	1
3	1	1	1	1	1

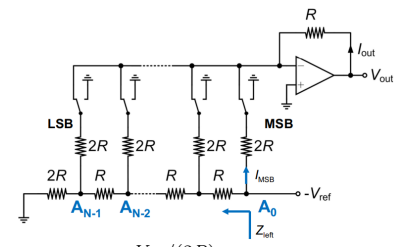
Same

topology as binary weighted but sources are not weighted just I_0

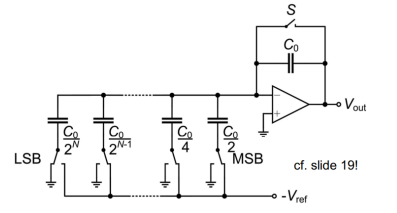
R2R-Ladder:



Branch current:
 $I = -\frac{V_{ref}}{2R + (2R \parallel 2R)} = -\frac{V_{ref}}{3R}$
 $V_{out,i} = -d_i \cdot \frac{I}{2^{i+1}} \cdot 3R = V_{ref} \cdot d_i \cdot \frac{1}{2^{i+1}}$
 $V_{out} = \sum_{i=0}^{N-1} V_{out,i} = V_{ref} \cdot \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}}$
Inverse R2R-Ladder:



$V_{out,i} = -d_i \cdot \frac{V_{ref}}{2^i} \cdot R = V_{ref} \cdot d_i \cdot \frac{1}{2^{i+1}}$
 $V_{out} = \sum_{i=0}^{N-1} V_{out,i} = \frac{V_{ref}}{R} \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}}$
 $V_{ref} \cdot \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}}$
Bin weighted Cap. volt divider:
 $V_{out,i} = d_i \cdot \frac{C_0 / 2^{i+1}}{C_0} \cdot V_{ref}, d_i \in \{0, 1\}, i = 0, \dots, N-1$
 $V_{out} = \sum_{i=0}^{N-1} V_{out,i} = V_{ref} \sum_{i=0}^{N-1} d_i \cdot \frac{1}{2^{i+1}}$
where $d_i \in \{0, 1\}$ Capas are weighted in the denominator, but otherwise topology same as



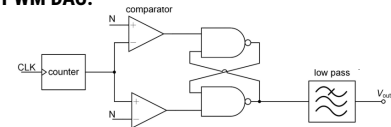
Realization with less capas:
- Phase 1: reset MSB
- Phase 2: load (here: 10000000) MSB
- All input capacitors (except the first one) have zero potential on both sides
cf. slide 19!

$$\sum_{i=1}^{N_{right}} d_{i,right} \text{ with } d_{i,right} \in \{0, 2^{-i}\}$$

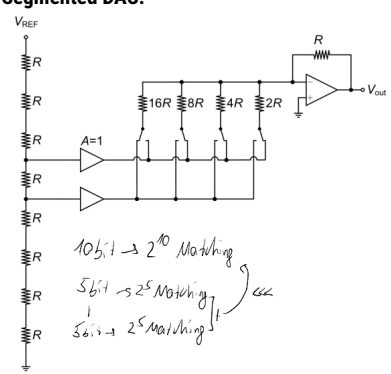
$$V_{out} = \frac{\alpha_{left}}{16} V_{ref}$$

$$\alpha_{left} = \sum_{i=1}^N d_{i,left} \text{ with } d_{i,left} \in \{0, 2^{-i}\}$$

PWM DAC:

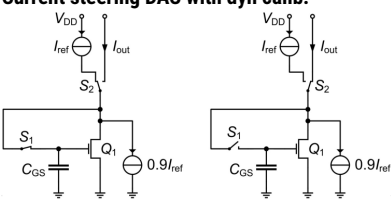


Segmented DAC:



Course DAC(MSBs) feeds fine DAC (LSBs), matching needs to be instead 2^{10} for 10 bit only $2 \cdot 2^5$

Current steering DAC with dyn calib:



Each source supposed to have I_{ref} , Assume one only has $0.9I_{Ref}$
 Calib phase: V_{GS1} settles so that Q_1 draws $0.1I_{Ref}$
 Operat. Phase: Q_1 calibs. so that $I_{d1} = I_{ref}$