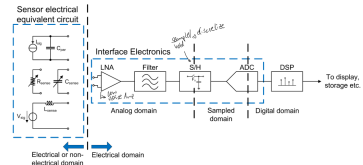


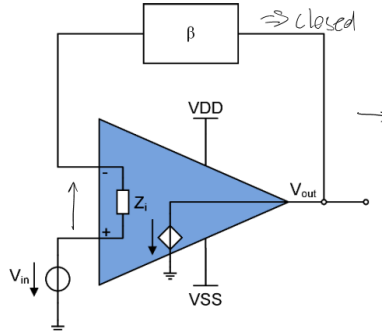
Sensor Principles

A generic sensor interface



OPAMP Basics

Opamps in feedback



Gain: $T(\omega) = \beta \cdot A_0(\omega)$
 Phase Margin:
 $PM = 180^\circ + \angle(T(\omega_1)) |_{|T(\omega_1)|=1}$ Gain margin: $GM = 20 \cdot \log_{10} \left(\frac{1}{|T(\omega_2)|} \right) |_{\angle(T(\omega_2)) = -180^\circ}$ **Generic**

Transfer function

$$V_x = \beta \cdot V_{out} = \beta \cdot A(\omega) \cdot (V_{in} - V_x) = \beta \cdot A(\omega) \cdot (V_{in} - \beta \cdot V_{out})$$

$$\frac{V_{out}}{V_{in}} = \frac{A(\omega)}{1 + \beta \cdot A(\omega)} \approx \frac{1}{\beta} \quad \text{Negative}$$

Feedback and linear operation

$A(\omega) \gg 1 \Rightarrow$ virtual short at the input
 $\Delta V \approx 0$

$Z_i \rightarrow \infty \Rightarrow i_{oa} \approx 0$

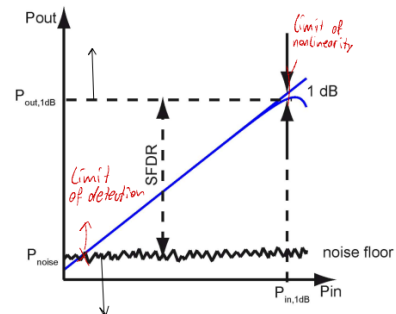
Voltage Drive \rightarrow negative feedback

Current drive \rightarrow positive feedback

Errors and Noise

Limit of detection (LOD): minimum measurable input amplitude ($SNR \approx 0$)

Dynamic range (DR): ratio of max and min amplitude within inaccuracy levels.



Error types:

Deterministic: source loading, offset, gain error
 Random: thermal noise, 1/f noise

Quantification:

$$\text{Absolute: } \Delta x = |\hat{x} - x_0|$$

$$\text{Relative: } \left| \frac{\Delta x}{x_0} \right| = \left| \frac{\hat{x} - x_0}{\hat{x}} \right|$$

Max inaccuracy:

$$\Delta x_{max} | x \in [\hat{x} - \Delta x_{max}, \hat{x} + \Delta x_{max}]$$

Error Propagation

$$y = f(x_1, x_2, \dots, x_N)$$

Deterministic fluctuations of $x_i \rightarrow$ total error:

$$\Delta y \approx \sum_{i=1}^N \frac{\partial f}{\partial x_i} \cdot \Delta x_i$$

Partial derivative $\frac{\partial f}{\partial x_i}$ is called **sensitivity**

Additive errors are best specified absolute and multiplicative errors are best specified relative

Interference:

Unwanted coupling of external signal

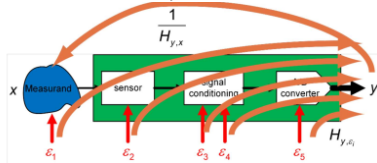
Noise: random fluctuations from setup \rightarrow can be modeled as error sources

Combining Error sources

Output referred noise

Effect of an error-source on the output

Input referred noise Equivalent effect of the error-source on the input



Find TF from ES to output

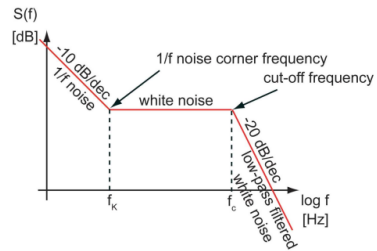
$$y: H_{y,ES} \Rightarrow y_{out,e2} = H_{y,e2} \cdot e_2$$

Refer result back to input wit

$$H_{y,x}: x_{e2} = \frac{y_{out,e2}}{H_{y,x}} = H_{y,e2} \cdot \frac{e_2}{H_{y,x}}$$

Lin. System noise:

$$PSD = S_y(f) = |H(f)|^2 \cdot S_x(f)$$



Noise Types

Thermal noise: excitation of charge carriers (white)

Shot noise: carriers randomly crossing the barrier, dependent on DC bias and white

Flicker Noise: due to traps in semiconduct. 1/f spectral density. MOS trans at low freq.

Thermalnoise Theorem: Every closed system at temp. T has average. Energy of $kT/2 \rightarrow$

$$S_u(f) = 2kTR \text{ (double-sided) } \text{ math}$$

$$S_u(f) = 4kTR \text{ (single-sided) } \text{ physics}$$

Langevin Approach kt/C noise

PSD noise voltage of V_n

$$= S_{v_n}(f) = 4kT \cdot \Re\{Z(j2\pi f)\}$$

$$\rightarrow \overline{V_n^2} = kT \left[\frac{1}{C_\infty} - \frac{1}{C_0} \right] = \frac{kT}{C}$$

MOS IRN

$$S_{\Delta V_{nG-tot}} = 4kT \cdot R_{nG-tot}, \quad R_{nG-tot} =$$

$$\frac{\rho}{W \cdot L \cdot f} + \frac{\gamma_n D}{G_m} \text{ with GateExcess Noise factor}$$

$$\gamma_n D = (n \cdot 1.3) \cdot [0.5 W I; 2/3 S I]$$

Bipolar Trans IRN:

$$S_{\Delta V_{nR}}^2 = 4kT \cdot R_B$$

Noise Analysis

small-signal-equivalent is valid.

Total ORN:

$$S_{n,out}(f) = \sum_{k=1}^N |H_k(f)|^2 \cdot S_{n_k}(f)$$

N uncorrelated NS.

$H_k(f)$ TF from NS ton output. IRN:

$$S_{V_{eq,IRN}}(f) = \frac{S_{V_{out}}(f)}{|A(f)|^2} \text{ with } A(f) \text{ is TF}$$

Sensor types

Information domain \rightarrow Electrical domain

Transduction: Converting a signal from the energy domain into another.

Sensors and actuators are transducers

Sources of error

Noise, sensitivity to unintended quantities,

Noise, EMI

$$\begin{pmatrix} E_{meas} \\ E_{sens} \\ E_{cond} \\ E_{conv} \\ E_{out} \\ E_{enc} \end{pmatrix} = \begin{pmatrix} \alpha_{meas,meas} & \alpha_{meas,sens} & \alpha_{meas,cond} & \alpha_{meas,conv} & \alpha_{meas,out} & \alpha_{meas,enc} \\ \alpha_{sens,meas} & \alpha_{sens,sens} & \alpha_{sens,cond} & \alpha_{sens,conv} & \alpha_{sens,out} & \alpha_{sens,enc} \\ \alpha_{cond,meas} & \alpha_{cond,sens} & \alpha_{cond,cond} & \alpha_{cond,conv} & \alpha_{cond,out} & \alpha_{cond,enc} \\ \alpha_{conv,meas} & \alpha_{conv,sens} & \alpha_{conv,cond} & \alpha_{conv,conv} & \alpha_{conv,out} & \alpha_{conv,enc} \\ \alpha_{out,meas} & \alpha_{out,sens} & \alpha_{out,cond} & \alpha_{out,conv} & \alpha_{out,out} & \alpha_{out,enc} \\ \alpha_{enc,meas} & \alpha_{enc,sens} & \alpha_{enc,cond} & \alpha_{enc,conv} & \alpha_{enc,out} & \alpha_{enc,enc} \end{pmatrix} \begin{pmatrix} E_{meas} \\ E_{sens} \\ E_{cond} \\ E_{conv} \\ E_{out} \\ E_{enc} \end{pmatrix} + \begin{pmatrix} E_{meas,ext} \\ E_{sens,ext} \\ E_{cond,ext} \\ E_{conv,ext} \\ E_{out,ext} \\ E_{enc,ext} \end{pmatrix}$$

Tandem transducers: Multiple steps to target domain.

cross-sensitivity, sensitivity to undesired quantity

Sensor classification:

Active Sensors:

Require external source of excitation

Passive / self-generating sensors:

Generate their own electrical output signal

Draws all required energy from the measurand (source loading) E: Potentiometer

for angle measurements. **Modulating sensors:**

Measure desired quantity by modulating

Additional source with modulated energy. Also

adds error. E: Noncontact displacement measurement (rotating disk)

Analog vs. Digital:

Analog: time and value continuous

Digital: Discrete outputs

Deflection mode sensors:

Response to an output is a deviation from the equilibrium position

Null mode sensors:

Sensor or instruments exert an influence the measured system opposing the effect of the measurand. Ideally the result is a 0

measurement, typically achieved by feedback.

The opposing influence is then the sensor

output. Slower than deflection, but more accurate.

Resistive sensors - strain gauges

Change in geometry under mechanical stress produces associated resistance change

Volum. = const. \rightarrow

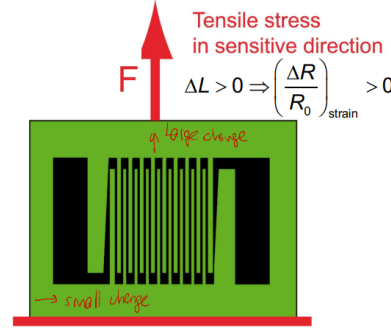
$$\frac{\partial V}{V_0} = \frac{L_0}{V_0} \cdot \partial A + \frac{A_0}{V_0} \cdot \partial L \wedge \frac{\partial V}{V_0} = 0 \Rightarrow$$

$$\frac{\partial A}{A_0} = -\frac{\partial L}{L_0} \rightarrow \frac{\partial R}{R_0} = \frac{\partial \rho}{\rho_0} + 2 \frac{\partial L}{L_0}$$

$$\rightarrow \frac{\partial R}{R_0} = \alpha \cdot \frac{\partial L}{L_0} + 2 \frac{\partial L}{L_0} = (\alpha + 2) \cdot \frac{\partial L}{L_0} \triangleq k$$

k: gauge factor

α proportionality factor $\frac{\partial \rho}{\rho_0} \propto \frac{\partial L}{L_0}$



Readout resistive sensors

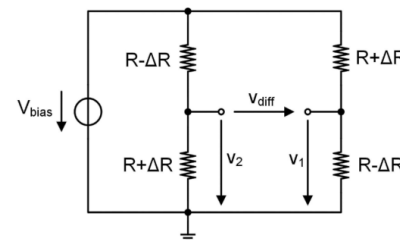
Use a half bridge resistive divider top element sensing resistor

$$\frac{v_{out}}{V_{bias}} = \frac{R}{2R + \Delta R} \Leftrightarrow \frac{\Delta R}{R} = \frac{V_{bias}}{v_{out}} - 2$$

Full bridge to remove offset, 2 sensing elements more sensitive,

remove nonlinearity by implementing differential measurements

Best use 4point full bridge



$$V_{diff} = v_2 - v_1 = \frac{\Delta R}{R} \cdot V_{bias}$$

Sensitivity $S = 45mV/V$ excitation for 1V input

$$\text{Accuracy } A = \frac{v_{diff}(\frac{\Delta R}{R}) - V_{diff,lin}(\frac{\Delta R}{R})}{V_{diff}(\frac{\Delta R}{R})}$$

Deviation from the ideal bridge

BridgeParameters

Bridge resistance: Unloaded R across the signal terminals

Offset error Output voltage at 0 input

Drift: Output change conditioned on environmental condition

Differential/ Commonmode signals

$$\text{Recall: } V_1 = \frac{1}{2} \cdot \left(1 - \frac{\Delta R}{R} \right) \cdot V_{bias}, \quad v_2 =$$

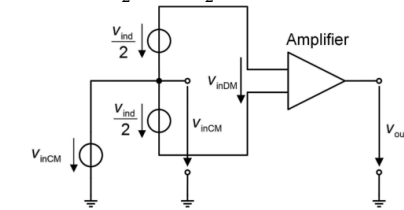
$$\frac{1}{2} \cdot \left(1 + \frac{\Delta R}{R} \right) \cdot V_{bias}$$

Differential signal:

$$V_{diff} = V_2 - V_1 = \frac{\Delta R}{R} \cdot V_{bias}$$

Common-mode signal:

$$V_{CM} = \frac{v_2 + v_1}{2} = \frac{V_{bias}}{2}$$



DiffGain: $V_{out} = A_{DM} \cdot V_{indDM}$

CMMGain: $V_{out} = A_{CM} \cdot v_{inCM}$

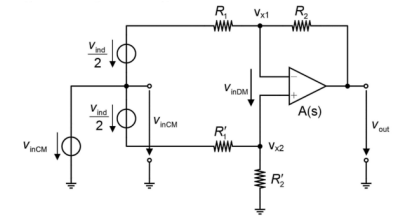
Ideally A_{cm} is 0 or $A_{cm} \ll A_{dm}$

Commonmode rejection ratio (CMRR):

$$CMRR \triangleq \left| \frac{A_{DM}}{A_{CM}} \right| = \left| \frac{dv_{out}/dv_{indDM}}{dv_{out}/dv_{inCM}} \right|$$

Often in dB

Opampbase difference aplifier:



1st: $A(s) = \infty, R_2 = R'_2 = \alpha \cdot R_1 = \alpha R'_1$

$A_{DM} = -\alpha, A_{CM} = 0, CMRR = \infty$

2nd assume $A(s) = \infty, R_2 = \alpha \cdot R_1,$

$R'_2 = (\alpha + \Delta\alpha) \cdot R'_1, R'_1 = R_1 + \Delta R$

$$A_{DM} = -\left(\alpha + \frac{\Delta\alpha}{2 \cdot (1 + \alpha + \Delta\alpha)} \right)$$

$$A_{CM} = \frac{\Delta\alpha}{1 + \alpha + \Delta\alpha}$$

$$CMRR = -\frac{1}{2} + \frac{\alpha \cdot (1 + \alpha + \Delta\alpha)}{\Delta\alpha}$$

$$3d: A(s) = A_{DC} / \left(1 + s \cdot \frac{A_{DC}}{GBW} \right)$$

$$A_{DM} = -\left(\alpha + \frac{\Delta\alpha}{2 \cdot (1 + \alpha + \Delta\alpha)} \right) \cdot$$

$$\frac{1}{1 + \frac{1}{A_{DC}} + \frac{\alpha}{A_{DC}} + (1 + \alpha) \cdot \frac{s}{GBW}}$$

$$A_{CM} = \frac{\Delta\alpha}{1 + \alpha + \Delta\alpha} \cdot \frac{1}{1 + \frac{1}{A_{DC}} + \frac{\alpha}{A_{DC}} + (1 + \alpha) \cdot \frac{s}{GBW}}$$

$$CMRR = -\frac{1}{2} + \frac{\alpha \cdot (1 + \alpha + \Delta\alpha)}{\Delta\alpha}$$

Magnetic Field Sensors

$$V_{hall} = G \cdot \frac{\mu \cdot \rho}{h} \cdot I \cdot B = G \cdot \frac{1}{e \cdot n \cdot h} \cdot I \cdot B =$$

$S_L \cdot I \cdot B$ G: geometry factor **Noise sources:**

- Thermal noise

- 1/f noise

- Noise of conditioning electronics

Noise power: $V_{noise} \sqrt{4kT \cdot R \cdot \Delta f}$

Signal power: $V_{hall} S_L \cdot I \cdot B$

Resolution: $B_{\min} \hat{A} \frac{V_{\text{noise}}}{V_{\text{hall}}/B} = \frac{\sqrt{4kT \cdot R \cdot \Delta f}}{S_I \cdot I}$

With $S_I = \frac{\mu \cdot \rho}{h \cdot e \cdot n} \cdot G$

At $B = 0$ Hall plate can be modeled as wheatstone bridge, non-uniform stress causes intrinsic offset(5 to 30mT)

Cancel intrinsic offset by coupling ° turned sensors, current and sense ports are swapped

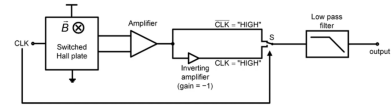
Idea: Hall voltage is in phase and gradient caused offsets are than 180° out of phase

Spinning current method:

Sense and current ports are switched every clock cycle

Idea: A DC component equal to the offset and a frequency component proportional to the hall voltage is produced (**Fourier Expansion of a square wave**)

By modulating the information to the clock frequency noise and offset are greatly mitigated.



Multiplying by 1 and -1 to demodulate back to low frequency.

Low pass stage to remove signal around the 2nd harmonic

This allow to see the base noise floor of the sensor not 1/f noise.

In the conventional implementation 2 bias currents are used in the low power configuration the bias form the plate is reused for the amplifier.

Temperature Sensors

Seebeck effect: Convert temp. gradient to electromotive force

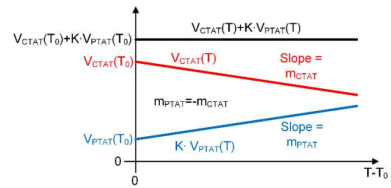
$$F(T) \doteq \int_{T_{\text{stand}}}^T (S_+(T') - S_-(T')) dT'$$

$$\rightarrow E_{\text{emf}} = F(T_{\text{sense}} - F(T_{\text{ref}}))$$

Temperature stable references:

$$V_{\text{REF}}(T) = V_{\text{CTAT}}(T) + K \cdot V_{\text{PTAT}}(T)$$

Bandgap reference:



PTAT: Proportional to absolute temperature

CTAT: Complementary to absolute temperature

Scale one slope and add the function for a temp. constant reference

This can be built with a diode or a BJT with a shorted base and collector

Butt current is also temperature dependent so use diff pair.

$$V_{\text{PTAT}} = \Delta V_{\text{EB}} = V_{\text{EB1}} - V_{\text{EB2}} \approx \frac{kT}{q} \cdot \ln \left[\frac{I_{q1} \cdot I_{S1} \cdot A_{E2}}{I_{S1} \cdot A_{E1} \cdot I_{q2}} \right] = \frac{kT}{q} \cdot \ln \left[\frac{A_{E2}}{A_{E1}} \right]$$

Scale currents or diodes to make equal bias

$$V_{\text{EB}} = \underbrace{E_g/q}_{V_G} \cdot \underbrace{\frac{kT}{q}}_{U_T} \cdot \ln \left(\frac{I'_S}{I_D} \right) = V_G - U_T \cdot \ln \left(\frac{K_1 \cdot T^\gamma}{I_D} \right)$$

$$V_G \approx V_{G0K} - a \cdot T$$

With $I'_S > I_D$ but also temperature dependent → curved CTAT.

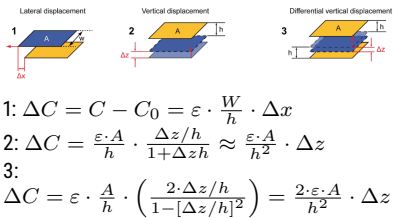
Combine CTAT and PTAT: Series:

$$V_{\text{REF}} = I_{\text{PTAT}} \cdot R_2 + V_{\text{CTAT}} = \frac{R_2}{R_1} \cdot V_{\text{PTAT}} + V_{\text{CTAT}}$$

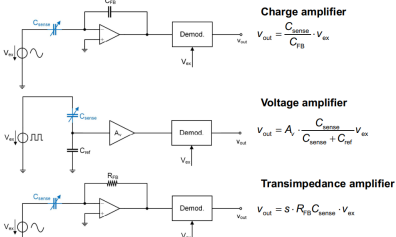
Parallel:

$$V_{\text{REF}} = (I_{\text{PTAT}} + I_{\text{CTAT}}) \cdot R_3 = \frac{R_3}{R_1} \cdot V_{\text{PTAT}} + \frac{R_3}{R_2} \cdot V_{\text{CTAT}}$$

Capa. Sensor Readout



OpenLoop readout:



ChargeAmp: $V_{\text{out}} = \frac{C_{\text{sense}}}{C_{\text{FB}}} \cdot V_{\text{ex}}$

Nonideal: $\frac{V_{\text{out}}}{V_{\text{in}}} = - \frac{j\omega \cdot R_{\text{FB}} C_{\text{in}}}{(1 + \text{GBW}) \cdot (1 + \frac{j\omega}{\omega_{\text{FB}}})}$

VoltAmp: $v_{\text{out}} = A_v \cdot \frac{C_{\text{sense}}}{C_{\text{sense}} + C_{\text{ref}}} V_{\text{ex}}$

TransImpAmp: $v_{\text{out}} = s \cdot R_{\text{FB}} C_{\text{sense}} \cdot v_{\text{ex}}$

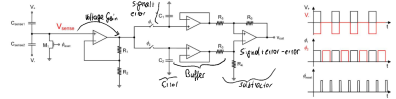
V_{Sense} not grounded sensitive to leakage currents

Bias resistor provides well defined DC-point.

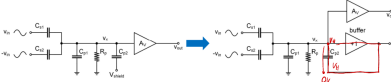
R_{Bias} needs to be large because of the highpass forming, corner frequency needs to be low enough compared to excitation freq.

Periodic reset can also be used where $V_{p+} = V_{p-} = 0$

Correlated double sampling can also be used:



Bootstrapping: VoltAmp



$C_{s1} = C_{s0} + \Delta C/2$ $C_{s2} = C_{s0} - \Delta C/2$

v_{out} becomes sensitive to parasitic capacitance

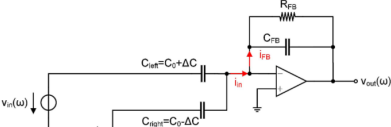
Can be bootstrapped out with voltage buffer driving V_{shield}

$$v_{\text{out}} = \frac{\Delta C}{2C_{s0} + C_{p1} + (C_{p2})} \cdot A_v \cdot v_{\text{in}} \quad C_{p2}$$

gets removed by V_{shield}

Can also use TIA, the virtual ground helps with the parasitic capacitance, this also works in the charge amplifier

DiffReadoutChargeAmp:



Req Vex:

Error will propagate to output directly

Accuracy more important for small ΔC

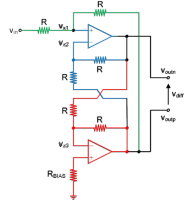
Mismatch or drift will introduce errors

Sinewave can be created with center-tapped transformer or active balun

Rectangular witch switched excitation schemes

DiffReadoutChargeAmp:

- Assuming ideal opamps ($A_{\text{op}} \rightarrow \infty$)
 $\Rightarrow V_{\text{d2}} = V_{\text{d1}}, V_{\text{d3}} = 0$
- The part of the circuit highlighted in red is a simple inverting amplifier with input voltage v_{out1} and output voltage v_{out2}
- The part of the circuit highlighted in blue is a simple non-inverting amplifier with input voltage v_{d1} and with its reference potential pulled to v_{out2}
- The part of the circuit highlighted in green is a voltage divider between v_{in} and v_{out2}

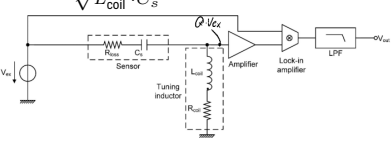


$$V_{\text{outp}} = V_{\text{in}} = -V_{\text{outn}}$$

Differential Capa sensing:

$$\Delta v_{\text{x, amp}} \approx - \frac{\Delta x}{x_0} \cdot V_{\text{ex}}$$

Resonant readout:



DoubleDiffReadout:

$$\Delta V_{\text{out}} = -\Delta V_{\text{ex}} \cdot \left(\frac{\Delta C_s}{C_i} \cdot \left[1 - \frac{C_s}{C_s + C_i + C_p} \right] - \frac{\Delta C_i + \Delta C_p}{C_i} \cdot \frac{C_s}{C_s + C_i + C_p} \right)$$

gain error offset error

Differential Measurements

- Inherent rejection to common-mode interference and noise
- Wider Signal swing for a given supply voltage

- Minimum effect of even order distortion including DC offset

- Can lead to improved sensor linearity

But respond to some degree to common mode signal

Ideal DiffPair:

$$\text{CMMR} \triangleq 20 \cdot \log \left(\left| \frac{A_{\text{DM}}}{A_{\text{CM-to-DM}}} \right| \right) = 20 \cdot \log \left(\frac{2G_{\text{mav}} \cdot r_{\text{out}}}{\Delta R_L / R_{\text{Lav}} + \Delta G_{\text{m}} / G_{\text{mav}}} \right)$$

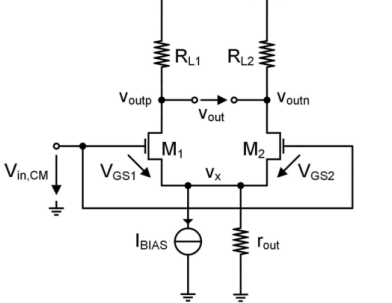
$$\text{PSRR} \triangleq 20 \cdot \log \left(\left| \frac{A_{\text{DM}}}{A_{\text{VDD}}} \right| \right) = 20 \cdot \log \left(\left| \frac{A_{\text{DM}}}{\Delta V_{\text{out}} / \Delta V_{\text{DD}}} \right| \right)$$

Process variation causes mismatch between resistors and current source is not ideal

Input referred offset is the voltage needed to get zero volts differential output

IRO: $\Delta V_G = V_{\text{OS}} = - \frac{I_{\text{DAV}}}{G_{\text{mav}}} \cdot \left(\frac{\Delta R_L}{R_{\text{Lav}}} + \frac{\Delta \beta}{\beta_{\text{AV}}} \right) - \Delta V_{\text{TO}}$

OffsetVoltage: 10mV **MOS**; 1mV **BJT**, 120 μ , trimmed BJT



$$\text{CMRR} \cdot V_{\text{os}} \approx G_{\text{mav}} \cdot r_{\text{out}} \cdot V_{\text{OV}}$$

$$V_{\text{OV}} = \frac{I_{\text{DAV}}}{G_{\text{mav}}}$$