Structural Analysis of Complex Geometries Using the Finite Cell Method

Jan Niklas Schmäke - jan.schmaeke@hs-duesseldorf.de

HSD University of Applied Sciences Düsseldorf
Mechanical & Process Engineering

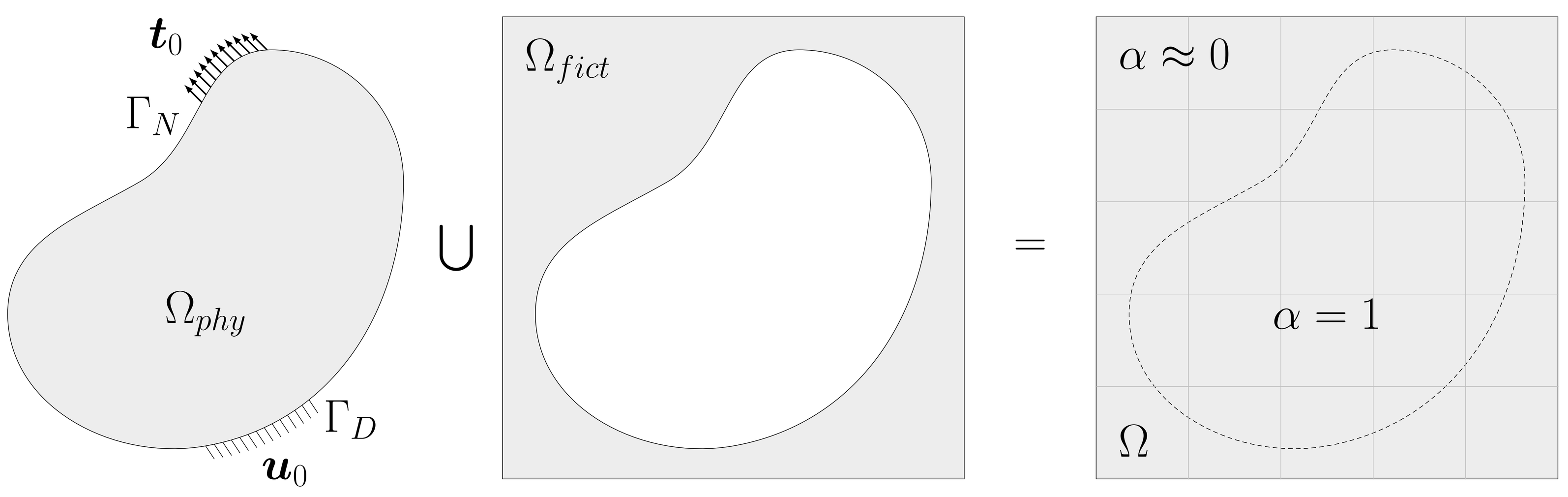
1. Introduction

Even though the **Finite Element Method** (FEM) has become one of the most essential tools of modern engineering and science, the current workflow contains a major **bottleneck**:

- **Pre-processing and meshing** of CAD geometries takes up to **80% of the analysis time** [2].
- Models which are not based on CAD data, like **CT-Scans**, **voxel geometries** or **point clouds** have to be **converted into a surface representation** first.

2. The Finite Cell Method

The Finite Cell Method (FCM), presented by Parvizian *et. al.* [5, 6], works around a complex mesh generation by **embedding the original problem** domain Ω_{phy} into a fictitious one Ω_{fict}. The result is a domain Ω, for which the mesh creation is trivial.

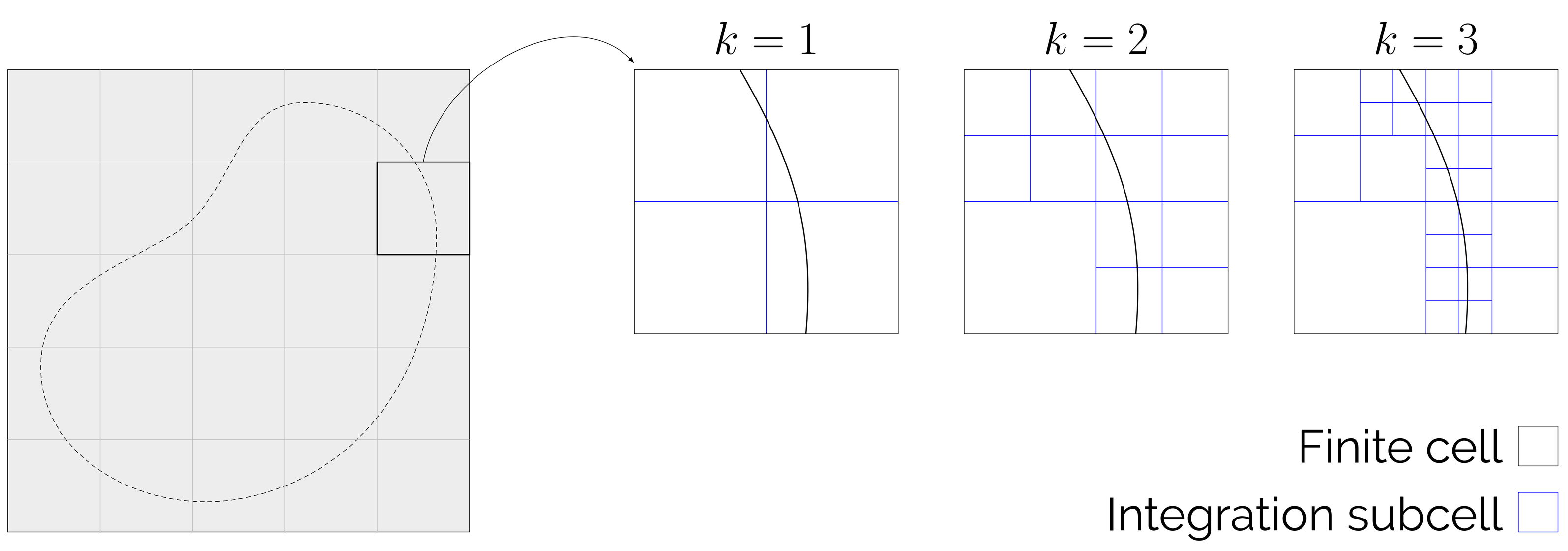


Concept of the Finite Cell Method

To diminish the influence of the fictitious part, the weak form of the problem is modified using an **indicator function** $\alpha(\boldsymbol{x})$ which is equal to one in the physical and approximately zero in the fictitious domain:

$$\mathcal{B}(\boldsymbol{u}, \boldsymbol{v}) = \int_{\Omega} \alpha(\boldsymbol{x}) \cdot \boldsymbol{\varepsilon}(\boldsymbol{v}) : \boldsymbol{C} : \boldsymbol{\varepsilon}(\boldsymbol{u}) \, d\Omega. \quad (1)$$

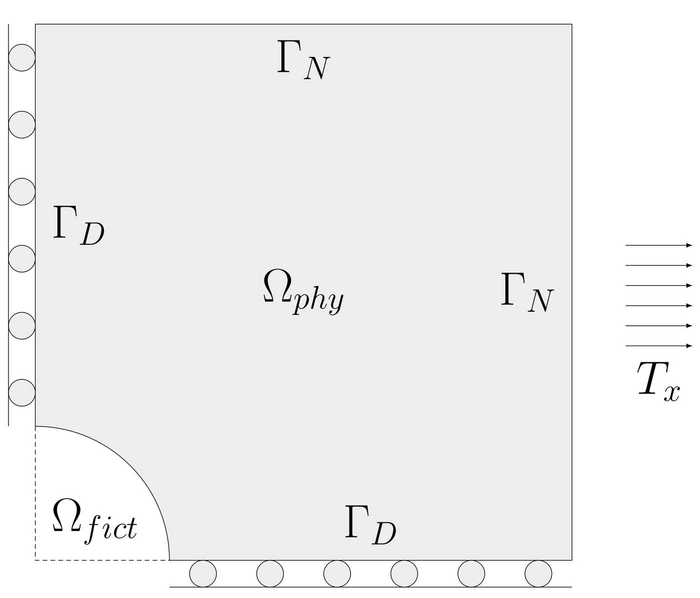
Since the integration is usually performed numerically and the integrand has become non continuous due to the modification, the boundary is approximated using a **recursive bisection** of the domain.

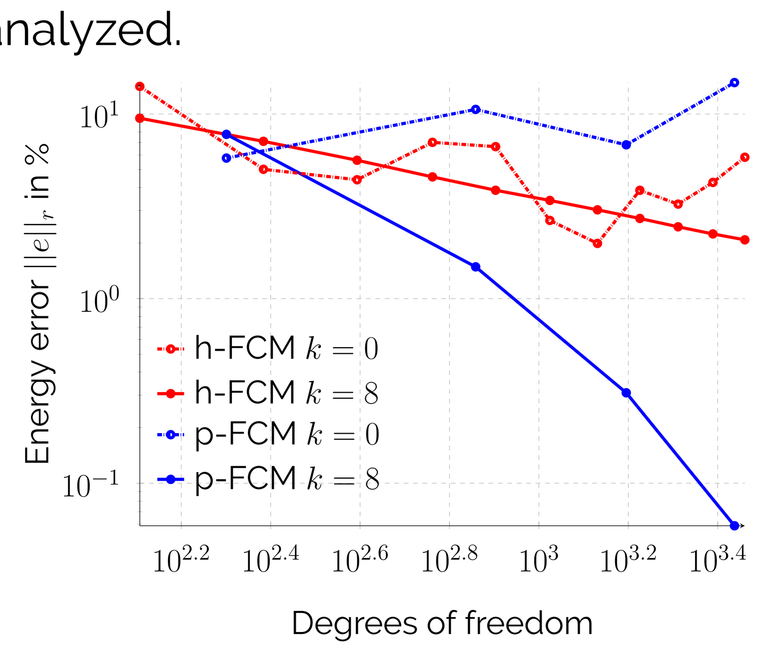


Recursive division of a cut cell into subcells

3. Validation

To examine the methods accuracy, a commonly used **benchmark problem** from [4] has been implemented and analyzed.





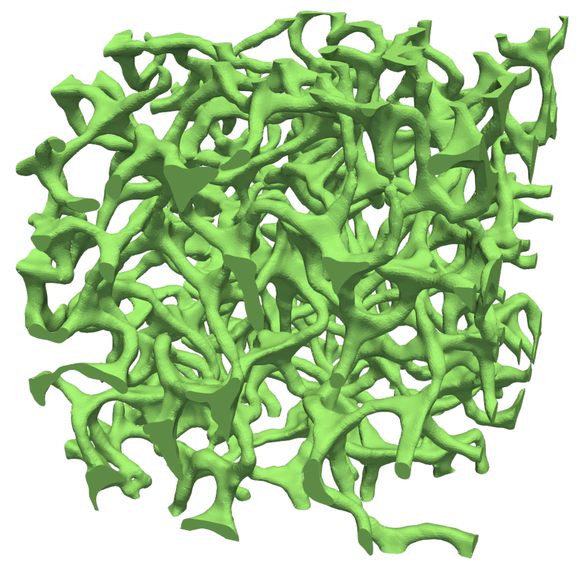
a) *Benchmark problem setup* **b**) *Error reduction with increasing DOF*

The results show, that the model **converges towards the analytical solution** if the number nodes is gradually increased and shows a similar convergence behavior compared to the classic FEM.

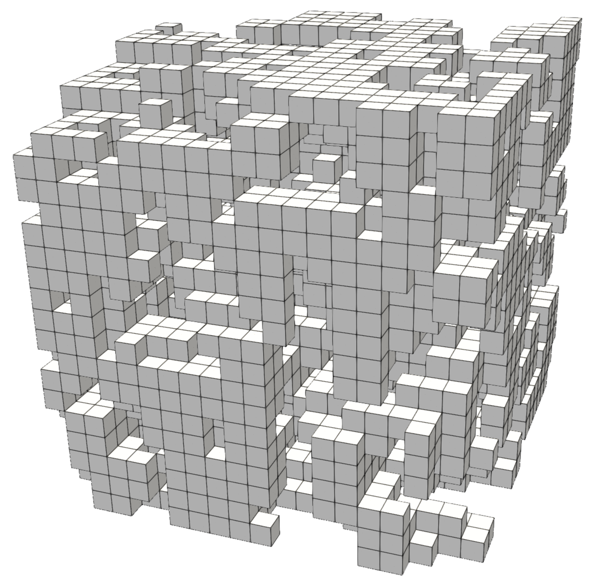
4. Application

To demonstrate the methods capabilities in the real world, a model of an very complex **metallic foam** is analyzed (available at [1]). The geometry is a $1\,\mathrm{mm}^3$ cut out from an larger plate and analyzed in the case of a **virtual compression test**. All faces except the top on are clamped, while to top boundary has a prescribed compression of 0.01 mm.

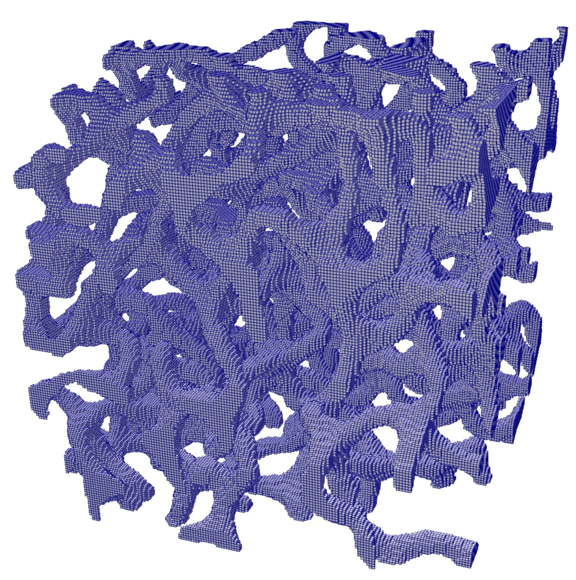
To analyze the model it is first embedded into a cubic domain which is then meshed using hexahedral cells **b** (The ones not in touch with the foam have been removed).



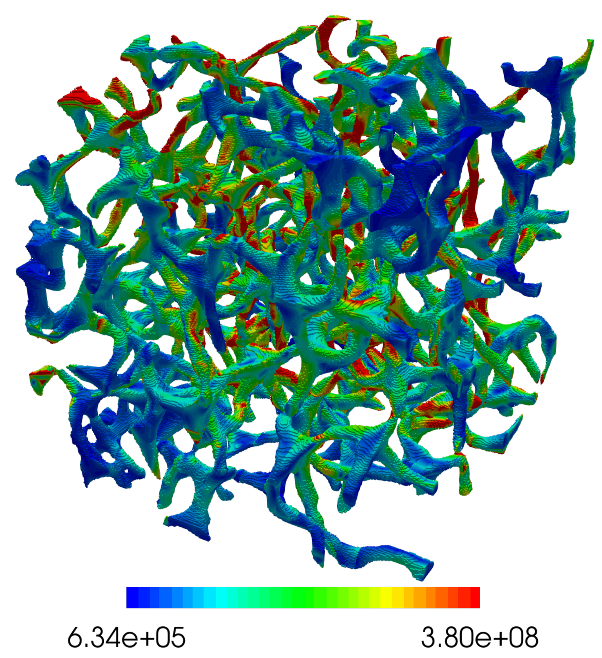
a) *STL from CT-Scan*



b) *Reduced Mesh*



c) *Integration sub-cells $k = 3$*



d) *Stressfield solution*

The final model contained about $50,000$ degrees of freedom which is about a third compared to which one would get with an classic FEM mesh.

5. Conclusion

Using the Finite Cell Method:

- Geometries **don't have to be meshed** to perform an analysis.
- Models which are not based on surface data **don't have to be converted**.
- **Similar error and convergence behavior** compared to the **classic FEM** is possible.

The concept already has a broad community in the field of research with applications in e.g. **solid**- [7, 9], **fluid**- [8] and **biomechanics** [3] and also sees the potential to gain further interest in the industry and commercial software.

References

[1] M. Chadband. Auxetic foam sample. GrabCAD, May 2014. last access 23.05.2022 18:54.

[2] J. A. Cottrell, T. J. Hughes, and Y. Bazilevs. *Isogeometric analysis: toward integration of CAD and FEA*. John Wiley & Sons, 2009.

[3] M. Elhaddad, N. Zander, T. Bog, L. Kudela, S. Kollmannsberger, J. Kirschke, T. Baum, M. Ruess, and E. Rank. Multi-level hp-finite cell method for embedded interface problems with application in biomechanics. *International journal for numerical methods in biomedical engineering*, 34(4):e2951, 2018.

[4] T. J. Hughes, J. A. Cottrell, and Y. Bazilevs. Isogeometric analysis: Cad, finite elements, nurbs, exact geometry and mesh refinement. *Computer methods in applied mechanics and engineering*, 194(39-41):4135–4195, 2005.

[5] J. Parvizian, A. Düster, and E. Rank. Finite cell method. *Computational Mechanics*, 41(1):121–133, 2007.

[6] D. Schillinger and M. Ruess. The finite cell method: A review in the context of higher-order structural analysis of cad and image-based geometric models. *Archives of Computational Methods in Engineering*, 22(3):391–455, 2015.

[7] D. Schillinger, M. Ruess, N. Zander, Y. Bazilevs, A. Düster, and E. Rank. Small and large deformation analysis with the p-and b-spline versions of the finite cell method. *Computational Mechanics*, 50(4):445–478, 2012.

[8] F. Xu, D. Schillinger, D. Kamensky, V. Varduhn, C. Wang, and M.-C. Hsu. The tetrahedral finite cell method for fluids: Immersogeometric analysis of turbulent flow around complex geometries. *Computers & Fluids*, 141:135–154, 2016.

[9] N. Zander, S. Kollmannsberger, M. Ruess, Z. Yosibash, and E. Rank. The finite cell method for linear thermoelasticity. *Computers & Mathematics with Applications*, 64(11):3527–3541, 2012.