Pool simulation concept

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0.1 Concept

My approach to solving this problem is based on fundamental concepts of analytical geometry in the Cartesian plane. It should be noted that the path taken by the ball after a strike is a line segment on a straight line. Knowing two points (the starting point and the point calculated using the starting point and a given vector), we can calculate the equation of this line using the following formula:

$$y - y_a = a(x - x_a)$$

where slope:

$$a = \frac{\Delta y}{\Delta x}$$

Since the table's edge is perpendicular to the coordinate system axes, the slope of the next line (in which the segment the ball will move after a bounce is contained) is the opposite of the slope of the previous line. Using transformed formulas derived from the line equation above:

$$y = a(x - x_a) + y_a$$

$$x = \frac{y - y_a}{a} + x_a$$

we can calculate from which edge the next bounce will occur, substituting either x or y accordingly, then checking if the calculated y or x lies within the bounds of the table calculated using:

$$P = \{(x, y) \in \mathbf{R}^2 : -2 * m \leq x \leq 2 * m, -1 * m \leq y \leq 1 * m\}$$

where m is the table size multiplier provided as a program parameter. By tracking the sign of the current vector coordinates, we know whether the bounce is from the upper or lower edge (for bounces off edges parallel to the X-axis) or from the right or left edge (for bounces off edges parallel to the Y-axis). After bouncing from a given edge, this sign is reversed. This can be represented by a boolean value in implementation.

Knowing two points, we can calculate the length of the segment between them using the formula:

$$|AB| = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2}$$

The calculated length should be subtracted from the current strength of the ball's strike after each bounce, considering a five percent loss of strength.

After each bounce calculation, we need to determine if the ball still has enough strength to cover the required distance. If not, we need to calculate where it will stop. Since we know the segment's length and slope, we can transform the above formulas in this way:

$$x_b^2 + (ax_b)^2 - 2x_a x_b - 2a^2 x_a x_b + (ax_a)^2 - |AB|^2 = 0$$

and

$$y_b = ax_b - ax_a + y_a$$

This quadratic equation will give us two valid points. One of them should be discarded based on the current sign of the relevant vector coordinate and the slope.

Each bounce, as well as the starting and ending points, is recorded in a list, which will later be used to display the ball's path in the terminal. To display each segment, Bresenham's algorithm is used.