USA TFR by sex

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Notation

Suppose that there are A reproductive ages $a = 1 \dots A$ in which either men or women may have children. At some of these ages, notably 50+, male fertility may be possible even though female fertility is not.

Call $\theta \in \mathbb{R}^A$ the vector of age-specific fertility rates for *women*. Define $\mathbf{W} = diag(W_1 \dots W_A)$ as a diagonal matrix with population counts of *women*, and $\mathbf{M} = diag(M_1 \dots M_A)$ as a diagonal matrix with population counts of *men*.

Define an $A \times A$ matrix $\mathbf{P}$ in which

$$P_{ij} = \text{fraction of births to mothers aged } j \text{ that have fathers aged } i$$

Notice that the column sums of $\mathbf{P}$ equal one by definition: $\mathbf{1}'\mathbf{P} = \mathbf{1}'$, where $\mathbf{1}$ is an $A \times 1$ vector of ones.

Age-Specific Births

The vector of births arranged by age of *mothers* is

$$b_{female} = \mathbf{W}\,\theta$$

and the vector of births arranged by age of *fathers* is

$$b_{male} = \mathbf{P}\,\mathbf{W}\,\theta$$

Age- and Sex-Specific Birth Rates

Birth rates for women are age-specific birth/women ratios:

$$f_{female} = \mathbf{W}^{-1}\,b_{female} = \mathbf{W}^{-1}\,\mathbf{W}\,\theta = \theta$$

Birth rates for men are age-specific birth/men ratios:

$$f_{male} = \mathbf{M}^{-1}\,b_{male} = \mathbf{M}^{-1}\,\mathbf{P}\,\mathbf{W}\,\theta$$

Age-Specific Birth Rates and TFRs

With single-year ages, TFR is simply the sum of age-specific rates, so

$$TFR_{female} = \mathbf{1}'\,\theta$$

and

$$TFR_{male} = \mathbf{1}'\mathbf{M}^{-1}\mathbf{P}\mathbf{W}\,\theta$$

The difference of the sex-specific TFRs is

$$TFR_{male} - TFR_{female} = \mathbf{1}'\left[\,\mathbf{M}^{-1}\mathbf{P}\mathbf{W} - \mathbf{I}_A\,\right]\theta$$

In very broad terms, we can see that this difference is more likely to be positive (higher male TFR) if there are more women and fewer men in the reproductive-age population – i.e. if the diagonal elements of $\mathbf{W}$ tend to be larger than the diagonal elements of $\mathbf{M}$. In that case the general fertility rate for males (total period births divided by total number of reproductive-age males) would be higher than the general fertility rate for females, and the TFRs would tend to have the same difference in favor of males.

More subtly, the age distributions of males and females also affect the difference in sex-specific TFRs.

```
# USA male fertility
# from Dudel, C. and S. Klüsener. Male fertility data for high-income
# countries [unpublished data]. Submitted to the HFC by C. Dudel
# on 15.05.2019.
#
# downloaded from Human Fertility Collection 2 Nov 2019
# https://www.fertilitydata.org/data/RAW_DATA/m_USA_51.zip




library(tidyverse)
```

```
## Warning: package 'tidyverse' was built under R version 3.5.3
```

```
## -- Attaching packages ----------------- tidyverse 1.2.1 --
```

```
## v ggplot2 3.1.0     v purrr   0.3.2
## v tibble  2.1.3     v dplyr   0.8.3
## v tidyr   0.8.2     v stringr 1.4.0
## v readr   1.3.1     v forcats 0.3.0
```

```
## Warning: package 'tibble' was built under R version 3.5.3
```

```
## Warning: package 'readr' was built under R version 3.5.3
```

```
## Warning: package 'purrr' was built under R version 3.5.3
```

```
## Warning: package 'dplyr' was built under R version 3.5.3
```

```
## Warning: package 'stringr' was built under R version 3.5.2
```

```
## -- Conflicts -------------------- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
# Male data from HFC, as described above
MM = read.table(file='m_ASFR_USA.txt', header=TRUE) %>%
      mutate(Y = factor(Year))

# Female data from HFD
FF = read.table(file='USAasfrTR.txt', skip=2, header=TRUE) %>%
       filter(Year %in% unique(MM$Year)) %>%
       mutate(Age = 11 + as.numeric(Age)) %>%
       group_by(Year,Age) %>%
       summarize(ASFR = mean(ASFR)) %>%
       mutate(Y=factor(Year))
```

```
## Warning in mutate_impl(.data, dots, caller_env()): Unequal factor levels:
## coercing to character
```

```
## Warning in mutate_impl(.data, dots, caller_env()): binding character and
## factor vector, coercing into character vector
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```

```
big = bind_rows(
         mutate(MM,Sex='Male'),
         mutate(FF,Sex='Female')
       )
```

```
## Warning in bind_rows_(x, .id): binding factor and character vector,
## coercing into character vector

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```

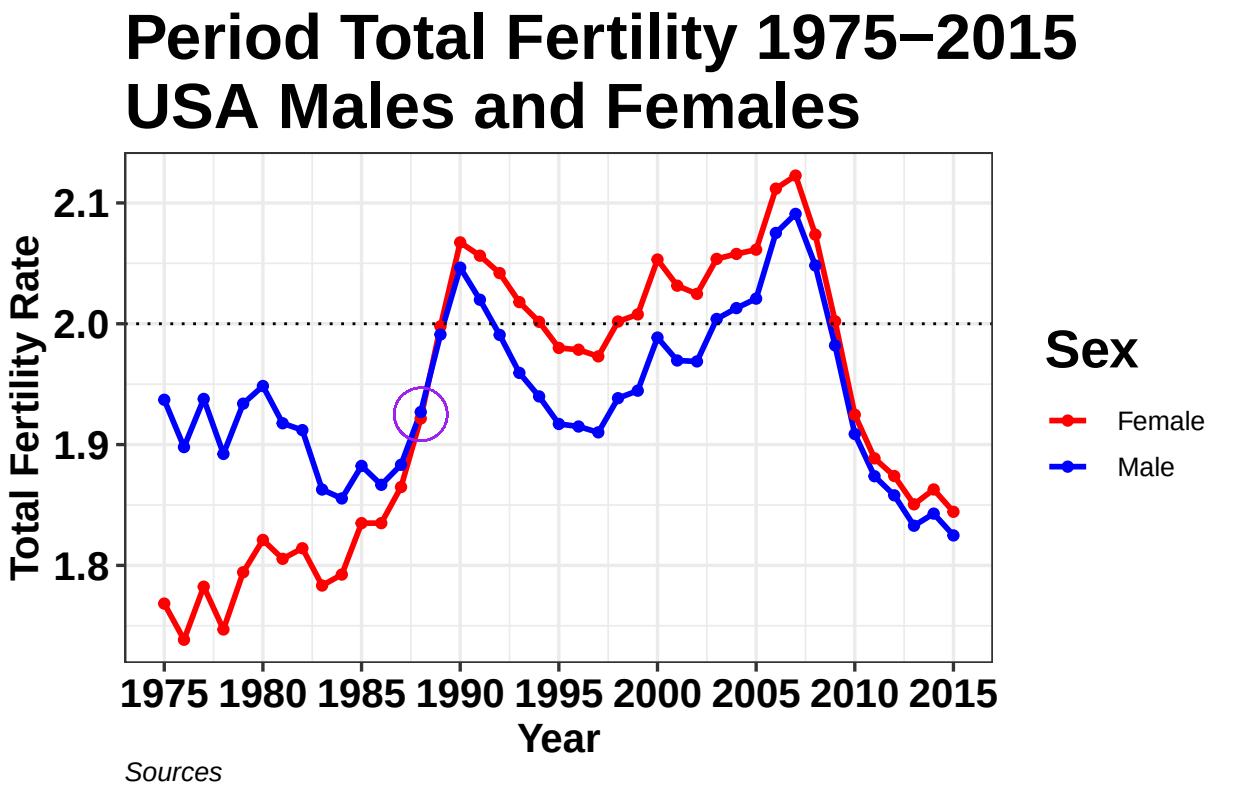
```
tmp = big %>%
           group_by(Sex,Year) %>%
           summarize(TFR=sum(ASFR))

theme_carl <- function () {
  theme_bw(base_size=13) %+replace%
    theme(
      title       = element_text(size=20, face='bold'),
      plot.caption  = element_text(size=10, face='italic',hjust=0),
      axis.text   = element_text(size=15, face='bold'),
      axis.title = element_text(size=15, face='bold')
    )
}
```

```
tmp = big %>%
  filter(Year > 1974) %>%
  group_by(Sex,Year) %>%
  summarize(TFR=sum(ASFR))

G = ggplot(data=tmp) +
      aes(x=Year, y=TFR, group=Sex, color=Sex) +
      geom_line(lwd=1) +
      geom_point() +
      geom_point(x=1988, y=1.925, shape=1, size=9, color='purple') +
      geom_hline(yintercept = 2, lty='dotted') +
      scale_color_manual(values=c('red','blue')) +
      scale_x_continuous(breaks=seq(1975,2015,5)) +
      labs(x='Year',
           y='Total Fertility Rate',
           title='Period Total Fertility 1975-2015\nUSA Males and Females',
           caption='Sources\nMALES: C Dudel and S Klüsener, Human Fertilty Collection https://www.ferti
      theme_carl()


print(G)
```



Sources
MALES: C Dudel and S Klüsener, Human Fertilty Collection https://www.fertilitydata.org/data/R
FEMALES: Human Fertility Database

```
#ggsave(G, file='USA-TFR-by-sex.png', height=8, width=11, units='in', dpi=300)
```