Comparing Period TFR for 2 groups

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TFR for one group

Suppose that there are A possible single-year reproductive ages $x = 1 \dots A$ in which individuals may have children. At some of these ages fertility rates may be zero for some groups (for example, for women at $x > 50$).

Suppose also that there are a total of B births in a given year to some group of interest (such as "all women", "all men", "Asian-American men", "residents of Montana" ...). Call π_x the fraction of the B births that happened to group members who were age x. That is, births disaggregated by age are

$$B_x = B\,\pi_x \quad x = 1, 2, \dots, A$$

If there were N_x group members at age x at mid-year, then period TFR for this group is

$$TFR = \sum_x \frac{B_x}{N_x} = B\,\sum_x \pi_x \left(\frac{1}{N_x}\right) = B\,E_\pi\left(N_x^{-1}\right) \tag{1}$$

where the expectation in (1) is taken over the observed distribution of parents' ages for this year's births.

Comparing TFR for 2 groups

If there are two groups then the ratio of their period TFRs is

$$\frac{TFR_1}{TFR_2} = \frac{B_1\;\cdot\;E_{\pi_1}\left(N_{1x}^{-1}\right)}{B_2\;\cdot\;E_{\pi_2}\left(N_{2x}^{-1}\right)} \tag{2}$$

In some cases (for example when the groups are "women" and "men") the total births are the same for both groups. For those cases:

$$\frac{TFR_1}{TFR_2} = \frac{E_{\pi_1}\left(N_{1x}^{-1}\right)}{E_{\pi_2}\left(N_{2x}^{-1}\right)} \tag{3}$$

Using the results derived in the Appendix below, the ratio in (3) can be approximated fairly well by

$$\frac{TFR_1}{TFR_2} \approx \frac{N_{2,m_2}}{N_{1,m_1}}$$

where m_1 and m_2 are the mean ages of parents in the two groups for this year's observed births.

USA data

USA *male* fertility data from Dudel, C. and S. Klüsener. Male fertility data for high-income countries [unpublished data]. Submitted to the HFC by C. Dudel # on 15.05.2019. downloaded from Human Fertility Collection 2 Nov 2019 https://www.fertilitydata.org/data/RAW_DATA/m_USA_51.zip

USA *female* fertility data from H**F**D

USA *male* and *female* population data from H**M**D

```r
# construct a data frame with annual USA data, from the listed sources

# rates for 1969-2015 x ages 15-59
asfr_male = read.table(file='m_ASFR_USA.txt', header=TRUE,
                       colClasses='numeric') %>%
            add_column(Sex='Male')

# rates for 1969-2015 x ages 12-55
asfr_female = read.table(file='USAasfrTR.txt', skip=2, header=TRUE) %>%
       mutate(Age = 11 + as.numeric(Age)) %>%
       filter(Year %in% unique(asfr_male$Year)) %>%
       group_by(Year,Age) %>%
       summarize(ASFR = mean(ASFR)) %>%
       add_column(Sex='Female')

ASFR = bind_rows(
         asfr_male,
         asfr_female
        )

pop = read.table('Population.txt',skip=2, header=TRUE) %>%
        mutate( Year = as.numeric(as.character(Year)),
                Age  = as.numeric(as.character(Age))) %>%
        filter(Age %in% unique(ASFR$Age),
               Year %in% unique(ASFR$Year)) %>%
        select(Year,Age,Female,Male) %>%
        gather(key=Sex, value=pop, -Year, -Age)
```

```
## Warning: NAs introduced by coercion

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```
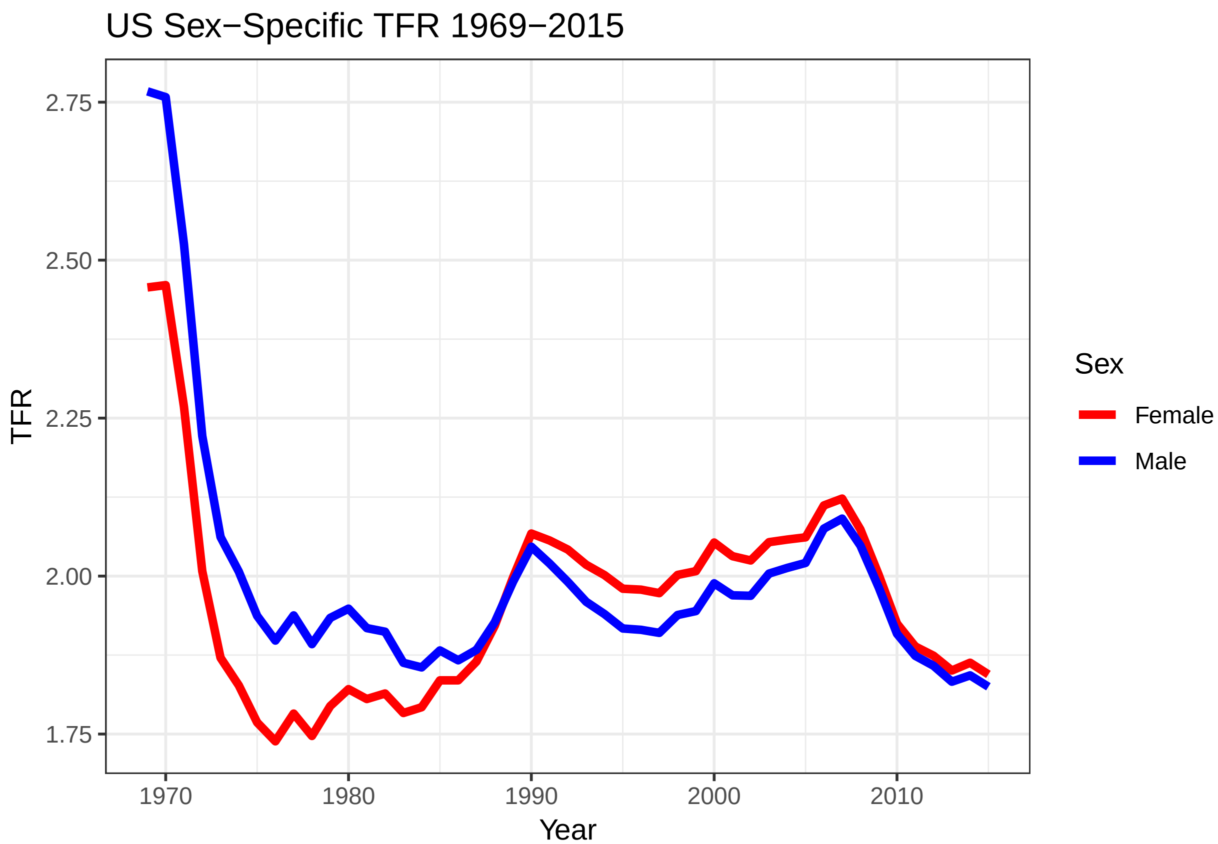
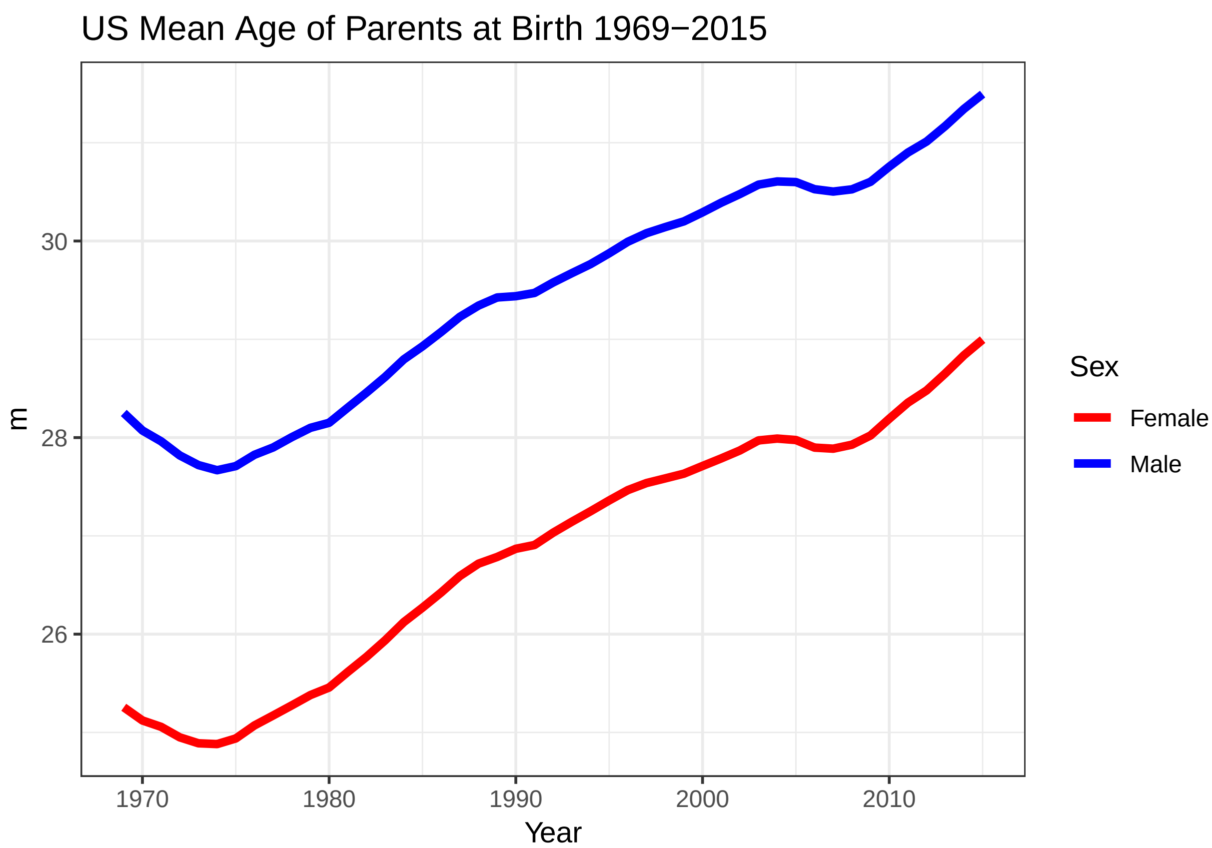

```r
## add estimated births, fraction of births by age, etc.

big = full_join(pop, ASFR, by=c('Year','Age','Sex')) %>%
         mutate(births = pop * ASFR)

big$births[is.na(big$births)] = 0

# summarize each year x sex

mean_expos = function(A,P,M) {
  approx(x=A, y=P, xout=M )$y
}

df = big %>%
       group_by(Year,Sex) %>%
       summarize( TFR = sum(ASFR, na.rm=TRUE),
                  B   = sum(births),
                  m   = weighted.mean( Age+.50   , births),
                  Nm  = mean_expos(Age+.50,pop,m),
                  approx_TFR = B/Nm)
```

```r
ggplot(data=df) +
   aes(x=Year, y=TFR, color=Sex) +
   geom_line(lwd=1.5) +
   scale_color_manual(values=c('red','blue')) +
   labs(title='US Sex-Specific TFR 1969-2015') +
  theme_bw()
```

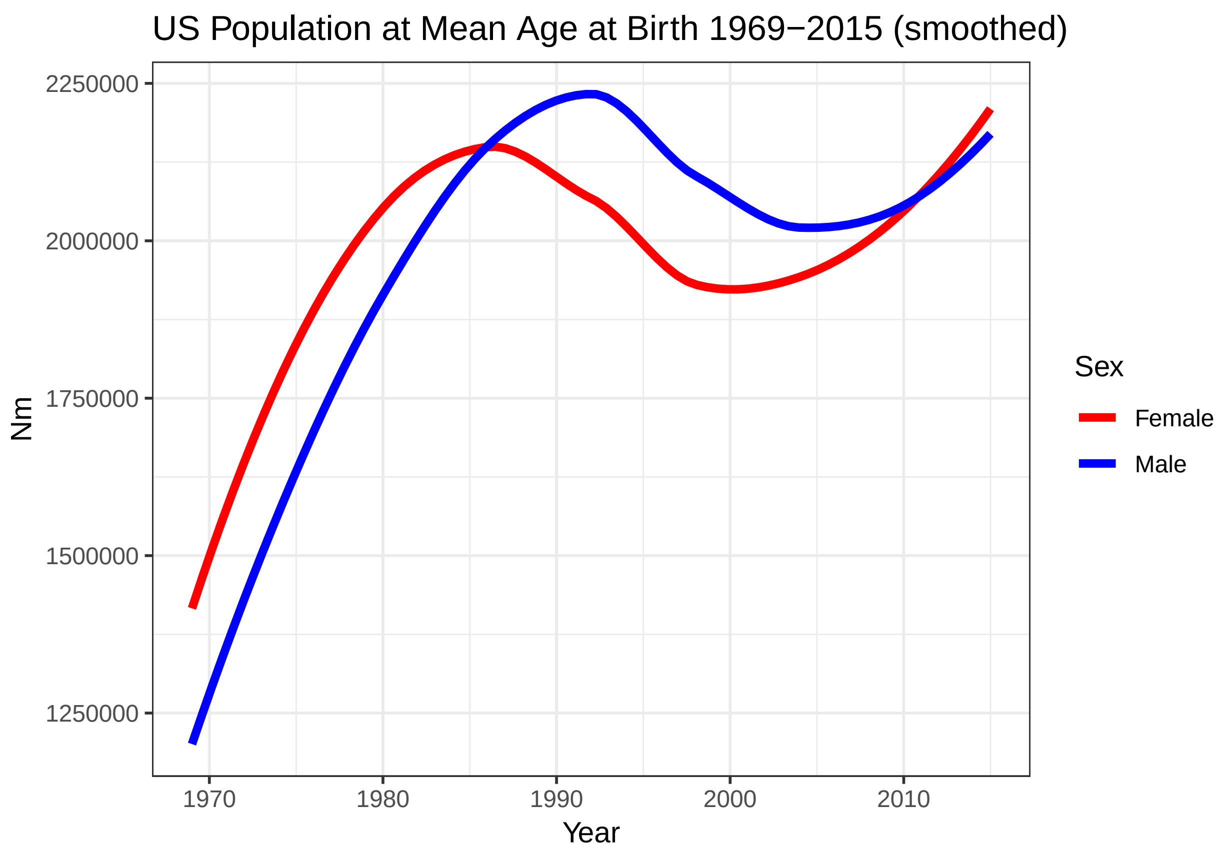



```r
ggplot(data=df) +
   aes(x=Year, y=m, color=Sex) +
   geom_line(lwd=1.5) +
   scale_color_manual(values=c('red','blue')) +
   labs(title='US Mean Age of Parents at Birth 1969-2015') +
  theme_bw()
```




```r
ggplot(data=df) +
   aes(x=Year, y=Nm, color=Sex) +
   geom_smooth(lwd=1.5,se=FALSE) +
   scale_color_manual(values=c('red','blue')) +
   labs(title='US Population at Mean Age at Birth 1969-2015 (smoothed)') +
  theme_bw()
```

```
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```



Example: US TFRs in 1995

Start by loading ASFR and population data for 1995. Calculate male and female TFRs.

USA male fertility from Dudel, C. and S. Klüsener. Male fertility data for high-income countries [unpublished data]. Submitted to the HFC by C. Dudel # on 15.05.2019.

downloaded from Human Fertility Collection 2 Nov 2019 https://www.fertilitydata.org/data/RAW_DATA/m_USA_51.zip

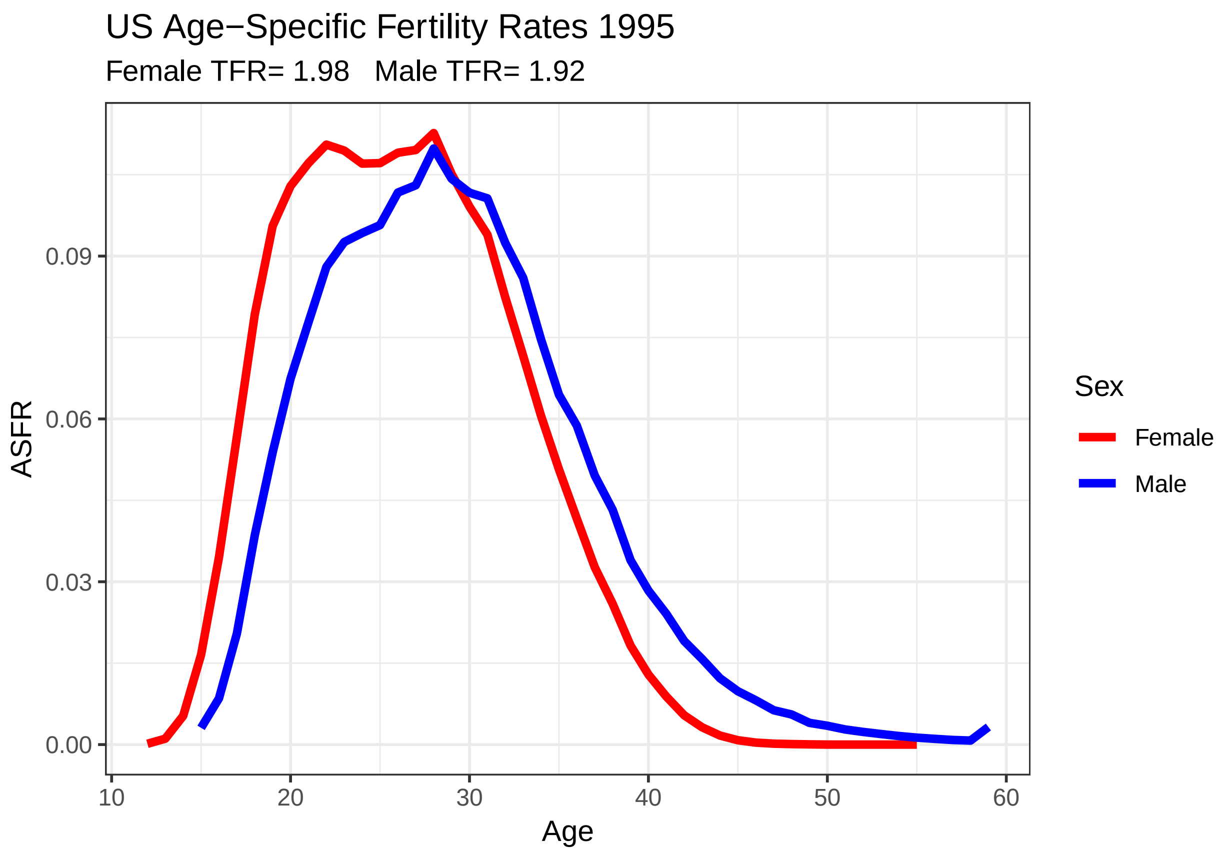
```
tmp.big = filter(big, Year == detail_year)
tmp.df  = filter(df,  Year == detail_year)

TFR        = tmp.df$TFR
approx_TFR = tmp.df$approx_TFR
m          = tmp.df$m
Nm         = tmp.df$Nm

B = mean(tmp.df$B) #avg of (slightly difft) male and female total births

names(TFR) = names(approx_TFR) = names(m) = names(Nm) = c('Female','Male')

ratio        = TFR['Male']/TFR['Female']
approx_ratio = approx_TFR['Male']/approx_TFR['Female']
```

For the US in 1995, female TFR was **1.98** and male TFR was **1.92**.

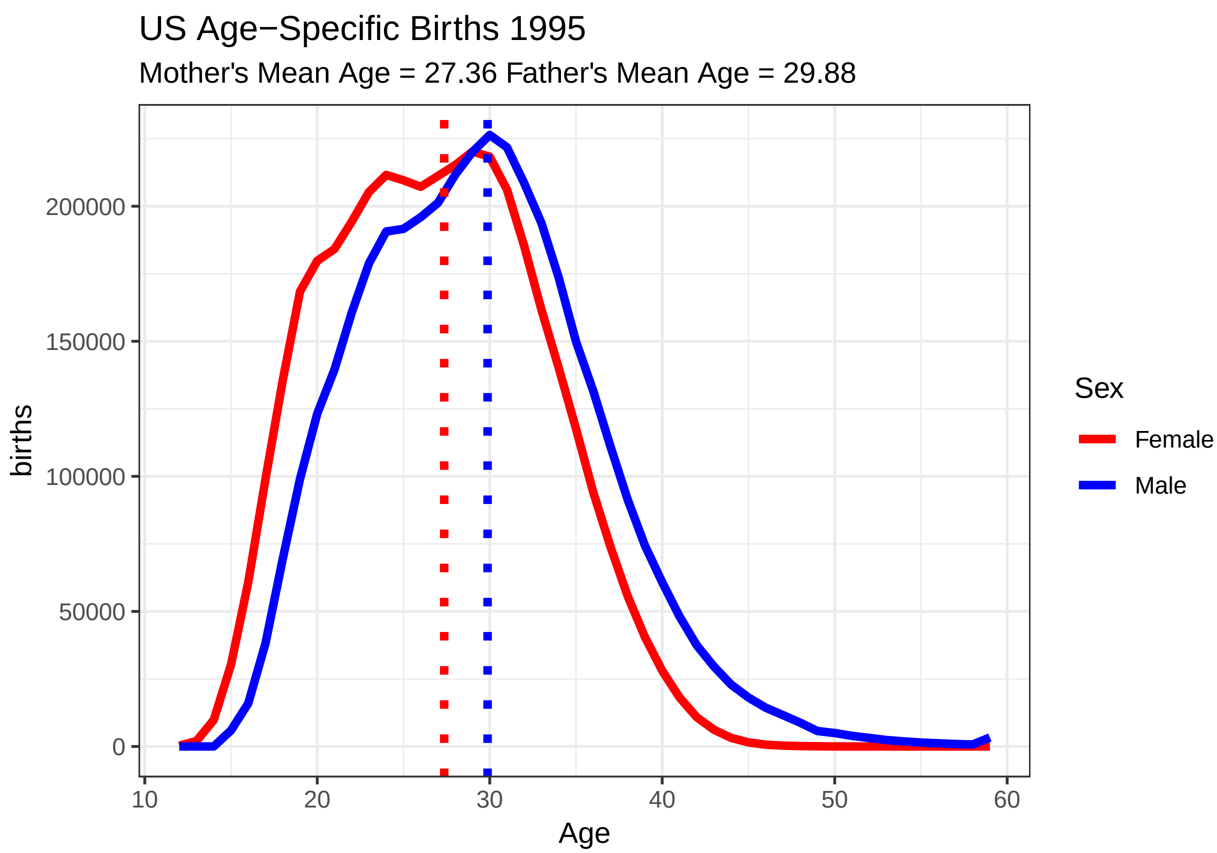
Now illustrate 1995 fertility rates, births, and populations, by age and sex.

```
ggplot(data=tmp.big) +
  aes(x=Age, y=ASFR, color=Sex) +
  geom_line(lwd=1.5) +
  scale_color_manual(values=c('red','blue')) +
  labs(title=paste('US Age-Specific Fertility Rates',detail_year),
       subtitle=paste('Female TFR=',round(TFR['Female'],2),
                      '   Male TFR=',round(TFR['Male']   ,2))) +
  theme_bw()
```

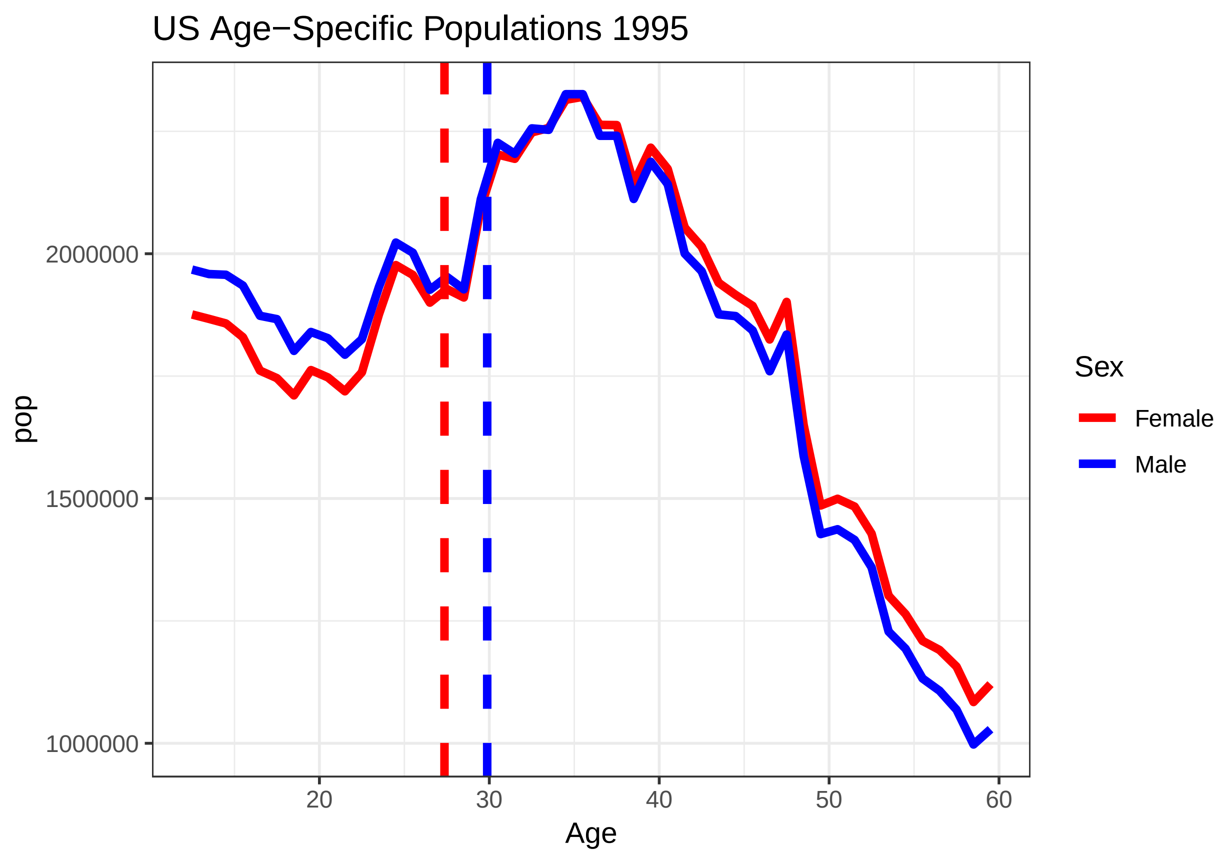
```
## Warning: Removed 7 rows containing missing values (geom_path).
```




```
ggplot(data=tmp.big) +
  aes(x=Age, y=births, color=Sex) +
  geom_line(lwd=1.5) +
  geom_vline(xintercept = m[c('Female','Male')], color=c('red','blue'),
             lwd=1.5, lty='dotted') +
  scale_color_manual(values=c('red','blue')) +
  labs(title=paste('US Age-Specific Births',detail_year),
       subtitle=paste('Mother\'s Mean Age =',round(m['Female'],2),
                      'Father\'s Mean Age =',round(m['Male'],2))) +
  theme_bw()
```




```
ggplot(data=tmp.big) +
  aes(x=Age+0.5, y=pop, color=Sex) +
  geom_line(lwd=1.5) +
  geom_vline(xintercept = m[c('Female','Male')], color=c('red','blue'),
             lwd=1.5,lty='dashed') +
  scale_color_manual(values=c('red','blue')) +
  labs(title=paste('US Age-Specific Populations',detail_year),
       x='Age') +
  theme_bw()
```



Now we will check the approximations based on

- mean parental ages for 1995 births: 27.36 for females and 29.88 for males
- 1995 populations at mean ages: 1924013 for females and 2154632 for males

Thus

Sex	True 1995 TFR	m	N_m (000s)	Approx TFR
Male	1.92	29.88	2155	1.81
Female	1.98	27.36	1924	2.03

and

True Male/Female TFR Ratio	Approximate Ratio
0.97	0.89

Appendix: Approximation

We can approximate the TFRs in (1) by considering a Taylor series approximation for N_x^{-1}. For the purposes of this derivation, temporarily ignore the discrete nature of ages in observed data, and treat N_x as a continuous function of exact age x.

Define the (period) mean age of parents in our group as m. The second-order Taylor approximation to N_x^{-1} is

$$N_x^{-1} \approx N_m^{-1} + (x-m)\left[-N_m^{-2}N_m'\right] + \tfrac{1}{2}(x-m)^2\left[2N_m^{-3}N_m' - N_m^{-2}N_m''\right]$$

Thus the expectation in (1) is

$$E(N_x^{-1}) \approx N_m^{-1} + \tfrac{\sigma_x^2}{2}\left[2N_m^{-3}N_m' - N_m^{-2}N_m''\right]$$

where σ_x^2 is the observed variance of parental ages. Typical values for the variance term are in $[25, 50]$, so when exposure N is measured in thousands or millions, the second-order term is many orders of magnitude smaller than the first-order term.

Using only the first term leads to a very simple approximation for TFR, based on (1) that is fairly robust:

$$TFR \;\approx\; \frac{B}{N_m} = \frac{\text{total annual births}}{\text{mid-year population at mean parental age}}$$