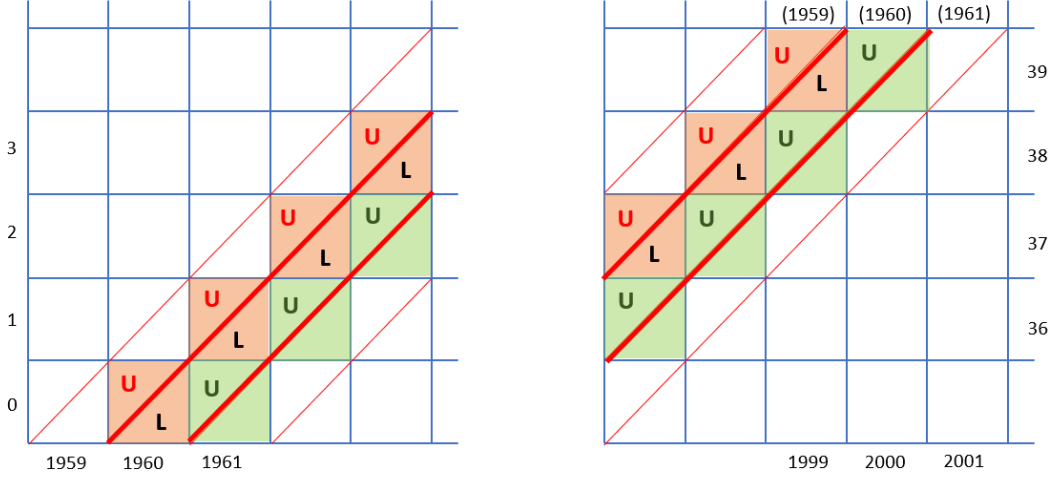


Figure 1: Lexis diagram for the 1960 birth cohort of women



Suppose that mortality is zero, and that there are N_{1960} women in the 1960 birth cohort. Define L_{xt} and U_{xt} as the number of births that happen in the lower and upper triangles, respectively, of the age-period square for age x , year t . The total births to the 1960 cohort before exact age 40 would then be

$$\begin{aligned} B_{1960} &= (L_{0,1960} + L_{1,1961} + \cdots + L_{39,1999}) \\ &\quad + (U_{0,1961} + U_{1,1962} + \cdots + U_{39,2000}) \\ &= L_{1960}^* + U_{1960}^* \end{aligned}$$

where we divide the births by the category of Lexis triangle that they fall into. With no mortality, true cohort completed fertility per woman would then be

$$F_{1960} = \frac{L_{1960}^* + U_{1960}^*}{N_{1960}}$$

If we used the orange age-period squares to approximate the fertility history there would be

$$\begin{aligned} B &= (L_{0,1960} + L_{1,1961} + \cdots + L_{39,1999}) + (U_{0,1960} + U_{1,1961} + \cdots + U_{39,1999}) \\ &= L_{1960}^* + U_{1959}^* \end{aligned}$$

and estimated completed fertility would be

$$\hat{F}_{1960} = \frac{L_{1960}^* + U_{1959}^*}{\frac{1}{2}(N_{1959} + N_{1960})}$$

So the error from summing along the diagonal of the age-period squares is

$$\hat{F}_{1960} - F_{1960} = \frac{L_{1960}^* + U_{1959}^*}{\frac{1}{2}(N_{1959} + N_{1960})} - \frac{L_{1960}^* + U_{1960}^*}{N_{1960}}$$

Defining

$$K = \frac{N_{1959}}{N_{1960}}$$

as the size of the 1959 cohort relative to the 1960 cohort, we can rewrite the error as

$$\begin{aligned} \hat{F}_{1960} - F_{1960} &= \frac{L_{1960}^* + U_{1959}^*}{\frac{1}{2}(K \cdot N_{1960} + N_{1960})} - \frac{L_{1960}^* + U_{1960}^*}{N_{1960}} \\ &= \frac{1}{N_{1960}} \left[\frac{L_{1960}^* + U_{1959}^*}{\frac{1}{2}(K + 1)} - (L_{1960}^* + U_{1960}^*) \right] \\ &= \frac{1}{N_{1960}} \left[\left(\frac{L_{1960}^*}{\frac{1}{2}(K + 1)} - L_{1960}^* \right) + \left(\frac{U_{1959}^*}{\frac{1}{2}(K + 1)} - U_{1960}^* \right) \right] \end{aligned}$$

If $K = 1$ the error simplifies to

$$\hat{F}_{1960} - F_{1960} = \frac{U_{1959}^* - U_{1960}^*}{N_{1960}}$$

which is the “rate effect”. If fertility rates fell between the 1959 and 1960 cohorts, then the error from “taking the AP diagonal” as a cohort estimate is positive, and vice versa.

On the other hand, if rates did not change but