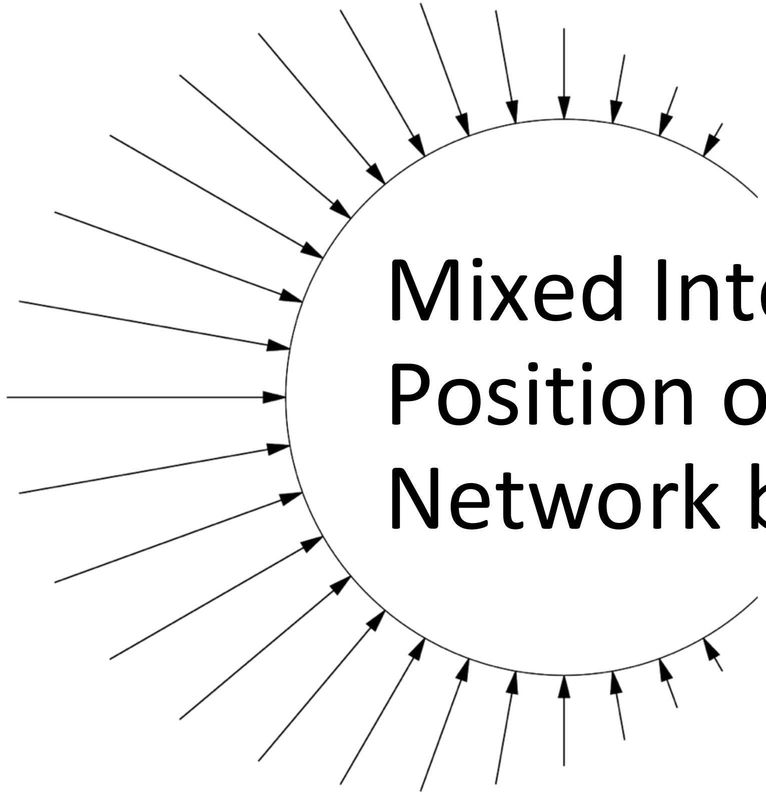


Mixed Integer Stochastic Optimization of relative Position of two Wind Turbines using using Neural Network based Constraint Learning

– Simon Schmetz –

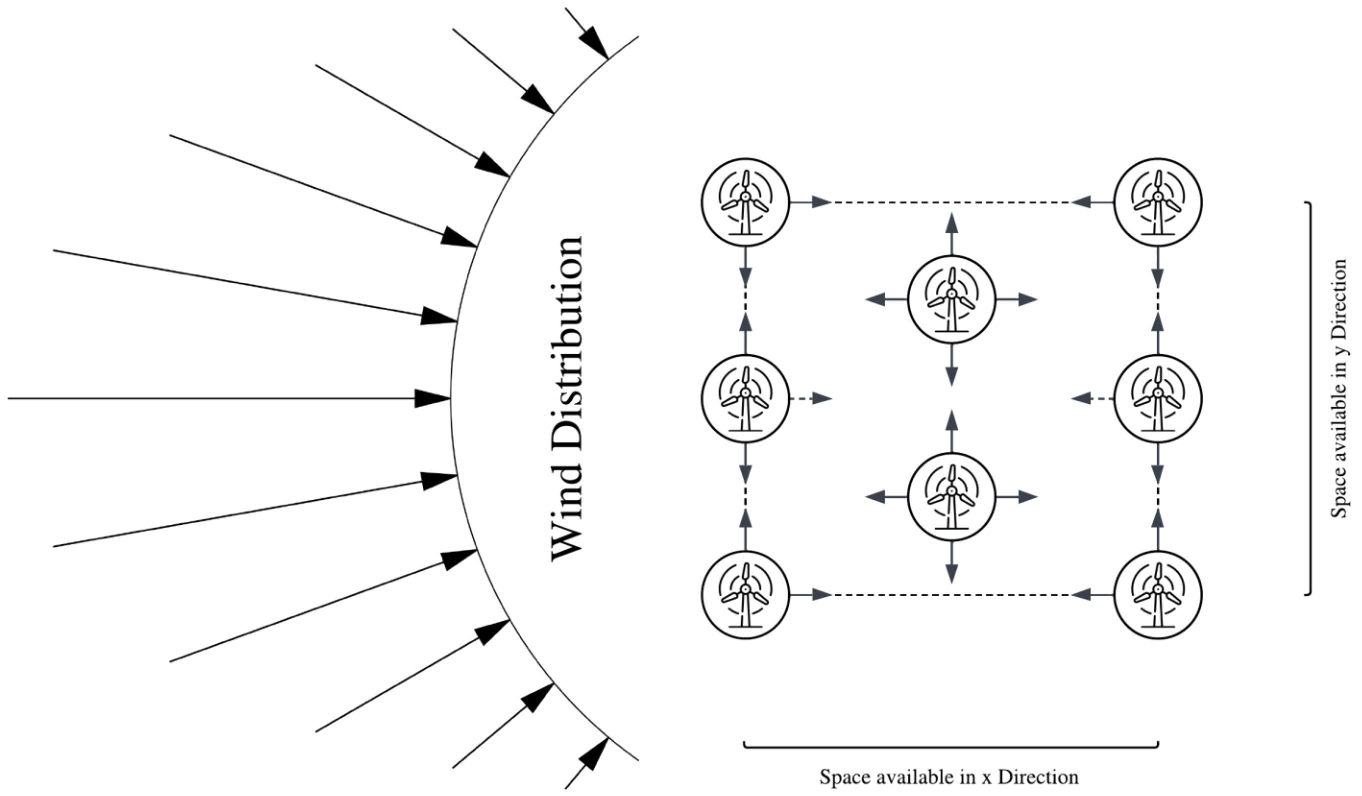
Universidad Carlos III de Madrid

Master Thesis Defense

Madrid the 14th of July 2025

Supervisor: Carlos Ruiz Mora

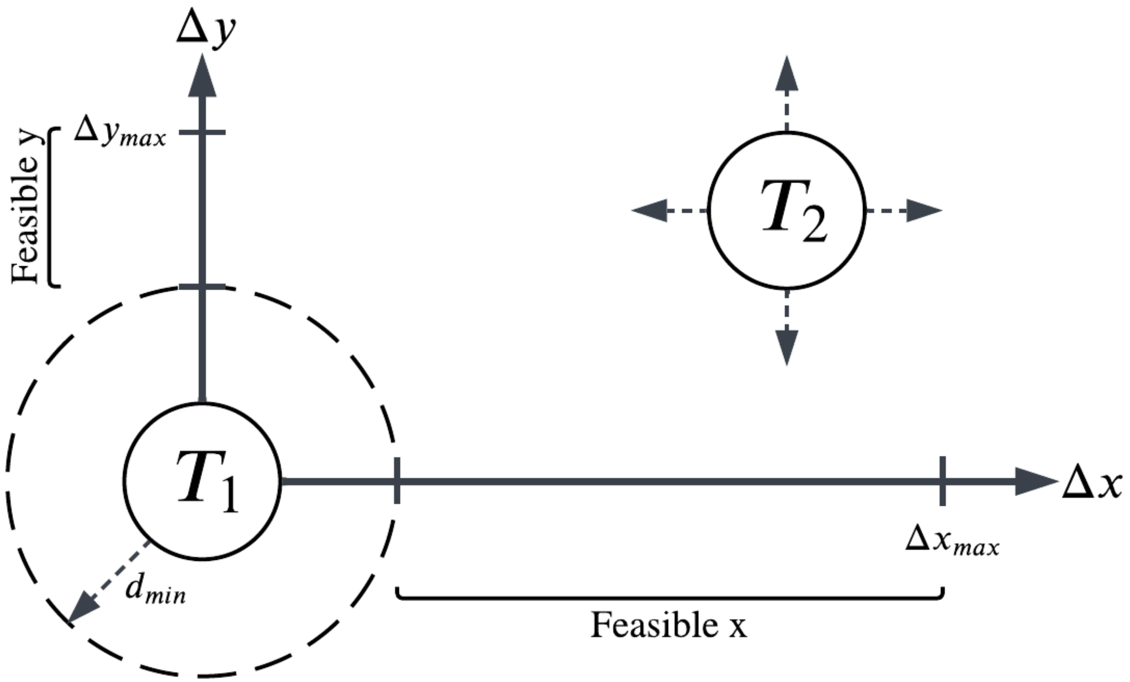
The Problem: Maximize Power



Required Components:

- Objective Function
- Constraints
- Solver

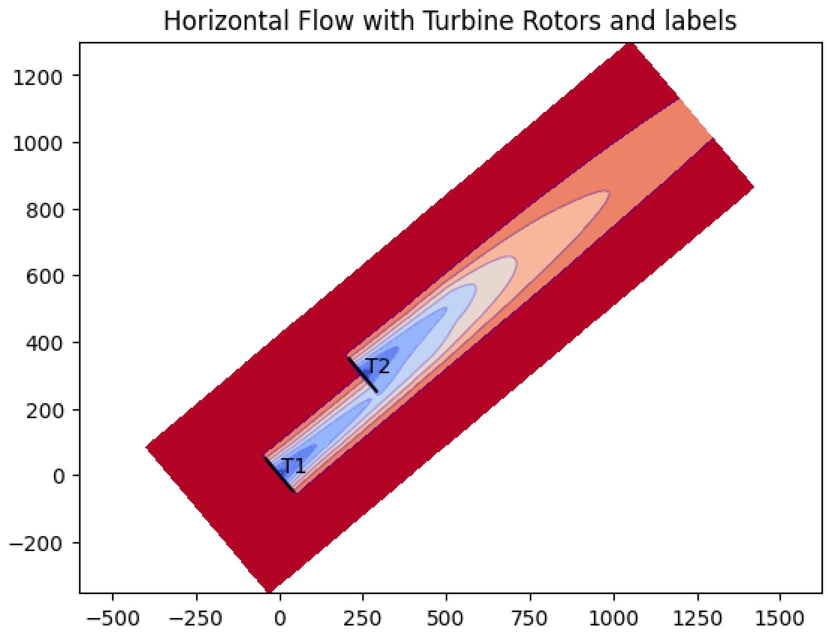
Problem Definition: General Two Turbine Problem

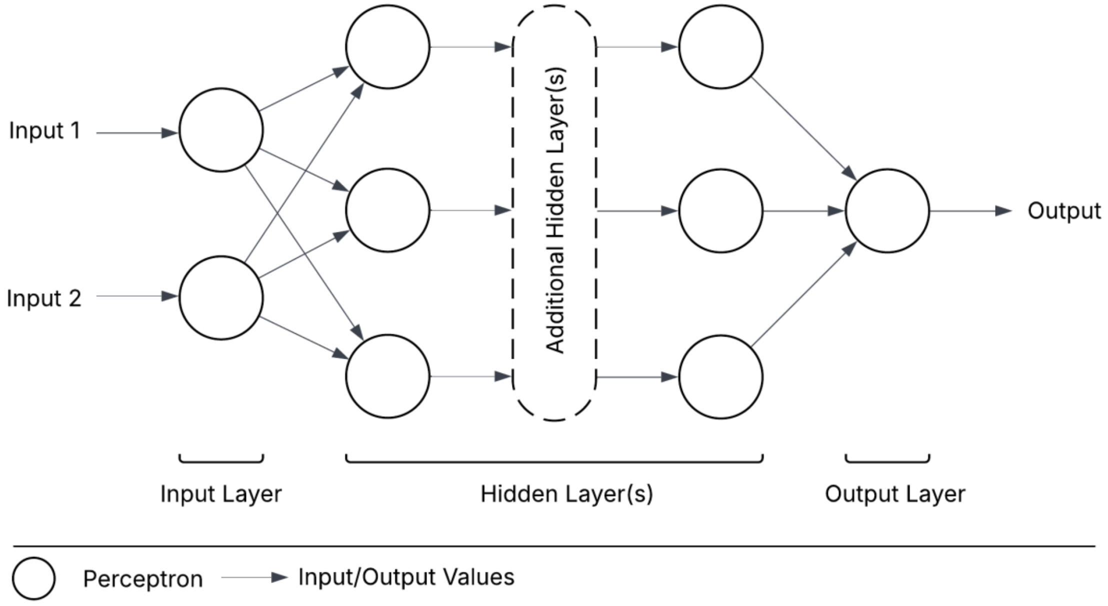


$$
\begin{aligned}
&\max_{\mathbf{x},\mathbf{y}} f_{Power,\mathrm{NN}}(\Delta x, \Delta y) \\
\text{s.t.}\quad & 0 \leq \Delta x \leq X_{\max} \\
& 0 \leq \Delta y \leq Y_{\max} \\
& \sqrt{(\Delta x)^2 + (\Delta y)^2} \geq d_{\min}
\end{aligned}
$$

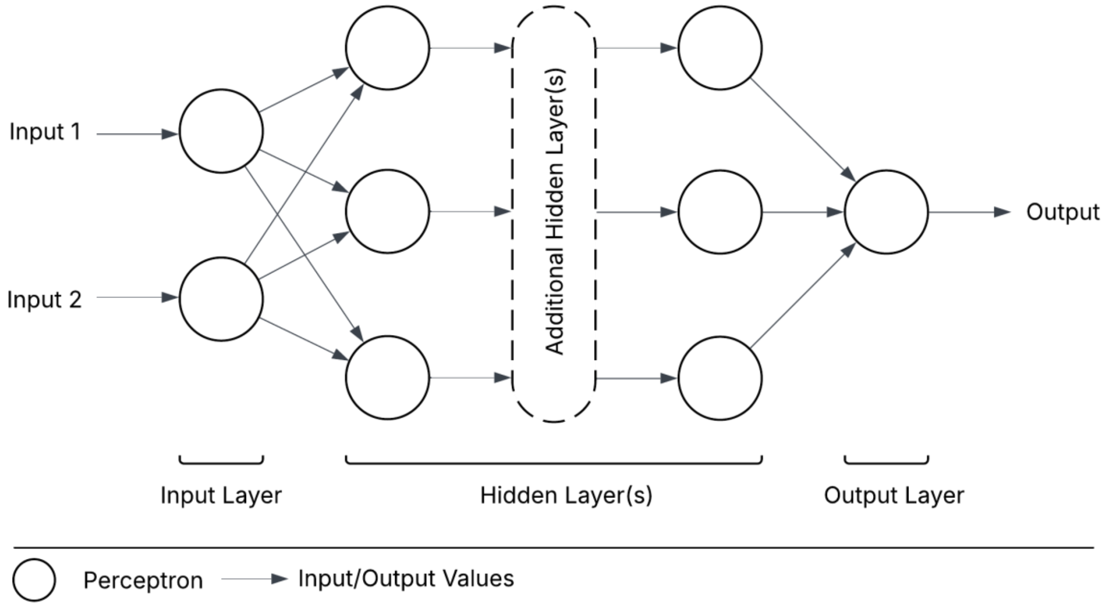
Objective Function: Physical Reality

Objective Function: Data to Surrogate Model



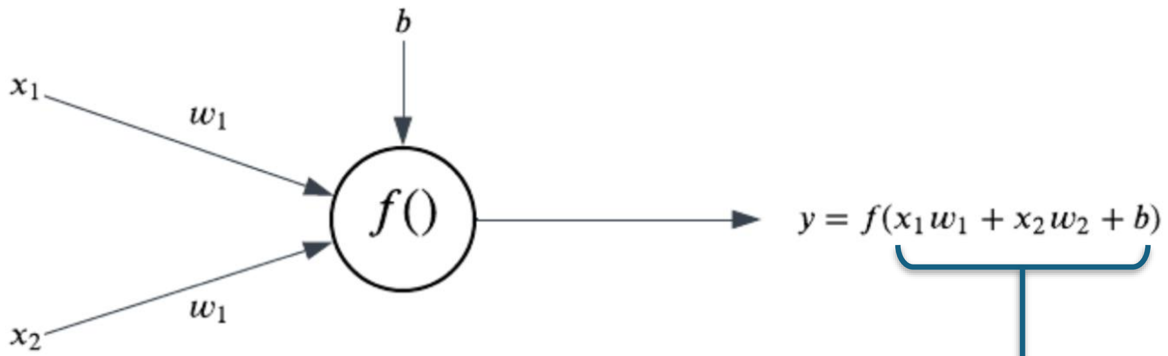


Objective Function: Constraint Learning (CL)



$$\max_{\mathbf{x},\mathbf{y}} f_{Power}(x_i, y_i, \text{wind conditions})$$
$$\text{s.t.} \quad 0 \leq x_i \leq X_{\max}$$
$$0 \leq y_i \leq Y_{\max}$$

CL-Approach: Neural Network Decomposition



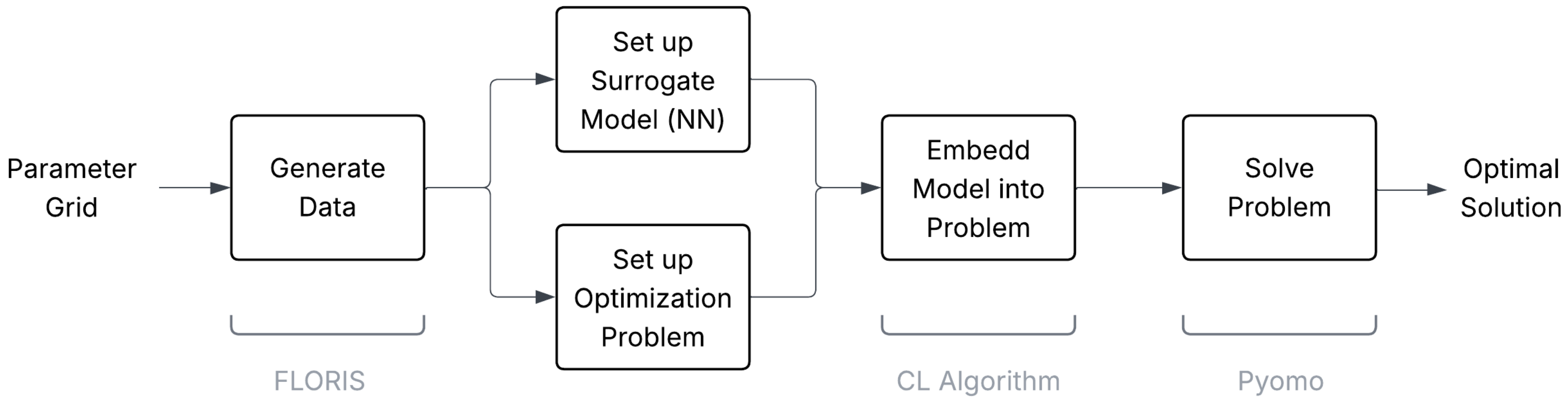
$$\mathrm{ReLU}(\tilde{v}_i^{\ell}) = \underbrace{v_i^{\ell}}_{\text{Neuron Output}} = \max(0, \underbrace{\boxed{\tilde{v}_i^{\ell}}}_{\text{Weighted Sum}}) = \begin{cases} 0 & \text{if } \tilde{v}_i^{\ell} < 0, \\ \tilde{v}_i^{\ell} & \text{if } \tilde{v}_i^{\ell} \geq 0. \end{cases}$$

$$v_i^{\ell} = \max(0, \tilde{v}_i^{\ell})$$

$$\boxed{\begin{aligned} v_i^{\ell} &\geq \tilde{v}_i^{\ell} \\ v_i^{\ell} &\leq \tilde{v}_i^{\ell} - M^{\text{low}}(1 - j_i) \\ v_i^{\ell} &\leq M^{\text{up}} j_i \end{aligned}}$$

$$j_i = \begin{cases} 0 & \text{if } \tilde{v}_i^{\ell} < 0 \\ 1 & \text{if } \tilde{v}_i^{\ell} > 0 \end{cases}$$

Steps to Solving the Two Turbine Problem



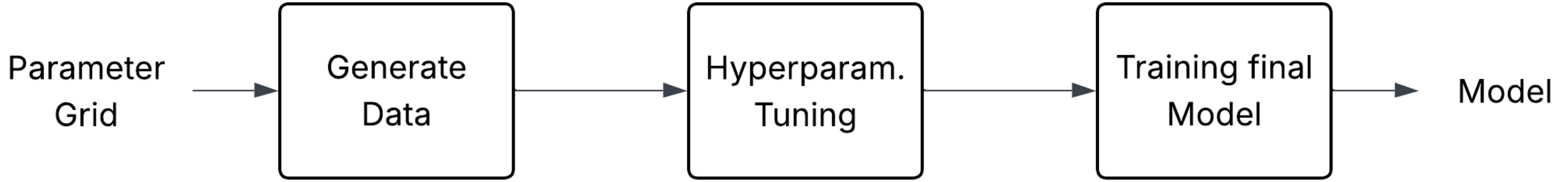
Approach: Wind condition Independent Neural Network

$$\max_{\mathbf{x},\mathbf{y}} \boxed{\sum_{i=1}^{n}} \boxed{f_{Power,\mathrm{NN}}(\Delta x, \Delta y, \text{wind condition})} \cdot \boxed{p_{n,\text{wind condition combination}}}$$

$$\begin{aligned} \text{s.t.} \quad & \Delta x \leq X_{\max} \\ & \Delta y \leq Y_{\max} \\ & \sqrt{(\Delta x)^2 + (\Delta y)^2} \geq d_{\min} \end{aligned}$$

Modeling Process

$$\max_{\mathbf{x},\mathbf{y}} \sum_{i=1}^{n} f_{Power,\mathrm{NN}}(\Delta x, \Delta y, \text{wind condition}) \cdot p_{n,\text{wind condition combination}}$$



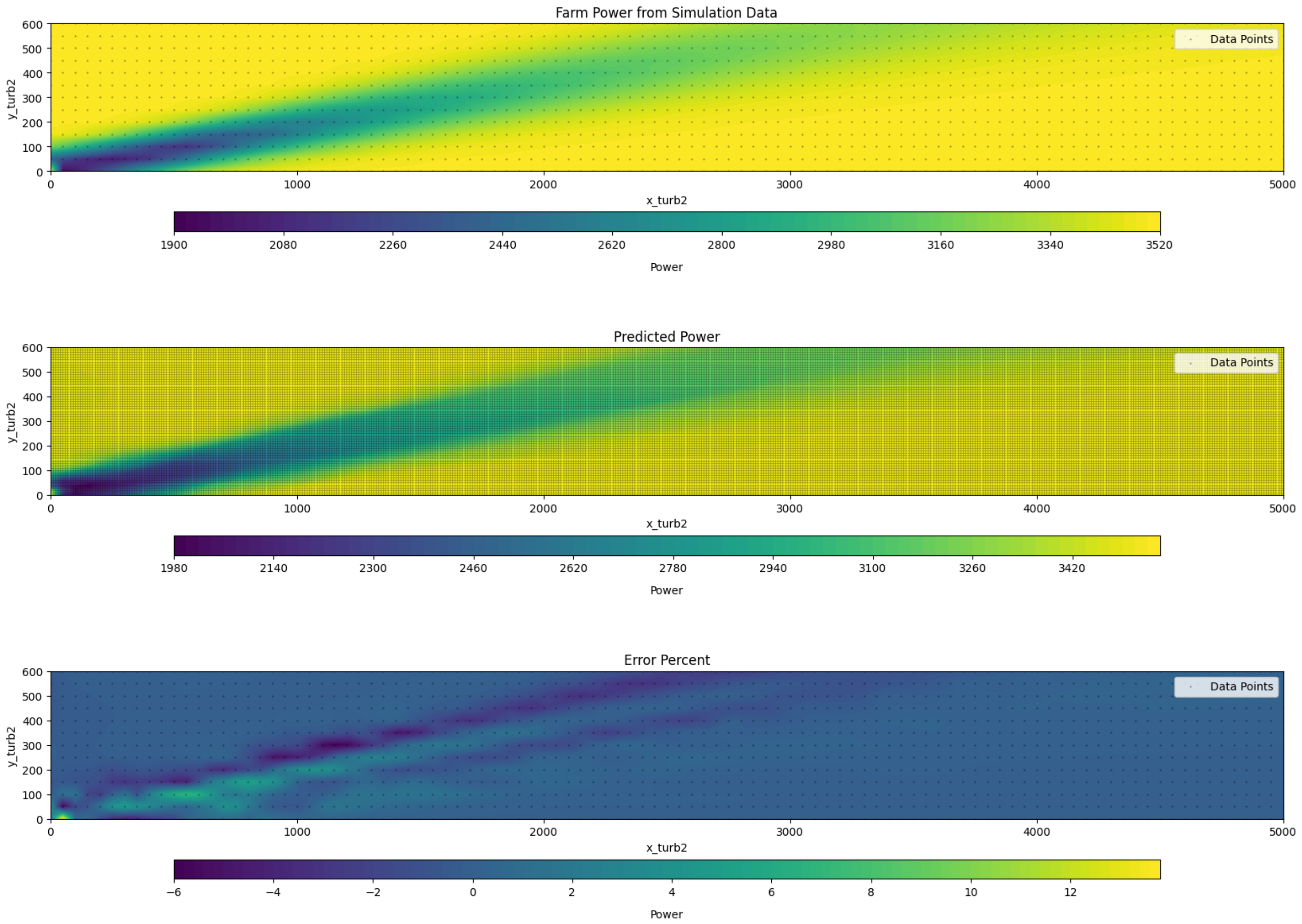
Parameter Grid

Variable	**Const/Variable**	**Value**	*Steplength*
Δx_{turb2}	Variable	[0, 5000] m	50 m
Δy_{turb2}	Variable	[0, 500] m	50 m
wind_speed	Constant	8 m/s	-
wind_direction	Variable	[180°, 270°]	10°
turbulence_intensity	Constant	0.06	-

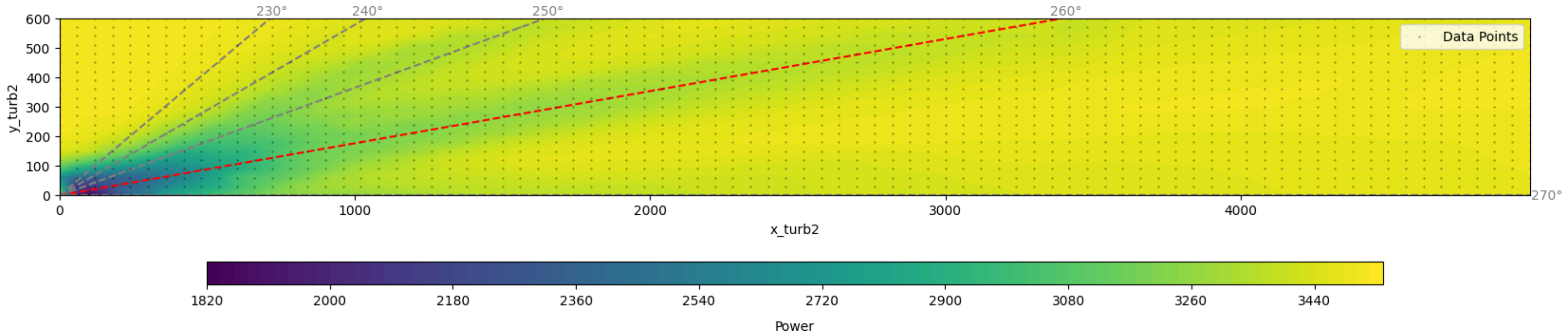
Generate Data Using FLORIS

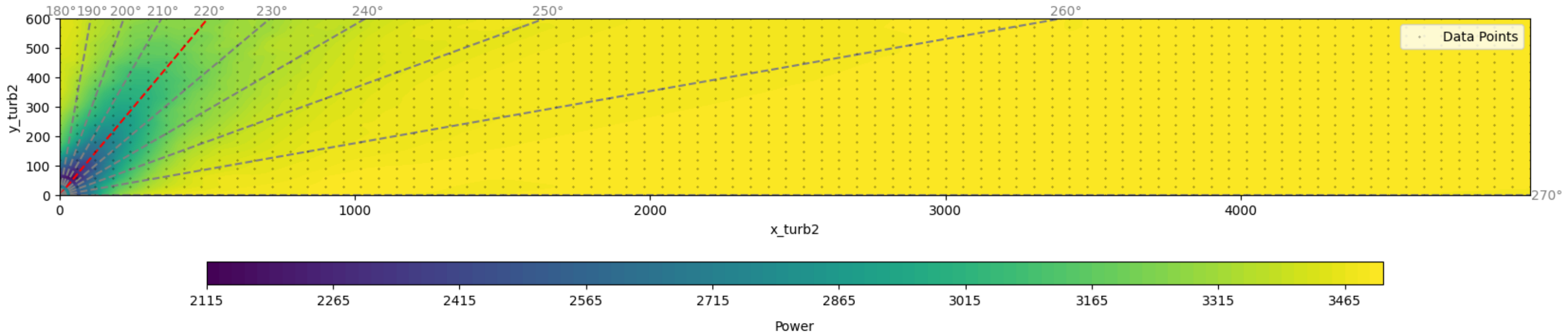
uc3m

Modeling (Using NN: $5 - 50^{\times 3} - 1$)



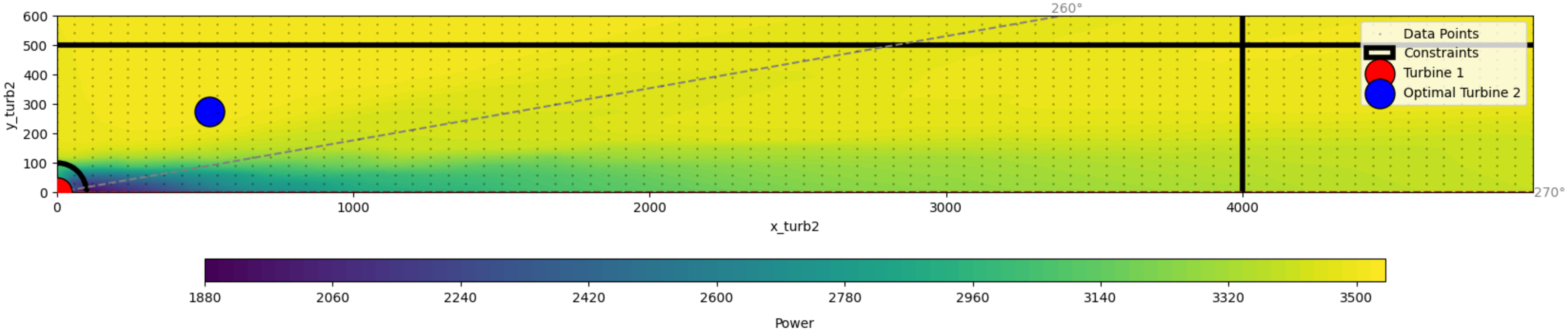
Modeling (Expectation)





Optimization

With two scenarios from a discretized $N(270°, 5°)$ as wind direction distribution

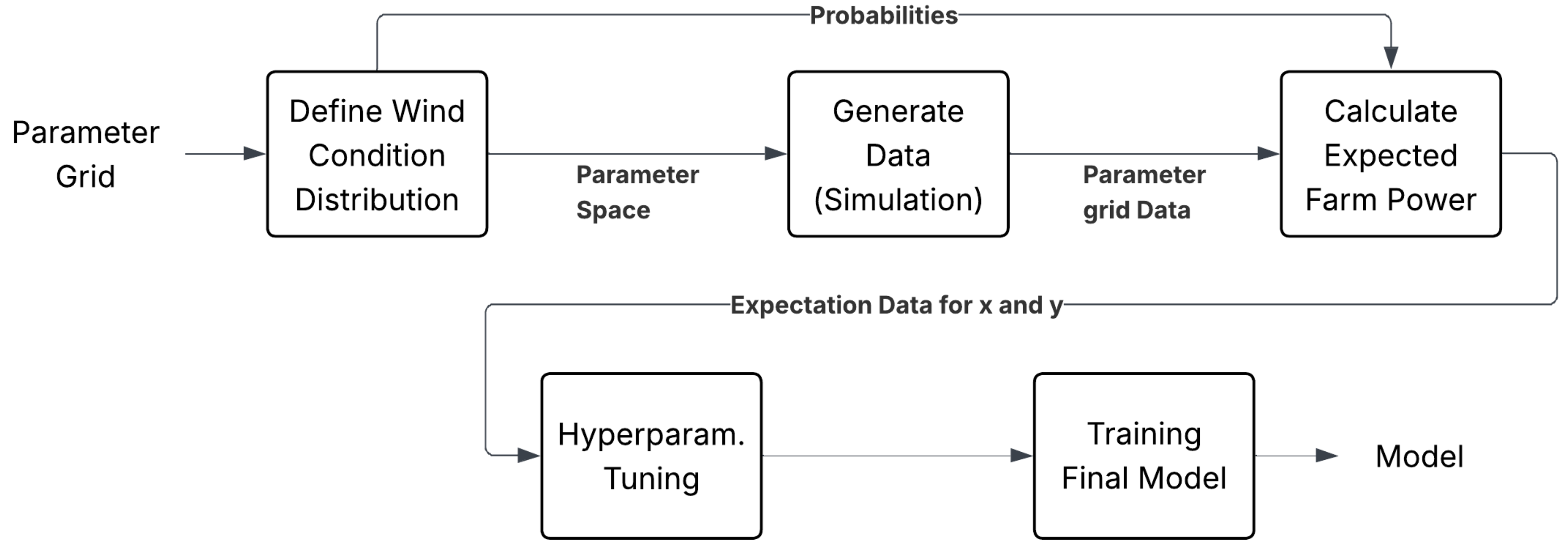


Wind Expectation Neural Network

$$
\begin{aligned}
\max_{\mathbf{x},\mathbf{y}} \quad & \boxed{\mathbb{E}\left[f_{Power}(\Delta x, \Delta y) \mid \text{wind condition distribution}\right]_{\text{NN}}} \\
\text{s.t.} \quad & 0 \le \Delta x \le X_{\max} \\
& 0 \le \Delta y \le Y_{\max} \\
& \sqrt{(\Delta x)^2 + (\Delta y)^2} \ge d_{\min}
\end{aligned}
$$

Modeling Process

$$\max_{\mathbf{x},\mathbf{y}} \mathbb{E}\left[f_{Power}(\Delta x, \Delta y) \mid \textbf{wind condition distribution}\right]_{\mathrm{NN}}$$



Parameter Grid

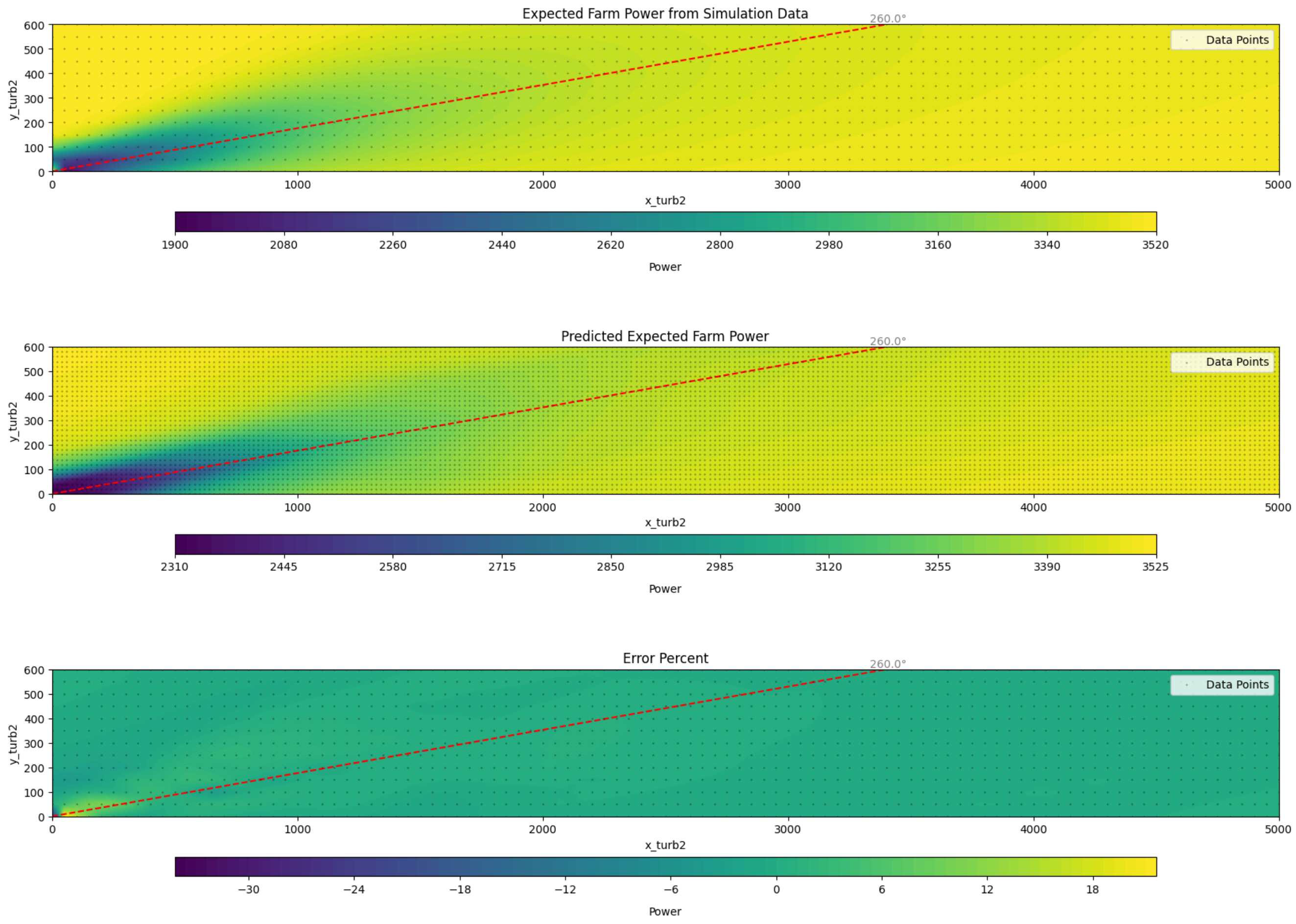
Variable	Const/Variable	Value	*Steplength*
Δx_{turb2}	Variable	[0, 5000] m	50 m
Δy_{turb2}	Variable	[0, 500] m	50 m
wind_speed	Constant	8 m/s	-
wind_direction	Variable	[180°, 270°]	35 Quantiles
turbulence_intensity	Constant	0.06	-

With wind direction distribution as $N(270°, 10°)$

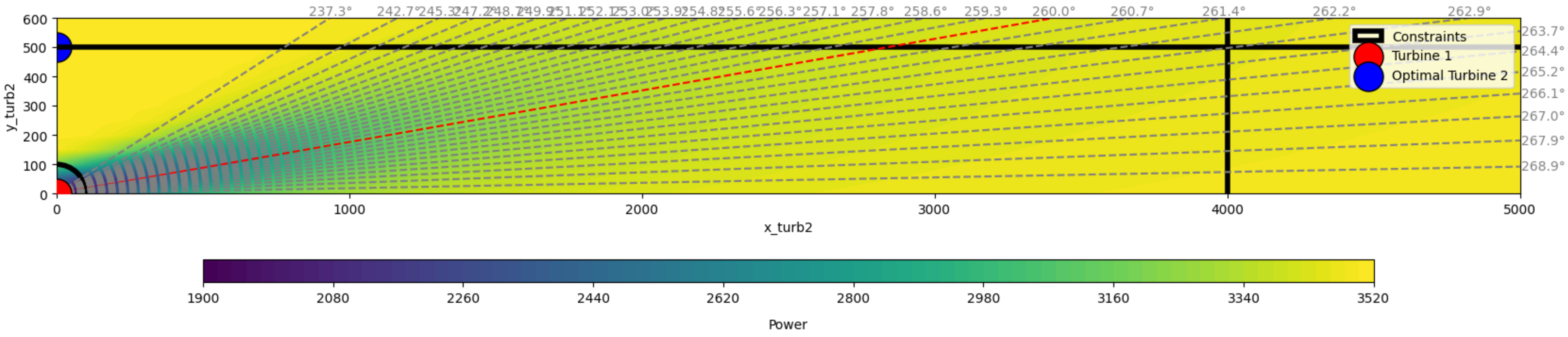
$\underbrace{\hspace{10em}}$

Generate Data Using FLORIS

Modeling (Using NN: $5 - 20^{\times 2} - 1$)



Optimization



Comparison

Wind Distribution independent Approach

- Delivers generally applicable solution independent of location/wind condition distribution
- Has limitations due to model complexity and number of possible scenarios

Direct Expectation Modeling Approach

- Delivers a Model conditional on a specific location/wind condition distribution
- Allows for a large number of scenarios using a small neural Network

Conclusion

Wind Distribution independent Approach

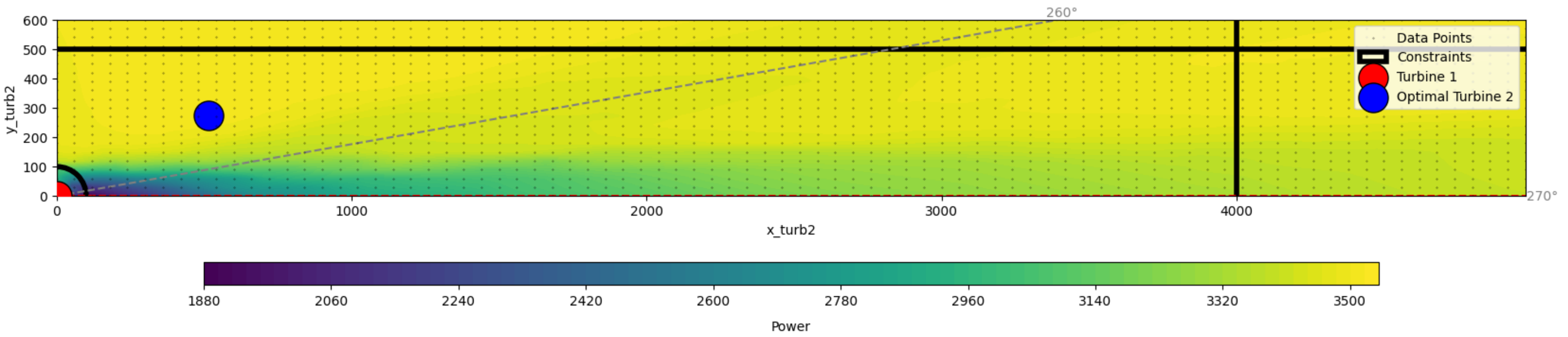
- Delivers generally applicable solution independent of location/wind condition distribution
- Has limitations due to model complexity and number of possible scenarios

Direct Expectation Modeling Approach

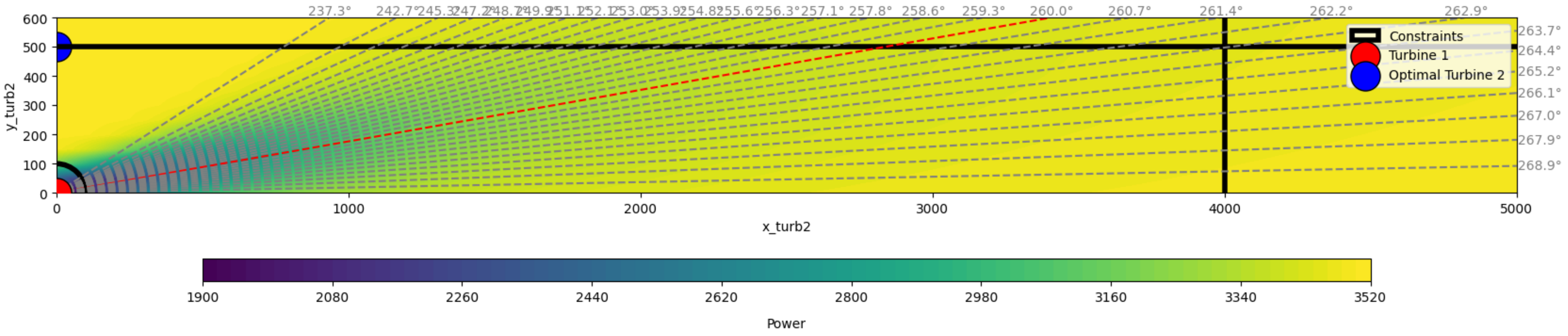
- Delivers a Model conditional on a specific location/wind condition distribution
- Allows for a large number of scenarios using a small neural Network

Questions?

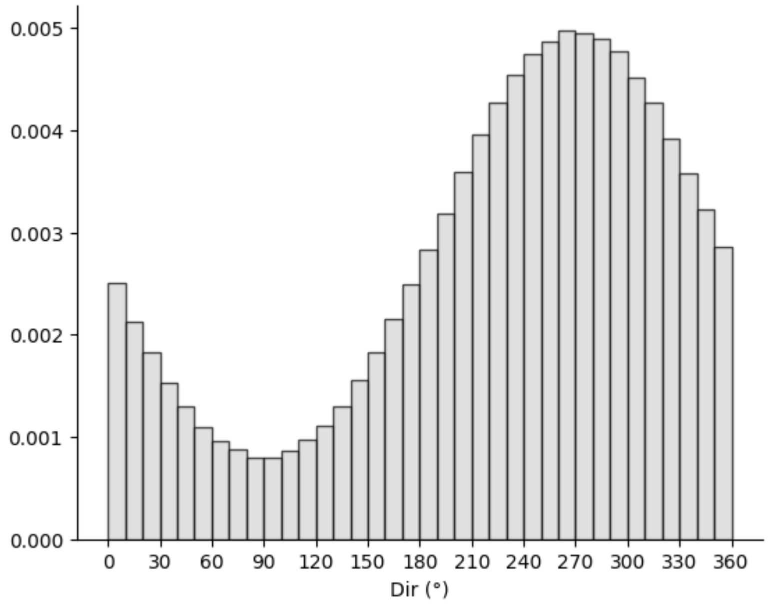
Wind Dist. Indip.

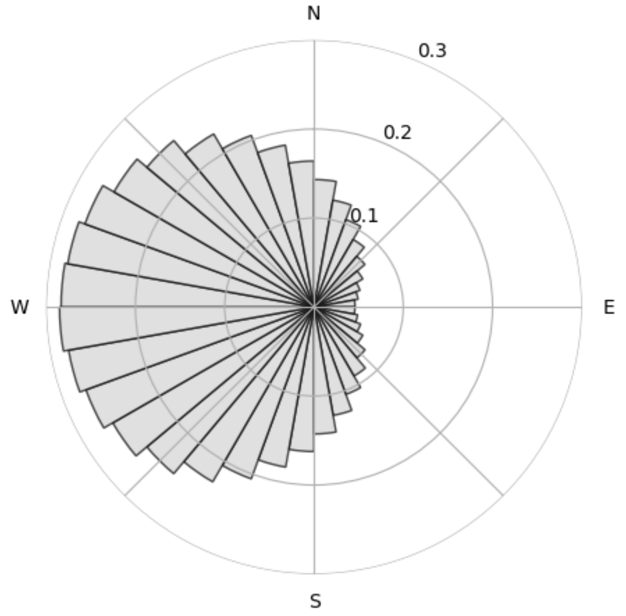


Expect. NN



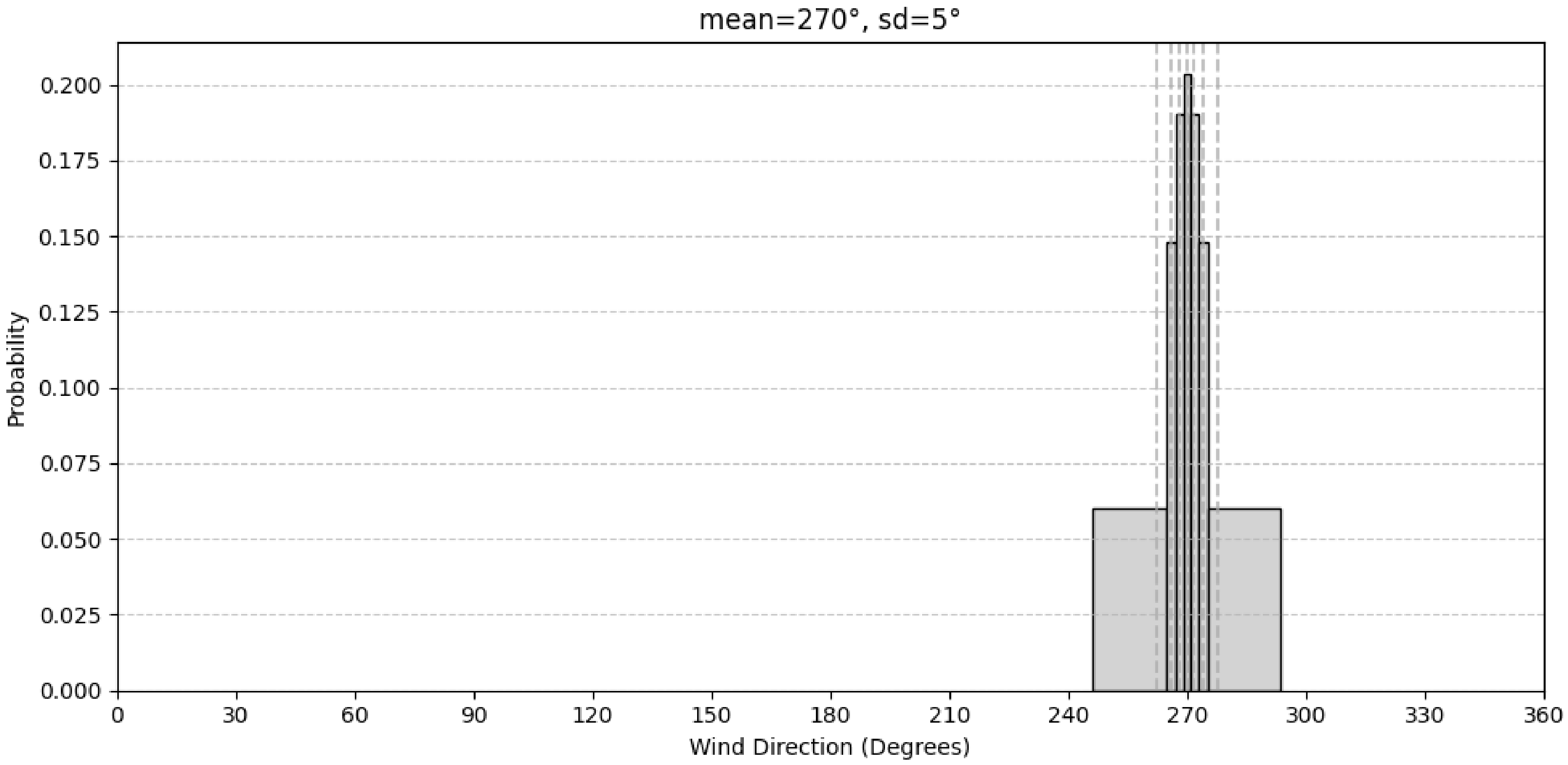
Expectation Maximization



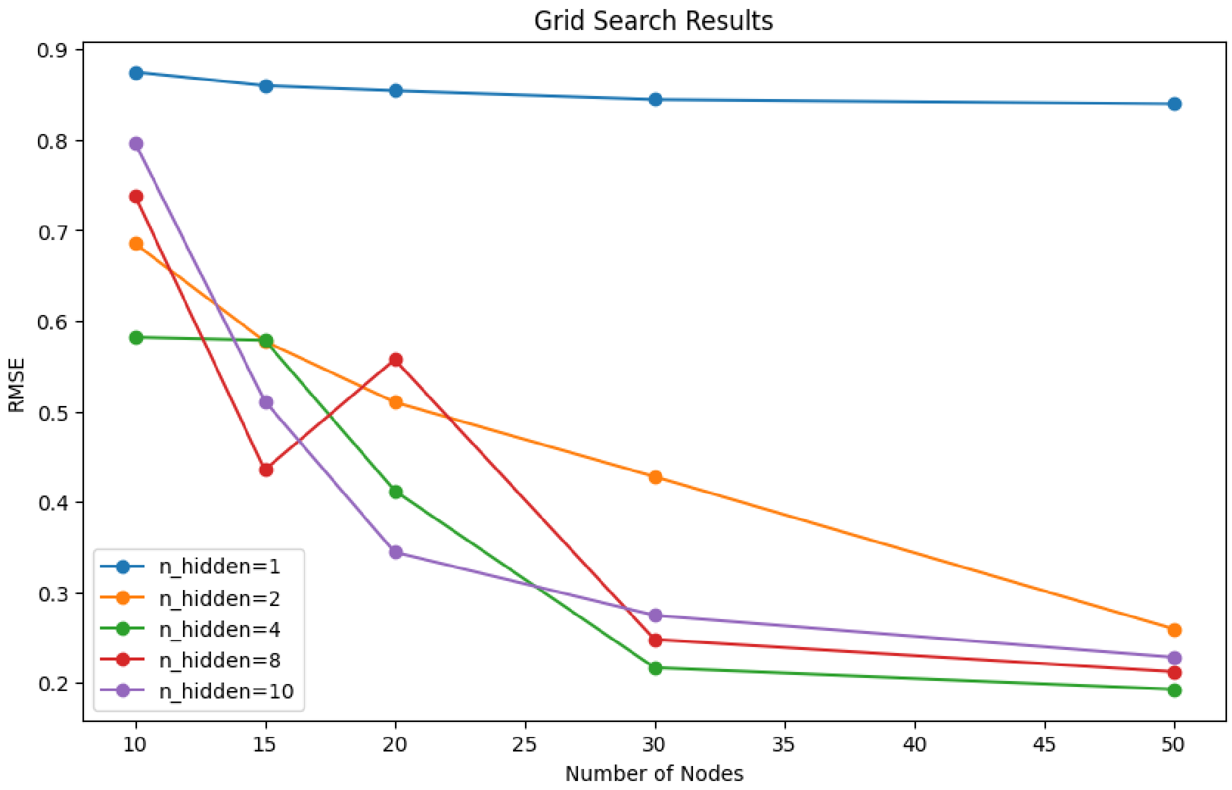


$$\mathbb{E}[X] = \sum_i x_i \cdot \mathbb{P}(X = x_i) = \sum_i x_i p_i$$

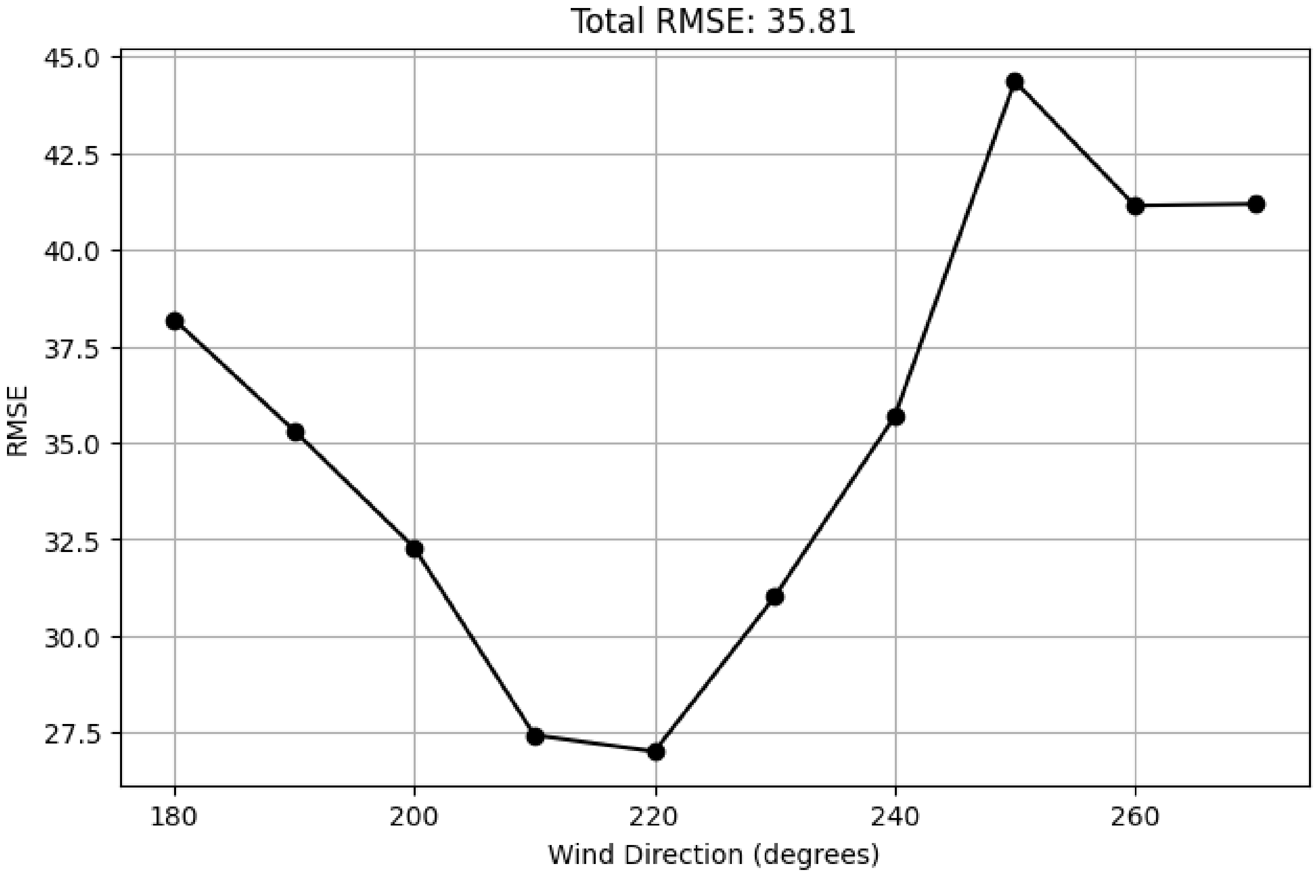
Probability density function discretization



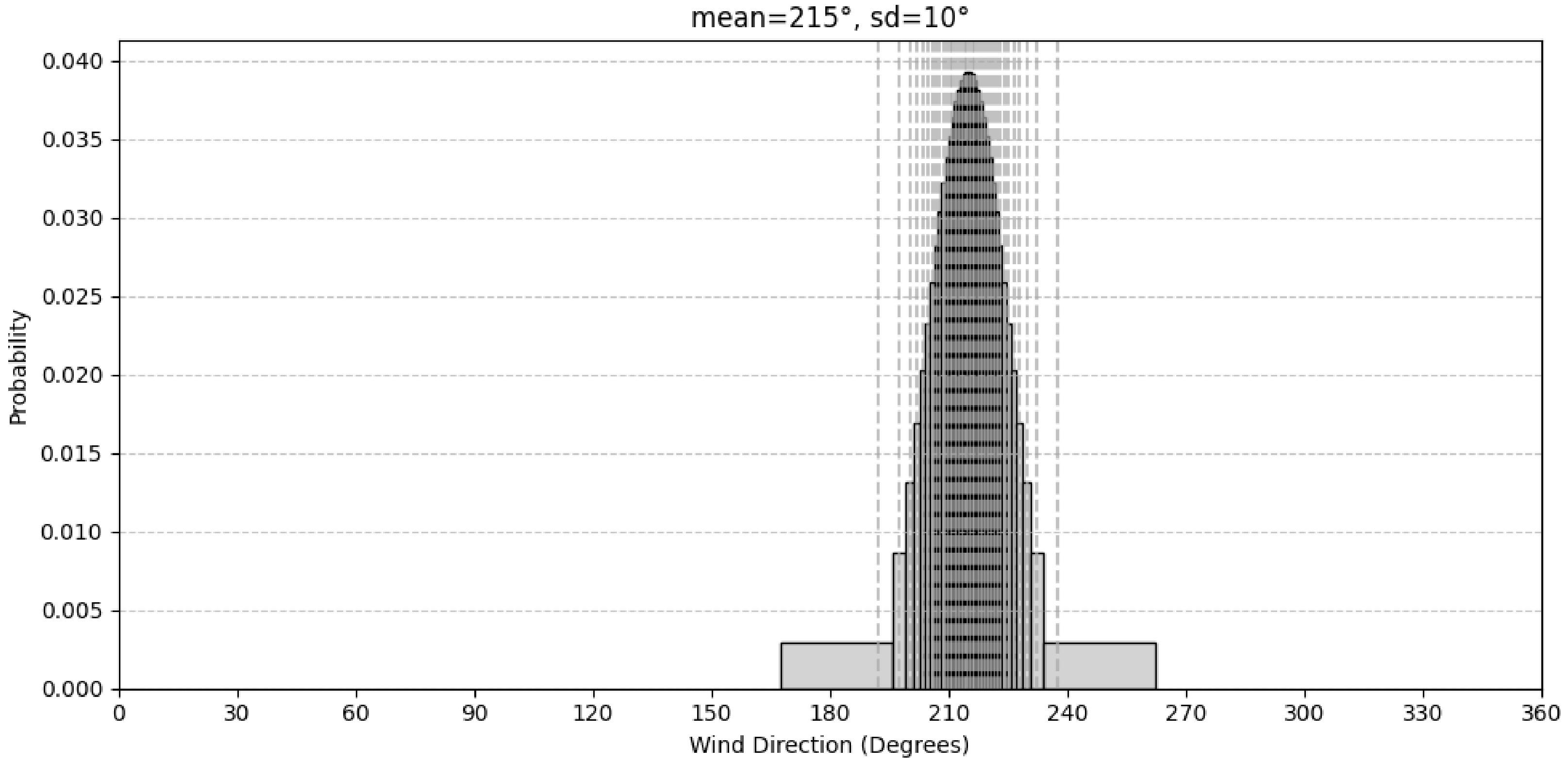
Appendix: Wind Condition indip NN



Appendix: Wind Condition indip NN



Appendix: Expectation Neural Network



Appendix: Expectation Neural Network