Graphical Models - Assignment 2

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```
library(readr)
library(dplyr)
library(knitr)
library(graph)
library(gRim)
library(gRain)
library(bnlearn)
library(Rgraphviz)
```

Taks

a) Provide a brief introduction to the data set

b) Try to define a possible network structure and give a brief explanation as to why this structure could make sense. Include a graph of the proposed model.

c) Fit the model and use it to make an out of sample prediction (like what we did in class to predict the probability a patient, who smokes and has not been to China, has tuberculosis).

d) Try to fit the graph structure of the model using one or more of the different approaches we saw in class. Do the fits make sense? If not you could use whitelists or blacklists to make certain links impossible.

e) Which of the proposed graphical structures is the best?

f) Provide a brief summary of your results.

a) Introduction

The following work on Bayesian Network was done as part of the Master in Statistics for Data Science at the Universidad Carlos III de Madrid and contains the design of a Graphical Bayesian Network model. The analysis was performed using the dataset titled "Weightlifting Injuries in Master Athletes" (https://zenodo.org/records/6679575), which comprises survey responses from 976 master weightlifters (aged 35–88, 51.1% female) across Australia, Canada, Europe, and the USA. Collected in June 2021, the data includes information on acute weightlifting-related injuries, chronic diseases, training practices, and sport history. Out a range of available variables, the following variables were extracted in the preprocessing of the data and then used for anaylsis:

- sex: sex of participant (m/f)
- agegrp3: age group of participant (35-49, 50-59, 60+)
- yrs_experience: years of experience in weightlifting (much = >2 years, little = <2 years)
- injury: sustained injuries in past (1/0) in any of the categories "shoulder", "knees", "back", "hips"
- train_warm: warm up before training in minutes (1/2+)
- coached: coached by a professional (1/0), either in person or online

the work begins with loading the data and some initial preprocessing. Then, graphical models are designed and fitted, first manually based on made assumptions and then based on design algorithms. The results are compared and a summary of the findings is provided.

```
### Data Preprocessing
## Load Data
data <- read_csv("data/wlinj_dryad.csv")
```

```
## Rows: 976 Columns: 44
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## chr  (3): sex, agegrp3, hips
## dbl (41): id, age, age_start, yrs_experience, shoulder, knees, back, wrist, ...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
## transform data

# aggregate yrs_experience
data <- data %>%
  mutate(across(all_of(c("yrs_experience")), ~ case_when(
    . <= 2 ~ "little",
    TRUE ~ "much"
  )))

# aggregate train_warm
data <- data %>%
  mutate(across(all_of(c("train_warm")), ~ case_when(
    . <= 1 ~ "1",
    TRUE ~ "2+"
  )))

# aggregate coached
data <- data %>%
  mutate(coached = if_else(pcoach == 1 | premote == 1, 1, 0))

# aggregate injury
data <- data %>%
  mutate(injury = if_else(shoulder == 1 | knees == 1 | back == 1 | hips == 1, 1, 0))

# select vars
data = data[c("sex", "agegrp3", "yrs_experience", "injury","train_warm","coached")]

# turn vars into factors
data <- data %>%
  mutate(across(everything(), as.factor))

# make sure data is a dataframe
data <- as.data.frame(data) # important dont know why but R is dogshiiiiiiiiit so I spend 20 min to fin

#print data
head(data)
```

```
##   sex agegrp3 yrs_experience injury train_warm coached
## 1   m   35-44         little      1         2+       1
## 2   m   45-59           much      1          1       1
## 3   m   35-44           much      1         2+       1
## 4   m   35-44           much      1         2+       1
## 5   f   45-59           much      0          1       1
## 6   m     60+           much      1          1       0
```

b&c) Manual Model Design

The following section contains the manual design of a graphical model, which is then fitted to the data and used to make predictions.

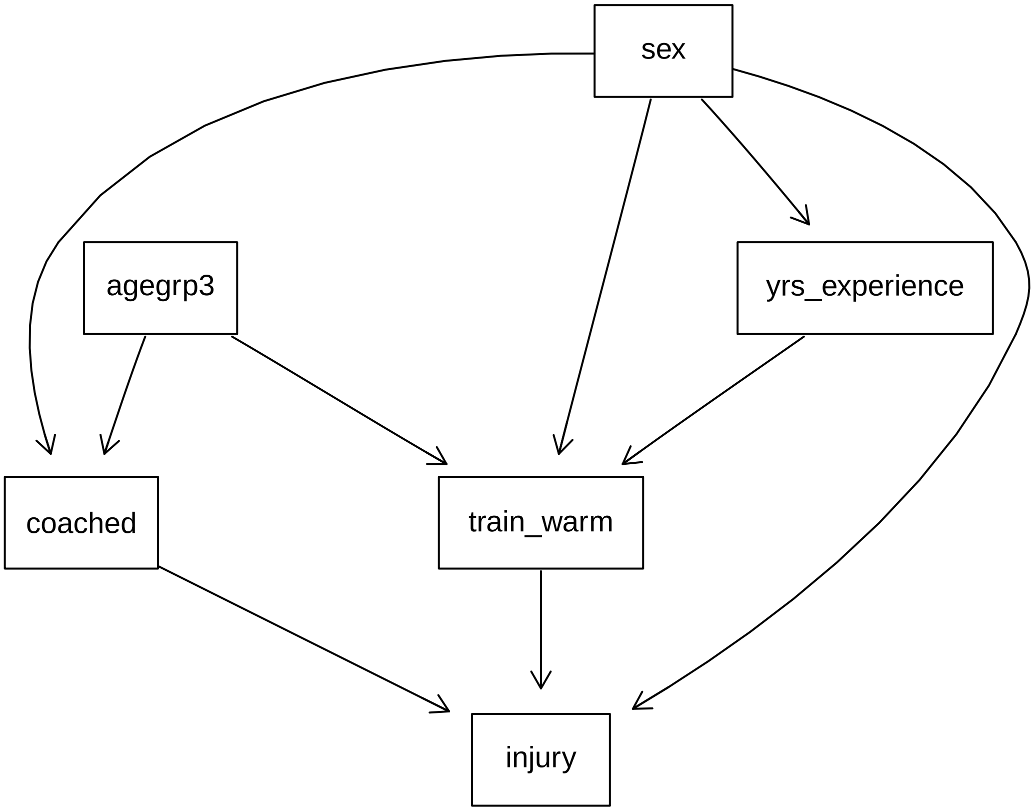
b) Design Graph (Manually)

We begin by designing a graphical model based on what variables we assume to be dependent. We begin by assuming that:

- **yrs_experience** is dependent on sex, as weight lifting appears to have started as a male dominated sport
- **train_warm** is dependent on yrs_experience, sex and agegrp3, as we assume that wiser are more likely to warm up before training and we expect that age, experience and not being male makes people wiser
- **coached** is dependent on agegrp3 and sex as we assume that older people prefer to do sport in groups and males to prefer to do sport alone
- **injury** is dependent on coached and train_warm, as we assume that people who are coached and/or properly warmed up are less likely to get injured, as well as dependent on sex as we expect men to tend to train more excessively than women

These assumptions yield the following model:

```
dag <- model2network("[sex][agegrp3][yrs_experience|sex][train_warm|sex:yrs_experience:agegrp3][coached
graphviz.plot(dag)
```



c) Fit Model and predict out of Sample

Next, we fit the model using the data and print out the probability tables of the resulting model.

```
# Using maximum likelihood.
model_fit = bn.fit(dag,data,method = "bayes")
model_fit
```

```
##
##   Bayesian network parameters
##
##   Parameters of node agegrp3 (multinomial distribution)
##
## Conditional probability table:
##     35-44     45-59       60+
## 0.3892869 0.4046401 0.2060730
##
##   Parameters of node coached (multinomial distribution)
##
## Conditional probability table:
##
## , , sex = f
##
##        agegrp3
## coached     35-44     45-59       60+
##       0 0.1208178 0.1108887 0.1501241
##       1 0.8791822 0.8891113 0.8498759
##
## , , sex = m
```

```
##
##        agegrp3
## coached     35-44     45-59       60+
##       0 0.3078975 0.3851291 0.6192547
##       1 0.6921025 0.6148709 0.3807453
##
##
##   Parameters of node injury (multinomial distribution)
##
## Conditional probability table:
##
## , , sex = f, train_warm = 1
##
##       coached
## injury         0         1
##      0 0.5383387 0.5522145
##      1 0.4616613 0.4477855
##
## , , sex = m, train_warm = 1
##
##       coached
## injury         0         1
##      0 0.3479638 0.3847162
##      1 0.6520362 0.6152838
##
## , , sex = f, train_warm = 2+
##
##       coached
## injury         0         1
##      0 0.4763314 0.4970782
##      1 0.5236686 0.5029218
##
## , , sex = m, train_warm = 2+
##
##       coached
## injury         0         1
##      0 0.2619962 0.3741659
##      1 0.7380038 0.6258341
##
##
##   Parameters of node sex (multinomial distribution)
##
## Conditional probability table:
##         f        m
## 0.511259 0.488741
##
##   Parameters of node train_warm (multinomial distribution)
##
## Conditional probability table:
##
## , , sex = f, yrs_experience = little
##
##           agegrp3
## train_warm     35-44     45-59       60+
##         1  0.7137767 0.5996678 0.7448980
##         2+ 0.2862233 0.4003322 0.2551020
##
## , , sex = m, yrs_experience = little
##
##           agegrp3
## train_warm     35-44     45-59       60+
##         1  0.6146497 0.4172414 0.7117647
##         2+ 0.3853503 0.5827586 0.2882353
##
## , , sex = f, yrs_experience = much
##
##           agegrp3
## train_warm     35-44     45-59       60+
##         1  0.6031291 0.6119709 0.6030383
##         2+ 0.3968709 0.3880291 0.3969617
##
## , , sex = m, yrs_experience = much
##
##           agegrp3
## train_warm     35-44     45-59       60+
##         1  0.5803727 0.5656830 0.6377049
##         2+ 0.4196273 0.4343170 0.3622951
##
##
##   Parameters of node yrs_experience (multinomial distribution)
##
## Conditional probability table:
##
##               sex
## yrs_experience          f          m
##         little 0.12862863 0.06753927
##         much   0.87137137 0.93246073
```

We then find a combination of variables that doesn't exist in our data

```
# find out of sample combination
all_combinations <- expand.grid(lapply(data, levels))
missing_combinations <- anti_join(all_combinations, data, by = names(data))
missing_combinations[1,]
```

```
##   sex agegrp3 yrs_experience injury train_warm coached
## 1   f     60+         little      0          1       0
```

```
# Why a combination? there are 18 out of the 96 possibles
```

and use that combination to perform a out of sample prediction for injury, yielding the following conditional probability of sustaining a injury for a female of 60+ years, with little experience who warms up only one minute before training.

```
# predict
cpquery(model_fit, (injury=="0"), (sex=="f" & agegrp3=="60+" & yrs_experience=="little" & train_warm  ==
```

```
## [1] 0.5547591
```

compared to that, a male who is 35-44 years old, has little experience and warms up for 1 minute before training has a larger probability of sustaining a injury while weightlifting

```
# predict
cpquery(model_fit, (injury=="1"), (sex=="m" & agegrp3=="35-44" & yrs_experience=="little" & train_warm
```

```
## [1] 0.6311922
```

d) Automatic Graph Design

With the manual model design done, we continue with automatically designing a model using two approaches: A score-based method and a hybrid method.

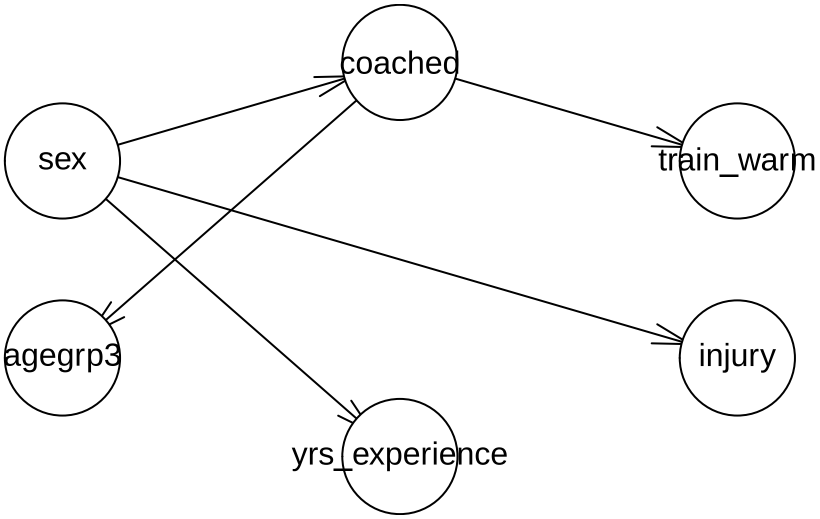
Scored Based Method

- **yrs_experience** is dependent on sex, as weight lifting appears to have started as a male dominated sport (same as we expected).
- **injury** apparently is dependent only on sex, this result is unexpected as we saw that other of the present variables can be related with having a injury.
- **coached** is dependent on sex, the theory of males to prefer to do sport alone may be right.
- **agegrp3** depends on coaching. This relation should not be in the model, because it does not make sense that the age change or not depending on how the person train.
- **train_warm** is dependent on coached, that can be clear to see that in a coached training a train warm usually is present.

```
scoredag<-hc(data)
scoredag
```

```
##
##   Bayesian network learned via Score-based methods
##
##   model:
##    [sex][yrs_experience|sex][injury|sex][coached|sex][agegrp3|coached]
##    [train_warm|coached]
##   nodes:                                 6
##   arcs:                                  5
##     undirected arcs:                     0
##     directed arcs:                       5
##   average markov blanket size:           1.67
##   average neighbourhood size:            1.67
##   average branching factor:              0.83
##
##   learning algorithm:                    Hill-Climbing
##   score:                                 BIC (disc.)
##   penalization coefficient:              3.441731
##   tests used in the learning procedure:  40
##   optimized:                             TRUE
```

```
plot(scoredag)
```



In general, as age and sex are characteristics proper of the weightlifters, it makes no sense that any of this two variables depend on any of the other nodes. For that reason a blacklist is created. Also, train warm is known to be really important to avoid injuries, that is why a white list is created with this relation.

- **Blacklist:**

From	To
coached	agegrp3, sex
injury	sex
sex	agegrp3
agegrp3	sex

- **Whitelist:**

From	To
train_warm	injury

With this changes we obtain the following results:

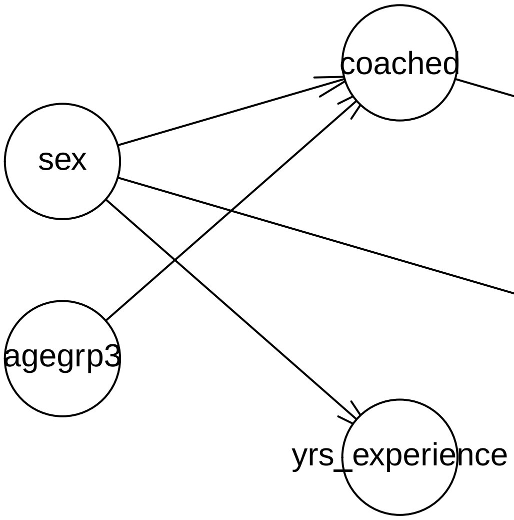
- **yrs_experience** is dependent on sex (same as before)
- **coached** is dependent on agegrp3 and sex, that is closer with what we assume at the beginning (older people prefer to do sport in groups and males prefer to do sport alone).
- **train_warm** is dependent on being coached, and indirectly, in the sex and age. That means that our initial thoughts have sense, but only passing through being coached, not because train warm depends on this two variables variables directly.
- **injury** is dependent on sex, but now also on train warm (as it is in the white list). This makes a more clear an logical relation of getting a injury as we obtain the relation

$$age \to coached \to warm \to injury$$

```r
blacklist <- data.frame(from = c("coached", "coached", "injury", "sex", "agegrp3"),
                        to = c("agegrp3", "sex", "sex", "agegrp3", "sex"), stringsAsFactors = FALSE)

whitelist <- data.frame(from = "train_warm", to = "injury", stringsAsFactors = FALSE)
```

```r
scoredag2 <- hc(data, blacklist = blacklist, whitelist = whitelist)
plot(scoredag2)
```



```r
scoredag2
```

```
##
##   Bayesian network learned via Score-based methods
##
##   model:
##    [sex][agegrp3][yrs_experience|sex][coached|sex:agegrp3][train_warm|coached]
##    [injury|sex:train_warm]
##   nodes:                                 6
##   arcs:                                  6
##     undirected arcs:                     0
##     directed arcs:                       6
##   average markov blanket size:           2.67
##   average neighbourhood size:            2.00
##   average branching factor:              1.00
##
##   learning algorithm:                    Hill-Climbing
##   score:                                 BIC (disc.)
##   penalization coefficient:              3.441731
##   tests used in the learning procedure:  41
##   optimized:                             TRUE
```

Hybrid Methods

First, we are going to make the graph structure without any restriction, and afterwards, we are going to take into consideration the white and black lists described before.

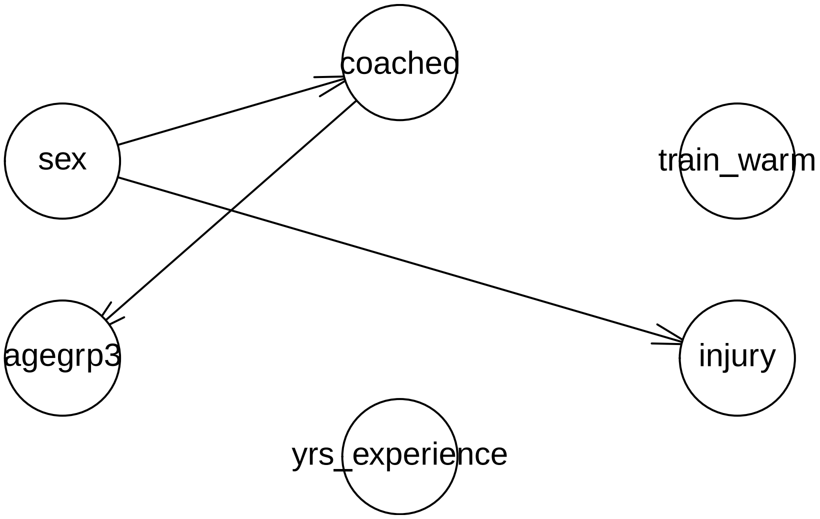
With this method we obtain only 3 relations between the 6 possible variables, and one of them is again counter intuitive ($coached \rightarrow agegrp3$).

```r
hybriddag<-mmhc(data)
hybriddag
```

```
##
##   Bayesian network learned via Hybrid methods
```

```
##
##   model:
##    [sex][yrs_experience][train_warm][injury|sex][coached|sex][agegrp3|coached]
##   nodes:                                 6
##   arcs:                                  3
##     undirected arcs:                     0
##     directed arcs:                       3
##   average markov blanket size:           1.00
##   average neighbourhood size:            1.00
##   average branching factor:              0.50
##
##   learning algorithm:                    Max-Min Hill-Climbing
##   constraint-based method:               Max-Min Parent Children
##   conditional independence test:         Mutual Information (disc.)
##   score-based method:                    Hill-Climbing
##   score:                                 BIC (disc.)
##   alpha threshold:                       0.05
##   penalization coefficient:              3.441731
##   tests used in the learning procedure:  135
##   optimized:                             TRUE
```

```
plot(hybriddag)
```



We include the same white and black lists that we use in the previous method. With this changes we obtain again a small number of relations:

- **coached** is dependent on agegrp3 and sex, that is closer with what we assume at the beginning (older people prefer to do sport in groups and males to prefer to do sport alone).
- **injury** is dependent on sex, but now also on train warm (as it is in the white list).

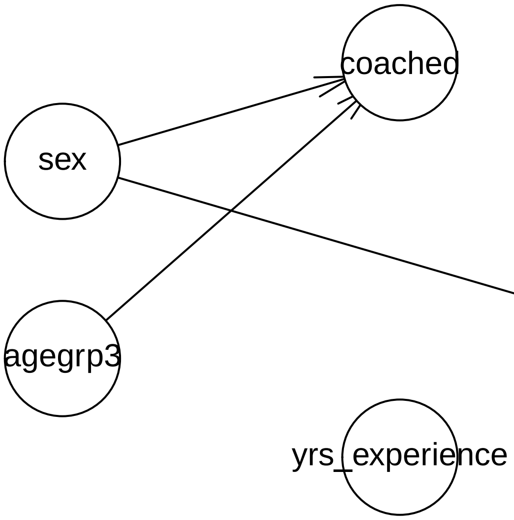
Nothing else is connected.

```
hybriddag2 <- mmhc(data, whitelist = whitelist, blacklist = blacklist)
hybriddag2
```

```
##
##   Bayesian network learned via Hybrid methods
##
##   model:
```

```
##    [sex][agegrp3][yrs_experience][train_warm][injury|sex:train_warm]
##    [coached|sex:agegrp3]
##   nodes:                                 6
##   arcs:                                  4
##     undirected arcs:                     0
##     directed arcs:                       4
##   average markov blanket size:           2.00
##   average neighbourhood size:            1.33
##   average branching factor:              0.67
##
##   learning algorithm:                    Max-Min Hill-Climbing
##   constraint-based method:               Max-Min Parent Children
##   conditional independence test:         Mutual Information (disc.)
##   score-based method:                    Hill-Climbing
##   score:                                 BIC (disc.)
##   alpha threshold:                       0.05
##   penalization coefficient:              3.441731
##   tests used in the learning procedure:  127
##   optimized:                             TRUE
```

```
plot(hybriddag2)
```



e) Compare Graphs

The model with smaller BIC and better coherence is the score-based algorithm with restrictions.

```
results <- data.frame(
  Model = c("Proposed", "Score", "Hybrid"),
  BIC = c(score(dag, data = data, type = "bic"),
          score(scoredag2, data = data, type = "bic"),
          score(hybriddag2, data = data, type = "bic"))
)

results
```

```
##      Model       BIC
## 1 Proposed -3923.119
## 2    Score -3875.815
## 3   Hybrid -3877.978
```

f) Summary and Conclusions

The in e) identified best model is in line with the main assumptions made in the initial manual model design like a dependence between sex and coaching, a overall dependence between sex and injury and a dependence between sex and years experience. Some of the assumptions however where not validated by the best automatic mode as for example the dependence between years of experience and injury. In general, most results fit well with the logical conclusion and are in line with what could be expected without the data. As shown in the previous section, the Score based model with restrictions is the best model would be recommended for practical application, delivering the best BIC score and a coherent model structure.

The res