Non-Parametric Statistics - Problem Sets

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Introduction

In the following document, the solutions to three problem sets from Prof. Eduardo García-Portugués book on nonparametric Statistics (https://bookdown.org/egarpor/NP-UC3M/) as final assignment for the Course in Nonparametric Statistics at Universidad Carlos III de Madrid. *Contributions made to this document are made are equal among each of the three authors*

Excercise 4.21

```
#TODO: Silvana
```

Excercises 5.10

Investigate the accuracy of the naive bootstrap confidence intervals implemented in *np::npplot*. To do so:

1. Simulate $M = 500$ samples of size $n = 100$ from the regression model $Y = m(X) + \varepsilon$, where $m(x) = 0.25x^2 - 0.75x + 3$, $X \sim N(0, 1.5^2)$, and $\varepsilon \sim N(0, 0.75^2)$.

```
library(np)
```

```
## Nonparametric Kernel Methods for Mixed Datatypes (version 0.60-18)
## [vignette("np_faq",package="np") provides answers to frequently asked questions]
## [vignette("np",package="np") an overview]
## [vignette("entropy_np",package="np") an overview of entropy-based methods]
```

```
set.seed(1234)
M = 500
n = 100
y_muestras = matrix(0,ncol = n,nrow = M)
m = function(a){
  0.25*a^2-0.75*a+3
}
for (i in 1:500){
  x = rnorm(100,0,1.5)
  epsilon = rnorm(100,0,0.75)
  y_muestras[i,] = m(x)+epsilon
}
```

2. Compute the 95% confidence intervals for $m(x)$ along $x \leftarrow seq(-5, 5, by = 0.1)$, for each of the M samples. Do it for the normal approximation and quantile-based confidence intervals.

```
x_grid = seq(-5,5,by=0.1)
```

3. Check if $m(x)$ belongs to each of the confidence intervals, for each x.
4. Approximate the actual coverage of the confidence intervals.

Excercise 6.8

```
#TODO: Simon
```