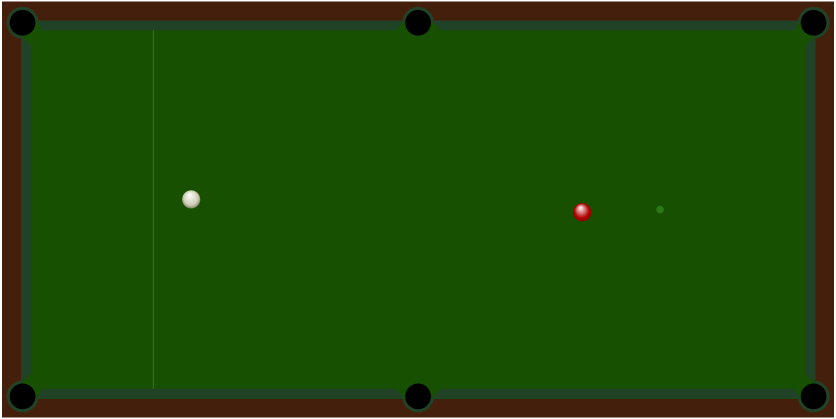
Proton form factors

Jan C. Bernauer

EINN preconference 2017

Massachusetts Institute of Technology

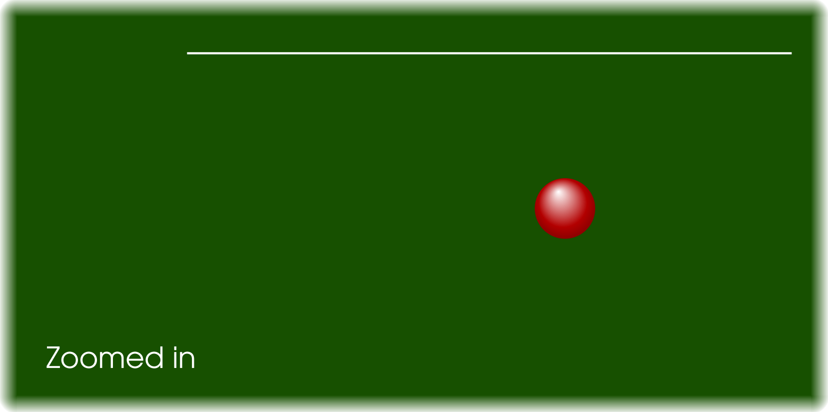
Electron scattering is like playing Billiards



Scattering is like playing Billiards. (And the holes are the detectors!)

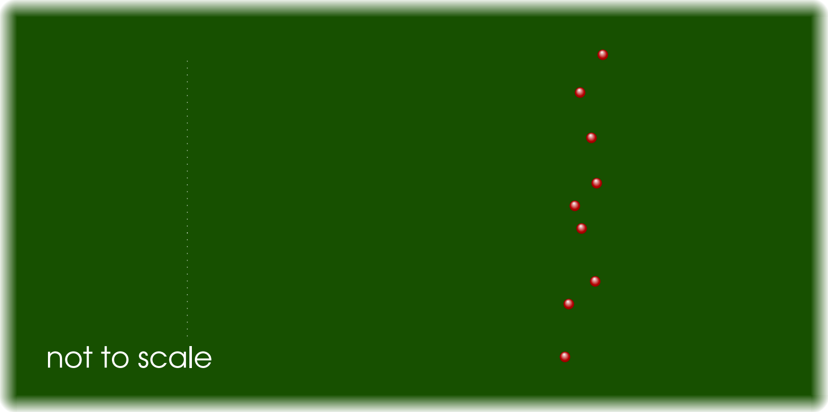
Electron scattering is like playing Billiards, but on an infinite table.

...with (many) point-like cue balls...



Electron scattering is like playing Billiards, but on an infinite table, with point-like, electrically charged cue-balls,

...and many target balls...

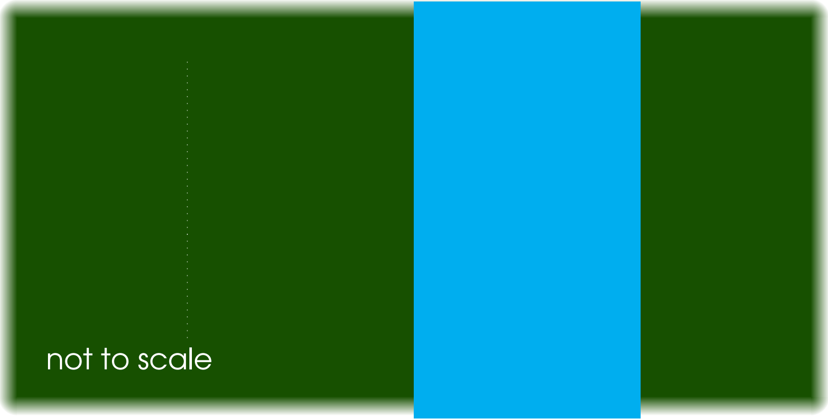


Electron scattering is like playing Billiards, but on an infinite table, with point-like, electrically charged cue-balls on 'zillions of targets, far apart,

...which are fuzzy...

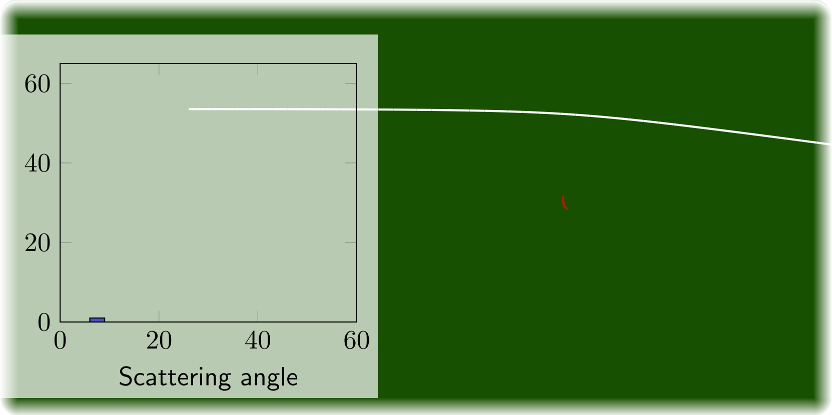
Electron scattering is like playing Billiards, but on an infinite table, with point-like, electrically charged cue-balls on 'zillions of targets, far apart, which are fuzzy, and have electric and magnetic charge,...

...and we don't know where they are...



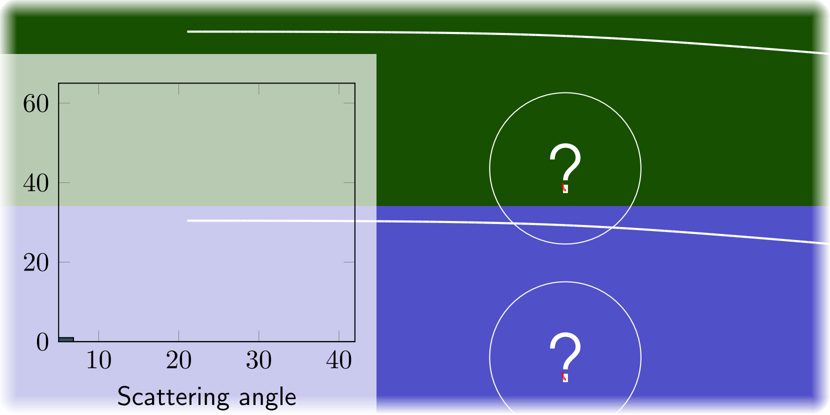
Electron scattering is like playing Billiards, but on an infinite table, with point-like, electrically charged cue-balls on 'zillions of targets, far apart, which are fuzzy, and have electric and magnetic charge, without knowing where exactly they are,

...recording in what direction they come out...



Electron scattering is like playing Billiards, but on an infinite table, with point-like, electrically charged cue-balls on ’zillions of targets, far apart, which are fuzzy, and have electric and magnetic charge, without knowing where exactly they are, recording in what direction they come out...

... to learn about the target's "shape"



Electron scattering is like playing Billiards, but on an infinite table, with point-like, electrically charged cue-balls on 'zillions of targets, far apart, which are fuzzy, and have electric and magnetic charge, without knowing where exactly they are, recording in what direction they come out, to learn about the target's shape.

The table

An experimentalists overview of the theory

In the interaction picture:

$$H = H_0 + V$$

$$\Psi(t) = U(t, t_0)\Psi(t_0)$$

$U(t, t_0)$ is the Dyson operator. It has to fulfill the Tomonga-Schwinger equation:

$$i\frac{d}{dt}U(t, t_0)\Psi(t_0) = V(t)U(t, t_0)\Psi(t_0)$$

$$\Longrightarrow U(t, t_0) = 1 - i\int_{t_0}^{t} dt_1 V(t_1)U(t_1, t_0)$$

Dyson series

$$U(t,t_0) = 1 - i\int_{t_0}^{t} dt_1 V(t_1) + (-i)^2 i\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 V(t_1)V(t_2) + \dots$$
$$= 1 + U_1 + U_2 + \dots$$

With the time ordering operator $\mathcal{T}$ we can rewrite U_n as:

$$U_n = \frac{(-i)^n}{n!}\int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \dots \int_{t_0}^{t} dt_n \mathcal{T} V(t_1)V(t_2)\dots V(t_n)$$

This gives us the Dyson series:

$$U(t,t_0) = \sum_{n=0}^{\infty} U_n(t,t_0) = \mathcal{T} e^{-i\int_{t_0}^{t} d\tau V(\tau)}$$

S matrix

Asymptotic $t_0 \to -\infty$, $t \to \infty$ gives us the S matrix:

$$S = U(\infty, -\infty) = \sum_{n=0}^{\infty} U_n(\infty, -\infty) = \mathcal{T} e^{-i \int_{-\infty}^{\infty} d\tau V(\tau)}$$

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This might not work after all!

PHYSICAL REVIEW VOLUME 85, NUMBER 4 FEBRUARY 15, 1952

Divergence of Perturbation Theory in Quantum Electrodynamics

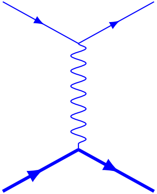
F. J. DYSON
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received November 5, 1951)

An argument is presented which leads tentatively to the conclusion that all the power-series expansions currently in use in quantum electrodynamics are divergent after the renormalization of mass and charge. The divergence in no way restricts the accuracy of practical calculations that can be made with the theory, but raises important questions of principle concerning the nature of the physical concepts upon which the theory is built.

Feynman graphs

Series elements can be calculated by drawing graphs and translating them with given set of rules.



First order Born scattering graph = single-photon exchange. Billiards.

Cross section for elastic scattering

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}} = \frac{1}{\varepsilon\left(1+\tau\right)}\left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$

with:

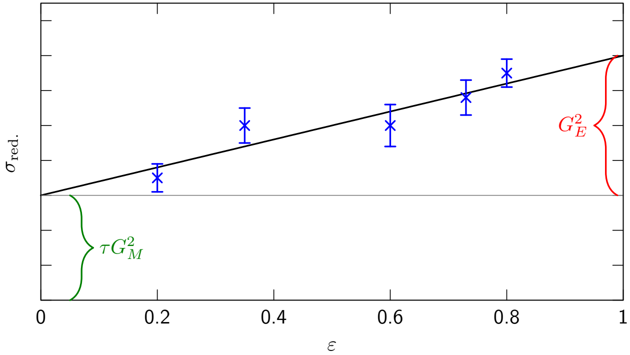
$$\tau = \frac{Q^2}{4m_p^2}, \quad \varepsilon = \left(1 + 2\left(1+\tau\right)\tan^2\frac{\theta_e}{2}\right)^{-1}$$

- Rosenbluth formula
- Electric and magnetic form factor encode the shape of the proton
- Fourier transform (almost) gives the spatial distribution, in the Breit frame

Rosenbluth separation to disentangle form factors

$$\sigma_{\text{red.}} = \varepsilon\,(1+\tau)\,\frac{\sigma_{\text{exp.}}}{\sigma_{\text{Mott}}} = \varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)$$

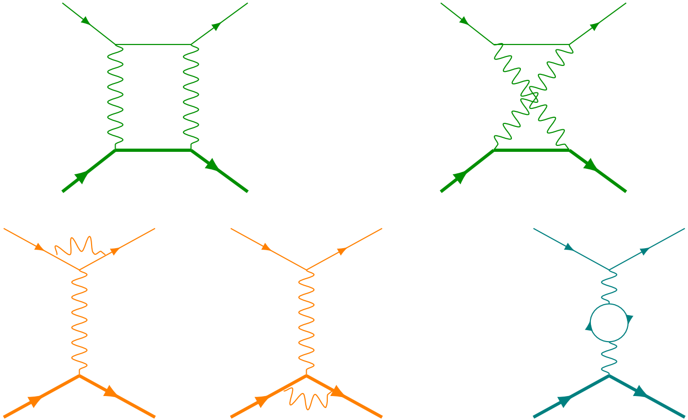
- Measure at constant Q^2 but different ε
- $\Rightarrow$ Different angle, beam energy!



Not so fast

- This is only the first term!
- What about higher orders?

TPE, vertex correction, vacuum polarization



Two photon exchange, vertex correction, vacuum polarization

Physicists are lazy

- We can calculate these graphs
    - proton side not easy, because of internal structure
    - loop integrals
    - some assumptions to make it workable

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Map back to first Born approximation

$$\frac{d\sigma}{d\Omega}_{\mathrm{exp.}} = \frac{d\sigma}{d\Omega}_{\mathrm{1st.\ Born}} (1 + \delta)$$

More graphs, more problems

- Vertex corrections are IR and UV divergent.
- Regularization removes UV divergence

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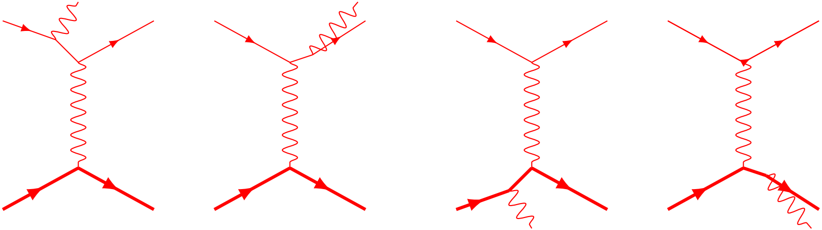
- Vertex corrections are IR and UV divergent.
- Regularization removes UV divergence
- Have to cancel IR divergence by IR divergence of bremsstrahlung graphs



- Correction now depends on acceptance of detector
- Simplest case:

$$\frac{d\sigma}{d\Omega}_{\text{exp.}} = \frac{d\sigma}{d\Omega}_{\text{1st. Born}}(1 + \delta(\Delta E))$$

How?

- In principle, these two processes are distinguishable—one has an extra photon
  - That's why the cancelation is on the cross section level, not on the amplitude level!

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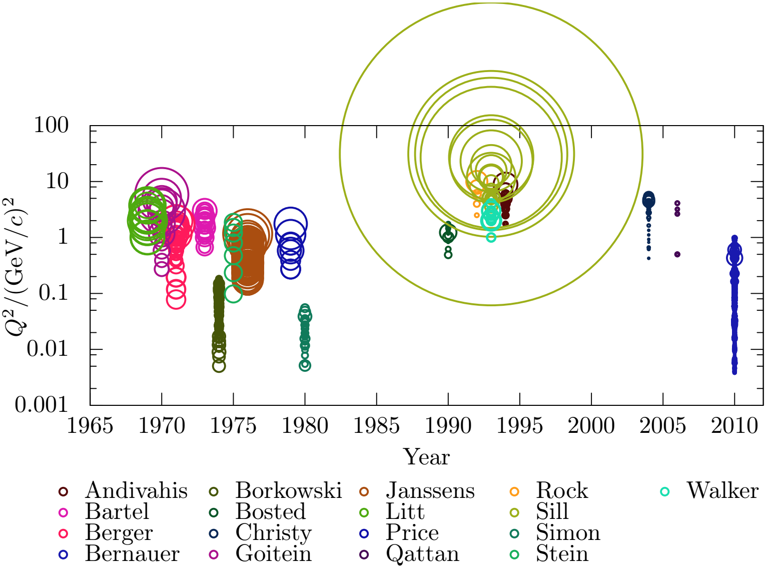
How?

- In principle, these two processes are distinguishable—one has an extra photon
  - That's why the cancelation is on the cross section level, not on the amplitude level!
- Standard argument: Any experiment has finite resolution and/or finite detectable photon threshold$\rightarrow$ indistinguishable
- My view:
  - Feynman graphs are a mathematical tool to calculate a series expansion of the real quantity.
  - Series terms do not directly relate to real physics processes.
  - There is no true elastic scattering. Will always produce infinite number of photons with vanishing momentum.

History of unpolarized electron-proton scattering



High-precision p(e,e′)p measurement at MAMI

High-precision p(e,e′)p measurement at MAMI

Mainz Microtron

- cw electron beam
- 10 μA polarized,
  100 μA unpolarized
- MAMI A+B: 180-855 MeV
- MAMI C: 1.6 GeV

A1 3-spectrometer facility

- 28 msr acceptance
- angle resolution: 3 mrad
- momentum res.: 10^{-4}

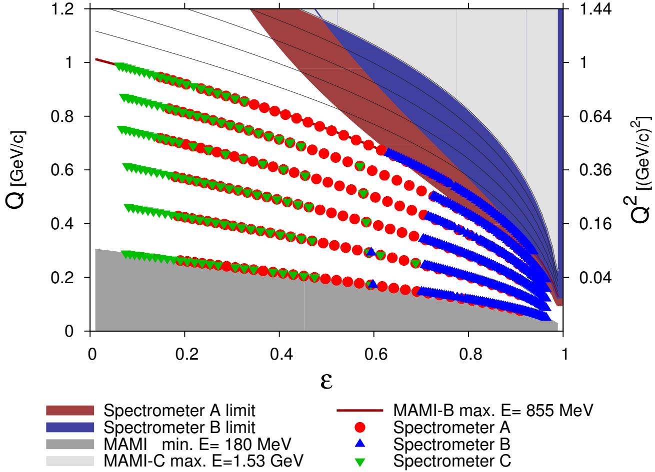
Johannes Gutenberg Uni

Image © 2015 GeoBasis-DE/BKG
Image © 2015 DigitalGlobe
Image © 2015 AeroWest

3.97 mi

Google earth

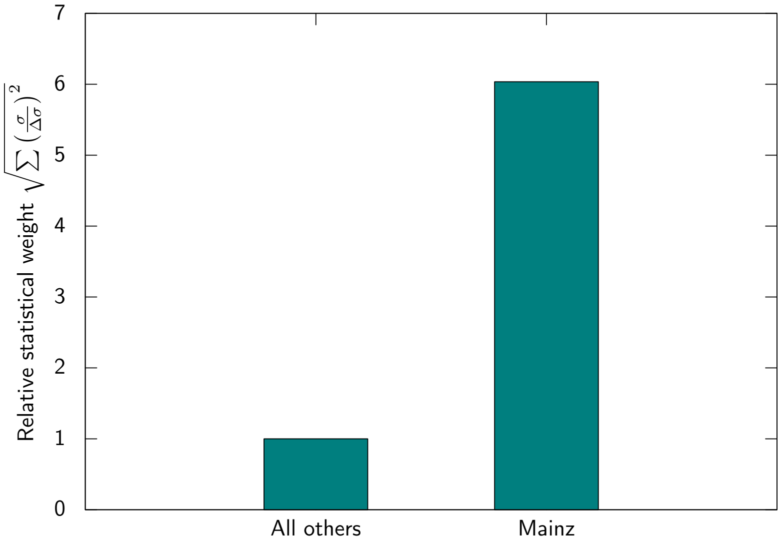
Measured settings



1422 settings

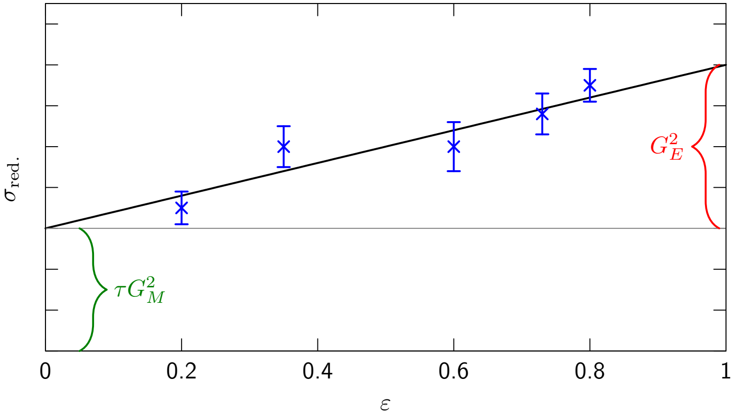
JCB et al., Phys. Rev. Lett. 105 (2010) 242001,
M. O. Distler, JCB, Th. Walcher, Phys. Lett. B 696, 343 (2011)
JCB et al., Phys. Rev. C90 (2014) 015206

Volume of Data



- Classic: Rosenbluth separation

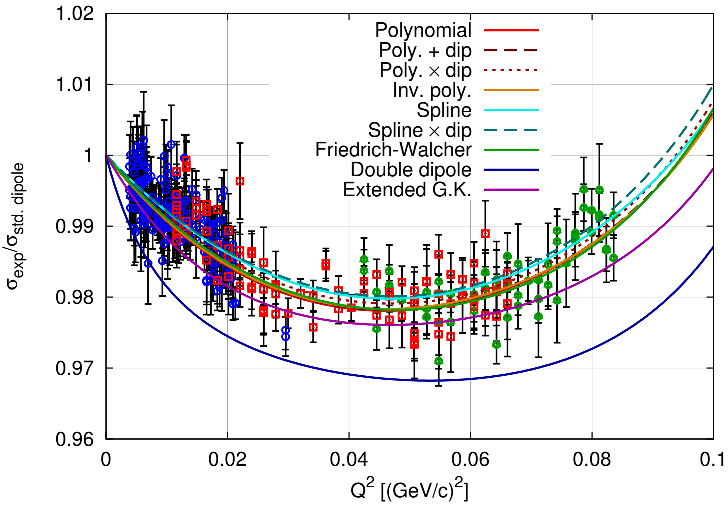
$$\sigma_{\text{red.}} = \varepsilon\,(1+\tau)\,\frac{\sigma_{\text{exp.}}}{\sigma_{\text{Mott}}} = \varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)$$



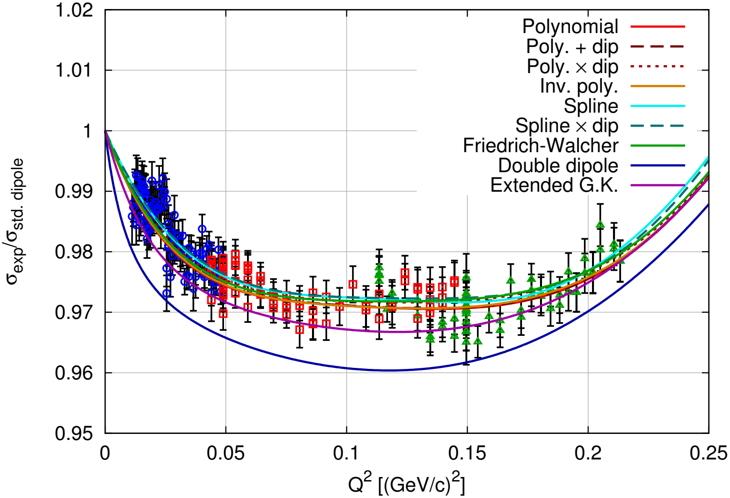
Novel techniques

- ~~Classic: Rosenbluth separation~~
- "Super-Rosenbluth separation": Fit of form factor models directly to the measured cross sections
  - All data contribute to the fit
  - No (explicit) projection needed
  - Easy fixing of normalization
- Comprehensive study of model dependence

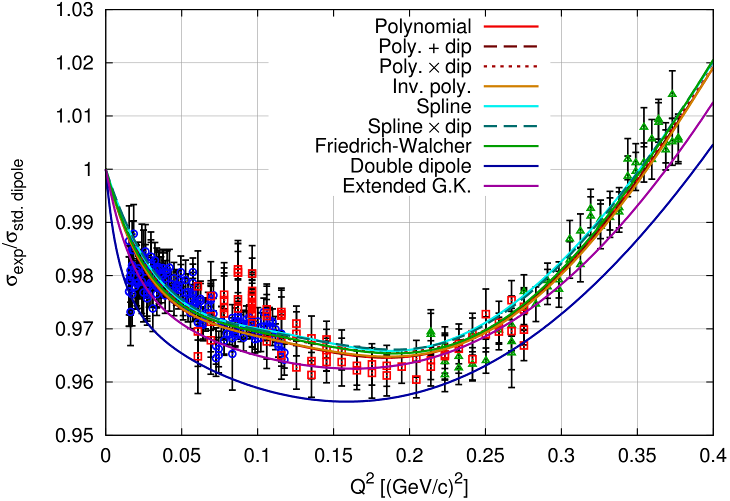
Cross sections: 180 MeV



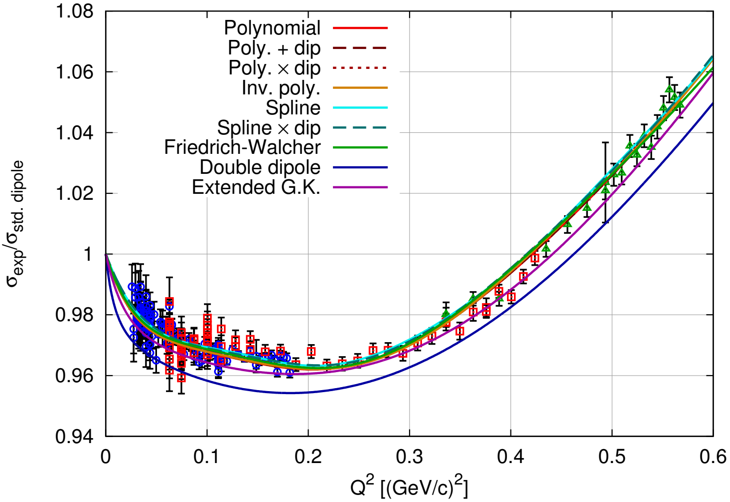
Cross sections: 315 MeV



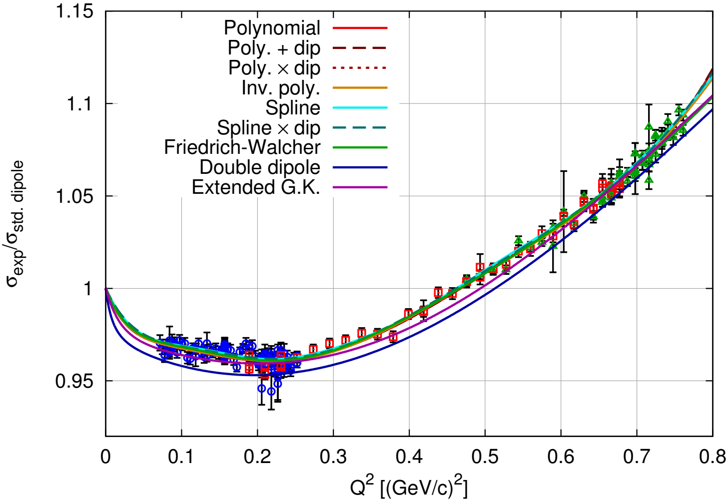
Cross sections: 450 MeV



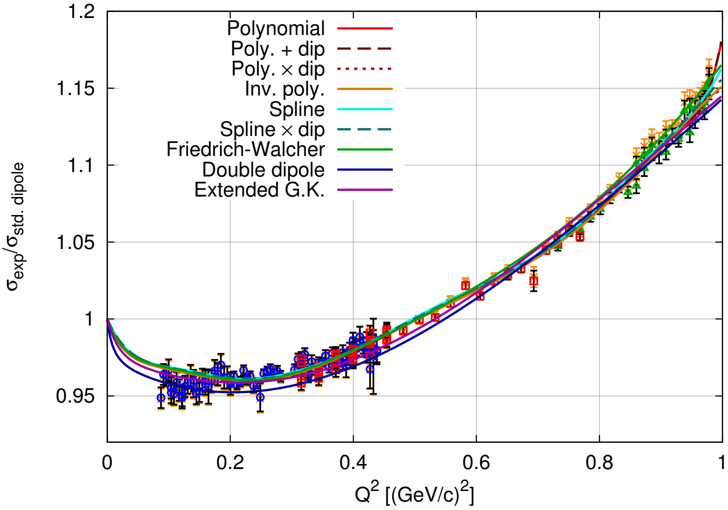
Cross sections: 585 MeV



Cross sections: 720 MeV



Cross sections: 855 MeV



Inclusion of world data

- Extend data base with world data
  $\Longrightarrow$ Cross check, extend Q^2 reach

Inclusion of world data

- Extend data base with world data
  $\Longrightarrow$ Cross check, extend Q^2 reach
- Take **cross sections** from Rosenbluth exp's
- Sidestep unknown error correlation
  - Update / standardize radiative corrections
  - One normalization parameter per source

L. Andivahis *et al.*,
Phys. Rev. D50, 5491 (1994).
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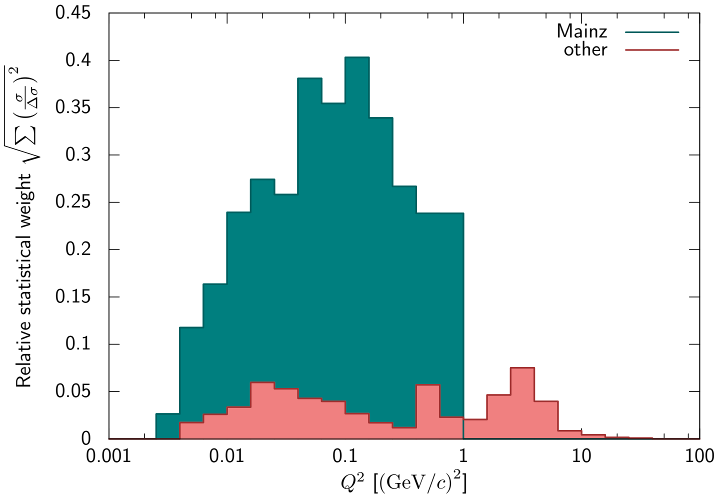
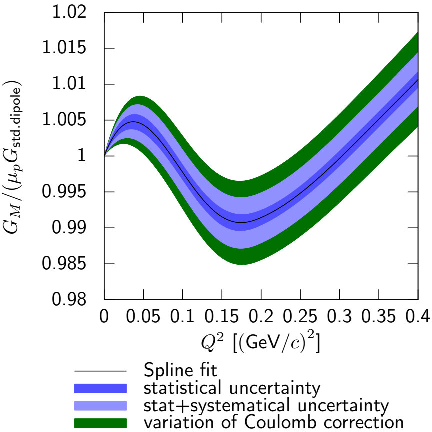
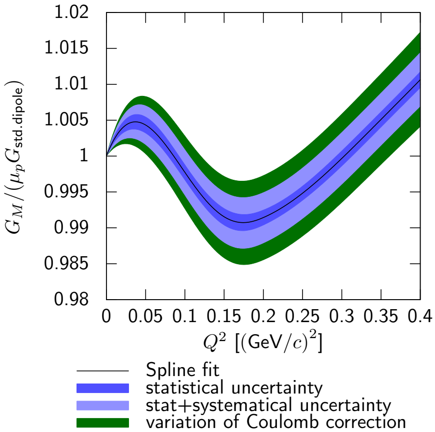

Relative statistical weight $\sqrt{\sum\left(\frac{\sigma}{\Delta\sigma}\right)^2}$
Q^2 [(GeV/c)^2]
Mainz
other
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.001
0.01
0.1
1
10
100

Magnetic form factor: Low-Q



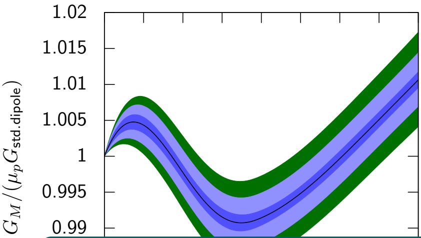
- Up-Down-Up structure
- Not seen before
  - Older fits approach from below
  - Lack of data

Magnetic form factor: Low-Q



- Up-Down-Up structure
- Not seen before
  - Older fits approach from below
  - Lack of data
- Gives rise to small r_m
- Calculations / physics motivated models typically do not reproduce
- Sensitive to radiative corrections

Magnetic form factor: Low-Q



- Up-Down-Up structure
- Not seen before
  - Older fits approach from below
  - Lack of data
- Gives rise to small r_m

Need new data

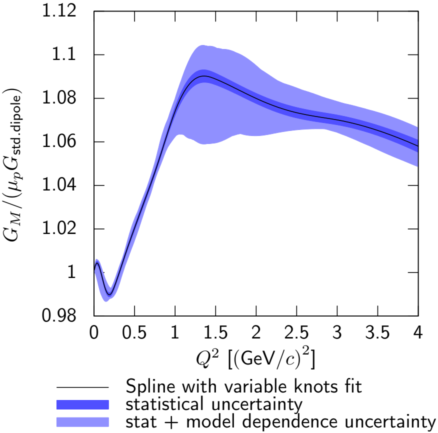
- Low beam energy at large angles
- Ideal: ERLs: JLab, Cornell, MESA
- Possibly measured with DarkLight

lly

variation of Coulomb correction

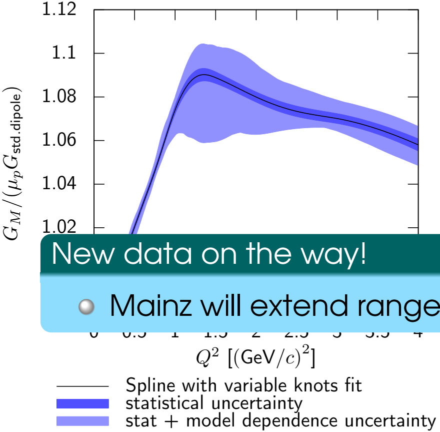
corrections

Magnetic form factor: High-Q



- Cusp between 1 and 1.5 (GeV/c)^2
- Seen in older fits
- Mainz data $<$ 1 (GeV/c)^2
- Should be visible in Lattice QCD

Magnetic form factor: High-Q



- Cusp between 1 and 1.5 (GeV/c)^2
- Seen in older fits
- Mainz data $<$ 1 (GeV/c)^2

New data on the way!

- Mainz will extend range to about 2 (GeV/c)^2

First puzzle: The form factor ratio

$$\sigma_{\text{red.}} = \varepsilon \left(1+\tau\right) \frac{\sigma_{\text{exp.}}}{\sigma_{\text{Mott}}} = \varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)$$

- Q^2 large $\rightarrow G_M$ term dominates, G_E term hard to measure
- Direct measurement via polarization degrees of freedom

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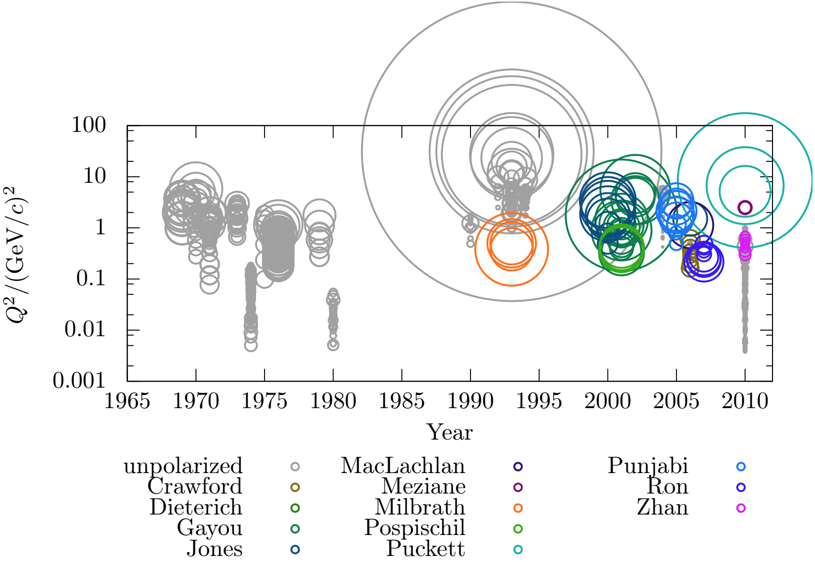
Polarization transfer

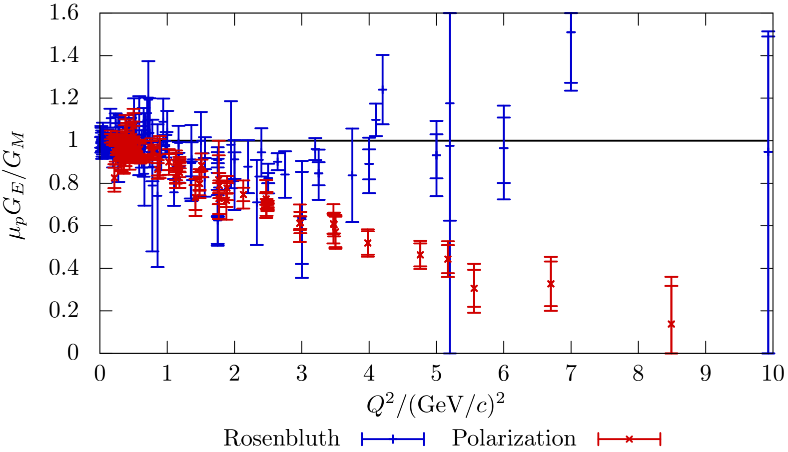
- Polarized beam
- Measure polarization of recoiling protons

$$\frac{G_E}{G_M} \propto \frac{P_T}{P_L}$$

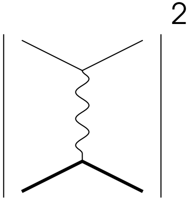
Measurements with polarization: FF ratio



Ratio: Difference!



Expected explanation: Two Photon Exchange



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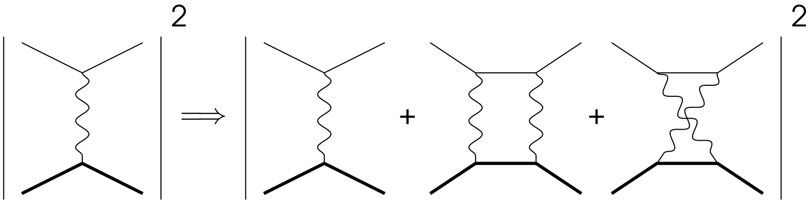

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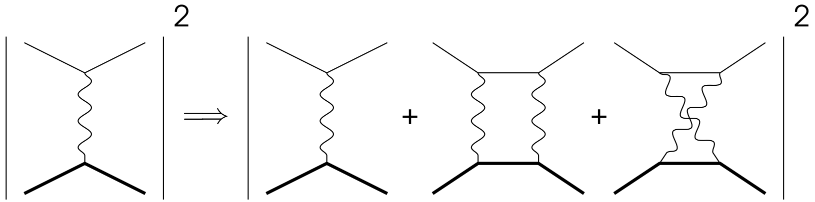
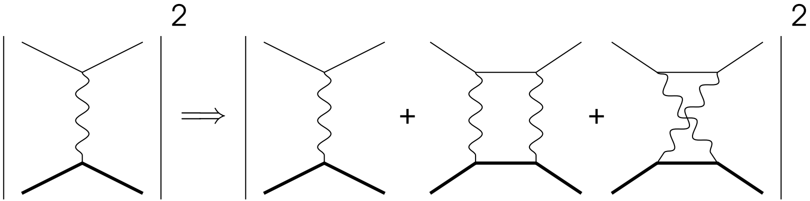


Impact of direct measurement

- Reconcile Rosenbluth and polarized data?
- How to treat (off-shell) hadron line?

Expected explanation: Two Photon Exchange



Method

- Mixed term changes sign with lepton sign:

$$\sigma(e^{\pm}p) \propto \left|M_{1\gamma}\right|^2 \pm 2\Re\left\{M_{1\gamma}^{\dagger}M_{2\gamma}\right\} + \ldots$$

$$R_{2\gamma} = \frac{N_{exp}^{e^+}}{N_{exp}^{e^-}} \Big/ \frac{N_{MC}^{e^+}}{N_{MC}^{e^-}}$$

- MC contains luminosity, acceptance, soft radiative corrections

Can TPE explain discrepancy? Extract effect size from data!

- Available polarization data is sparse
- Mostly Q^2 dependence
- Few data points on ε dependence

Can TPE explain discrepancy? Extract effect size from data!

- Available polarization data is sparse
- Mostly Q^2 dependence
- Few data points on ε dependence
- Add polarization data to Mainz global fit
- Include correction $1 + \delta$ on top of Feshbach ($Q^2 = 0$ limit), with:

$$\delta = a \cdot (1 - \varepsilon) \cdot \log\left(1 + b \cdot Q^2\right)$$

Direct measurement: Three modern experiments

CLAS

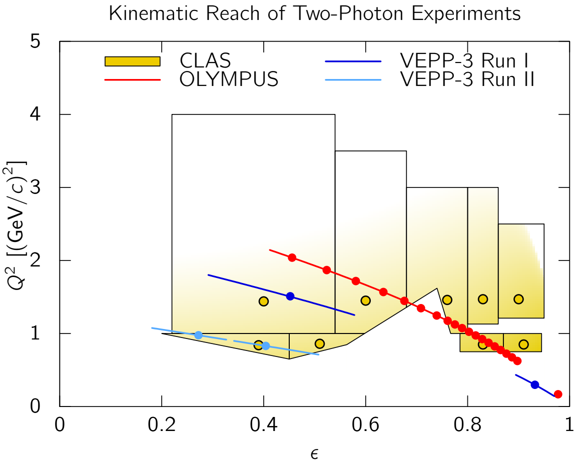
- e^- to γ to $e^{+/-}$ -beam
- Phys. Rev. C 95, 065201 (2017)
- PRL 114, 062003

VEPP-3

- 1.6/1 GeV beam
- no field
- Phys. Rev. Lett. 114, 062005 (2015)

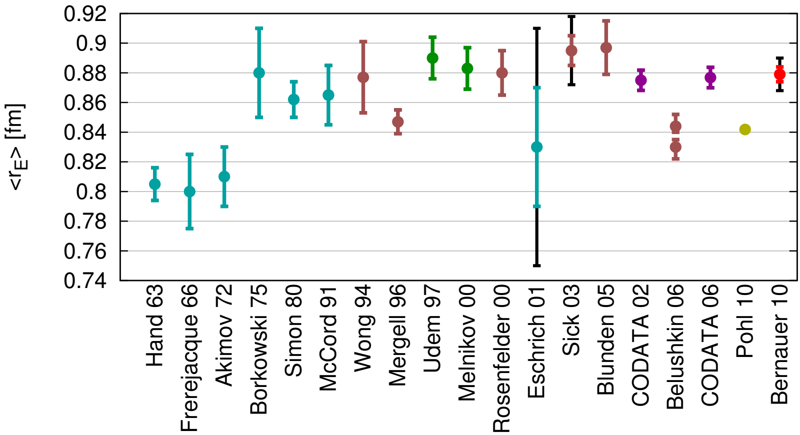
OLYMPUS

- DORIS @ DESY
- 2 GeV beam
- Phys. Rev. Lett. 118, 092501 (2017)



Second puzzle: The Proton Radius

$$\left\langle r_E^2 \right\rangle = -6\hbar^2 \left. \frac{dG_E}{dQ^2} \right|_{Q^2=0} \qquad \left\langle r_M^2 \right\rangle = -6\hbar^2 \left. \frac{d\left(G_M/\mu_p\right)}{dQ^2} \right|_{Q^2=0}$$



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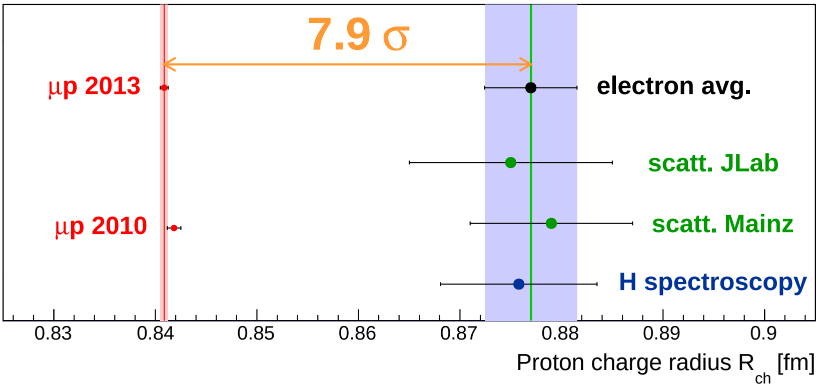

Second puzzle: The Proton Radius

What does TPE mean for the radius

- Findings so far in agreement with "Feshbach" :)
- But still off from calculations!

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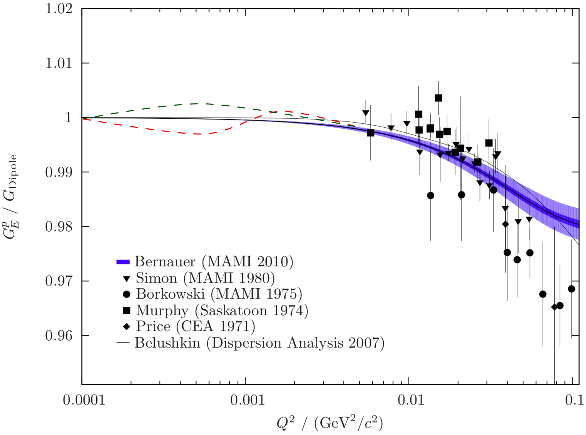
- But still off from calculations!

	r_e (fm)	r_m (fm)
(ours) McKinley/Feshbach	0.879	0.777
Borisyuk/Kobuskin	0.876	0.803
Arrington/Sick	0.875	0.769
Blunden et al.	0.875	0.799

Probably not important for electric radius.
Very important for magnetic radius!
$\Longrightarrow$ Measure TPE at low Q^2 / small ε too!

Extrapolation problematic? Structures at low Q^2?



Multiple ways to lower Q^2

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = \frac{1}{\varepsilon(1+\tau)}\left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$

$$\tau = \frac{Q^2}{4m_p^2}, \quad Q^2 = 4EE'\sin^2\frac{\theta}{2}$$

- For smaller Q^2 :
  - smaller θ (PRad)
  - smaller E (Rosenbluth / ISR)

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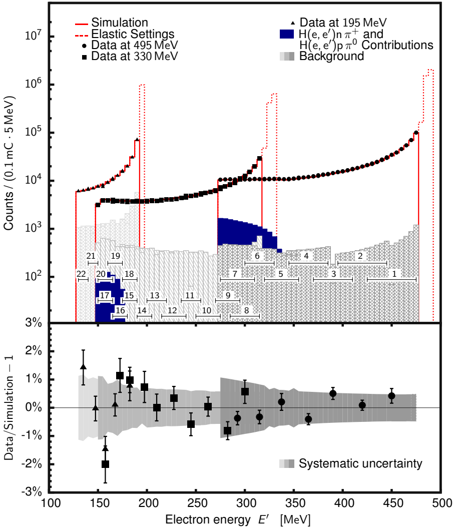
- For smaller Q^2 :
  - smaller θ (PRad)
  - smaller E (Rosenbluth / ISR)
- At small Q^2, small $\theta \implies \varepsilon >> \tau$, can use model for G_M

ISR at MAMI

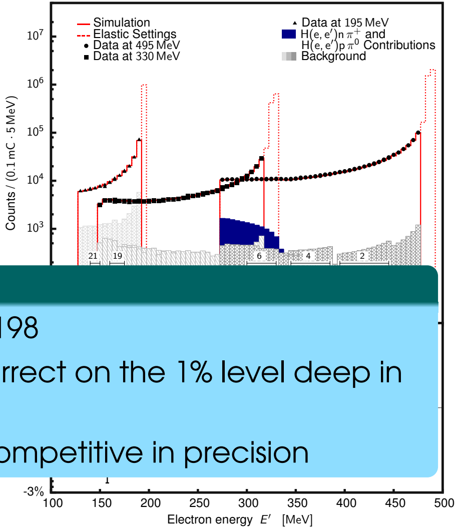
- ISR $\longrightarrow$ small E $\longrightarrow$ small Q^2
- Extract F.F. from radiative tail
- Or: test radiative tail description



See: arXiv:1612.06707

ISR at MAMI

- ISR $\longrightarrow$ small E $\longrightarrow$ small Q^2
- Extract FF from

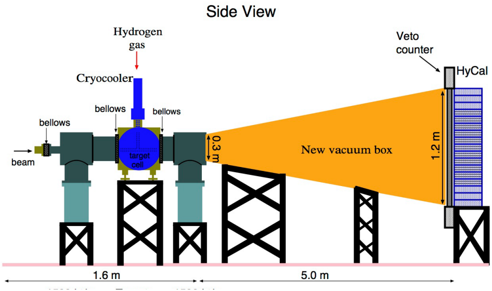


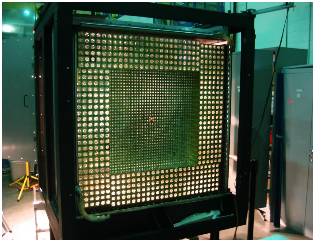
See: arXiv:1612.06707

Status

- Published: PLB 771:194-198
- Radiative correction correct on the 1% level deep in the tail!
- Radius extraction not competitive in precision

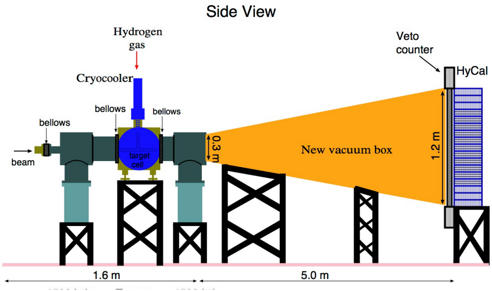
JLAB: PRoton RADius

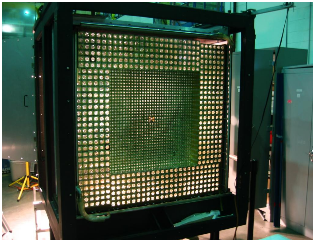




- High resolution, large acceptance hybrid calorimeter
- Windowless target
- Simultaneous measure $ep \to ep$ and Møller scattering
- Q^2 range: 2×10^{-4} to 2×10^{-2} $(\mathrm{GeV}/c)^2$

JLAB: PRoton RADius



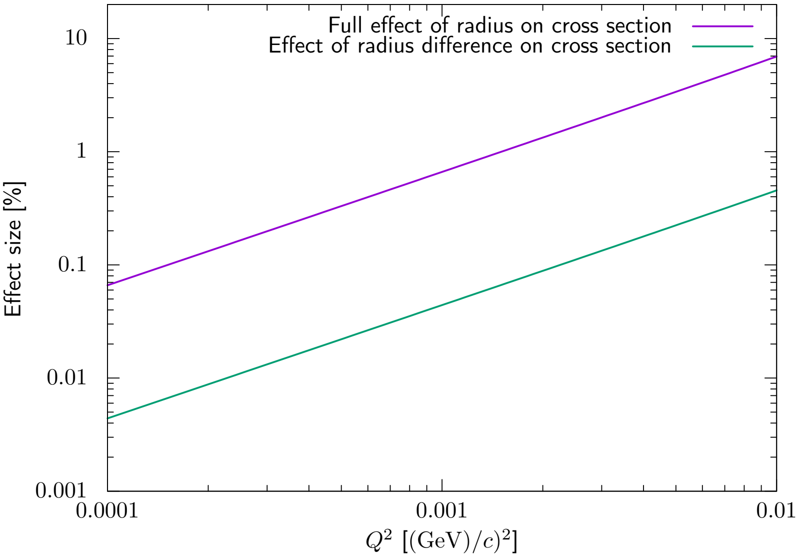


- High resolution, large acceptance hybrid calorimeter

Status

- Data taken successfully
- Analysis ongoing

The false promise of low Q^2



Will we (be able to) see structures?

- Simplest model: Kink, i.e. linear-kink-linear
- Assume 30 data points between 10^{-4} and 10^{-2} $(\mathrm{GeV}/c)^2$, 0.05% precision
- Kink just below the available data

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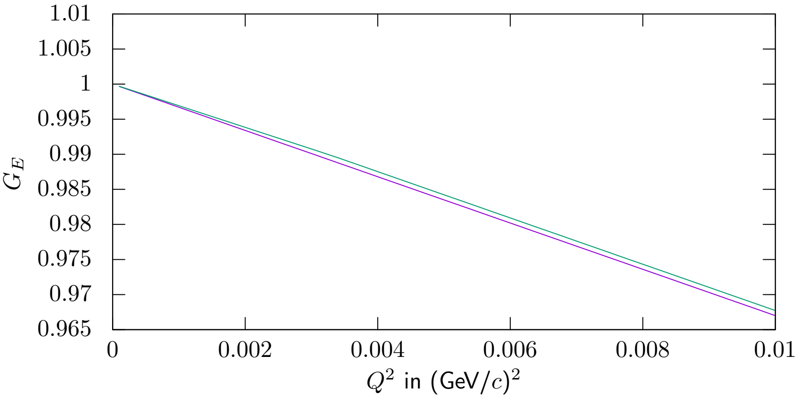

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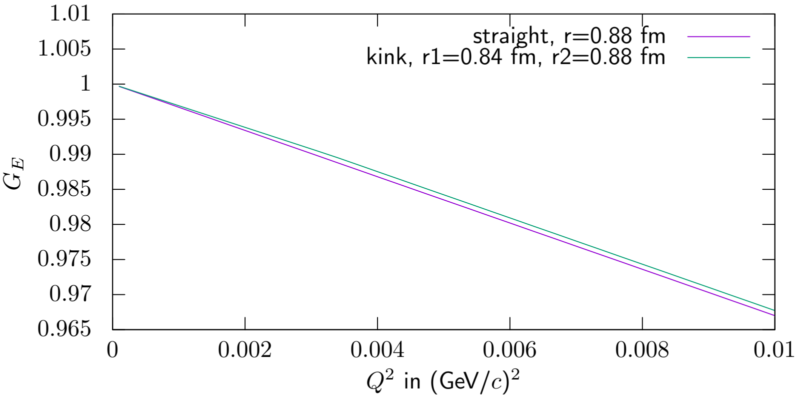

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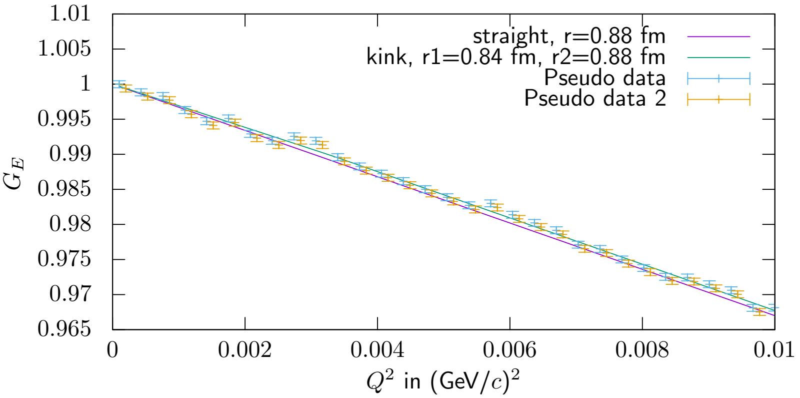

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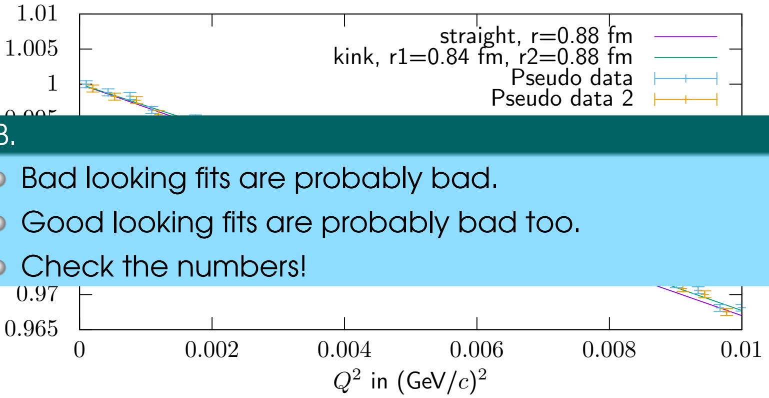
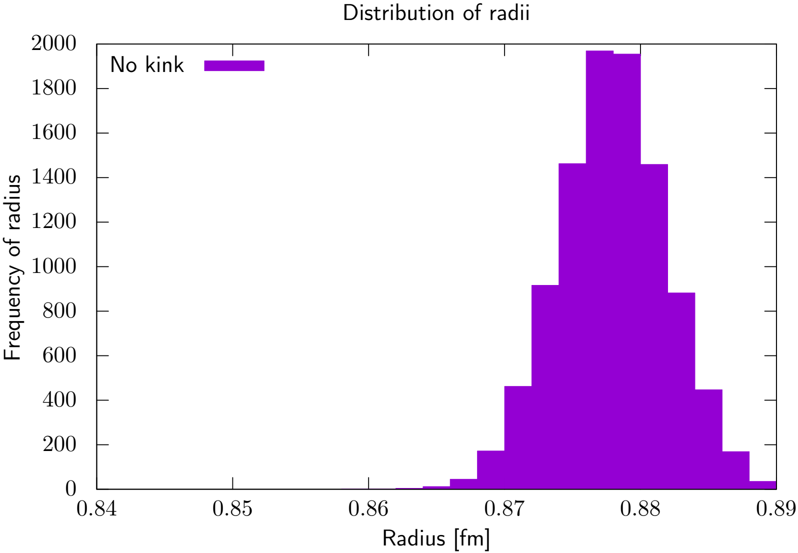


N.B.

- Bad looking fits are probably bad.
- Good looking fits are probably bad too.
- Check the numbers!

Structures, in numbers



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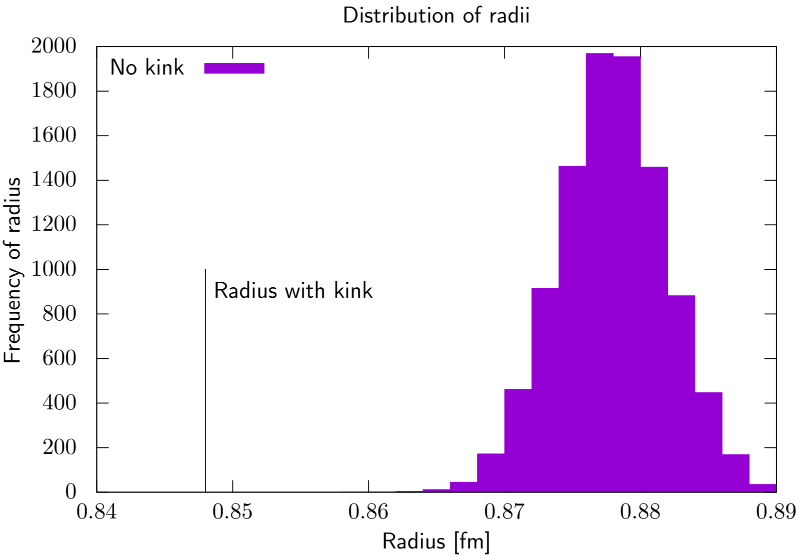

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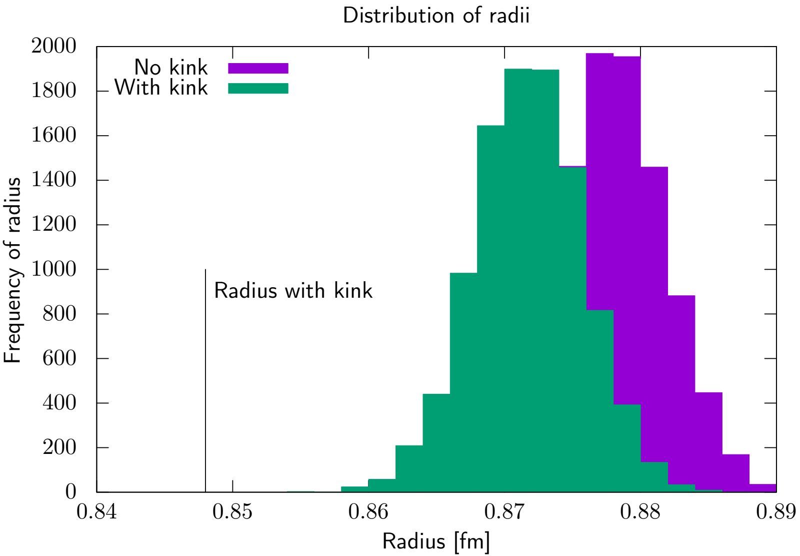

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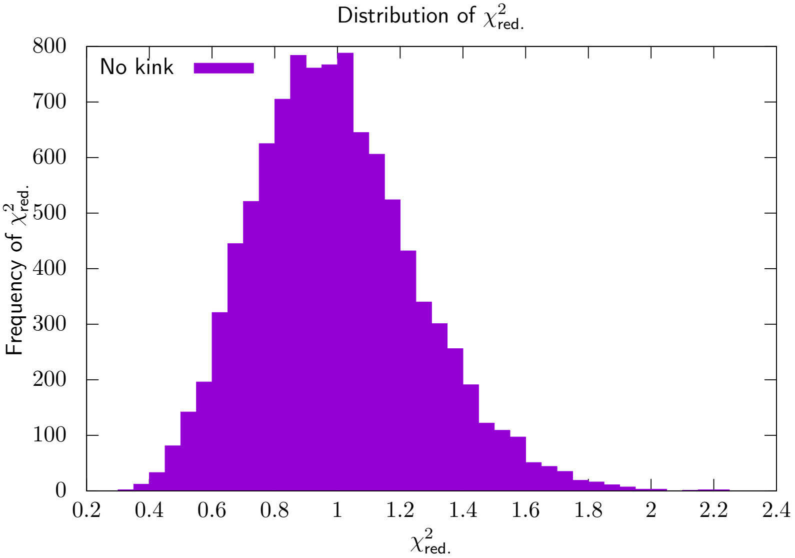

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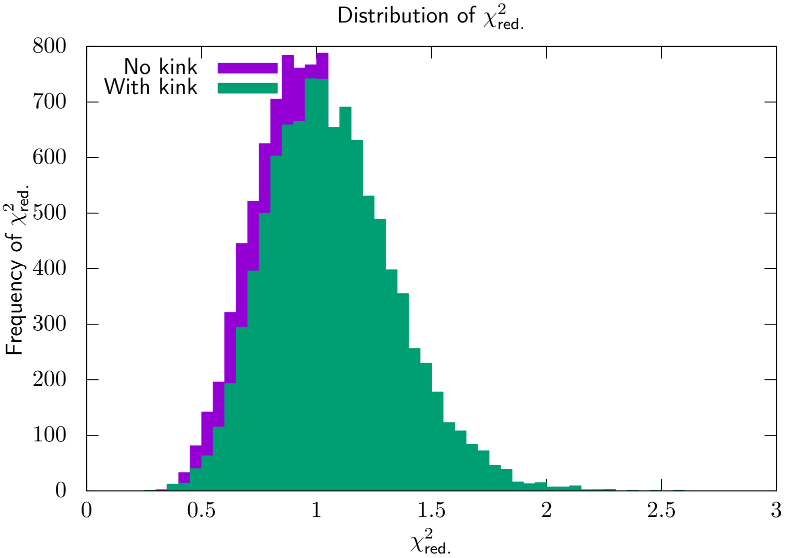

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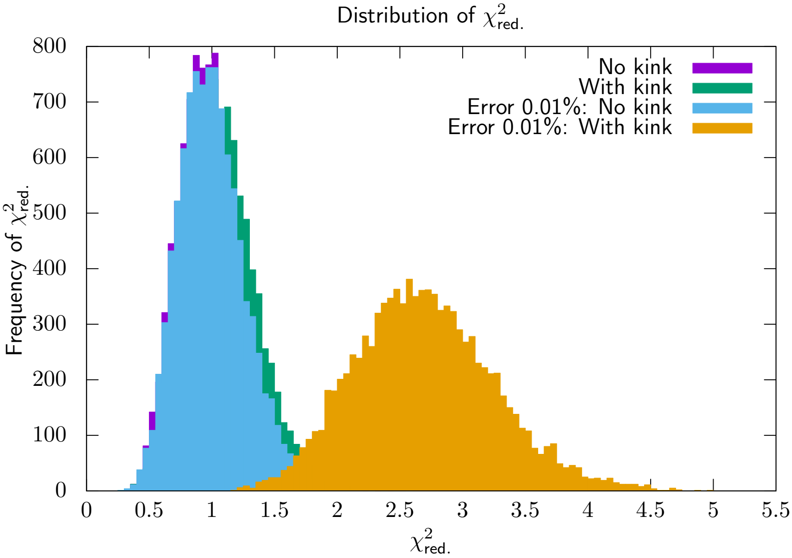

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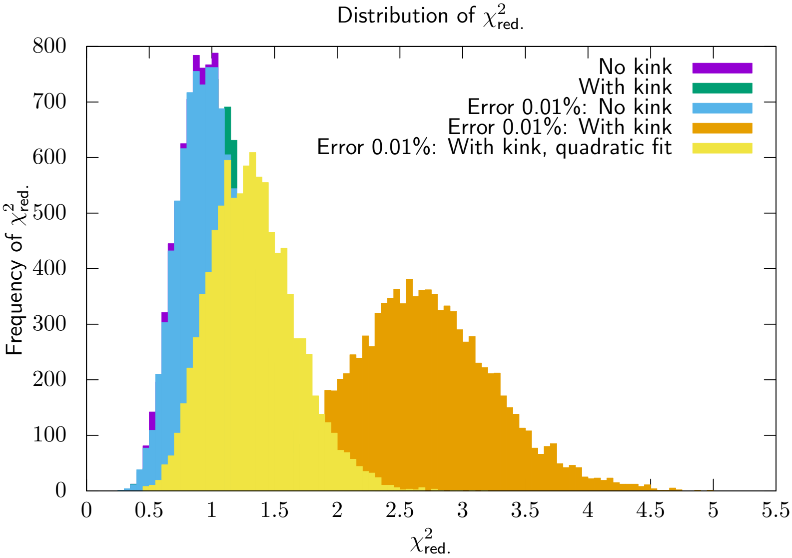

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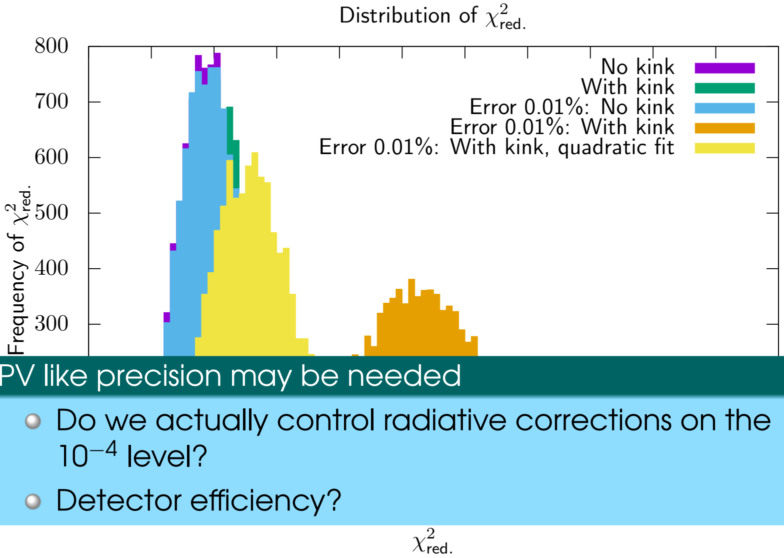

PV like precision may be needed

- Do we actually control radiative corrections on the 10^{-4} level?
- Detector efficiency?

MUSE: The missing piece

r_E (fm)	ep	μp
Spectroscopy	0.8758 ± 0.077	0.84087 ± 0.00039
Scattering	0.8770 ± 0.060	????

Measure radius with muon-proton scattering

MUSE - Muon Scattering Experiment at PSI

World's most powerful low-energy $e/\pi/\mu$-beam:

Direct comparison of *ep* and μp!

- Beam of $e^+/\pi^+/\mu^+$ or $e^-/\pi^-/\mu^-$ on liquid H_2 target
  - Species separated by ToF, charge by magnet
- Absolute cross sections for *ep* and μp
- Charge reversal: test TPE
- Momenta 115-210 MeV/c $\Rightarrow$ Rosenbluth G_E, G_M

It's all Billiards!