QED corrections to the Bethe-Heitler process in the $\gamma p \to l^+ l^- p$ reaction

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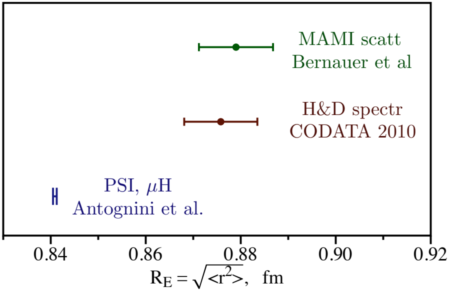
Table of content

Proton radius puzzle

- discrepancy between measurements from muonic spectrocospy and electron data
- at least 4 standard derivations difference
- physics beyond the Standard model?



Lepton universality

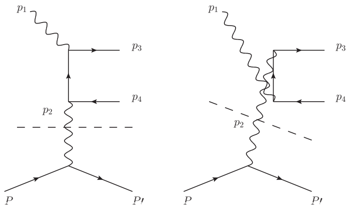
- Assume universal proton form factor for all leptons
  $\rightarrow$ results in same proton radius for muons and electrons
- Broken universality could be an explanation for proton radius puzzle
- Test this with upcoming experiments:
  - MUSE @ PSI (electron vs muon scattering)
  - MAMI $\gamma p \rightarrow e^- e^+ p$ vs. $\gamma p \rightarrow \mu^- \mu^+ p$

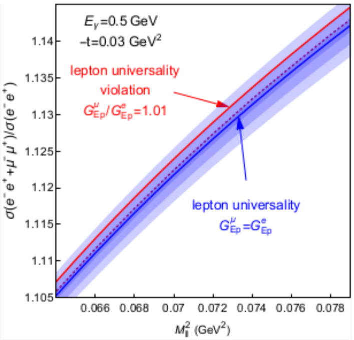
Bethe-Heitler process

- phtotproduction of lepton pair on a proton target

$$\gamma p \to l^+ l^- p$$

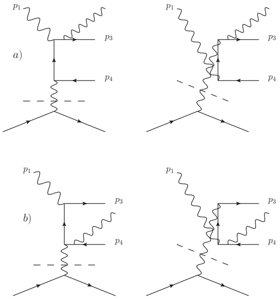
- comparsion of cross sections for $l = e$ and $l = \mu$ can be used to test lepton universality



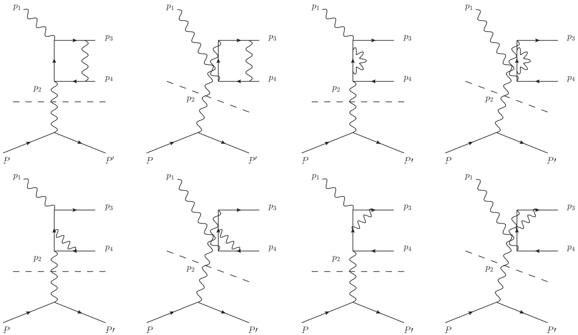


QED corrections

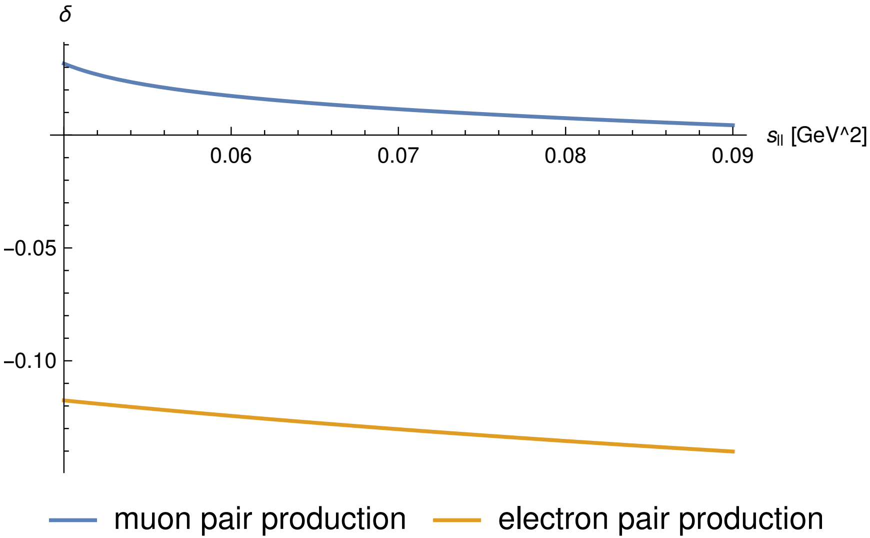
Real corrections



Virtual corrections

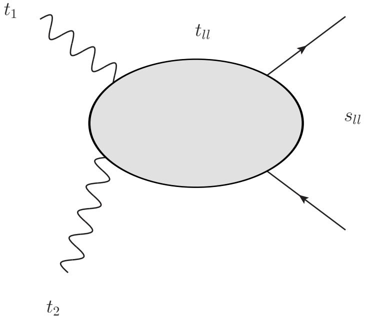


First result in Soft-photon approximation

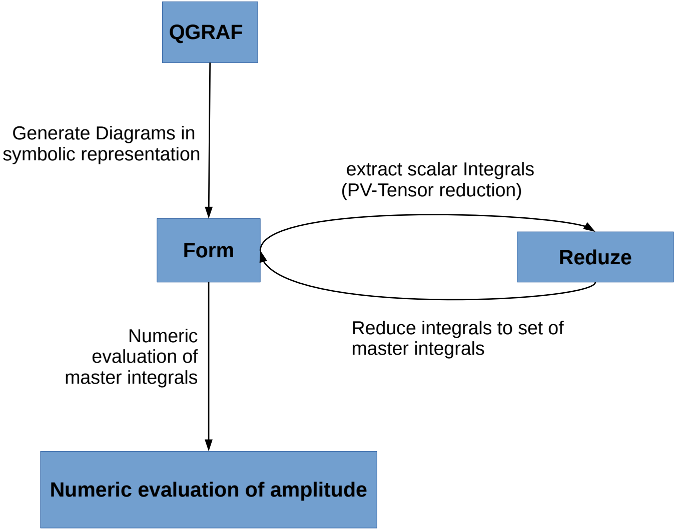


Difficulties in the full calculation

- many scales
  - 4 external scales
  - 1 internal mass (lepton)
- due to massive Dirac chains huge expressions
- Passarino-Veltman tensor reduction up to rank 3



Overview of full calculation



IBP relations

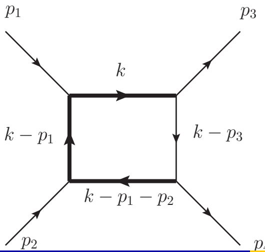
- relations between scalar integrals based on Integration-By-Part
- in dimensional regularisation, any scalar integra obeys

$$\int d^d k_i \frac{\partial}{\partial k_i^\mu} \left[q^\mu I(p_1, ..., p_n, k_1, ...k_m) \right] = 0 \tag{1}$$

- leads to rational system of equations, that can be solved for a set of so called master integrals

Integral families

- set of scalar propagators $P_1, \dots P_n$, such that any scalar product between external momenta and loop momenta or between two loop momenta can be re-expressed by these
- member of family can be labeled by integer exponents of the P_i
- for Bethe-Heitler process we have



Example for labeling:

$$\int \frac{d^d k}{(2\pi)^d} k^2 \frac{1}{(k-p_1)^2 - m^2} \frac{1}{(k-p_1-p_2)^2 - m^2} \frac{1}{(k-p_3)^2}$$
$$= \int \frac{d^d k}{(2\pi)^d} (P_1^{-1} + m^2) P_2 P_3 P_4$$
$$= I(-1,1,1,1) + m^2 I(0,1,1,1)$$

Solving master integrals

- master integrals can be solved numerically or analytically
- for numerical solution, one can use Sector Decomposition and MC-Integration
- for analytic solution, use method of differential equations

$$\partial_x \vec{I} = R(x, \epsilon)\vec{I} \tag{2}$$

- by particular change of master integrals, one can get *canonical* form, where solution gets significant simpler

$$\partial_x \vec{I} = \epsilon\, R(x)\vec{I} \tag{3}$$

Conclusion and Outlook

- As a first step, we presented the first order QED calculation for the Bethe Heitler process in the Soft-photon limit
- Full calculation by method of master integrals is work in progress
- Upcoming experiment at MAMI (Mainz) aims to test lepton universality
- Complementary $\gamma p \to l^+ l^- p$ experiment could shed light on the proton radius puzzle

Example: Bubble-Integral

Define the integral family:

$$I(a,b) = \int \frac{d^d k}{(2\pi)^d} \left(\frac{1}{k^2}\right)^a \left(\frac{1}{(k-p)^2 - m^2}\right)^b \tag{4}$$

IBP identities:

$$\frac{\partial}{\partial k^\mu} \left(\frac{1}{k^2}\right)^a \left(\frac{1}{(k-p)^2 - m^2}\right)^b \tag{5}$$

$$= -2a \left(\frac{1}{k^2}\right)^{(a+1)} \left(\frac{1}{(k-p)^2 - m^2}\right)^b k_\mu - 2b \left(\frac{1}{k^2}\right)^a \left(\frac{1}{(k-p)^2 - m^2}\right)^{(b+1)} (k_\mu - p_\mu) \tag{6}$$

This can be contracted with k_μ or p_μ (2 identities).

Any integral of the family can be related with these identities to two master integrals, e.g. $I(0,1)$ and $I(1,1)$.

Example: Bubble-Integral

Choose

$$\vec{I}_1 = \begin{pmatrix} I(0,1) \\ I(1,1) \end{pmatrix}, \tag{7}$$

then

$$\partial_m \vec{I}_1 = \frac{2}{m\,(m^2 - s)} \begin{pmatrix} -(m^2 - s)(-1+\epsilon) & 0 \\ (-1+\epsilon) & -m^2(-1+2\epsilon) \end{pmatrix} \vec{I}_1, \tag{8}$$

$$\partial_s \vec{I}_1 = \frac{1}{s\,(m^2 - s)} \begin{pmatrix} 0 & 0 \\ -(-1+\epsilon) & m^2 + (-2+\epsilon)s + m^2(-2+\epsilon) + 2s \end{pmatrix} \vec{I}_1. \tag{9}$$

"Better" choice:

$$\vec{I}_2 = \begin{pmatrix} \epsilon m^{2\epsilon}\; I(0,2) \\ \epsilon(-s)\; m^{2\epsilon} I(1,2) \end{pmatrix}, \quad x = \frac{(-s)}{m^2}. \tag{10}$$

Then:

$$\partial_x \vec{I}_2 = \epsilon \left[\frac{1}{x} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{x+1} \begin{pmatrix} 0 & 0 \\ -1 & -2 \end{pmatrix} \right] \vec{I}_2 \tag{11}$$