Effects of Irregular Topology in Non-Planar SOM Variants

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Abstract

The development of the spherical SOM has been driven by the border effects observed in traditional SOM. Two problems exist with the Spherical SOM. The first is the level of control over the network size. The second is the topologically induced errors caused by the arrangement of neurons on the sphere. Both of these problems stem from the problem of uniformly distributing points on a sphere. These problems will be investigated through the introduction of a new method for testing topologically induced errors. The method first analyzes the neural network to find topological mis-matches, next we train the network with an overwhelming about of synthetic data. Through a series of simple plots we can then compare each neurons internal variance as defined by the variance of the observations that best fit that neuron with suchs metrics as neighborhood influence, number of child observations, etc.

1 Introduction

Using a spherical lattice is widely suggested as a solution to the boundary effect found in the traditional Self-Organizing Maps (Ritter, 1999; Boudjemai et al., 2003; Sangole and Knopf, 2003; Wu and Takatsuka, 2006; Nishio et al., 2006). However, the use of the spherical lattice introduces a new problem. Save the five platonic solids, distributing points on a sphere will always result in irregular topology (Ritter, 1999). The classic method for generating a spherical lattice is to tessellate the sides a platonic solid. When tessellating the icosahedron, as described by Wu and Takatsuka (2006), the resulting topology will always consist of 12 pentagons and $N-12$ hexagons. Where N is the total number of sides, or neurons. The main drawback of this method is the finite control over N, which grows at a rate of $f^2 * 10 + 2$, where f is the frequency of the tessellation (Wu and Takatsuka, 2006). There has been little discussion on the effects of irregular topology in Spherical SOM. The goal of this research is to investigate these effects in order to determine the usefulness of slightly more irregular topologies which offer greater control over N.

2 Background and Lit Review

The Self-Organizing Map (SOM) is an unsupervised competitive learning process developed by Teuvo Kohonen as a technique to analyze high dimensional data sets. The SOM algorithum uses an artificial neural network to organize high dimensional data onto a low dimensional lattice, or map, of neurons. Each neuron contains a reference vector that can be considered to model a portion of the input space. Before training these neurons are initialized, most commonly to random values. During the training process a randomly selected input vector searches the map for its best matching unit (BMU), that is the neuron to which it is most similar. The BMU and its neighborhood, as defined by a neighborhood function, are than adjusted to better match that observation (Kohonen, 2000). The training process is repeated a predefined number of times, or ideally until the map converges. The traditional SOM is laid out on a two dimensional plane using either a rectangular or hexagonal topology. According to Wu and Takatsuka (2006) the hexagonal structure is more uniform and generally preferred.

2.1 Boundary Effect

One obvious drawback of building the neural lattice in a discrete euclidean plane is the boundary of the resulting lattice. A neuron located on the boundry has fewer neighbors and thus fewer chances of being updated (Wu and Takatsuka, 2006). As observed in Figure 1, neurons in the center of the map tend to better represent the mean of the input-space. This is arguably caused by outliers being pushed to the edges of the map, where they encounter fewer competing signals.

The toroidal som was introduced by Li et al. (1993) and removes this drawback. However, the torus is not hugely effective for visualization, as maps generated from a torus are not very intuitive (Ito et al., 2000; Wu and Takatsuka, 2006). Ritter (1999) describes the torus as being topologically flat and suggests that a curved topology, such as that of a sphere, may better reflect directional data. A sphere also results in a more intuitive map, since we are accustomed to looking at maps based on a sphere.

2.2 Spherical SOM

Ritter (1999) first introduced the spherical SOM and several enhancements have since been suggested (Boudjemai et al., 2003; Sangole and Knopf, 2003; Nishio et al., 2006; Wu and Takatsuka, 2006). A good comparison of these enhancements can be found in (Wu and Takatsuka, 2006). All of these methods derive their spherical structure through the tessellation of a polyhedron as originally proposed by Ritter. Wu and Takatsuka (2006) point out the importance of a uniform distribution on the sphere and that it is preferable for all neurons to have an equal number of neighbors and to be equally spaced. They find generally that the tessellation method best satisfies these conditions and specifically that the icosahedron is the best starting point (Wu and Takatsuka, 2005).

2.2.1 Network Size

The literature offers little theoretical guidance on network size (Cho et al., 1996). Vesanto (2005) suggests simply using a network size of $5 * \sqrt{n}$, where n is the number of observations. The tessellation method used by the class of spherical SOMs based on Ritter's work results in a network size that is a function of the tessellation frequency and therefore grows exponentially. In practice 2D Euclidean SOMs also offer limited control over network size, as it is undesirable to have one dimension dramatically larger then the other. Nishio et al. (2006) try to address the issue of network size granularity by departing from the tessellation method and suggesting the use of a partitioned helix to uniformly distribute any number of neurons on a sphere. A similar method was dismissed by Wu and Takatsuka (2005) for failing to satisfy the uniformity conditions. Nishio et al. seem to have addressed variance in nearest neighbor distances, but the issue of having a uniform number of direct neighbors is still not addressed.

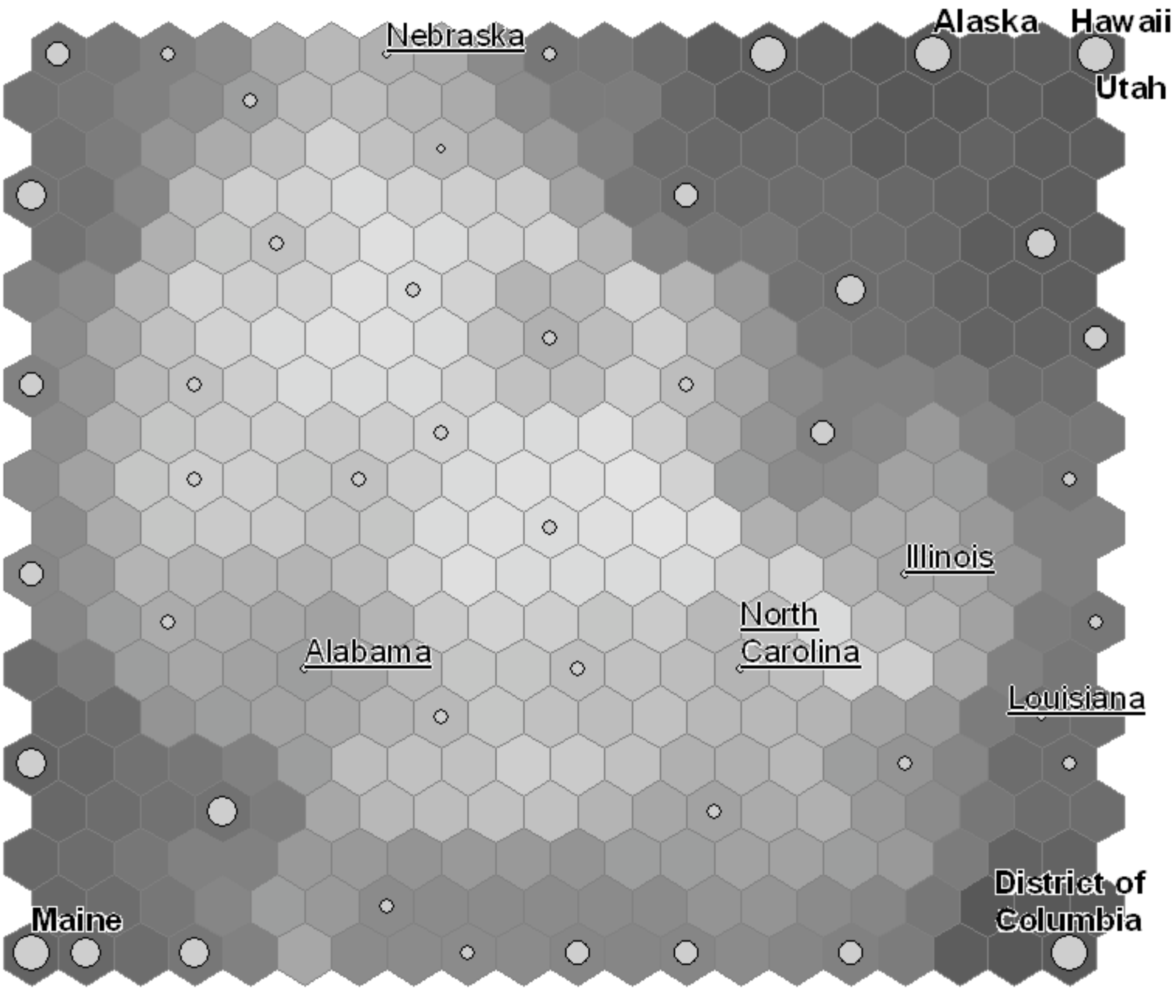

Figure 1: Fifty states plus D.C. mapped onto SOM trained with the first thirty-two census variables. Darker neurons have a relatively large differences from the mean of the states, while lighter neurons are relatively closer. Larger dots represent inputs that were poorly fit to the map, small smaller dots show inputs that were better bit to the map.

2.3 Conclusion

Tessellation of the icosahedron results in a network of neurons, each of which have exactly six neighbors, save the original twelve which each have five. This is very close to the ideal structure in which every neuron would have exactly six neighbors. This structure has very low variances in both neuron spacing and neighbor size. The literature has attached too much importance to the variance of neuron spacing, specifically as a metric to compare different network topologies.

Based solely on measures of neuron spacing Wu and Takatsuka (2005) dismissed a method proposed by Rakhmanov et al. (1994) for distributing points on a sphere. Similarly Nishio et al. (2006) use these variance measures to support their helix algorithm for distributing points on a sphere. Table 1 shows that these metrics can be misleading and may not be comparable across topologies. The traditional rectangular and hexagonal topologies have no variance in neuron spacing and the generally preferred hexagonal structure displays greater variance in neighborhood size than the rectangular structure. The torus by comparison would have variance in neuron spacing, yet no variance in neighborhood size. The distance between two neurons is only considered during the formation of the neuronal network. At this stage the spacing is significant as it plays a part in determining neuron adjacency. However using this measure to evaluate potential topologies for use in SOM may be misleading.

Table 1: Variances in Topologies

Topology	Grid Size	Neuron Spacing	Variance in Neighborhood Size
Rectangular	9x18	1	0.2716
Hexagonal	9x18	1	1.2138
Tessellation	162	0.25319 - 0.31287	0.0686
Rakhmanov	162	0.15779 - 0.30069	0.2908

Methods, for distributing points on the sphere, which allow for fine grained control over network size produce slightly more irregular topologies. However, no discussion of these irregularities or their effects on SOM training has occurred in the literature. Given that limited theoretical guidance is available for choosing network size the desire for finer control over the network size, should not be overlooked. In particular for a larger SOM the ideal network size may not be achievable via tessellation of the icosahedron.

3 Data and Methodology

It is useful to represent our neural lattice as a graph, G. We know that any arrangement of neurons onto the surface of a sphere will

(Wu and Takatsuka, 2006, pp. 900) state that "[f]or SOM, it is desirable to have all neurons receive equal geometrical treatment." To satisfy this constraint two conditions must be met. Firstly, each neuron should occupy the same amount of space on the given surface. Secondly, each neuron should be bordered to the same number of surrounding neurons. The first condition in hugely important for visualization, but irrelevant for training. During the training of the SOM only the topology of the neurons is considered, as such the network can be effectively treated as a graph. Doing so enables us to use the properties of the graph and the principles of graph theory to help us understand the relationship between neurons during training.

the variance in neuron spacing should be minimized. The first condition ensures that each neuron will occupy is only of concern when visualizing the SOM.

The get at the second condition we must first define the degree of a neuron, deg(n) to be number of its direct neighbors. Whit that in mind the second condition is that the variance in the degree of the neurons must also be minimized.

This statement may be somewhat misleading to those investigating alternative

For the purpose of SOM visualization it is important for each neuron to receive equal geometrical treatment.

It is useful to represent the neural network as a graph, G, in order use the... In order to examine the irregularites in the neural network it is useful ....

The basic problem of the "boundary effect" is that neurons on the edge have fewer neighbors. Yet there are only five possible arrangements of points on a sphere such that all points have the same number of neighbors. Any spherical lattice consisting of more then twenty (dodecahedron) neurons will contain topological irregularities. This is to say that not all neurons will have the same number of neighbors. The importance of these irregularities and the magnitude of their effects on SOM training is not known. This goal of this research is to determine whether more flexible network structures may be used in spherical in SOM without introducing significant errors. To accomplish this goal, basic methods in network analysis with be combined with the result from several empirical training runs each utilized different topology.

3.1 Synthetic Data

In order to have a basis for comparison

References

Boudjemai, F., Enberg, P. B., and Postaire, J. G. (2003). Surface modeling by using self organizing maps of kohonen. In *Systems, Man and Cybernetics, 2003. IEEE International Conference on*, volume 3, pages 2418–2423 vol.3.

Cho, S., Jang, M., and Reggia, J. A. (1996). Effects of varying parameters on properties of self-organizing feature maps. *Neural Processing Letters*, V4(1):53–59.

Ito, M., Miyoshi, T., and Masuyama, H. (2000). The characteristics of the torus self organizing map. In *Proceedings of 6th International Conference ON Soft Computing (IIZUKA2000)*, volume A-7-2, pages pp.239–244, Iizuka, Fukuoka, Japan.

Kohonen, T. (2000). *Self-Organizing Maps*. Springer, 3 edition.

Li, X., Gasteiger, J., and Zupan, J. (1993). On the topology distortion in self-organizing feature maps. *Biological Cybernetics*, V70(2):189–198.

Nishio, H., Altaf-Ul-Amin, M., Kurokawa, K., and Kanaya, S. (2006). Spherical som and arrangment of neurons using helix on sphere. *IPSJ Digital Courier*, 2:133–137.

Rakhmanov, E. A., Saff, E. B., and Zhou, Y. M. (1994). Minimal discrete energy on the sphere. *Mathematical Research Letters*, 1:647–662.

Ritter, H. (1999). Self-organizing maps on non-euclidean spaces. In Oja, E. & Kaski, S., editor, *Kohonen Maps*, pages 97–110. Elsevier, Amsterdam.

Sangole, A. and Knopf, G. K. (2003). Visualization of randomly ordered numeric data sets using spherical self-organizing feature maps. *Computers & Graphics*, 27(6):963–976.

Vesanto, J. (2005). Som toolbox: implementation of the algorithm.

Wu, Y. and Takatsuka, M. (2005). Geodesic self-organizing map. In Erbacher, R. F., Roberts, J. C., Grohn, M. T., and Borner, K., editors, *Proc. SPIE Vol. 5669*, volume 5669 of *Visualization and Data Analysis 2005*, pages 21–30. SPIE.

Wu, Y. and Takatsuka, M. (2006). Spherical self-organizing map using efficient indexed geodesic data structure. *Neural Networks*, 19(6-7):900–910.