EFFECTS OF IRREGULAR TOPOLOGY
IN
SPHERICAL SELF-ORGANIZING MAPS

A Thesis

Presented to the

Faculty of

San Diego State University

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

in

Geography

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by

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May 2008

SAN DIEGO STATE UNIVERSITY

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Effects of Irregular Topology in Spherical Self-Organizing Maps

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CHAPTER 1

INTRODUCTION

The Self-Organizing Map (SOM) is an unsupervised competitive learning process developed by Teuvo Kohonen as a technique to analyze and visualize high dimensional data sets. The applications of SOM are far reaching; Kohonen (2001) provides a thorough review of the SOM literature including applications of SOM. SOM has been used in applications ranging from speech recognition and image classification to breast cancer detection and gene expression clustering. Skupin and Agarwal (2008) outline the growing interest of SOM to the GISciences, and propose that the relationship between SOM and GIScience should be bidirectional. The SOM offers a powerful method for exploring and visualizing geographic data and GIScience offers a wide array of tools and methods to enable the exploration of the SOM itself. The exploration of spatial relationships has always been of great interest to geographers, and as Ritter (1999) states, the goal of SOM is "to translate *data similarities* into *spatial relationships*" (Ritter, 1999, p. 1). This thesis leverages GeoVisualization and GeoCompution in order to explore some of the basic properties of the SOM.

The SOM is a type of artificial neural network in which neurons are "organized" in such a way as to project the high-dimensional relationships of a set of training data onto a low-dimensional network structure. The traditional SOM uses a rectangular or hexagonal network topology (Kohonen, 2001). These topologies create a well-known problem in SOM called the boundary or edge effect. Neurons on the boundary of the hexagonal and rectangular lattices have fewer neighbors, which reduces their ability to interact with other neurons during the self-organizing process. Using a spherical lattice has been widely suggested as a solution to the problem (Ritter, 1999; Boudjemai et al., 2003; Sangole and Knopf, 2003; Nishio et al., 2006; Wu and Takatsuka, 2006). The use of the spherical lattice, however, does not

completely overcome the boundary problem, and the choice of which spherical topology to use for the network can be difficult to make.

A regular network topology is one in which every node on the network has exactly the same number of adjacent nodes. Any topology involving an edge is irregular. Arranging our lattice on the surface of a sphere seems to be an obvious way to overcome the edge. However, there exist only five arrangements on the sphere which are completely regular; these are the five platonic solids (Ritter, 1999; Harris et al., 2000). Any other arrangement of neurons on the surface of the sphere will result in an irregular topology, as not all neurons will have the same number of neighbors. The classic method for minimizing this irregularity is to generate the spherical lattice by tessellating the sides of the icosahedron (Nishio et al., 2006). While this method will always result in a highly regular spherical topology, the main drawback is that the number of neurons in the network (the network size), N, grows exponentially as tessellations are applied. That results in only very coarse control over network size. Other methods for arranging neurons on the sphere allow for unlimited control over network size, but yield topologies with increased irregularity (Harris et al., 2000; Wu and Takatsuka, 2005; Nishio et al., 2006). To date the literature has largely ignored the more irregular methods in favor of the aforementioned tessellation-based methods. A topology which yields a more flexible network size may be desirable. However, in order to address this issue of network size, we must first determine the degree to which irregularity effects the SOM.

1.1 Research Objectives

The objective of this research is to determine the utility of certain irregular spherical topologies beyond offering greater control over network size. Toward that end, we develop and test new diagnostics to measure and visualize topology-induced errors in SOM. The following diagnostics are developed and implemented:

1. Compare the internal heterogeneity of observations captured by a given neuron to that neuron's first-order neighborhood size.

2. For different topologies, compare the internal heterogeneity of each neuron against a composite measure of topological regularity.

3. Develop a SOM-based visualization of the internal heterogeneity.

These diagnostics help facilitate the evaluation of both traditional and spherical SOMs. To satisfy the objective of this research, we apply these diagnostics to a series of comparable SOMs. Each SOM is trained using the same synthetic data and training parameters, but utilize different network topologies. By formally testing for difference of means and variance in the results of the diagnostics, the following questions are addressed:

1. Does the internal heterogeneity of a neuron decrease as its first-order neighborhood size, or degree, increases?

2. Is the average internal heterogeneity of a SOM higher when a more irregular topology is used?

3. Which insights, if any, can be gained from a SOM-based visualization of internal heterogeneity?

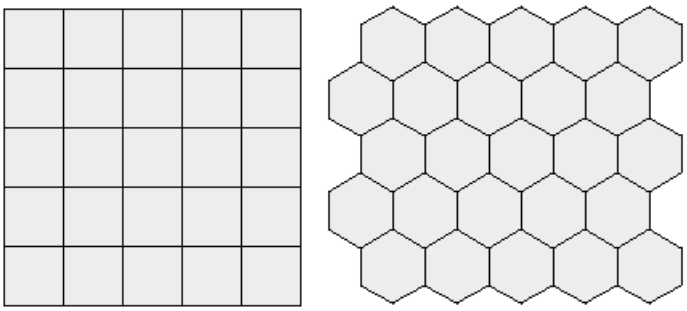
CHAPTER 2

BACKGROUND

This chapter is divided into six sections. Section 2.1 introduces the SOM and some of its basic properties. Next, section 2.2 discusses the training process which allows the SOM to represent our input data. In section 2.3 we explore the importance of topology in the SOM. The following two sections, 2.4 and 2.5, expand on this discussion by introducing some potential problems and possible solutions found with the traditional topologies used in SOM. Specifically, section 2.4 describes edge effects and section 2.5 describes the use of spherical topologies which attempt to overcome these edge effects. Finally, section 2.6 highlights the importance of the network size in SOM.

2.1 SELF-ORGANIZING MAPS

The SOM is a type of artificial neural network developed by Teuvo Kohonen. In the SOM an input-space is organized onto a set of neurons through an unsupervised competitive learning process (Kohonen, 2001). The neurons, arranged on a lattice, compete for input signals. The input signals are samples from an input-space. During training the winning neurons and those around them are updated to better model these signals. In each step of the training process the winning neuron is the one most similar to a given input signal. Traditionally, the neural lattice, or network, has either a rectangular or hexagonal topology. As shown in Figure 2.1 these topologies have different properties. In the rectangular topology, Figure 2.1(a), neurons are bounded by four neighbors. In the hexagonal topology, Figure 2.1(b), neurons are bounded by six neighbors. In both topologies, neurons on the edge have fewer neighbors. As the learning progresses, *data similarities* in the high-dimensional input-space are translated into *spatial relationships* in the low-dimensional neural network (Ritter, 1999).



(a) Rectangular Topology (b) Hexagonal Topology

Figure 2.1. In traditional SOM either a rectangular or hexagonal topology is used.

The SOM has a number of applications, but is primarily used for data reduction and data visualization. The SOM is often compared with other data reduction techniques, such as principle components or multi-dimensional scaling. Like SOM, these techniques reduce the dimensionality of the input-space (Kohonen, 2001; Skupin and Agarwal, 2008). However unlike SOM, these techniques do not directly preform clustering. The SOM on the other hand has the ability to do both simultaneously. That is the SOM can collapse a high dimensional input-space into two dimensions, *and* collapse observations from the input-space into groups or clusters. The degree to which clustering occurs is controlled by the size of the SOM. In smaller SOMs, as the neurons try to model the input-space, observations are "collected" by the neuron which models it most accurately. In larger SOMs, perhaps even where there are more neurons than observations, observations are still collected by their best model. However, other neurons map the intermediate areas between observations, providing a low-dimensional spatial layout of otherwise high-dimensional data.

Skupin and Agarwal (2008) demonstrate these properties when they use state level data from the U.S. Census Bureau to train two SOMs of different sizes. In the three-by-three (9) case the neurons act as containers clustering similar states, while in the twenty-by-twenty (400) case relationships are expressed with much finer granularity. In the second case the SOM proves to be a very useful visualization tool. The similarities and dissimilarities among the states are represented as spatial orientations and distances. Even in this larger case, where neurons outnumber observations, clustering may still occur. This happens because the SOM

tries to represent the entire input-space. If some observations are sufficiently different, while others are sufficiently similar; the network may be better utilized by allowing some neurons to represent dissimilarities, while clustering of observations occurs at other neurons.

A useful property of the SOM is that the network structure between the neurons allows us to create meaningful visualisations. Observations used in the training, as well as new observations from the input space, can be mapped onto the trained surface in order to show higher dimensional relationships in a familiar map-like form. The SOM's component planes capture the spatial layout of each dimension, these are often visualized in a series of maps. These maps can provided useful information about the relationships between the different attributes of your input-space. In terms of information visualization, the spatial layout of observations on the network provides far more information than traditional methods, such as ordered lists or scatter plots.

2.2 TRAINING

As with other artificial neural networks, the SOM has to be trained with samples from the input-space. These samples, or observations, are represented as input vectors. During the training process neurons compete for inputs; with each training step winning neurons are adjusted to better match the signals they receive. Feedback between the neurons allows the entire network to eventually coverage to a final state. After training, each neuron in the SOM will represent a portion of the input space. To accomplish this representation each neuron is associated with a parametric reference vector, m_i, referred to as a model vector (Kohonen, 2001). The length of the model vectors are equal to the length of the input vectors, such that each element within a model vector represents a dimension of the input-space. The initial values of the elements are most commonly randomized, such that a mapping of the input-space onto the initial SOM would have no meaning. Other initializations are possible and may reduce the time required for the map to coverage (Kohonen, 2001).

Our implementation follows the "Original Incremental SOM Algorithm" as laid out by Kohonen (2001). In each step of the algorithm, a randomly select observation (input vector x)

searches for its best model (reference vector m_i) among the neurons. The best model is defined as the m_i with the smallest distance to x. These distances are referred to as quantization errors (QErrors), and they measure the distance between two vectors in attribute space (Kohonen, 2001). Our implementation uses Euclidean distances, however, any reasonable distance measure can be used here. The "winning" neuron is termed the Best Matching Unit (BMU c). The neighborhood (N_c) around the BMU (c) is found and all m_i within N_c are updated. The size of the neighborhood and the magnitude of the updates are controlled by the neighborhood function. In our implementation, the width of the neighborhood decreases as the training progresses, and the magnitude of the updates decrease, with a Gaussian function, toward the edge of the neighborhood. A learning-rate factor is used to further reduce the magnitude of the updates as training progresses. Combined these create the neighborhood kernel function h_{ci} which defines a scaler used to adjust the magnitude of each update in a given training step. This function always evaluates to zero for neurons outside the neighborhood. The update is defined as,

$$m_i(t+1) = m_i(t) + h_{ci}(t)[x(t) - m_i(t)] \tag{2.1}$$

where, t is the current training step. The training process is repeated a predefined number of times, or ideally until the map converges.

2.3 TOPOLOGY

Wu and Takatsuka state that "[f]or SOM, it is desirable to have all neurons receive equal geometrical treatment" (Wu and Takatsuka, 2006, p. 900). To satisfy this constraint, two conditions must be met. First, each neuron should occupy the same amount of space on the given surface. Second, each neuron should be bordered by the same number of surrounding neurons, and we should maximize that number. The first condition is largely irrelevant in the training of the SOM. However, visualizations that do not have uniformly sized and spaced neurons could potentially mislead an untrained viewer, as larger neurons may appear to be more significant. Of greater importance to training is the SOM's topology,

as it describes how the neurons are connected within the network. In training the topology defines, N_c, the neighborhood around the winning neurons, and irregular topologies directly impact the size and shape of these neighborhoods.

We believe the regularity of a given topology is a better metric for evaluating different topologies for use in SOM. In graph theory a regular graph, is simply a graph in which every node has the same number of neighbors (Harris et al., 2000). A measure of regularity tells us how uniform neurons are in terms of their connections to other neurons in the network. In network theory, nodes with more connections are thought to be more central to the network and have a larger influence than nodes with fewer connections (Wasserman and Faust, 1994). A simple measure for capturing this is degree centrality. The degree (number of adjacent neurons) is measured for each neuron in the network. The variance in these measurements tells how regular the network is, a perfectly regular topology should have a variance equal to zero. Other methods, such as closeness centrality compare nodes based on their connectedness to every node in the network.

2.4 THE BOUNDARY EFFECT

Traditionally the SOM is laid out on a two-dimensional plane using either a rectangular or hexagonal topology. Both of these topologies are irregular, because neurons on the boundary of the network have fewer neighbors. Neurons with fewer neighbors have fewer chances of being updated (Wu and Takatsuka, 2006). As observed in Figure 2.2, neurons in the center of the map tend to better represent the mean of the input-space. In this SOM we used the same data as Skupin and Agarwal (2008) to map the fifty states and the District of Columbia onto a SOM trained with thirty-two population census variables. After training, we measured the distance between each neuron's model vector, m_i, and the mean of the input vectors, $\bar{x}$. Darker neurons have a relatively larger difference from the mean of the input-space, while lighter neurons are relatively closer. We also measured the difference of each observation to the mean $\bar{x}$, these distances are represented by the size of the point symbols. Larger symbols are farther from the mean. The five observations furthest from the

mean: Alaska, District of Columbia, Hawaii, Maine, and Utah are highlighted with bold labels. The five observations closest to the mean: Alabama, Illinois, Louisiana, Nebraska and North Carolina have underlined labels. Arguably the outliers are being pushed to the edges of the map, where they encounter fewer competing signals. Edge effects are also common in spatial analysis, for example in point patterns the edge of a study unit may hide the true distribution of an observed pattern. In SOM the edge of the neural lattice represents a true boundary, which effects its ability to represent *data similarities* as *spatial relationships*.

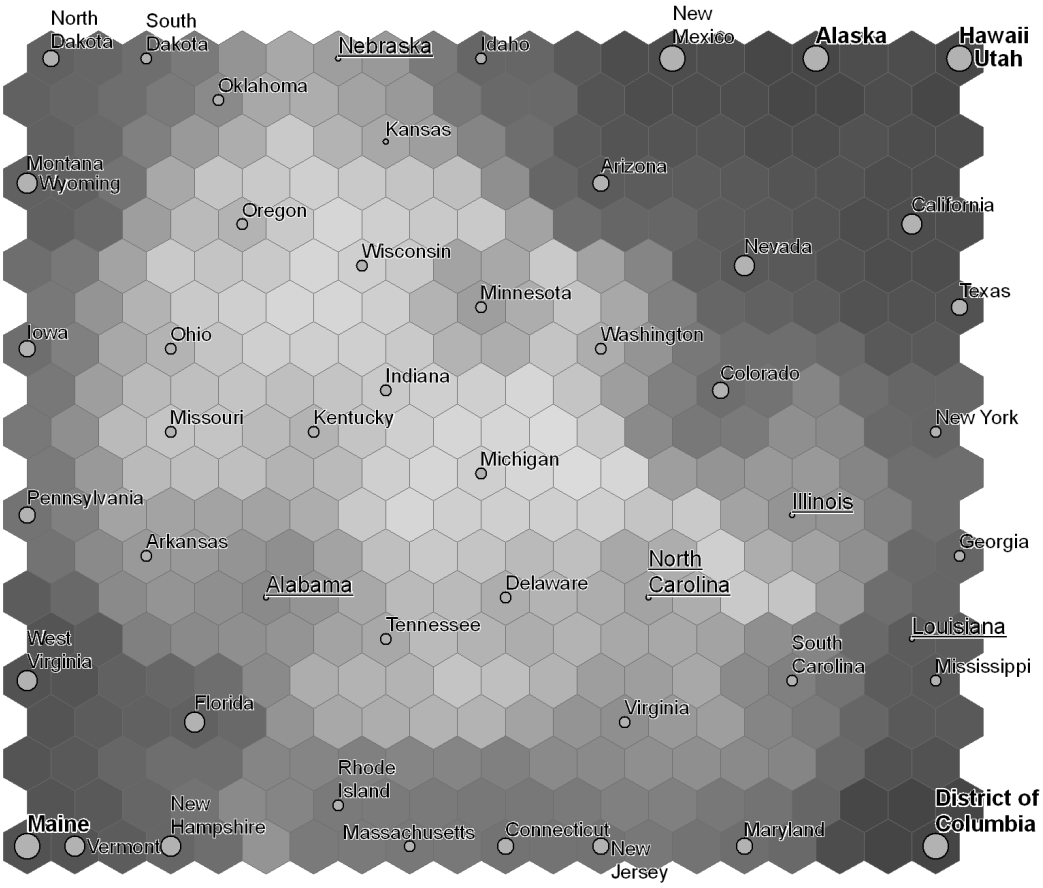


Figure 2.2. An example of the boundary effect in SOM.

One way to eliminate the edge effect is to wrap the lattice around a three-dimensional object such as a sphere or torus, thereby removing the edge entirely. The toroidal SOM was introduced by Li et al. (1993), however the torus is not effective for visualization, as maps

generated from a torus are not very intuitive (Ito et al., 2000; Wu and Takatsuka, 2006). Ritter (1999) describes the torus as being topologically flat and suggests that a curved topology, such as that of a sphere, may better reflect directional data. A sphere also results in a more intuitive map, since we are accustomed to looking at geographic maps based on a sphere.

2.5 SPHERICAL SOM

Ritter (1999) first introduced the spherical SOM, and several enhancements have since been suggested (Boudjemai et al., 2003; Sangole and Knopf, 2003; Nishio et al., 2006; Wu and Takatsuka, 2006). A good comparison of these enhancements can be found in Wu and Takatsuka (2006). All of these methods derive their spherical structure through the tessellation of a polyhedron as originally proposed by Ritter (1999). Wu and Takatsuka (2006) point out the importance of a uniform distribution on the sphere, and that it is preferable for all neurons to have an equal number of neighbors and to be equally spaced. They find generally that the tessellation method best satisfies these conditions, and specifically that the icosahedron is the best starting point (Wu and Takatsuka, 2005). Tessellation of the icosahedron results in a network of neurons, each having exactly six neighbors, save the original twelve which each have five neighbors. This is very close to the ideal structure in which every neuron would have exactly six neighbors. Wu and Takatsuka (2006) prefer these this structure, because it has very low variances in both neuron spacing and neighborhood size.

Based solely on measures of neuron spacing, Wu and Takatsuka (2005) dismissed the usefulness of a method proposed by Rakhmanov et al. (1994) for distributing points on a sphere. Similarly Nishio et al. (2006) use these variance measures to support their helix algorithm for distributing points on a sphere. Table 2.1 shows that these metrics can be misleading and comparison across topologies may not be consistent. The traditional rectangular and hexagonal topologies have no variance in neuron spacing, and the generally preferred hexagonal structure displays greater variance in neighborhood size than the rectangular structure. The torus, by comparison, would have variance in neuron spacing, yet no variance in neighborhood size. The distance between two neurons is only considered

during the formation of the neural network. At this stage the spacing is significant as it plays a part in constructing the network's topology by determining neuron adjacency. However, using this measure to evaluate potential topologies for use in SOM may be misleading.

Table 2.1. Variances in Topologies

Topology	Grid Size	Neuron Spacing	Variance in Neighborhood Size
Rectangular	9x18	1	0.2716
Hexagonal	9x18	1	1.2138
Tessellation	162	0.25319 - 0.31287	0.0686
Rakhmanov	162	0.15779 - 0.30069	0.2908

As spherical (and other alternative) topologies become increasingly more common it is necessary to investigate how the choice of topology effects the SOM. In this thesis the effect of irregularity within topologies is studied as an attempt to investigate not only the edge effect, but also to help facilitate the comparison of topologies. It is important to note that spherical topologies may not be appropriate for all applications. Removing the edge may reduce the SOM's ability to converge. As outliers are forced to interact they introduce more competition among the neurons. We would also expect outliers to occupy more space in the final map as their dissimilarity in attribute space should translate to more distant spatial relationships in the trained SOM. More research will be needed to help researches determine the most appropriate topology for their data and research objectives.

2.6 NETWORK SIZE

The number of neurons used in the SOM is a decision the research must make and is both a function of the size of their dataset and the purpose for which the SOM is to be used. Generally speaking fewer neurons would be used in a clustering application while larger SOMs are commonly used for visualizing high-dimensional datasets. The literature offers little theoretical guidance on choosing an appropriate network size for a given dataset (Cho et al., 1996). Vesanto (2005) suggests that the network size should be "as big as possible," but

also states that this becomes computational impractical for larger problems. As a general rule-of-thumb, Vesanto suggests using a network size of $5\sqrt{n}$, where n is the number of observations. The application for which this network size would be most relevant is unclear.

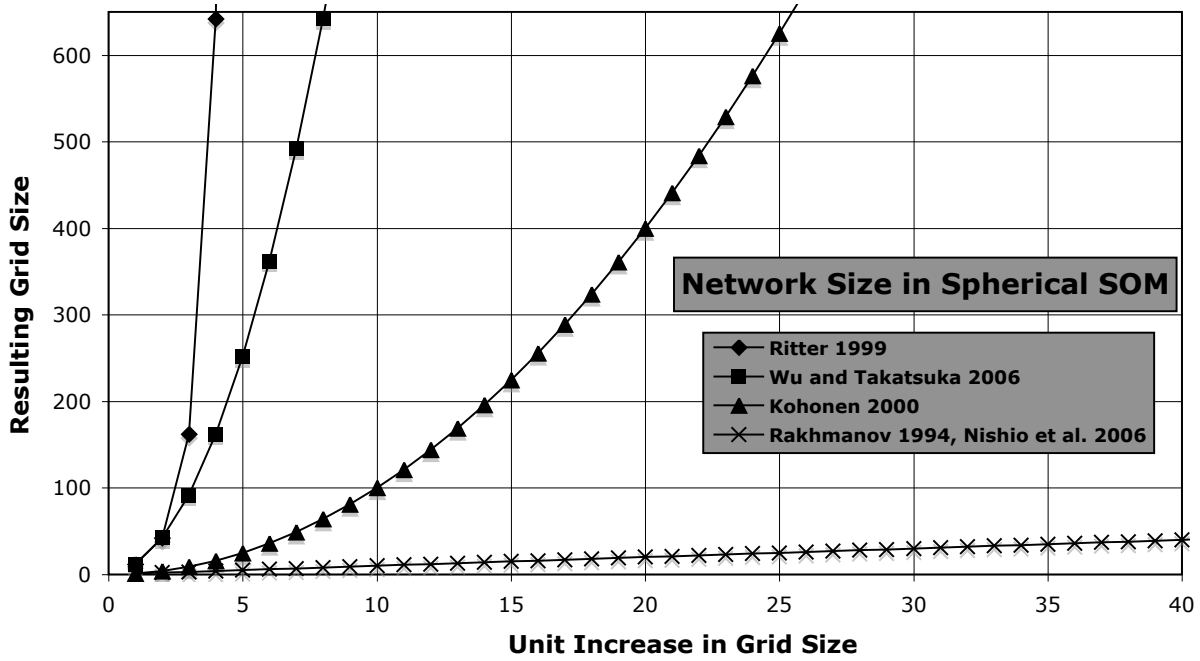


Figure 2.3. This figure demonstrates the achievable network size using various spherical topologies, in comparison with the traditional SOM. The Y-axis represents the achievable network size, while the X-axis represents the smallest increase in grid size for each topology.

Given this lack of theoretical development, researchers should be cautious when using methods that limit the control of network size. Having a high level of control over network size allows support for such very different SOM applications as clustering versus low-dimensional spatial layout. Figure 2.3 shows how the achievable network size varies between topologies. In rectangular and hexagonal SOMs it is undesirable to have one dimension drastically larger than another, as such there are practical limitations to the size of these networks. As an aside, the preferred ratio between these dimensions depends on the data being represented, and should generally not equal one (Kohonen et al., 1996; Vesanto, 2005). In Figure 2.3 we used a ratio of one only for comparison purposes with the other topologies.

Ritter's tessellation method results in a network size that grows at a rate of $N = 2 + 10 * 4^f$, where f is the frequency of tessellation. Wu and Takatsuka (2006) offer a slight improvement. Rather than recursively subdividing the faces, they redivide the original icosahedron with each step, resulting in $N = 2 + 10 * f^2$. Methods for arranging an arbitrary number of points on a sphere provide the highest degree of flexibility when choosing a network size. For example, the method proposed by Rakhmanov et al. (1994) can distribute any number of points onto the surface of a sphere. Strictly speaking, this is not a topology in itself, as no connections are defined between the points. In our implementation we create a spherical topology be applying Delaunay triangulation to these points (Ranka, 1997). We refer to this topology simply as "spherical". Using Ritter's method with a tessellation frequency of three, would result in 642 neurons, the next smallest size is 162 neurons. The geodesic topology offers three additional levels between 162 and 642 neurons (252, 362, 492). Using the spherical topology however, we are not limited to these network sizes.

Similarly Nishio et al. (2006) try to address the issue of network size granularity by departing from the tessellation method and suggesting the use of a partitioned helix to uniformly distribute any number of neurons on a sphere. The method proposed by Rakhmanov et al. (1994) was dismissed by Wu and Takatsuka (2005) for failing to satisfy the uniformity conditions described above. Nishio et al. (2006) suggest their method for distributing points satisfies these uniformity constraints, however they do not describe a network topology. It is unclear how they define neighbor relationships and without clarification we cannot implement their topology. Methods for distributing points on the sphere, which allow for fine-grained control over network size, produce slightly more irregular topologies. However, no substantive discussion of these irregularities or their effects on SOM training exists in the literature. Network size plays an import role in the SOM and given that limited theoretical guidance is available for choosing network size, researches should be cautious when using topologies that limit control over this parameter. Particularly for larger SOMs, the desired network size may not be achievable via tessellation of the icosahedron.

CHAPTER 3

METHODOLOGY

This chapter is composed of four sections. Section 3.1 describes PySOM, our own graph based implementation of SOM. PySOM provides the necessary inputs to our three diagnostics as described in section 3.2. These diagnostics were developed to help understand how irregularities in topology effect the SOM. To use these diagnostics we must train several SOMs with comparable training data and parameters. The specifics of our training data are explained in section 3.3. Section 3.4 describes the training parameters used across all of our SOMs.

3.1 GRAPH BASED IMPLEMENTATION OF SOM

The most widely available implementation of SOM is Kohonen's own SOM_PAK (Kohonen et al., 1996). SOM_PAK implements both the traditional rectangular and hexagonal topologies. However, implementations of the geodesic and spherical SOMs were not readily available at the time this thesis was written. In order to test these topologies it was necessary to implement our own version of SOM, PySOM. Because the goal of this thesis was to study different topologies for use in SOM, we created PySOM to be topologically agnostic. That is, our implementation is not explicitly aware of the topology. Rather, we represent the topology of the SOM as a graph. The graph provides the necessary information to determine neuron adjacency and construct neighborhoods. The nodes in the graph are pointers to the reference vectors (m_i).

To allow for rapid development and cross-platform support we choose to write PySOM in the Python programing language. As noted by Rey and Janikas (2006) Python is an object-oriented programing language that is becoming increasingly popular in scientific computing. PySOM is maintained as an open-source project is hopes of facilitating

collaboration and further research. In its present form PySOM has no user interface, however it is written as a Python library which allows all of its functionality to be accessed programmatically. Python also provides an interactive interpreter which may be used to access PySOM in a command line environment.

By abstracting the topology, PySOM can train a SOM using any topology for which a graph structure can be created. We leverage an existing graph library, NetworkX, to represent our topologies (Hagberg et al., 2008). Our neighborhood functions are built on top of this library. Apart from our neighborhood search functions, our implementation follows the original incremental SOM algorithm that is described by Kohonen (2001). We store our trained SOM in a similar fashion as SOM_PAK. The reference vectors are represented in a simple text file. These "codebook" files contain a description of the topology and other training parameters in the first line of the file. Each subsequent line lists the values for the parametric reference vectors, m_i, of the SOM's neurons (Kohonen et al., 1996). In addition to this codebook file we also store the graph as a serialized python object. Creating the graph structures for the spherical topologies is considerably more complex and requires more computation than the traditional topologies.

We provide utility functions to create the graph structures for both the rectangular and hexagonal topologies. To create the graph structure for the geodesic topology we use the "dome" software package, which outputs the point coordinates in "XYZ" format for each node (Bono, 1996). These coordinates are fed into the STRIPACK software program. STRIPACK computes both the Voronoi cells and their complement, the Delaunay triangulation, on the surface of the sphere (Ranka, 1997). The Delaunay triangulation provides the graph structure between the neurons of the geodesic SOM. A similar process is used for the spherical topology. In this case we wrote a python implementation of the method for distributing points on a sphere that was introduced by Rakhmanov et al. (1994). Once again the coordinates are fed into STRIPACK. An additional utility program reads the output from STRIPACK and creates the NetworkX graph structure. PySOM has no direct graphical

output, however several utility functions are provided to assist in the creation of visualizations. These functions create files that are compatible with popular GIS software packages, namely ESRI's ArcGIS.

3.2 DIAGNOSTICS

In traditional SOMs, outlying observations are pushed to the edge of the map where they encounter fewer competing signals. A prime example of this is the "Utah-Hawaii" case shown in Figure 2.2. Relying only on the SOM, one would be left to believe that the two states are similar. Recalling that the QError measures the distance between two vectors in attribute space, we see that the QError from Utah to the neuron is 1.509, the QError from Hawaii to the neuron in 1.505, but the QError from Utah to Hawaii is 3.014. In this case only Utah and Hawaii were mapped to that neuron. In a case where multiple observations land on the same neuron, it is possible to measure the average pairwise QErrors between those observations. This gives us a notion of internal heterogeneity, H, for each neuron. We define the internal heterogeneity of neuron i as,

$$H_i = \frac{2}{{n_i}^2 - n_i} \sum_{j=1}^{n_i} \sum_{k=j+1}^{n_i} ||x_{ij} - x_{ik}|| \tag{3.1}$$

where, n_i is the number of observations mapped to i, and x_i are the input vectors mapped to i. For any neuron that captures more then one observation, this measure tells how dissimilar those observations are.

The edge effects in SOM make it clear that the compression of the input-space is not uniform through out a trained map. While outliers being pushed to the edge of the SOM is not necessarily an undesirable outcome, it is important understand when and where information is being compressed. This variable compression of the input-space is what allows the SOM to represent high-dimensional data, but it can also mislead the viewer. Observing the internal heterogeneity of neurons may shed light on the patterns of compression.

More specifically we have developed diagnostics that explore how irregularities in the topology of the SOM effect this internal heterogeneity. We compare the internal heterogeneity

at the scale of the neuron and the overall map. Each diagnostic was designed to answer a specific research question. The first diagnostic addresses the research question regarding the internal heterogeneity and neighborhood size. The second diagnostic addresses the question concerning internal heterogeneity and topological irregularity. The third helps visualize the patterns between internal heterogeneity; the usefulness of which is examined in the next chapter.

3.2.1 Internal heterogeneity vs. first-order neighborhood size

This diagnostic compares the internal heterogeneity of each neuron against the neuron's first-order neighborhood size. It would be expected that in traditional SOMs neurons closer to the edge, those with fewer neighbors, will have higher internal heterogeneity. Neurons on the edge of the traditional (rectangular and hexagonal) topologies are considered more irregular, because they have few neighbors then neurons inside the edge. This can be extended to spherical SOMs by considering the degree of any given neuron. The degree of a neuron $deg(m_i)$ measures the number of adjacent neurons. If the relationship between H_i and $deg(m_i)$ is consistent across topologies, we would expect neurons with lower degrees to display higher internal heterogeneity.

To implement this diagnostic we calculate the internal heterogeneity (H_i) and degree ($deg(m_i)$) of each neuron. The neurons are then separated into a small number of groups based on the degree. For most topologies the number of different degrees will be limited to three or four. The variance and mean is calculated for each of these groups. The expected result is that variances and means of the groups will decrease as the degree increases. This hypothesis is tested using random labeling as described by Rey (2004). In random labeling, we randomly assign our calculated H values to the neurons and recompute the mean and variance of the groups. We do this many times, 9999 in our case, in order to approximate the true distribution. Finally we calculate pseudo p-values by comparing our observed mean and variance values with the simulated distributions. The results are also visualized using

box-and-whisker diagrams. Box-and-whisker diagrams, or box plots, show the properties of a distribution. The diagram shows the mean, first and second standard deviations, and outliers that extend beyond the second deviation.

One problem that we face in this diagnostic is a small sample size when the neurons of a given SOM are grouped by their degree. For example, the four corners of the rectangular topology are the only neurons that have a degree of two. The rest of the neurons have three or four neighbors depending on whether or not they are on the edge. To address the problem of small sample size for topologies with relatively few neurons of a particular degree, we will increase the sample size by combining the results of many SOMs.

3.2.2 Internal heterogeneity vs. topological regularity

This diagnostic compares the internal heterogeneity of each neuron against a measure of regularity for its associated topology. As mentioned above the degree of each neuron measures the number of adjacent neighbors. A completely regular network topology (i.e. the torus) has no variance between these measures. For irregular networks the variance between these degrees gives us a measure of irregularity. This particular measure is known as degree centrality. The degree of a node on a network is a measure of its centrality, or importance. Nodes with more connections are thought to be more central to the network and have a larger influence than nodes with fewer connections.

This holds with our understanding of the edge effect. Neurons on the edge are less central to the network and have less influence than other nodes. These edge nodes are also less influenced by the network, allowing outliers take root. During the training process observations that are more average than others tend to be centralized. The observations that surround them tend to be more extreme. If you refer back to figure 2.2, you'll notice that observations with smaller symbols are closer to the mean of the input-space and that these observations have been centralized in the network. Using the degree as a measure of centrality does not capture this picture well, as neurons near the edge can still have a large degree. A

better way to capture this effect is to look at closeness centrality, which is the inverse of the average distance of a neuron to every other neuron on the network (Wasserman and Faust, 1994).

Closeness centrality provides a more complete measure of connectedness in a given topology than degree centrality. In this diagnostic we compare the internal heterogeneity of the neurons against the average closeness centrality of their respective topologies. This results in one group of internal heterogeneity measurements for each topology tested. We evaluated this diagnostic in much the same way as the last. We compare the variances and means of each group, testing for differences with random labeling. It is expected that the distribution of internal heterogeneity will be narrower for groups trained on more regular topologies. It is further hypothesized that the mean of internal heterogeneity will decrease when the network is more regular.

3.2.3 Visualize internal heterogeneity mapping

Visualizing the internal heterogeneity may yield insight into how irregular topology effects the SOM. Creating these visualization for many different topologies however, offers a number of challenges. While the rectangular and hexagonal topologies are rather straight forward to visualize, the spherical and geodesic topologies are significantly more involved. In order to leverage the utility of existing GIS software we represent our topologies in a form that these software packages understand. Toward that end we create polygon layers in which each polygon represents a neuron. Shared borders represent connections between neurons. Creating the polygons for these topologies required that we first compute the Voronoi diagram on the surface of the sphere. This is done using STRIPACK, a software program created by Ranka (1997). Despite the prevalence of spherical coordinates in GIS, modern GIS software packages have their roots in CAD software. As such they all assume Cartesian distances and thus can not handle polygons that cross the 180^{th} meridian. To accommodate this we split each polygon at the 180^{th} meridian and redraw it as two parts.

Once the GIS layers have been created and the internal heterogeneity of each neuron has been calculated, a number visualizations become possible. We visualize the internal heterogeneity by shading the corresponding polygons. These visualizations allow for the exploration of patterns in internal heterogeneity with relation to the underlaying topology. Further we can visualize the component planes, which show how the higher dimensions are represented in the various SOMs. We also map our synthetic data back onto the SOM in order to calculate cluster membership of the neurons. Visualizing cluster membership clearly shows how the various topologies perform clustering.

3.3 DATA

Our internal heterogeneity measure is sensitive to both the properties of the SOM and the properties of the training data. Therefore, a dataset with uniform properties is needed. We follow the method for generating uniform synthetic data used by Wu and Takatsuka (2006). Their method creates seven clusters in three dimensions. Each cluster is normally distributed and has a standard deviation of one. The clusters are centered at the origin and ten units out in each directions on the x, y and z axes. The uniform clusters generated by this method allow us to systematically compare the diagnostics under several different topologies. To ensure that we can calculate an internal heterogeneity for as many neurons as possible, we create approximately $25,000$ observations in each dataset. As described in the next section, our SOMs have either 642 or 644 neurons (depending on the topology). Having a large number of observation relative to the number of neurons will force the SOM to preform clustering, increasing the number of neurons for which the internal heterogeneity can be computed.

As mentioned in section 3.2.1 we will need to combine the results of multiple SOMs in order to ensure a large enough sample size. To accomplish this we will create ten synthetic datasets as described above. Each of these datasets can be thought of as samples from the same input space. That is, the data generating process remains the same for each synthetic dataset created. After training ten SOMs for each topology we take the average internal heterogeneity of each to ensure that the results are comparable.

3.4 TRAINING PARAMETERS

Before we can go on to address the research questions we need to train a series of SOMs. We train SOMs using four different topologies: *rectangular, hexagonal, geodesic sphere* and *spherical*. The spherical topology is based on a method, developed by Rakhmanov et al. (1994), for distributing an arbitrary number of points on to the surface of a sphere. Delaunay triangulation is then applied to these points, producing a topological structure. To yield meaningful results these SOMs must be trained with comparable parameters. The literature provides many rules of thumb for training a SOM: each SOM is trained in two stages, the first of which uses a larger initial learning rate and neighborhood search radius with a small number of training steps; the second stage uses a lower initial learning rate and neighborhood search radius, but extends the length of training.

First Stage Parameters:

- Initial neighborhood search radius of 50%, which decreases during training.
- Initial learning rates of 0.04 which decreases during training.
- 100,000 training steps.

Second Stage Parameters:

- Initial neighborhood search radius of 33%, which decreases during training.
- Initial learning rates of 0.03 which decreases during training.
- 1,000,000 training steps.

As shown in Figure 2.3, topologies differ in terms of achievable network size. For comparability, the network size of each SOM needs to be as close as possible. The achievable network size for the geodesic SOM is the most limiting of the topologies we test. We chose the eighth frequency geodesic sphere, which has 642 nodes, which is relatively close to the 644-node hexagonal and rectangular topologies achieved when the dimensions are set to $28x23$. Finally, the spherical topology was set to 642 nodes.

CHAPTER 4

RESULTS AND DISCUSSION

This chapter presents the results from our diagnostics. Section 4.1 reviews the training process. Section 4.2 discusses the application of the first diagnostic, which looks at how internal heterogeneity changes with a neuron's first order neighborhood size, or degree. In section 4.3 the second diagnostic, which compares the mean internal heterogeneity across topologies, is applied. Finally, section 4.4 applies the third diagnostic, which visualizes the internal heterogeneity of each SOM.

4.1 TRAINING

Before appling the diagnostics we must first train a series of SOMs. The training is acomplished using our graph based implementation of SOM, PySOM. Using the ten synthetic datasets we train ten SOMs for each topology. The mean internal heterogeneity of these SOMs is summarized in Table 4.1. We find that the mean internal heterogeneity remains fairly stable, suggesting that the results of each simulation can be combined within a given topology. For the rectangular topology, we now have forty neurons with a degree of two for which an internal heterogeneity can be calculated. It should be noted that we can only measure the internal heterogeneity when a neuron captures two or more observations from the training data. Therefore, it is still possible to have less than forty measurements.

We used several machines of varying configurations to train the forty different SOMS. On the fastest of those machines the training one SOM took around forty-five (45) minutes. In contrast SOM_PAK takes only ninety (90) seconds to train the hexagon SOM with the same training parameters. This significant difference is largely do to differences in the underling data structures. We traded speed for the ability to represent a SOM with any topology.

Table 4.1. Mean internal heterogeneity for each simulation, by topology.

Simulation	Geodesic	Spherical	Hexagonal	Rectangular
1	0.0277	0.0277	0.0285	0.0289
2	0.0281	0.0281	0.0291	0.0295
3	0.0278	0.0280	0.0286	0.0292
4	0.0280	0.0282	0.0286	0.0293
5	0.0279	0.0280	0.0289	0.0296
6	0.0278	0.0274	0.0285	0.0290
7	0.0286	0.0283	0.0294	0.0297
8	0.0284	0.0285	0.0294	0.0298
9	0.0283	0.0282	0.0293	0.0295
10	0.0285	0.0285	0.0293	0.0298
Combined	0.0281	0.0281	0.0290	0.0294

Differences between the Python and C programing languages may also account for significant difference in runtime.

4.2 INTERNAL HETEROGENEITY VS. FIRST-ORDER NEIGHBORHOOD SIZE

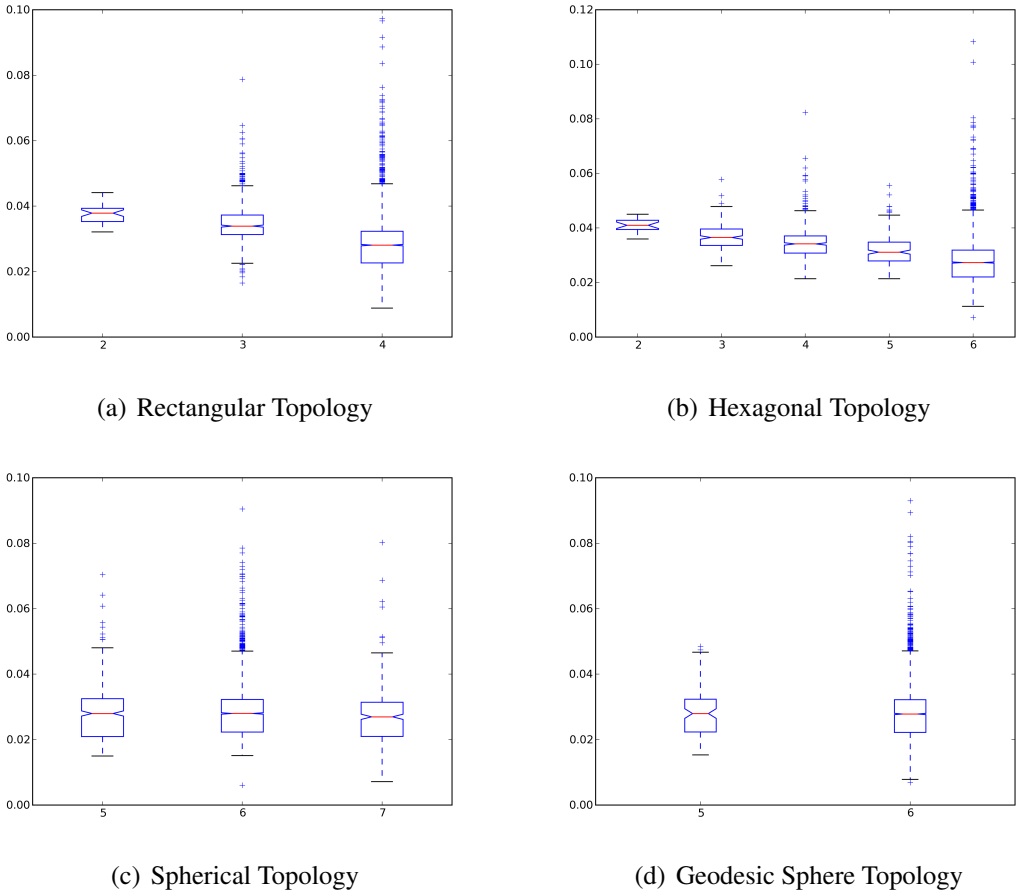
We hypothesized that outlying observations would migrate to less central neurons on the map, where they encounter less competition. We expected the internal heterogeneity as measured by equation 3.1 to increase at these locations, demonstrating that topological irregularity affects the placement of outliers in the SOM.

Table 4.2. Size, mean and variance of each sample

Degree	Geodesic		Spherical		Hexagonal		Rectangular	
	N	Mean (Var)	N	Mean (Var)	N	Mean (Var)	N	Mean (Var)
2					20	0.0409 (5.66E-06)	40	0.0378 (7.59E-06)
3					218	0.0371 (2.18E-05)	880	0.0348 (3.59E-05)
4					489	0.0347 (4.05E-05)	4926	0.0284 (6.28E-05)
5	113	0.0283 (5.28E-05)	526	0.0279 (6.43E-05)	206	0.0319 (3.50E-05)		
6	5598	0.0281 (6.05E-05)	4758	0.0282 (6.05E-05)	4954	0.0278 (6.09E-05)		
7			417	0.0273 (6.65E-05)				

For each topology, we calculated the internal heterogeneity and degree of all the neurons. We then grouped the internal heterogeneity measures by degree. These groups are treated as representative samples from a larger distribution. Table 4.2 shows the details of each sample, including its size, mean and variance. The size is the number of observations for

which we were able to calculate an internal heterogeneity. The mean and variance capture the centrality and dispersion of the samples. We create a box-and-whisker diagram (box plot) for each sample, as shown in figure 4.1, to visualize the samples summarized in table 4.2.



(a) Rectangular Topology

(b) Hexagonal Topology

(c) Spherical Topology

(d) Geodesic Sphere Topology

Figure 4.1. Box-and-whisker diagrams representing samples derived from forty trained SOMs. The samples within each topology were created by grouping the internal heterogeneity of each neuron of a given degree size. The diagrams how the centrality and dispersion of each sample.

We see that the means of the samples seem to respond as expected for the rectangular and hexagonal, or "flat," topologies, but not in the spherical topologies. While this is not what we anticipated, it may suggest that the spherical and geodesic topologies are effectively overcoming the edge problem. The variance of the samples, however, did not respond as expected. In the flat topologies the variance increased as the degree increased. This is

possibly due to the large difference in sample size. To verify these conclusions we formally test for differences in means and variance between the samples.

Table 4.3. Results of Difference of Mean Testing Within Each Topology

(a) Rectangular Topology

Degree	2	3	4
2	(0.000008)	**0.002700**	**0.000100**
3	0.002969	(0.000036)	**0.000100**
4	0.009364	0.006396	(0.000063)

(b) Hexagonal Topology

Degree	2	3	4	5	6
2	(0.000006)	**0.000800**	**0.000200**	**0.000100**	**0.000100**
3	0.003810	(0.000022)	**0.000100**	**0.000100**	**0.000100**
4	0.006253	0.002443	(0.000040)	**0.000100**	**0.000100**
5	0.009008	0.005197	0.002755	(0.000035)	**0.000100**
6	0.013067	0.009257	0.006814	0.004059	(0.000061)

(c) Spherical Topology

Degree	5	6	7
5	(0.000064)	0.485500	0.197400
6	0.000250	(0.000060)	**0.020000**
7	0.000674	0.000924	(0.000067)

(d) Geodesic Topology

Degree	5	6
5	(0.000053)	0.783900
6	0.000203	(0.000060)

The results of the means test are presented in table 4.3. This table shows the difference in means below the diagonal, the p-value above the diagonal (with significant values in bold), and the variance of each sample along the diagonal. In the rectangular and hexagonal topologies, we observe that all sample means are significantly different. We also observed significant differences in the variance of the samples, except the case of degree size four and five in the hexagonal topology, where they were not significantly different. No variance in the spherical and geodesic topologies were significantly different. In those topologies, the only

significant difference in means was between the sample with degree size six and seven in the spherical topology.

4.3 INTERNAL HETEROGENEITY VS. TOPOLOGICAL REGULARITY

Above we saw that changes in a neuron's degree was related to changes in internal heterogeneity in topologies with edges, but not in the two sphere-based topologies. The next step is to see if internal heterogeneity changes between topologies. To do this we will first order our topologies by a summary measure of the topological regularity. For this diagnostic we use the average closeness centrality for each topology as the summary measure. The summary measure and the sorting of our topologies is shown in table 4.4.

Table 4.4. Measure of Topological Regularity and Sample Mean and Variance

Topology	Closeness Centrality	Mean	Variance
Rectangular	0.0603	0.0294	0.0001
Hexagonal	0.0739	0.0289	0.0001
Geodesic	0.0890	0.0281	0.0001
Spherical	0.0906	0.0281	0.0001

In this diagnostic we once again group the neurons of our trained SOMs. This time we group them based on their topology, collapsing the neurons of the forty SOMs into four groups. Resulting in one sample for each of the four topologies. We test for a difference in mean and variance between each sample using the same method of random labeling that was applied in the previous diagnostic. No significant differences were found in the variances; the difference in means are presented in table 4.5. It was expected that the mean and variance of the samples would decrease for the more regular topologies. These results generally support this hypothesis. We see that the rectangular topology has the highest mean internal heterogeneity and is the least regular as measured by closeness centrality. The geodesic and spherical topologies are the most regular and have the lowest internal heterogeneity. These two groups display very similar measures of closeness centrality and show no significant

difference in mean internal heterogeneity. This suggests that even though the spherical topology is more irregular than the geodesic topology, similar levels of quality may be achieved.

Table 4.5. Results of Difference of Mean Testing Across Topologies

Topology	Rectangular	Hexagonal	Geodesic	Spherical
Rectangular	(0.000064)	**0.001000**	**0.000100**	**0.000100**
Hexagonal	-0.000479	(0.000064)	**0.000100**	**0.000100**
Geodesic	-0.001329	-0.000850	(0.000060)	0.505600
Spherical	-0.001328	-0.000849	0.000001	(0.000061)

4.4 VISUALIZATION OF INTERNAL HETEROGENEITY

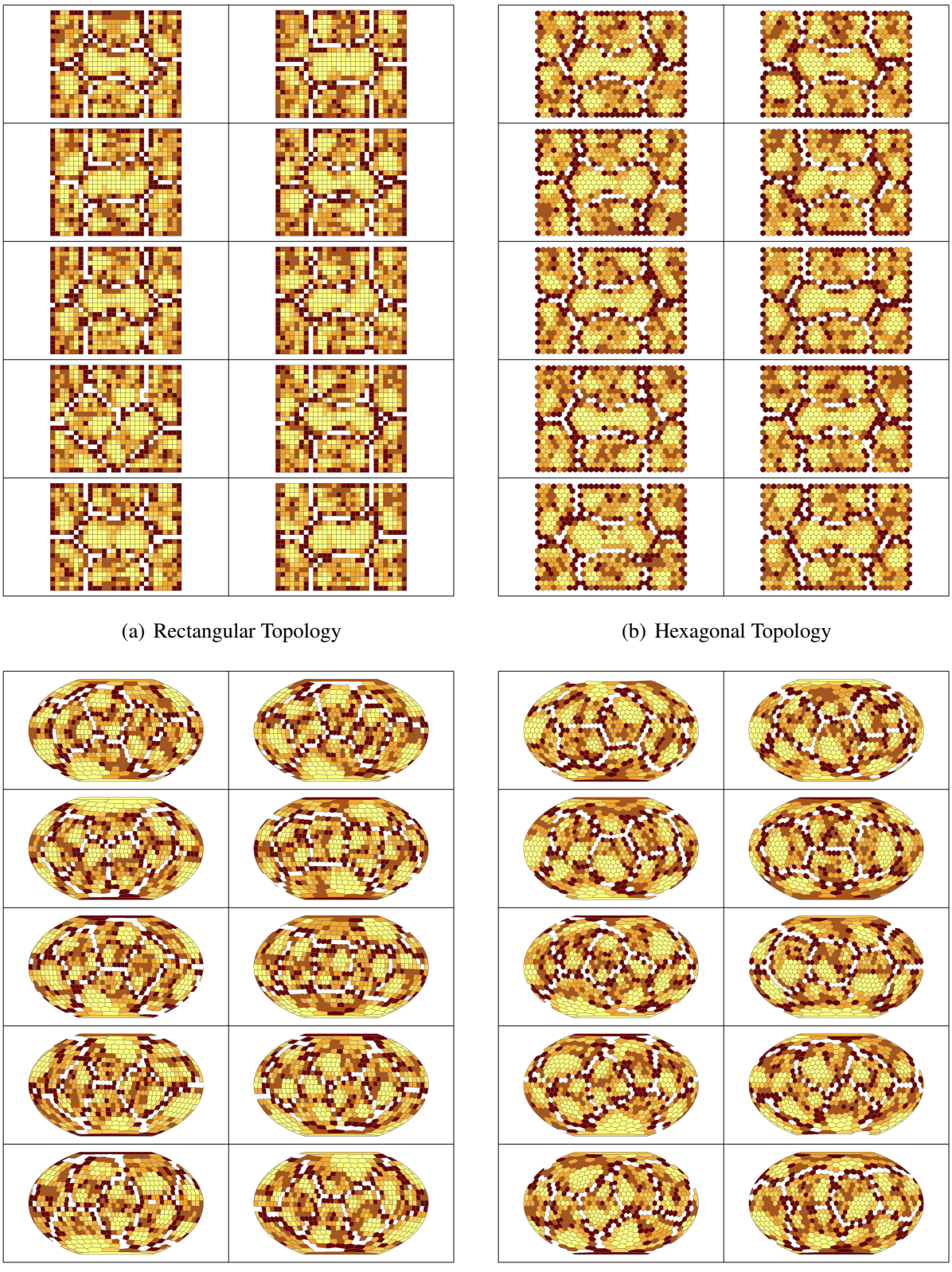
In order to address our research questions we first created ten synthetic datasets. Each dataset was created by sampling from the same data generating process that was provided by Wu and Takatsuka (2006). We used these datasets to train ten SOMs for each of our topologies. The purpose of training ten SOMs was to produce large enough samples sizes for the difference of means and variance testing that was necessary to formally evaluate the results of our first two research questions. In this section we verify that combining those simulations was appropriate by visualizing the similarities between them. In the remainder of this section we focus our efforts on comparing internal heterogeneity across topologies. We see that little variation exists between the ten simulations, as such we choose the first of those simulations for each topology and explore them in more depth.

As expected, figure 4.2 reveals little variation between simulations of a given topology. This homogeneity demonstrates that our ten synthetic datasets adequately increase our sample size without introducing bias. Because of the apparent homogeneity between the simulations *within* a given topology, we move on to compare the first simulation *across* topologies. Our synthetic training data consisted of seven clusters located in three dimensional space. We examine how the SOM treats the original data through a series of

visualizations. The component planes in figure 4.3 show how the SOM represents each dimension of the training data. The variations we see in the placement of high and low values, between topologies, is caused by the random initialization of the SOM. However, the relationships between the dimensions remain constant across topologies.

Figure 4.4 shows how the SOM detected the clustering of the input data. We compute the cluster each neuron represents by looking at the observations mapped to it. Knowing which cluster each observation belongs to allows us to see where the clusters are being mapped to on the trained SOM. For a given neuron we classify it by the cluster it captured most frequently.

It is interesting to note that the edges of the cluster exhibit higher internal heterogeneity. This is somewhat intuitive, as our clusters are normally distributed; the majority of our observations will fall well within the regions observed in figure 4.4. Each cluster's outliers will be pushed toward the *edges* of these regions. An observation that is on the edge of a cluster in the original input-space is further away from the other observations in the input-space. Therefore, one would expect that observation to also be near the edge of a cluster in the SOM space. In order to represent a three dimensional cluster in two dimensions the SOM must compress the edges of the clusters more than their centers. This explains the higher internal heterogeneity near the edges of the clusters.



(a) Rectangular Topology

(b) Hexagonal Topology

(c) Spherical Topology

(d) Geodesic Sphere Topology

Figure 4.2. Internal heterogeneity mapping for each of the forty SOMs. Darker colors represent neurons that display larger internal heterogeneity. Neurons for which an internal heterogeneity could not be calculated are not displayed.

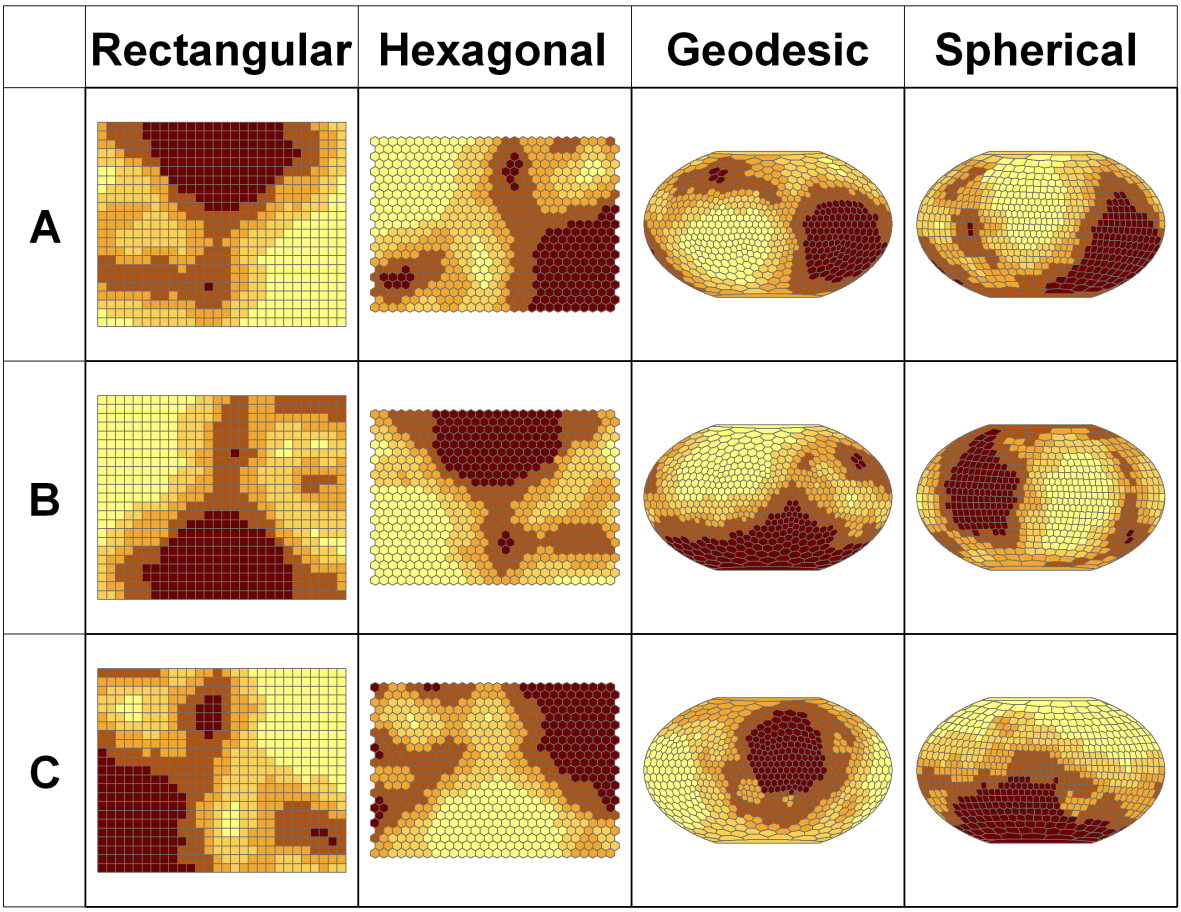
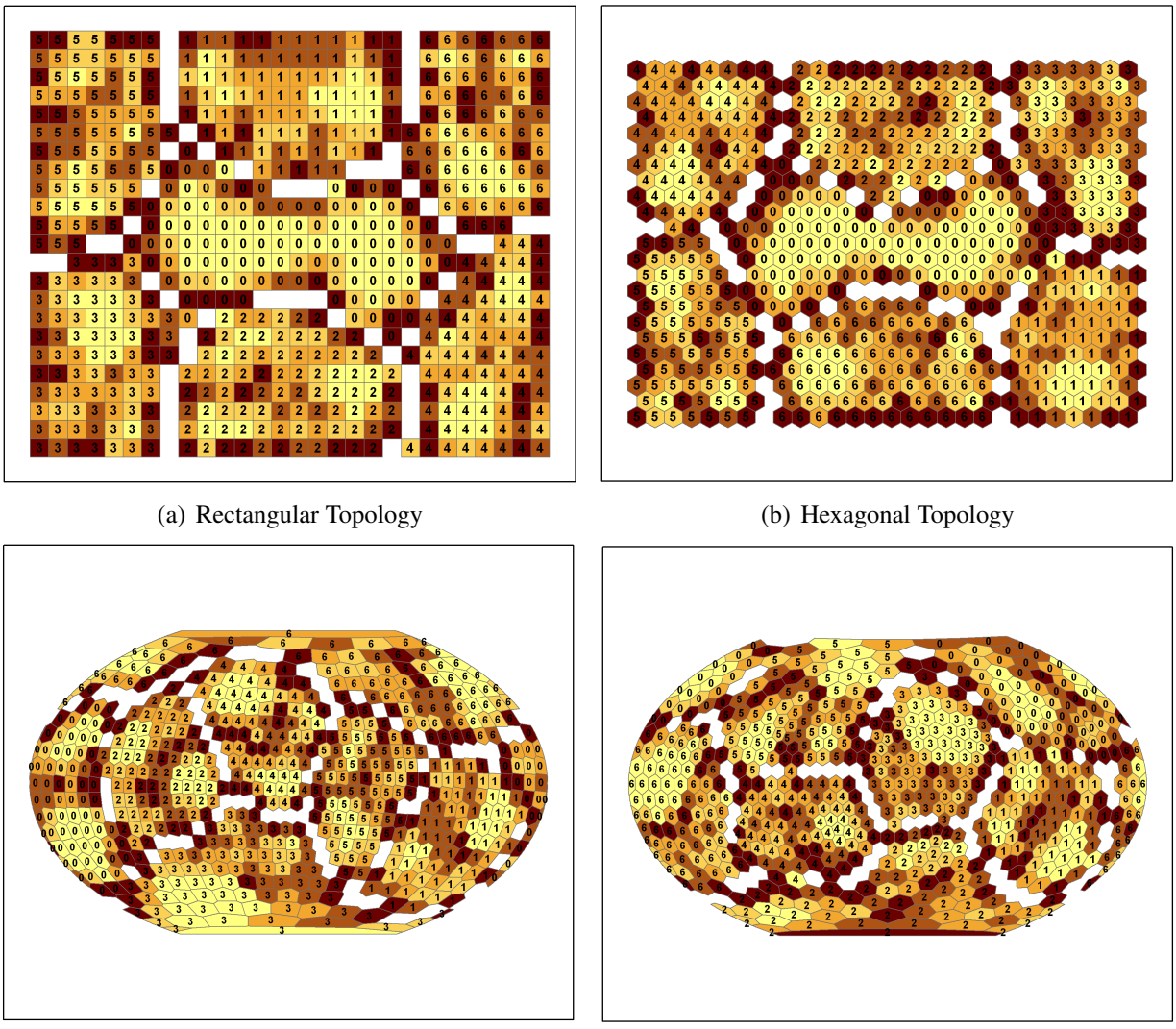


Figure 4.3. The first (A), second (B) and third (C) component planes are shown for the first simulation of each topology. These component planes show how the original dimensions are represented in the trained SOMs.



(a) Rectangular Topology

(b) Hexagonal Topology

(c) Spherical Topology

(d) Geodesic Sphere Topology

Figure 4.4. Detailed internal heterogeneity mapping for each topology. Darker colors represent neurons that display larger internal heterogeneity. Neurons for which an internal heterogeneity could not be calculated are not displayed. The numbers represent primary cluster mapped at that neuron.

CHAPTER 5

CONCLUSIONS

5.1 SIGNIFICANCE

The commonly used tessellated icosahedron based topology offers the most regular topology. However, the main disadvantage of this topology type is that it offers a limited control over network size. Alternative methods for generating the spherical topology, which can create a network of any size, have been reviewed or suggested by Wu and Takatsuka (2005) and Nishio et al. (2006). These alternative methods produce network structures that are more irregular. This research will take a closer look at the impact that the irregularity has on the training process in an attempt to address the suitability of these topologies for use in SOM.

5.2 LIMITATIONS

This research will look at the relationship between regularity in neuron connectedness and the training of a SOM. The relationship between topology and SOM visualization is not addressed. The topology chosen for a SOM has a direct link with how that SOM is visualized. When representing the topology on the surface of a sphere issues arise with the uniformity in neuron spacing and sizing. Future work may be needed to address these issues in visualization.

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APPENDIX

THE CODE

THE CODE

A.1 SOM.PY

```
"""
Python Self-Organizing Maps with Spherical Lattice
--------------------------------------------------------------------
AUTHOR(S):      Charles R. Schmidt cschmidt@rohan.sdsu.edu
--------------------------------------------------------------------
Copyright (c) 2006-2008  Charles R. Schmidt
====================================================================
This source code is probably licensed under the GNU General Public License,
Version 2, you should check.
====================================================================
"""
import random,math,time,sys,os
from numpy import array,empty,take,put,zeros
import numpy as N
from math import acos,sqrt,pi,degrees,sin,cos,asin
import networkx as NX
#import pylab
import pickle
from utils import *

class som:
    ''' Base class for the Self-Organizing Map,
```

```
    Each topology will inherit from this class.
    A template is provied in 'Topology'
'''
def __init__(self):
    self.Dims = 0
    self.X = 0
    self.Y = 0
    self.Size = 0
    self.Type = 'none'
    self.tSteps = 0
    self.maxN = 0
    self.alpha0 = 0
    self.nodes = []
    self.daMap = {}

def load(self,path='',name=None):
    if not name:
        name = '%ds_%dd_%dr_%fa'%(self.Size,self.Dims,self.tSteps,self.alpha0)
    codname = path+name+'.cod'
    mapname = path+name+'.map'
    if os.path.exists(mapname):
        try:
            f = open(mapname,'r')
            self.daMap = pickle.load(f)
            f.close()
        except:
            self.daMap = {}
```

```
        dataf = open(codname,'r')
        header = dataf.next()
        Dims,Type,x,y,nType= header.split()
        self.Dims = int(Dims)
        self.Type = Type
        self.X, self.Y = int(x),int(y)
        self.Size = self.X*self.Y
        self.nodes = array([[0.0 for i in xrange(self.Dims)] for j in xrange(self.Size)])
        for i in xrange(self.Size):
            data = dataf.next()
            data = data.split()
            data = map(float,data)
            self.nodes[i] = data
        self.diffs = array([[0.0 for i in xrange(self.Dims)] for j in xrange(self.Size)])

    def save(self,path='',name=None):
        if self.X == 0 or self.Y == 0:
            self.X = self.Size
            self.Y = 1
        if not name:
            name = '%ds_%dd_%dr_%fa'%(self.Size,self.Dims,self.tSteps,self.alpha0)
        codname = path+name+'.cod'
        cxdname = path+name+'.cxd'
        mapname = path+name+'.map'
        if self.daMap:
            mapfile = open(mapname,'w')
            pickle.dump(self.daMap,mapfile)
```

```
        mapfile.close()
    outf = open(codname,'w')
    outx = open(cxdname,'w')
    outf.write("%d %s %d %d gaussian\n"%(self.Dims,self.Type,self.X,self.Y))
    outx.write("%d %s %d %d gaussian CUM DIFF FILE\n"%(self.Dims,self.Type,self.X,self.Y))
    for i in xrange(self.Size):
        outf.write(' '.join(str(self.nodes[i].tolist())[1:-1].split(', '))+'\n')
        outx.write(' '.join(str(self.diffs[i].tolist())[1:-1].split(', '))+'\n')
    outf.close()
    outx.close()

def randInit(self):
    self.nodes = array([[random.random() for j in xrange(self.Dims)] for i in xrange(self.Size)])
    # This diffs will define your walls!
    self.diffs = array([[0.0 for i in xrange(self.Dims)] for j in xrange(self.Size)])

def findBMU(self,ind,v,ReturnDist = False):
    d = ((self.nodes-v)**2).sum(1)
    minI = d.argmin()
    if ReturnDist:
        minD = d[minI]
        return minI,minD
    else:
        return minI

def diff(self,nodeid,ind,v):
    node = take(self.nodes[nodeid],ind)
```

```
        return sum((node-v)**2)


    def alpha(self,t):
        r = self.alpha0 * (1 - (t/float(self.tSteps)))
        if r < 0: r = 0
        return r
    def hci(self, t, dist):
        sigma = self.kernalWidth(t)
        a = self.alpha(t)
        top = dist**2
        bottom = (2*(float(sigma)**2))
        return a * math.exp(-top/bottom)
    def kernalWidth(self,t):
        """Returns the number of neurons to include at time t
           this should probably return the current order instead,
           which means it should be moved to Topology."""
        r = round((self.Size*self.maxN) * (1 - (t/float(self.tSteps))))
        r = int(r)
        if r == 0: r = 1
        return r


    def merge(self,t,ind,v):
        bmu = self.findBMU(ind,v)
        sigma = self.kernalWidth(t)
        results = self.neighborhood( bmu , sigma )
        alteredNodes = [(results[i],self.hci(t,self.odist(i))) for i in xrange(len(results))]
        for nodeID,hc in alteredNodes:
```

```
            if len(ind) == self.Dims:
                part = self.nodes[nodeID]
                delta = hc*(v-part)
                self.nodes[nodeID] = part+delta
                self.diffs[nodeID] += abs(delta)
            else:
                part = take(self.nodes[nodeID],ind)
                delta = hc*(v-part)
                put(self.nodes[nodeID],ind,part+delta)
                self.diffs[nodeID] += abs(delta)


    def run(self,obsf):
        self.neighborhoodCache = {}
        print "####################################"
        print "###        Configuration         ###"
        print "####################################"
        print "  N = %d, maxN = %f"%(self.Size,self.maxN)
        print "  Total runs: %d"%self.tSteps
        print "  initial learning rate = %f"%self.alpha0
        print "  Obs File:%s"%obsf.filename
        print "####################################"
        print "###          End Config          ###"
        print "####################################"
        print ""
        print "Running..."
        t1 = time.time()
        #s = ''
```

```
        T = self.tSteps
        for t in xrange(self.tSteps):
            id,ind,v = obsf.stream()
            self.merge(t,ind,v)
            ###
            #if t%100 == 0:
            #    if len(s) > 60:
            #        print '\r'+s
            #        s = ''
            #    s += '%d...'%t
            #sys.stdout.write("\r%s%d"%(s,t))
            sys.stdout.write("\r%.2f%%"%(100*float(t)/T))
            sys.stdout.flush()
        print "\nRun compleated in %f seconds"%(time.time()-t1)


    def map(self,obsf):
        """ This function needs an overhall #5"""
        qerror = 0
        counter = 0
        daMap = {} # keys are node ID's, values are lists of observation ID's.
        for id,dimIds,obs in obsf:
            bm,err = self.findBMU(dimIds,obs,ReturnDist=True)
            qerror += err
            if not bm in daMap:
                daMap[bm] = []
            daMap[bm].append(id)
            sys.stdout.write(".%d,%f.\r"%(counter,err))
```

```
            sys.stdout.flush()
            counter += 1
        print ""
        qerror = qerror/counter
        self.daMap = daMap
        return qerror

class Topology(som):
    """ Template class for topology Copy this class to create a new topology for som"'
    def __init__(self):
        som.__init__(self)
    def save(self,path,name):
        som.save(self,path,name)
    def load(self,path,name):
        som.load(self,path,name)
    def randInit(self):
        som.randInit(self)
    def kernalWidth(self,t):
        """
        You should overwrite this, see note above...
        kernalWidth returns the width of the neighborhood in terms of order
        """
        pass
    def odist(n):
        """
        n is the nth neuron in the in neighborhood, return's order
        example the 3rd neuron in the set is 1 order from the 0th.
```

```
        """
        pass
    def neighborhood(self,bmu,kernalWidth):
        """
        This function must return the ID's of the nodes inside the neighborhood,
        NumNeighbors is defined by kernalWidth and is expressed as an order.
        bmu is the id of the best match, or neighborhood center.
        """
        pass

class GraphTopology(som):
    """ Template class for topology Copy this class to create a new topology for som"'
    def __init__(self,G=None,Type='Graph'):
        som.__init__(self)
        self.Type = Type
        if G:
            self.G = G
            self.Size = G.order()
            # if findWidth is not given a seed it will brute force the total network
            # width, this could take a long time. For the spherical network, one of
            # the polls should yield the correct width. Or possibly the node with
            # lowest degree.
            self.Width = nf.findWidth(G,G.nodes()[-1])
            #WHY WHY WHY WHY WHY!!!!
            #self.maxN = 0.5

    def save(self,path,name):
```

```
        som.save(self,path,name)
        if not name:
            name = '%ds_%dd_%dr_%fa'%(self.Size,self.Dims,self.tSteps,self.alpha0)
        graphFile = path+name+'.graph'
        f = open(graphFile,'w')
        pickle.dump(self.G,f)
        f.close()
    def load(self,path,name):
        som.load(self,path,name)
        if not name:
            name = '%ds_%dd_%dr_%fa'%(self.Size,self.Dims,self.tSteps,self.alpha0)
        graphFile = path+name+'.graph'
        if os.path.exists(graphFile):
            f = open(graphFile,'r')
            self.G = G = pickle.load(f)
            self.Size = G.order()
            self.Width = nf.findWidth(G,G.nodes()[-1])
            f.close()


    def randInit(self):
        som.randInit(self)
    def kernalWidth(self,t):
        """
        kernalWidth returns the width of the neighborhood in terms of order
        """
        r = round((self.Width*self.maxN) * (1 - (t/float(self.tSteps))))
        r = int(r)
```

```
        if r == 0: r = 1
        return r
    def odist(n):
        """
        n is the nth neuron in the in neighborhood, return's order
        example the 3rd neuron in the set is 1 order from the 0th.
        """
        raise "not implemented"
    def neighborhood(self,bmu,kernalWidth):
        """
        This function returns a dictionary containing the neighbors of bmu as
        keys and their dist (as an order) as values.
        """
        return nf.neighborhood(self.G,bmu,kernalWidth)


    def hci2(self, sigma, a, dist):
        top = dist**2
        bottom = (2*(sigma**2))
        return a * math.exp(-top/bottom)
    def merge(self,t,ind,v):
        """
        imporved neighborhood function eliminates the need for odist function.
        """
        bmu = self.findBMU(ind,v)
        a = self.alpha(t)
        sigma = self.kernalWidth(t)
        results = self.neighborhood( bmu , sigma )
```

```
            sigma = float(sigma)
            alteredNodes = [(node,(v-self.nodes[node])*(a*math.exp(-(odist*odist)/(2*sigma*sigma)))) for
            #alteredNodes = [(node,(v-self.nodes[node])*self.hci2(sigma,a,odist)) for node,odist in results.
            for nodeID,node in alteredNodes:
                self.nodes[nodeID] += node
            # trackking diffs slows us down. a lot.
            #     self.diffs[nodeID] += abs(delta)

class Sphere(som):
    def __init__(self):
        som.__init__(self)
        # The Cache cache only works if the Neighborhood size is decreasing!
        # If you increase the maxN clear the cache!
        self.neighborhoodCache = {}
        # WHY WHY WHY WHY!!!!!
        # self.maxN = 0.5
    def clearCache(self):
        self.neighborhoodCache = {}
    def randInit(self):
        som.randInit(self)
        N = self.Size
        points = []
        for i in xrange(1,N+1):
            i = float(i)
            N = float(N)
            h = (-1)+((2*(i-1)) / (N-1))
            theta = acos(h)
```

```
            if i == 1 or i == N:
                phi = 0
            else:
                phi = (points[int(i)-2][0] + (3.6/sqrt(N)) * (1/sqrt(1-h**2)) ) % (2*pi)
            points.append((phi,theta))
        points = array([(phi-pi,theta-(pi/2)) for phi,theta in points])
        self.grid = points

        size = self.Size * self.maxN
        maxW = 0
        W = 0
        while size >= (1+(3*maxW)*(maxW-1)):
            maxW += 1
        self.maxW = maxW

    def odist(self,n):
        d = 1
        while n >= (1+(3*d)*(d-1)):
            d+=1
        return d-1
    def save(self,path,name):
        som.save(self,path,name)
        geof = open(path+name+'_geo.txt','w')
        geof.write("%d %d\n"%(self.Dims,self.Size))
        for i in xrange(self.Size):
            geof.write("%f %f\n"%(degrees(self.grid[i][0]),degrees(self.grid[i][1])))
        geof.close()
```

```
def load(self,path,name):
    som.load(self,path,name)
    if os.path.exists(path+name+'_geo.txt'):
        geof = open(path+name+'_geo.txt','r')
        header = geof.next()
        self.grid = zeros((self.Size,2),'float')
        for i in xrange(self.Size):
            geo = geof.next()
            geo = geo.split()
            geo = (math.radians(float(geo[0])),math.radians(float(geo[1])))
            self.grid[i] = geo
    else:
        pass
def sdist(self,pt1,pt2):
    phi1,theta1 = pt1
    phi2,theta2 = pt2
    dphi = phi2 - phi1
    dtheta = theta2 - theta1
    a = sin(dtheta/2)**2 + (cos(theta1) * cos(theta2) * sin(dphi/2)**2)
    c = 2 * asin(min(1,sqrt(a)))
    return c
def kernalWidth(self,t):
    r = round((self.maxW) * (1 - (t/float(self.tSteps))))
    r = int(r)
    if r == 0: r = 1
    return r
def neighborhood(self,bmu,sigma):
```

```
        NumNeighbors = (1+(3*sigma)*(sigma-1))
        try:
            return self.neighborhoodCache[bmu][:NumNeighbors]
        except:
            pt0 = self.grid[bmu]
            dists = {}
            for i in xrange(self.Size):
                dists[self.sdist(self.grid[i],pt0)] = i
            keys = dists.keys()
            keys.sort()
            pts = [dists[keys[i]] for i in xrange(NumNeighbors)]
            self.neighborhoodCache[bmu] = pts
            return pts


class ObsFile:
    def __init__(self,filename,fileType = 'complete'):
        self.filename = filename
        self.fileObj = open(filename,'r')
        self.fileType = fileType
        self.reset()
    def __iter__(self):
        return self
    def listolists(self,comments=False):
        self.fileObj.seek(0)
        lines = self.fileObj.readlines()
        dims = lines.pop(0)
        dims = int(dims)
```

```
        lines = [line.split() for line in lines]
        if not comments:
            lines = [line[:dims] for line in lines]
            lines = [array(map(float,line),'float') for line in lines]
        else:
            lines = [line[dims:] for line in lines]
        return lines
    def Snext(self):
        line = self.fileObj.next()
        id,line = line.split(':')
        line = line.split(',')
        Num = len(line)/2
        indices = empty(Num,'int16')
        values = empty(Num,'float')
        c = 0
        for n in xrange(0,Num*2,2):
            indices[c] = int(line[n])-1
            values[c] = float(line[n+1])
            c += 1
        return id,indices,values
    def Cnext(self):
        line = self.fileObj.next()
        line = line.split()
        id = self.nextLine
        for n in xrange(0,self.Dims):
            self.values[n] = float(line[n])
        self.nextLine+=1
```

```
        return id,self.indices,self.values
    def reset(self):
        self.fileObj.seek(0)
        self.nextLine = 0 #Zero Base
        if self.fileType == 'complete':
            self.next = self.Cnext
            self.Dims = int(self.fileObj.next())
            self.indices = array(range(self.Dims),'int16')
            self.values = empty(self.Dims,'float')
        elif self.fileType == 'sparse':
            self.next = self.Snext
        else:
            raise "fileTypeError"
    def stream(self):
        try:
            return self.next()
        except:
            self.reset()
            return self.next()
    def close(self):
        self.fileObj.close()
```