R2: Boosting Liquidity in PCNs
with Online Admission Control

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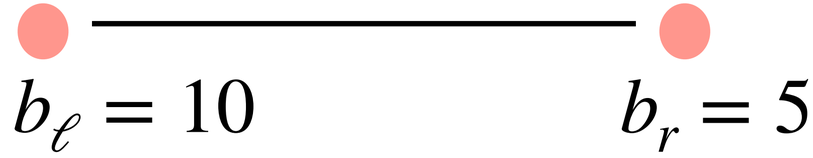
Payment Channel Networks

- Payment Channel Networks (PCNs) improve scalability of blockchains

Initialisation:



Blockchain

Payment Channel Networks

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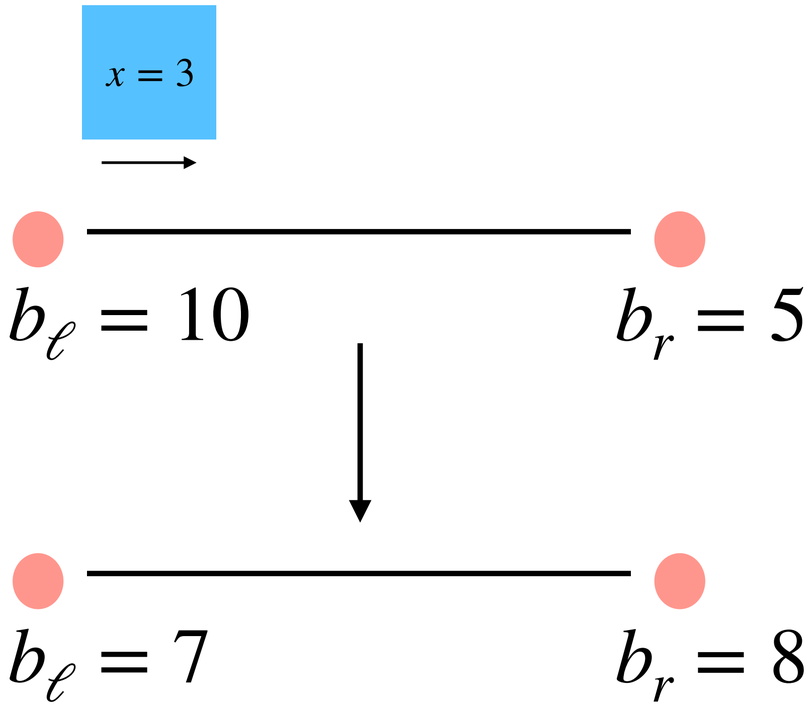

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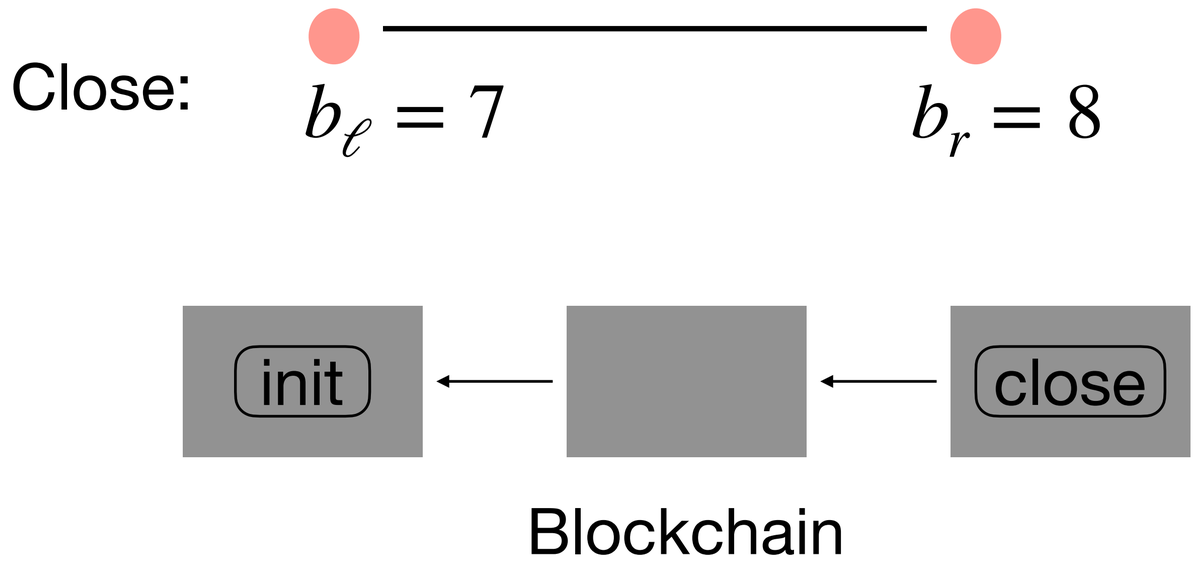
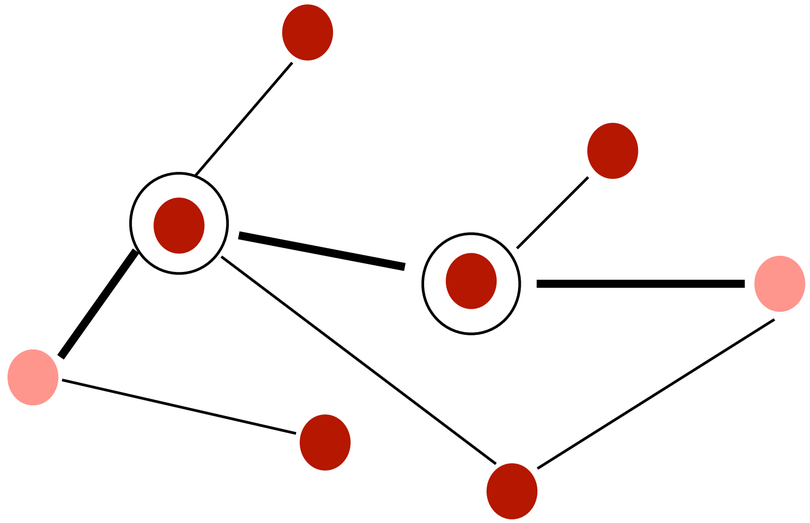


Payment Channel Networks

- Payment Channel Networks (PCNs) improve scalability of blockchains
- Intermediary nodes charge fees to forward payments
- Need to balance greedily forwarding payments and channel depletion



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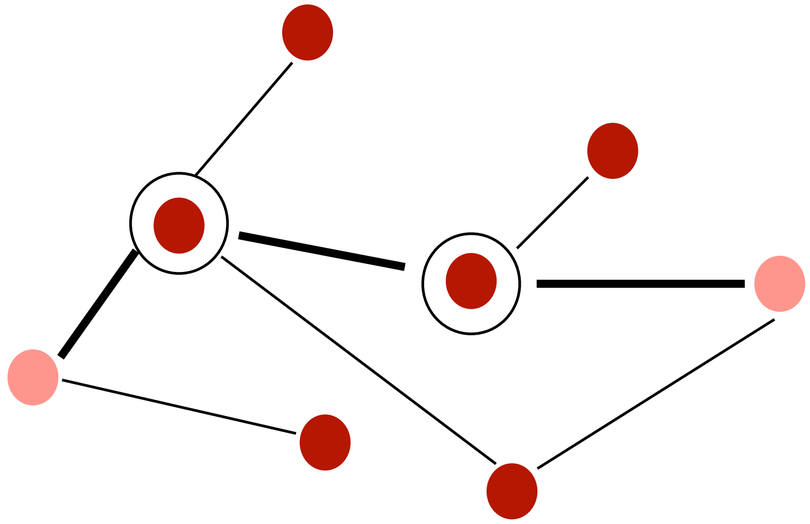

Can we design efficient algorithms to handle transactions as well as channel depletion?

Related Work

Transaction Handling

Online payment network design, Avarikioti et al. 2019

Channel depletion

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This work: transaction handling and channel depletion

Problem setting

- Online decision making
- Input: transaction sequence $X_t = \{x_1, \ldots, x_t\}, x_i \in \mathbb{R}^+$
- **sequential** access to the transaction sequence

Action Space

- Actions:
  1. accept transaction
  2. reject transaction
  3. recharge channel
  4. rebalance channel

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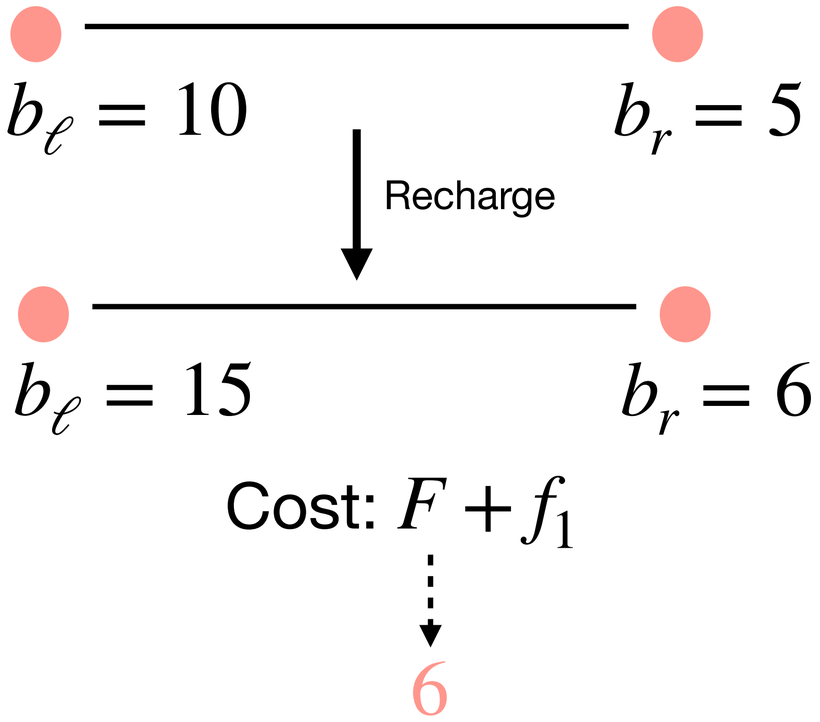

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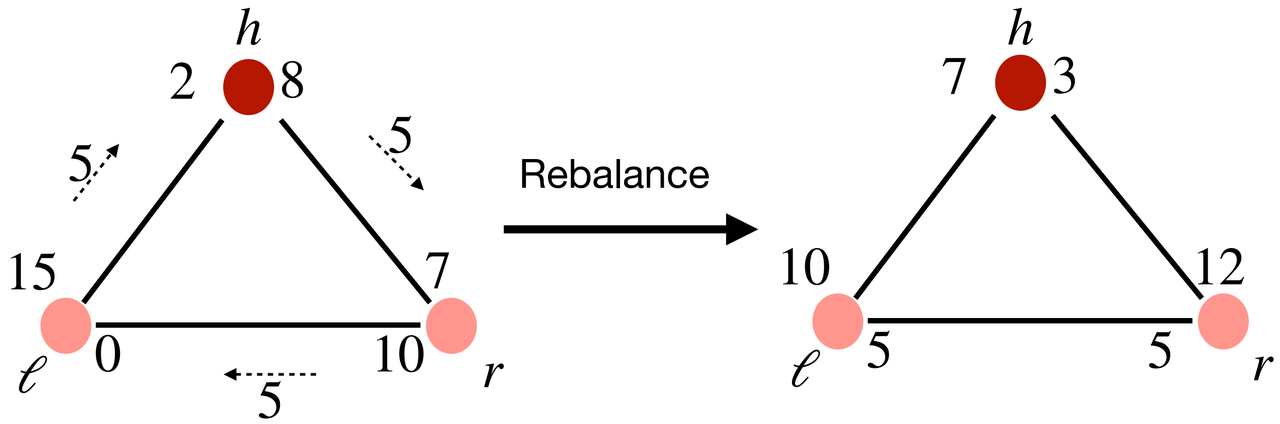

Cost: $C(Rx + f_2)$

Competitive Ratio

- Optimal offline solution (OFF): access to **entire** transaction sequence in advance
- Competitive ratio: online algorithm (ON) is c-competitive compared to OFF if for every transaction sequence X_t,

$$COST_{ON}(X_t) \leq c \cdot COST_{OFF}(X_t)$$

- Deterministic algorithms

Warm Up 1

- Unidirectional transaction stream (wlog left to right)
- ℓ can only **accept** transactions, and **recharge** channel when channel is depleted

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Theorem 1

There exists an optimal 2-competitive online algorithm

Warm Up 1

- Observation: OFF knows entire sequence so will recharge only once at the start

$$COST_{OFF}(X_t) = \sum_{i=1}^{t} x_i + f_1$$

- Assume ON has access to an oracle Funds(X_i) that returns the amount of funds OFF uses in order to process sequence X_i[1]

- Tracking algorithm sequentially processes X_t calls Funds(X_i) for each prefix $X_i \subset X_t$, recharges when Funds(X_i) goes above some threshold

[1]In general this is NP-hard, but can be efficiently approximated to constant factor, see https://arxiv.org/pdf/2204.13459.pdf

Warm Up 2

- Unidirectional transaction stream (wlog left to right)
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Theorem 2

There exists a $\left(2 + \frac{\sqrt{5}-1}{2}\right)$-competitive online algorithm

Warm Up 2

Big: $x > Rx + f_2$

Small: $x \leq Rx + f_2$

Warm Up 2

- Observations:

    1. OFF rejects big transactions

    2. If there are sufficiently many small transactions to offset the cost of recharging, OFF performs a single recharging action at the beginning of size

$$\sum_{x \in X_t, x \text{ small}} x$$

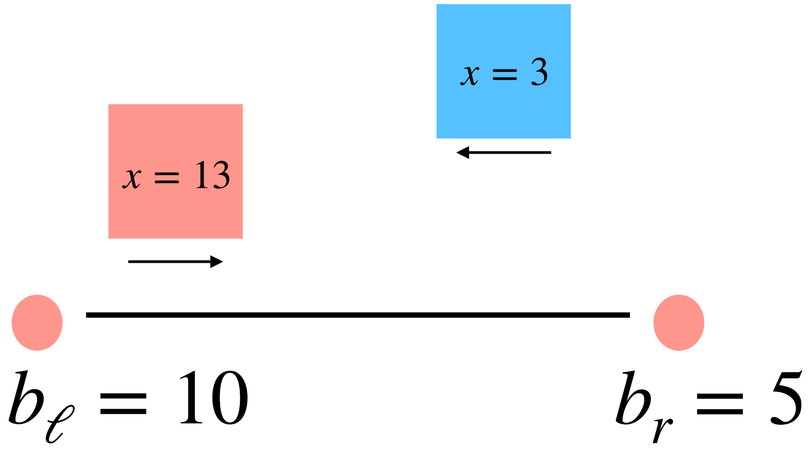
- ON similarly rejects big transactions, performs tracking to recharge channel

Big: $x > Rx + f_2$

Small: $x \leq Rx + f_2$

Main Problem

- Bidirectional transaction stream
- ℓ and r can **accept** and **reject** transactions, and **recharge** or **rebalance** their channel when the channel is depleted
- Want to minimise the cost over the **entire** channel (cost of **both** users)
- Real-life example: 2 routers collaborating to minimise total cost over their channel



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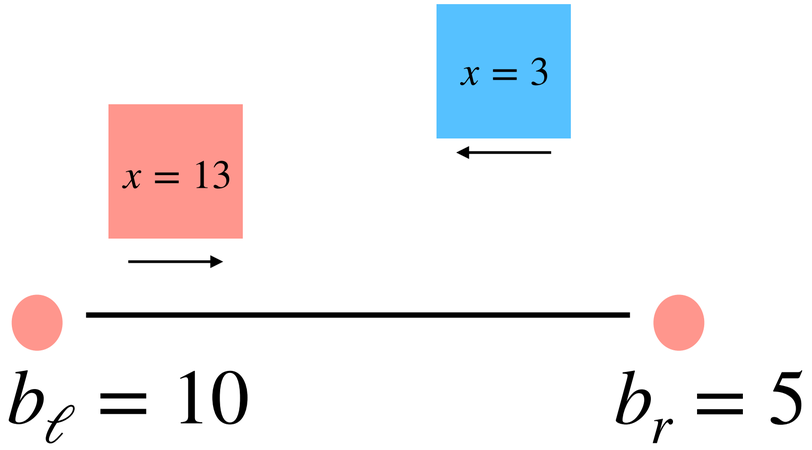
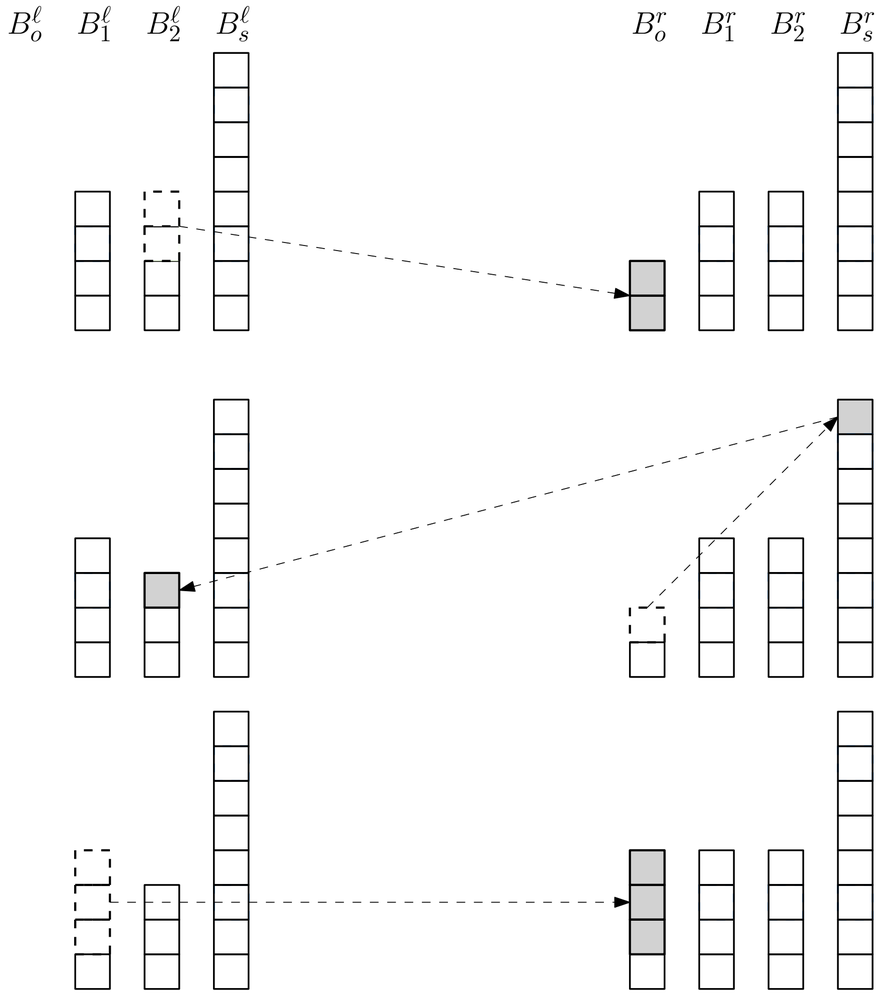


Theorem 3

There exists a $(7 + 2\lceil \log C \rceil)$-competitive online algorithm

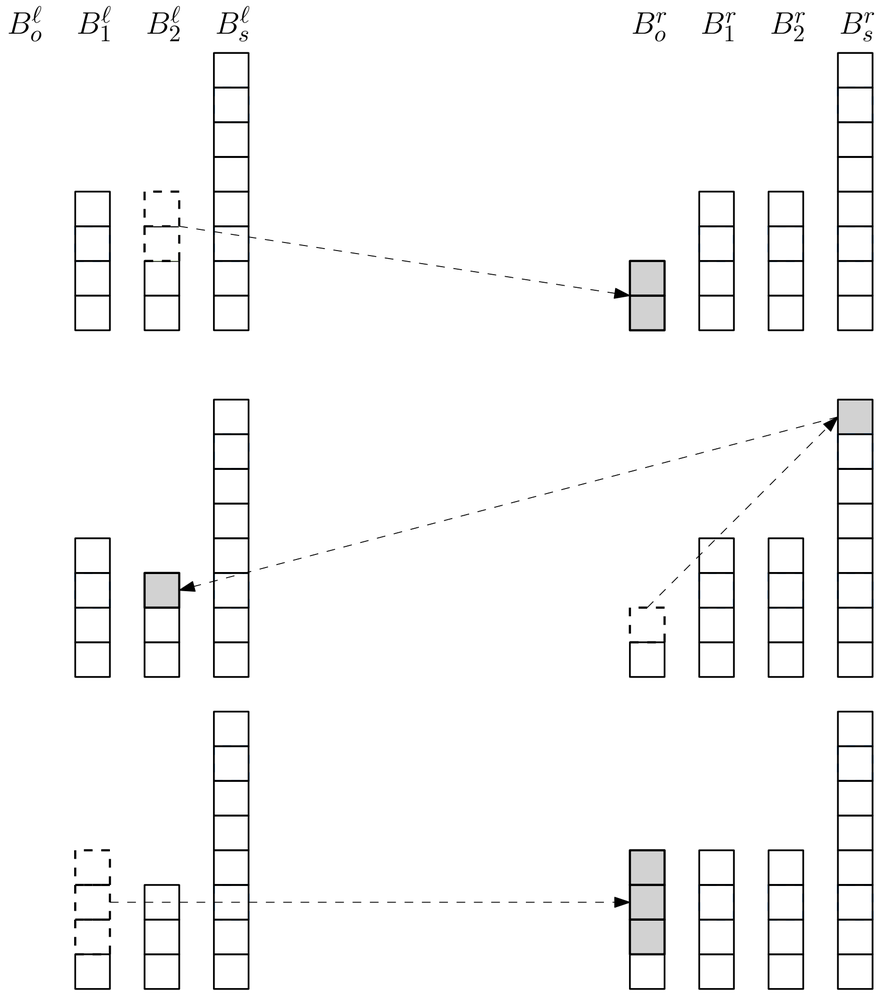
Main Algorithm

- Similar tracking algorithm to track funds of OFF in the entire channel
  - Recharges the channel whenever Funds(X_i) goes above a threshold



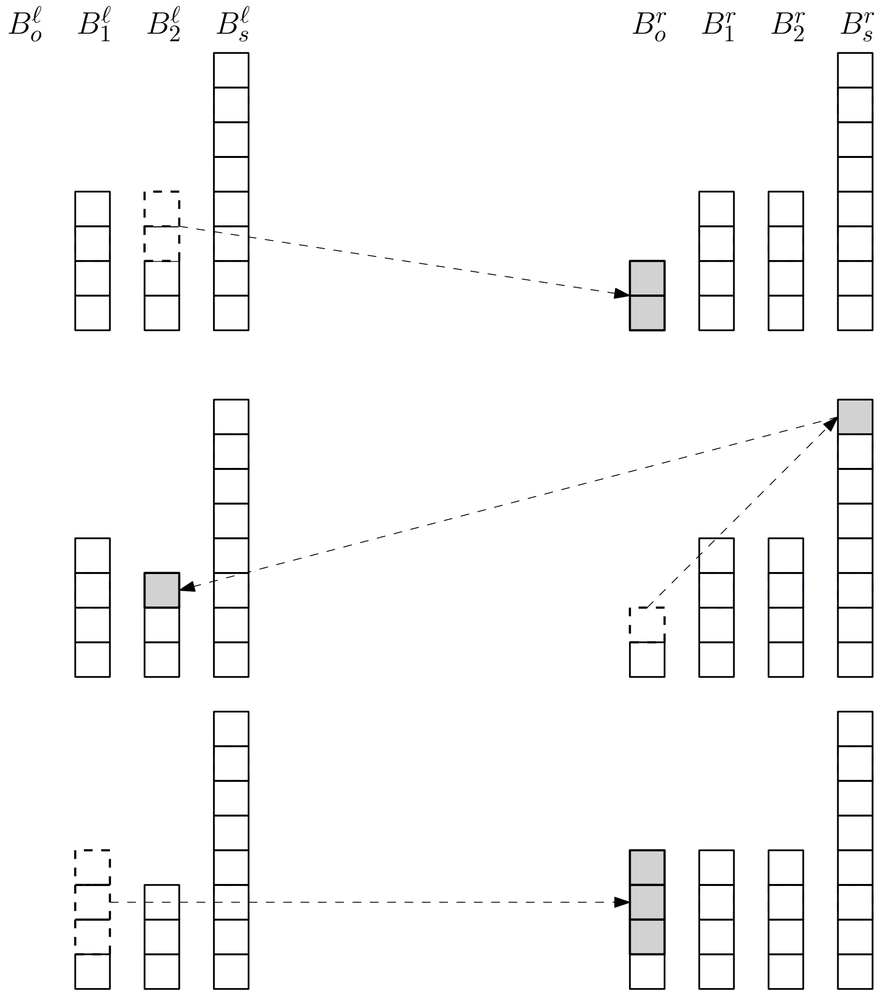
Main Algorithm

- Funds on each end of the channel divided into buckets
  - Each fund bucket is used to handle transactions of a certain size
  - If funds in a given bucket is insufficient, transaction is either rejected or off-chain rebalancing is performed



Main Algorithm

- Incoming funds are distributed to prioritise handling smaller transaction sizes
  - Rejecting smaller-sized transactions costs more



Simulations

- Compare average performance of main online algorithm on randomly generated transaction sequences (50 sequences of length 50) to our theoretical worst case bounds

Params		**OFF**	**ON**	
C	f_2	Cost	Cost	$(7+2\lceil\log(C)\rceil)OFF$
2	0.5	15.02	**63.3**	135.18
8	0.5	15.21	**87.79**	197.73
2	2	23.6	**127.02**	212.4
8	2	24.5	**184.32**	318.5

Conclusion and Future Directions

- Study the problem of online transaction handling and channel depletion in the general setting
- Provide algorithms with worst-case guarantees
- Simulations show significantly better average case performance
- Future work:
  - Consider randomised algorithms
  - Incorporate fees into the decision problem

Questions?
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