

Misleading stars: What cannot be measured on the Internet ?

Yvonne-Anne Pignolet, Stefan Schmid, Gilles Trédan





How accurate are network maps ?

Why ?

- To develop/adapt protocols to Internet *PaDIS*, *RMTP*
- To understand the impact of uncertainty: *networks metrology* ?

How ?

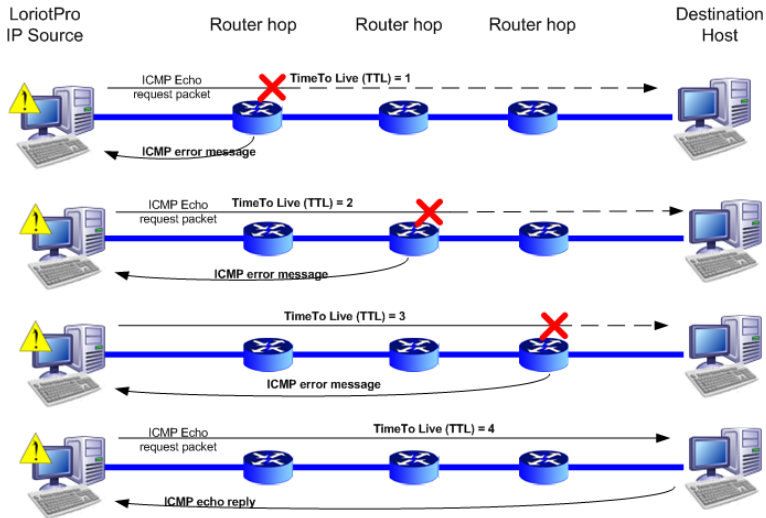
- multicast [*Marchetta* et al. , *JSAC'11*]
- network tomography
- **Traceroute**

Takeaway

- Compare internet topologies instead of counting them
- Local properties suffer
- Global properties suffer less

Traceroute principle

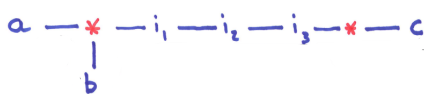
Idea= send « buggy » TTL packets, collect error messages



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Traceroute Limitations

- **Sampling bias**: Impact of sources location
- **Aliasing**: How to map IPs to the equipment
- **Load-Balancing**: Traceroute assumes all packets follow the same path...
- **Stars**: Disabled/Filtered ICMP messages



Trace:
 $t_1 = a, \star_1, b$
 $\rightarrow t_2 = b, \star_2, i_1, i_2, i_3, \star_3, c$

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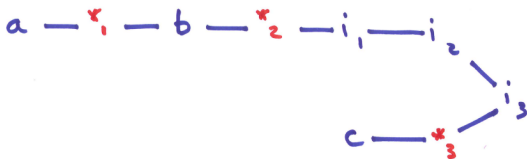
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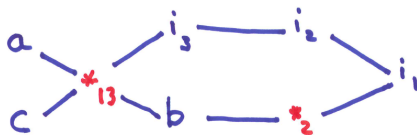
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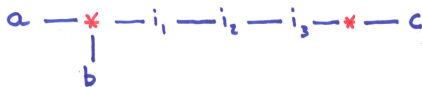
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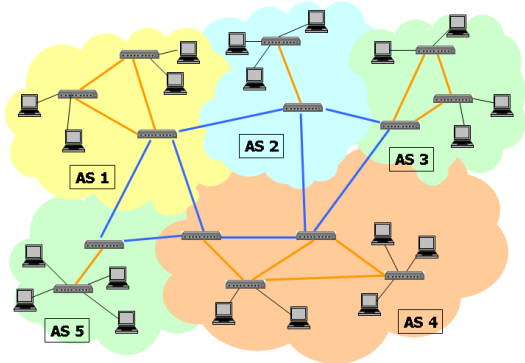


Works initiated by Acharya and Gouda
[SSS09, ICDCN10, ICDCN11]

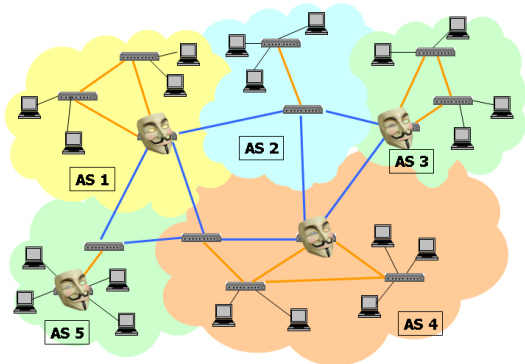
- Models to capture *irregular* nodes
- Irregular nodes \rightarrow anonymous nodes
- Central concept of *minimal* topologies
- Counting the number of generable topologies

Our approach: Lots of generable topologies is not a problem if they
are all **similar**

- $G_0(V_0, E_0)$: static undirected graph = **Target Topology**.
- $v \in V_0$ is
 - either **named**: always answers with its only name (no aliasing)
 - either **anonymous**: always answers \star .
- Not necessarily minimal!
- $d_{G_0}(u, v)$ is the shortest path distance.



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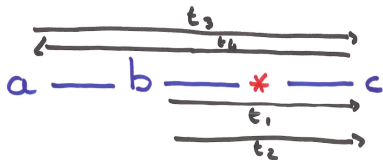


We don't know G_0 , but we know a set of traces \mathcal{T} of G_0 .

- Anonymous nodes appear as stars (\star) in \mathcal{T}
- Each star has a unique number ($\star_i, i = 1..s$) in \mathcal{T}
- $d_{\mathcal{T}}(u, v)$ is the number of symbols in $T \in \mathcal{T}$ between u and v .
- No Assumption on the number of traces, on path uniqueness nor symmetry.



$t_1 = a, \star_1, b$
 $t_2 = b, \star_2, i_1, i_2, i_3, \star_3, c$

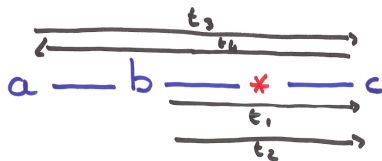


$t_1 = b, \star_1, c$
 $t_2 = b, \star_2, c$
 $t_3 = a, b, \star_3, c$
 $t_4 = c, \star_4, b, a$

Complete cover: Each edge of G_O appears at least once in some trace to \mathcal{T}

Reality sampling: For every trace $T \in \mathcal{T}$, if the distance between two symbols $\sigma_1, \sigma_2 \in T$ is $d_T(\sigma_1, \sigma_2) = k$, then there exists a path (i.e., a walk without cycles) of length k connecting two (named or anonymous) nodes σ_1 and σ_2 in G_0 .

No assumption on coverage



~~a, b, c~~

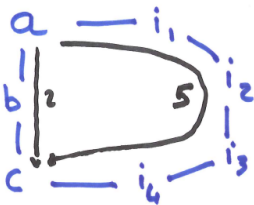
~~a, c, b~~

~~a, *, c~~

~~a, d, c~~

α -(Routing) Consistency: There exists an $\alpha \in (0, 1]$ such that, for every trace $T \in \mathcal{T}$, if $d_T(\sigma_1, \sigma_2) = k$ for two entries σ_1, σ_2 in trace T , then the shortest path connecting the two (named or anonymous) nodes corresponding to σ_1 and σ_2 in G_0 has distance at least $\lceil \alpha k \rceil$.

Note: $\alpha > 0 \Leftrightarrow$ **loop-less** routing



a, b, c

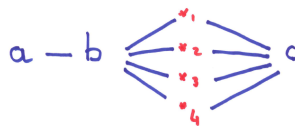
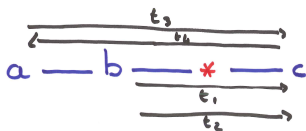
$a, i_1, i_3, i_4, c \Rightarrow \alpha \leq \frac{2}{5}$

Some more definitions

A topology G is α -consistently **inferred** from \mathcal{T} if it respects the 3 previous rules.

Let $\mathcal{G}_{\mathcal{T}} = \{G, \text{ s.t. } G \text{ is inferred from } \mathcal{T}\}$ We study the **properties** of the set $\mathcal{G}_{\mathcal{T}}$ of inferred topologies.

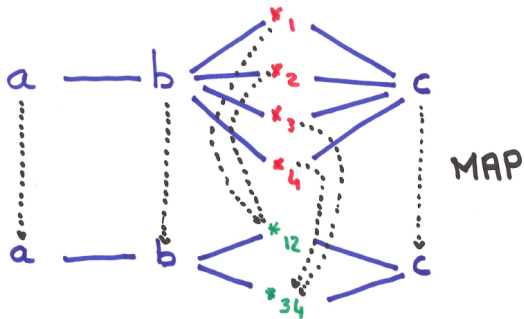
We define the **canonic graph** G_c as the straightforward graph that treats each star as unique.



How to infer topologies ?

1 inferrable topology = 1 mapping of stars to anonymous routers.

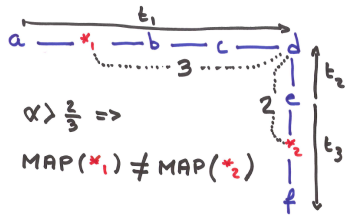
Let Map be such function.



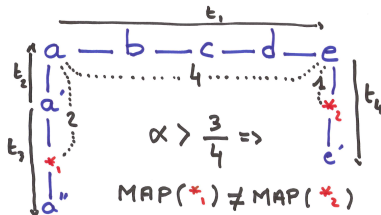
How to infer topologies -2

- G_c is inferrable. $\text{Map} = \text{Id}$.

- (i) if $\star_1 \in T_1$ and $\star_2 \in T_2$, and
 $[\alpha \cdot d_{T_1}(\star_1, u)] > d_C(u, \star_2) \Rightarrow$
 $\text{Map}(\star_1) \neq \text{Map}(\star_2)$.



- (ii) $\star_1 \in T_1$ $\star_2 \in T_2$, and $\exists T$ s.t.
 $[\alpha \cdot d_T(u, v)] > d_C(u, \star_1) +$
 $d_C(v, \star_2) \Rightarrow \text{Map}(\star_1) \neq \text{Map}(\star_2)$.



Algorithm constructive part

We construct the *Star Graph* $G_*(V_*, E_*)$:

- Vertices=stars in the trace
- Edges=if stars cannot be merged:
 $(\star_1, \star_2) \in E_* \Leftrightarrow \text{Map}(\star_1) \neq \text{Map}(\star_2).$

1 proper coloring of $G_* \leftrightarrow$ 1 Map function

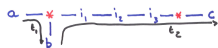
- minimal coloring \rightarrow minimal topology
- 'maximal' coloring $\rightarrow G_c$

$$\sum_{k=\gamma(G_*)}^{|V_*|} P(G_*, k)/k! \geq |\mathcal{G}_T|,$$

$\gamma(G_*)$ = chromatic number of G_*

$P(G_*, k)$ = *chromatic polynomial* of G_* .

Star Graph /2



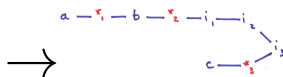
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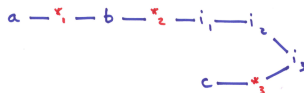
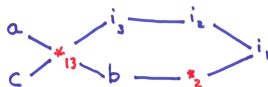
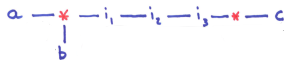
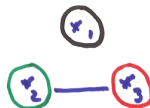
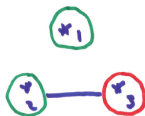
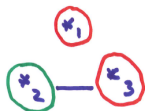
$$t_2 = b, \star_2, i_1, i_2, i_3, \star_3, c$$

$$\alpha = 0.5$$

G_c :



G_* Colorings



Results are **bad!**

- Connected components**

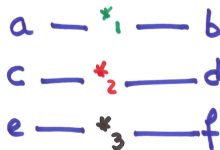
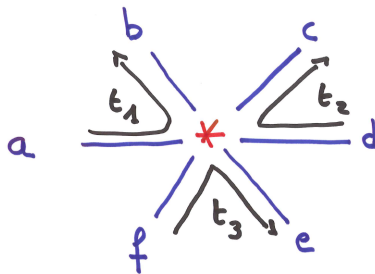
No assumption on « coverage » \Rightarrow
Stars disconnect the graph !

$$|cc(G_1)/cc(G_2)| \leq \frac{n}{2}$$

- Stretch**

Even if we only consider connected topologies.

$$|stretch(G)| \leq \frac{n+s-1}{2}$$

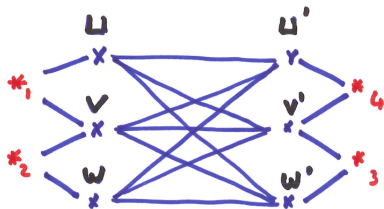


It's worse !

- Triangles**

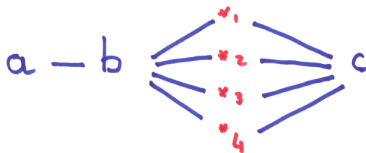
Bipartite complete graph worst case

$$|C_3(G_1)/C_3(G_2)| \leq \infty$$



- Degree**

Worst case is on anonymous nodes $|\text{DEG}(G_1) - \text{DEG}(G_2)| \leq 2(s - \gamma(G_*))$



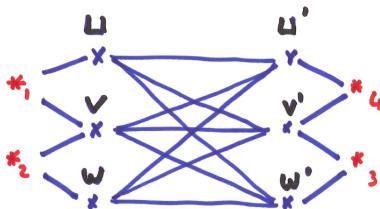
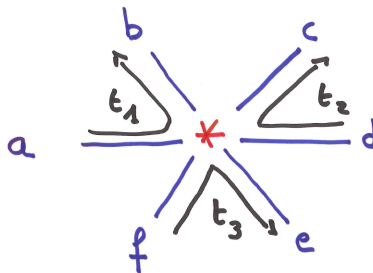
Best case: Fully explored topologies

Strong assumptions:

- $\alpha = 1$: shortest path routing
- $\forall u, v \in V_0, \exists T \in \mathcal{T}$ such that $u \in T \vee v \in T$
- We don't see any stronger case

Results:

- Global properties conserved
- Local properties: still bad!



Overall Results

Absolute difference: $G_1 - G_2$

Property	Arbitrary	Fully Explored ($\alpha = 1$)
# of nodes	$\leq s - \gamma(G_*)$	$\leq s - \gamma(G_*)$
# of links	$\leq 2(s - \gamma(G_*))$	$\leq 2(s - \gamma(G_*))$
# of CC	$\leq n/2$	$= 0$
Diameter	$\leq (s - 1)/s \cdot (N - 1)$	$s/2$ (¶)
Max. Deg.	$\leq 2(s - \gamma(G_*))$	$\leq 2(s - \gamma(G_*))$
Triangles	$\leq 2s(s - 1)$	$\leq 2s(s - 1)/2$

Relative difference: G_1/G_2

# of nodes	$\leq (n + s)/(n + \gamma(G_*))$	$\leq (n + s)/(n + \gamma(G_*))$
# of links	$\leq (\nu + 2s)/(\nu + 2)$	$\leq (\nu + 2s)/(\nu + 2)$
# of CC	$\leq n/2$	$= 1$
Stretch	$\leq (N - 1)/2$	$= 1$
Diameter	$\leq s$	2
Max. Deg.	$\leq s - \gamma(G_*) + 1$	$\leq s - \gamma(G_*) + 1$
Triangles	∞	∞

Conclusion

- Constructive proofs
- Complex algorithm
- Don't count, compare !
- Huge dissimilarities
- Worst case approach

A practical part:

- compare with reality
- develop property estimation algorithms



Thanks!