Weighted Packet Selection for Rechargeable Links

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Motivation

Problem

Sending transactions in blockchain is expensive (consensus, security).

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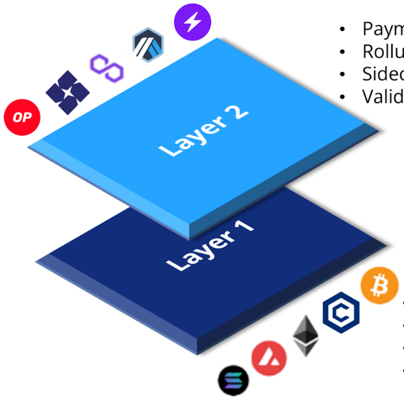
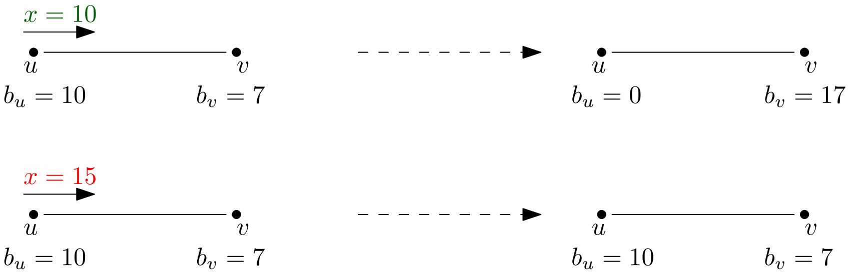
Methods are either Layer 1 or Layer 2 according to their focus (i.e., On-Chain or Off-Chain)



- Payment Channels
- Rollups
- Sidechains
- Validium

- Block Reparameterisation
- Sharding
- Consensus Mechanism
- DAG

Payment channels

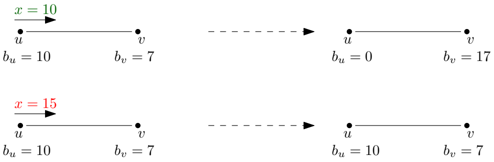


Goal

Create efficient algorithm for a link (two nodes) in PCN that cooperate, but do not trust each other.

Actions for link in PCNs

Action	Capacity/Size	Cost
Create link	c	c
Forward transaction	x	0^*
Reject transaction	x	$fx + m$



Definitions

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Channel creation + Cost for rejection.

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Solution
Initial capacity of the channel and which transactions to reject.

Outline of the talk

NP-hardness

Algorithm

- Linear program
- Approximating the channel capacity
- Tracking the linear program

Hardness

Problem is NP-hard

For set of numbers $x_1, x_2, \ldots$ and X, subset sum problem, we create transactions $(\rightarrow, x_1), (\rightarrow, x_2), \ldots, (\leftarrow, X)$. Problem is yes instance if the optimal solution has small cost.

Linear program

Constants: Fixed capacity M, Input $(\leftarrow, x_i)$.

Variables:

- amount of i-th transaction accepted y_i.
- balance on the left (right) after processing i-th transaction $S_{L,i}$ ($S_{R,i}$).

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Variables:

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$$
\begin{aligned}
\text{minimise} \quad & \sum_i f \cdot (x_i - y_i) + m\frac{x_i - y_i}{x_i} \\
\text{subject to} \quad & \forall i : y_i, S_{L,i}, S_{R,i} \geq 0 \\
& \forall i : y_i \leq x_i \\
& \forall i : S_{L,i} + S_{R,i} = M \\
\forall x_i \in \rightarrow &: S_{L,i} = S_{L,i-1} - y_i \\
\forall x_i \in \rightarrow &: S_{R,i} = S_{R,i-1} + y_i \\
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$$

Optimal capacity

If optimal capacity is M', we have

$$M' + \mathit{reject}_{M'}$$

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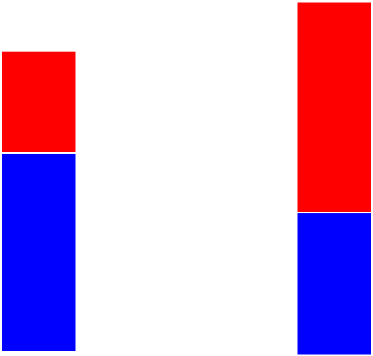
We try all capacities M of the form $(1+\varepsilon)^k$.

$$\min \left(\frac{M}{1+\varepsilon} + \mathit{reject}_M \right)$$

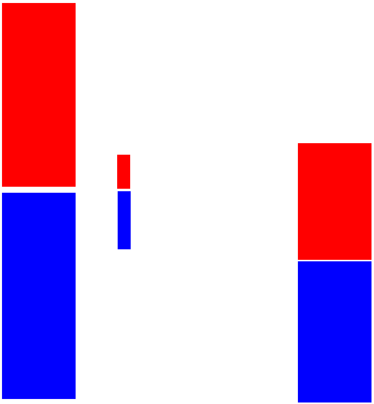
Accepting fully accepted transaction

Transaction where $y_i = x_i$ is *fully accepted*.

We track S_L and S_R, we add reserves R_L and R_R, such that $R_L + R_R = M$.



Accepting fully accepted transactions



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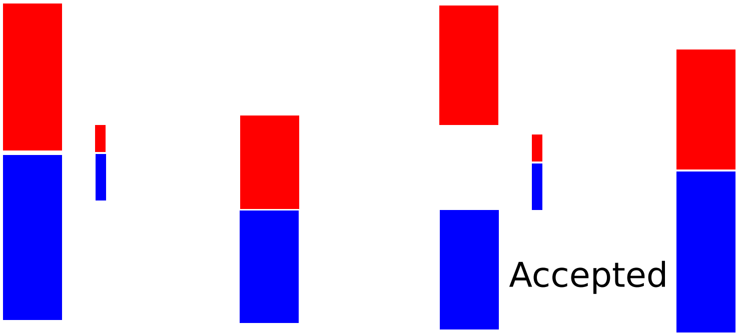

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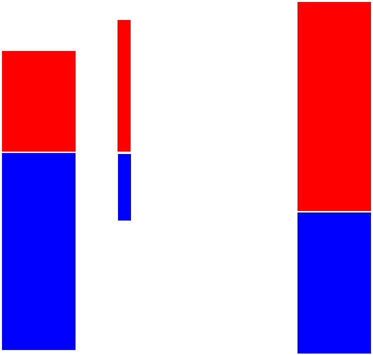

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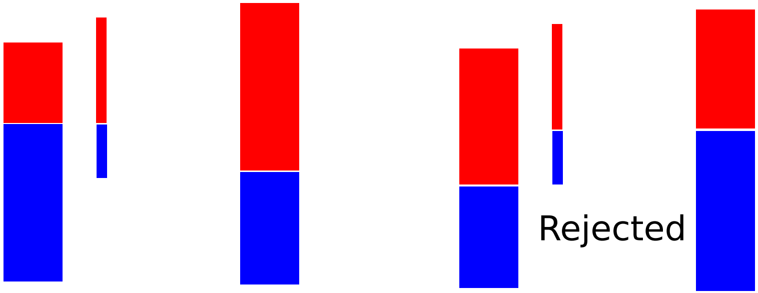

$R_L < x_i - y_i$

$R_R \geq y_i$

cannot accept

$x_i < M; R_L + R_R = M$

Main idea of the algorithm

Transactions are **almost-accepted** if

$$\frac{y_i}{x_i} \geq \frac{\sqrt{3}}{\sqrt{3}+1}\,.$$

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Summary

Problem definition

Hardness

$(1+\sqrt{3})(1+\varepsilon)$-approximation algorithm