Demand-Aware Small-World Networks on Clustered Demands

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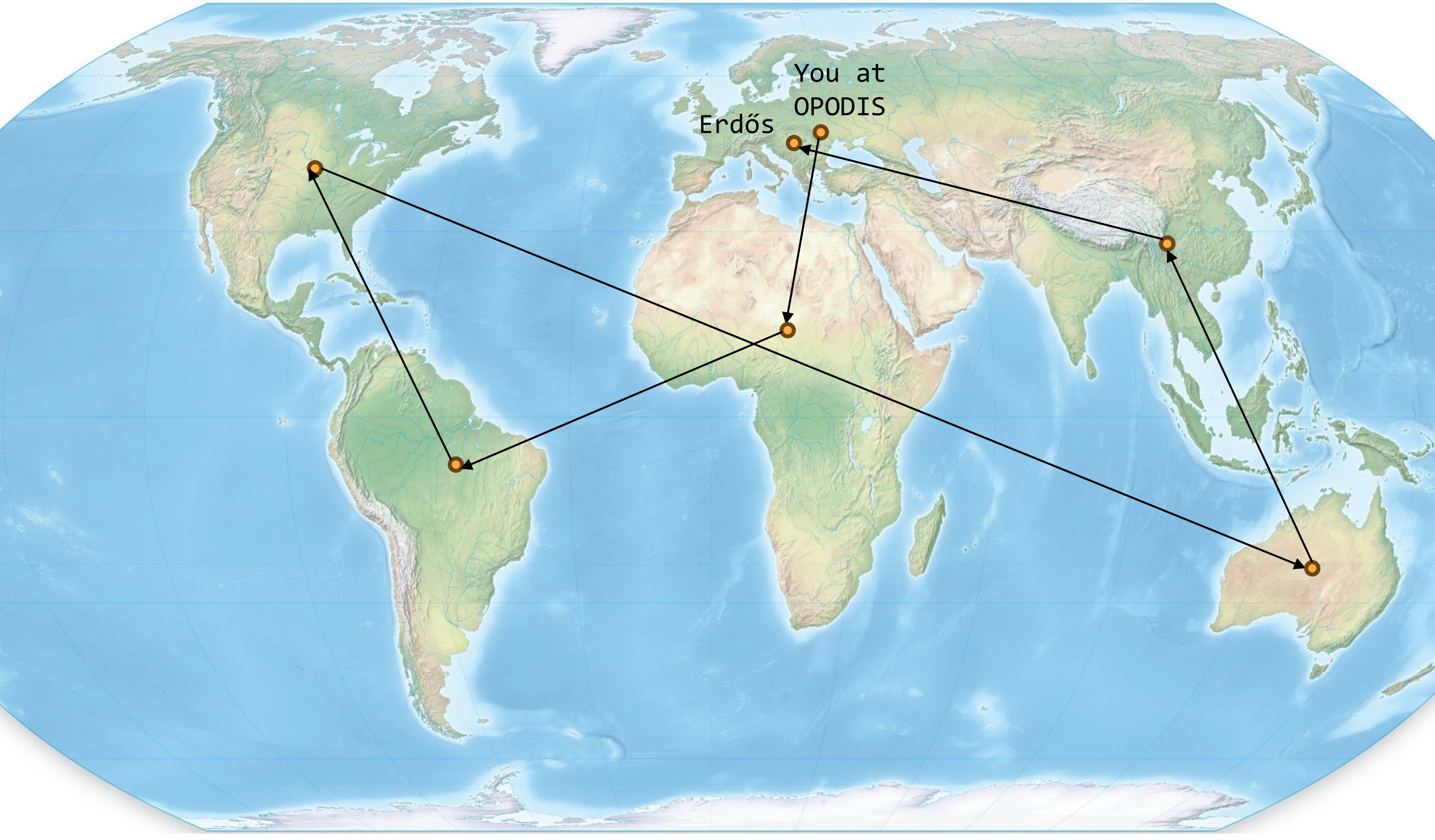
Acknowledgements:

Context: “Small World Phenomenon”

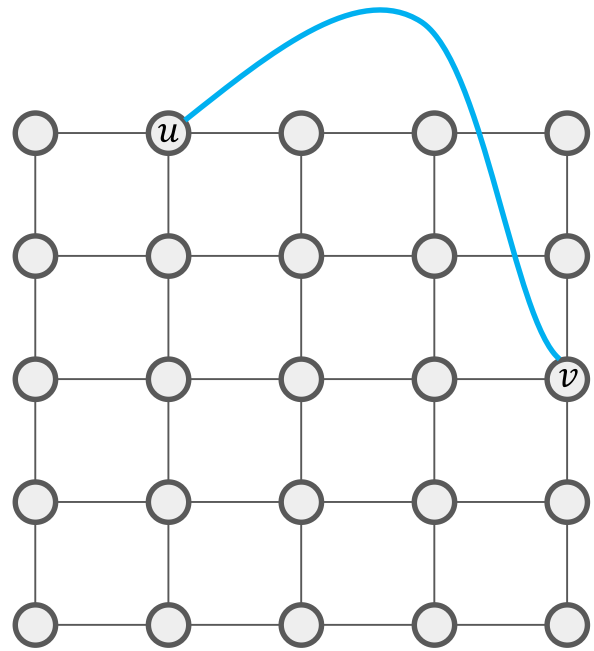
Six Degrees of Separation



Popular Model

- Grid + long-range contact
- Kleinberg [STOC '00]: Certain powerlaw gives polylog routes which can even be *found*!
- Also inspired distributed system designs, e.g., p2p networks
- Works for "any demand" (all-to-all)

Add edge with probability $\sim \mathrm{d}(u,v)^{-\alpha}$



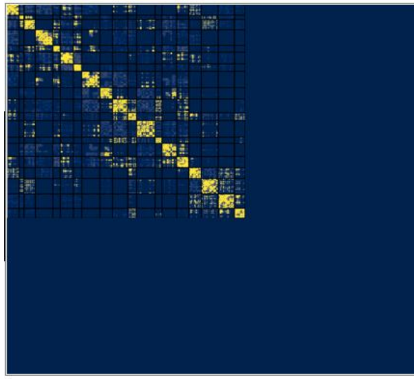
However, in practice:

Demand more structured

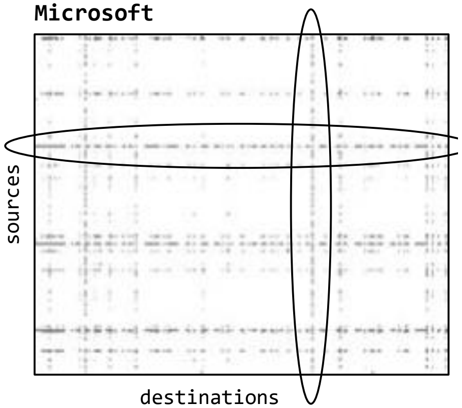
Empirical studies:

traffic matrices sparse and skewed

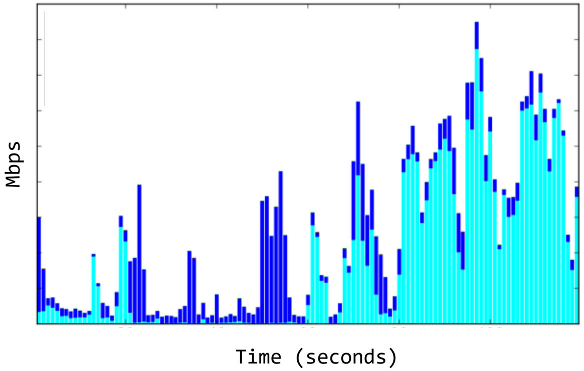
Facebook



Microsoft

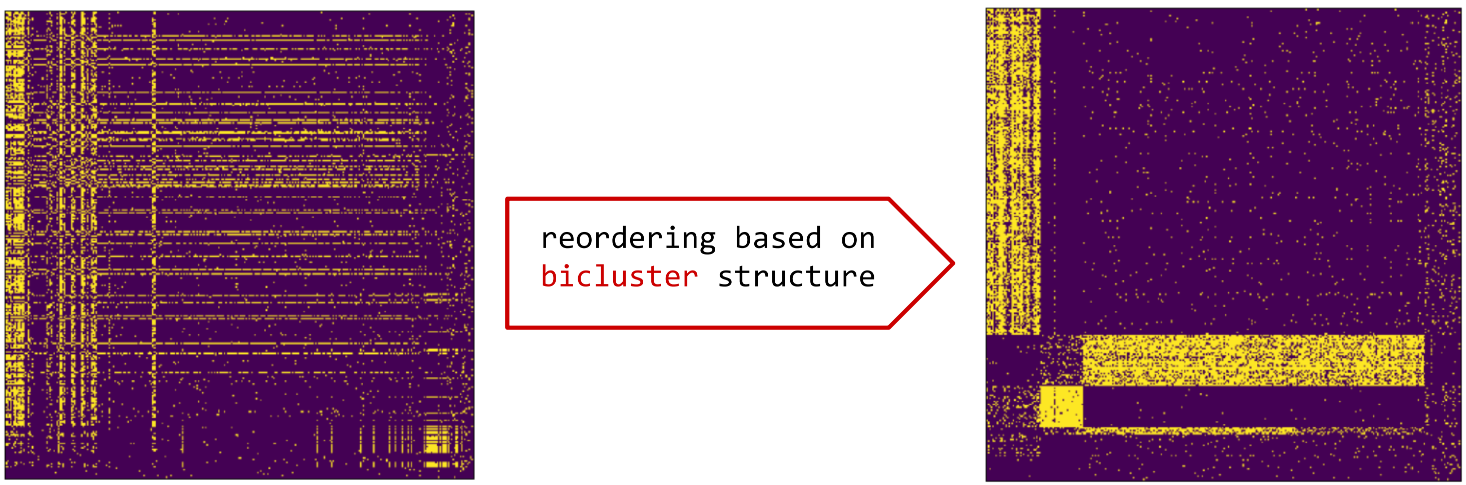


Facebook



Traffic is also clustered: bi-clustering results

Small Stable Clusters



Förster et al., Analyzing the Communication Clusters
in Datacenters. WWW 2023

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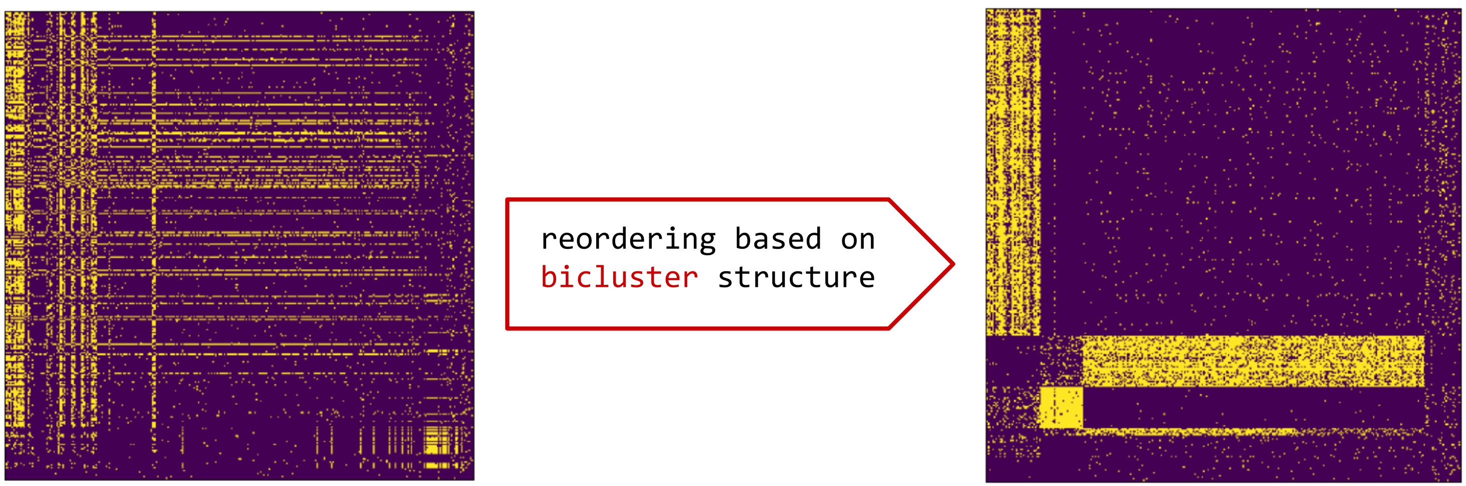
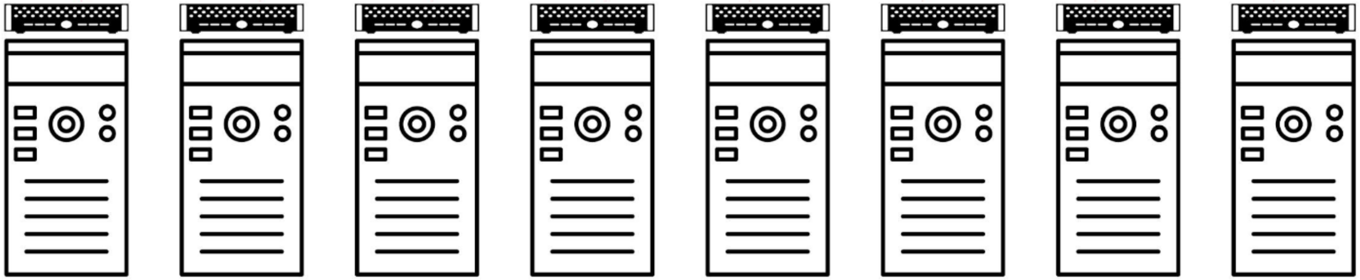


Can we exploit this?

Förster et al., Analyzing the Communication Clusters in Datacenters. WWW 2023

Trend:

Demand-Aware Graphs



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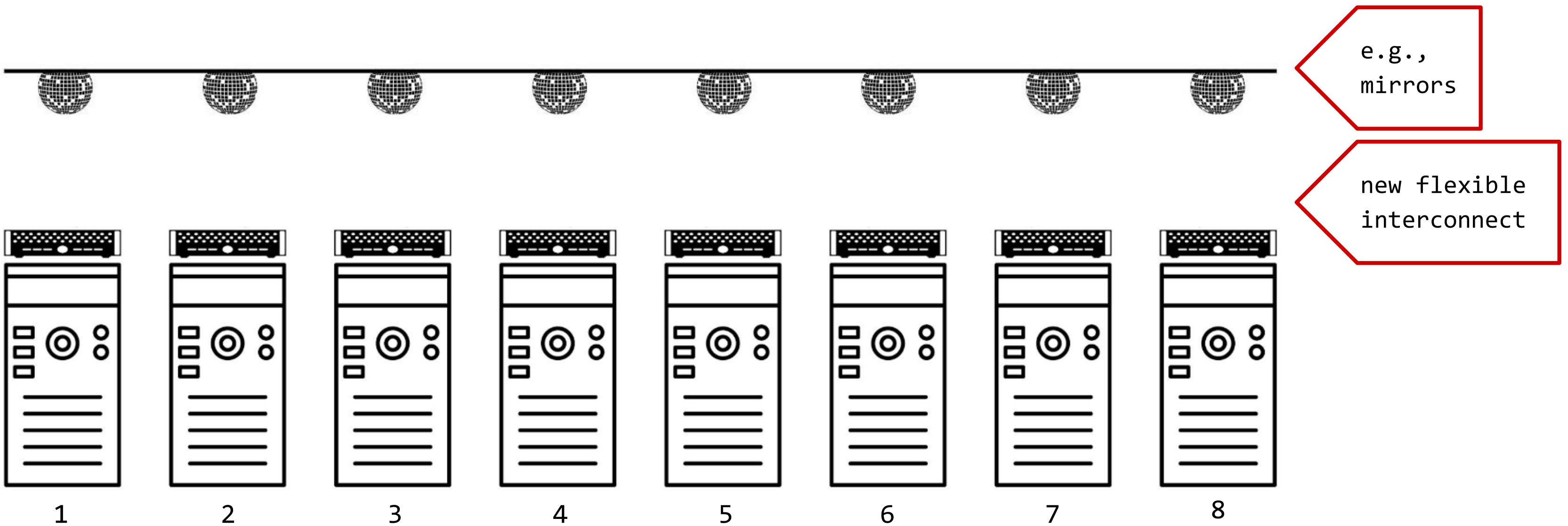

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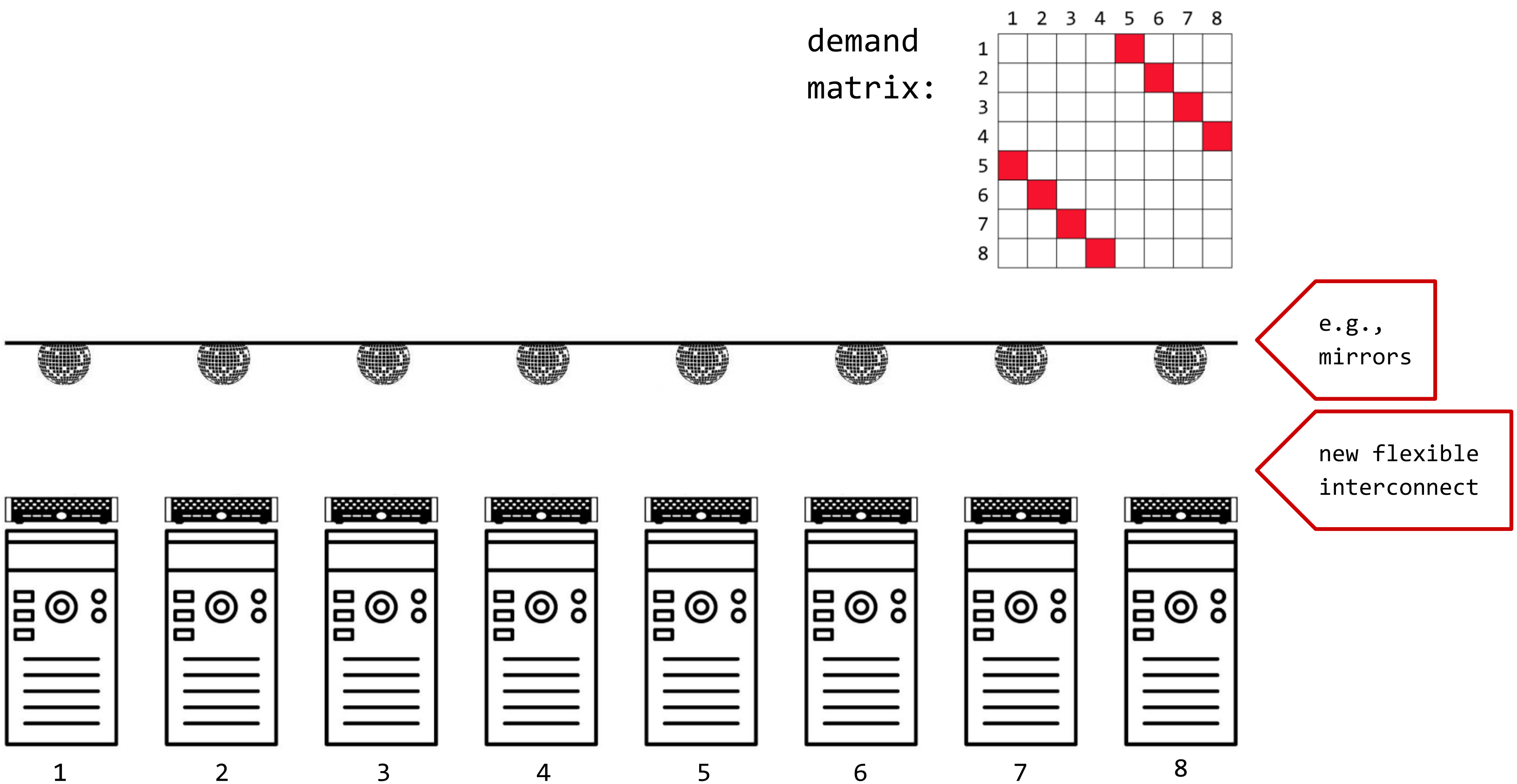
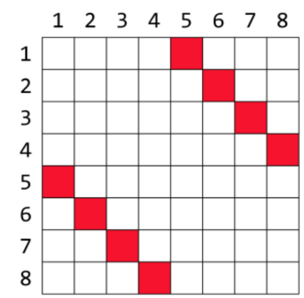


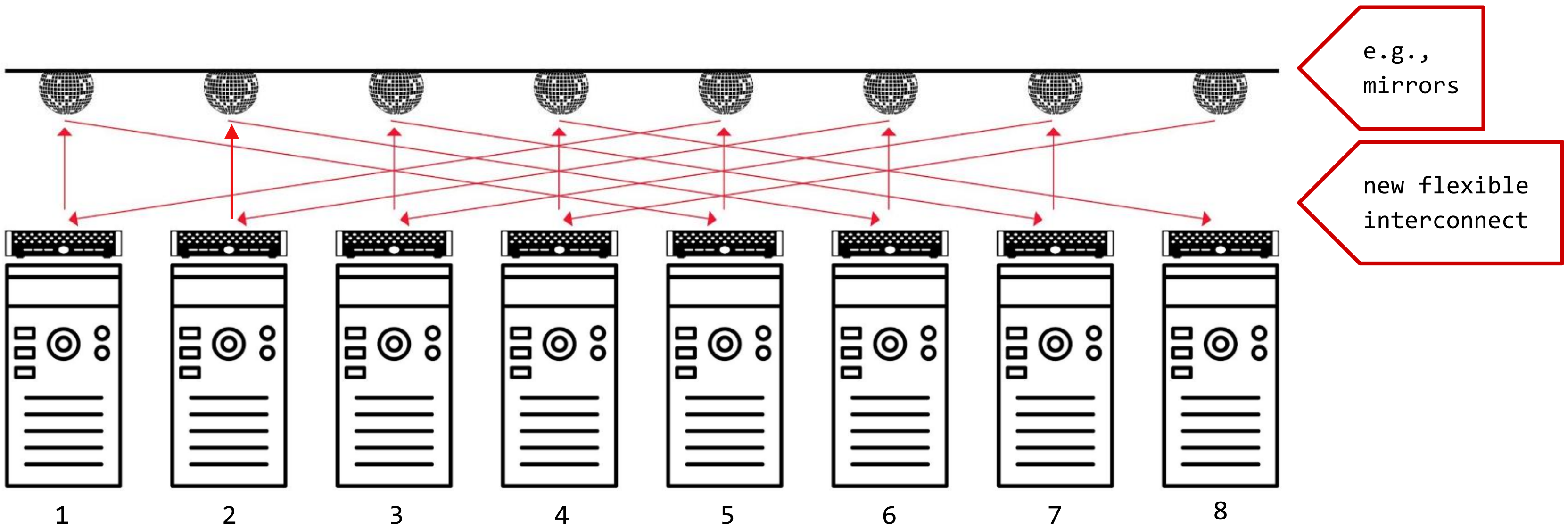
Trend:

Demand-Aware Graphs

Matches demand

demand matrix:





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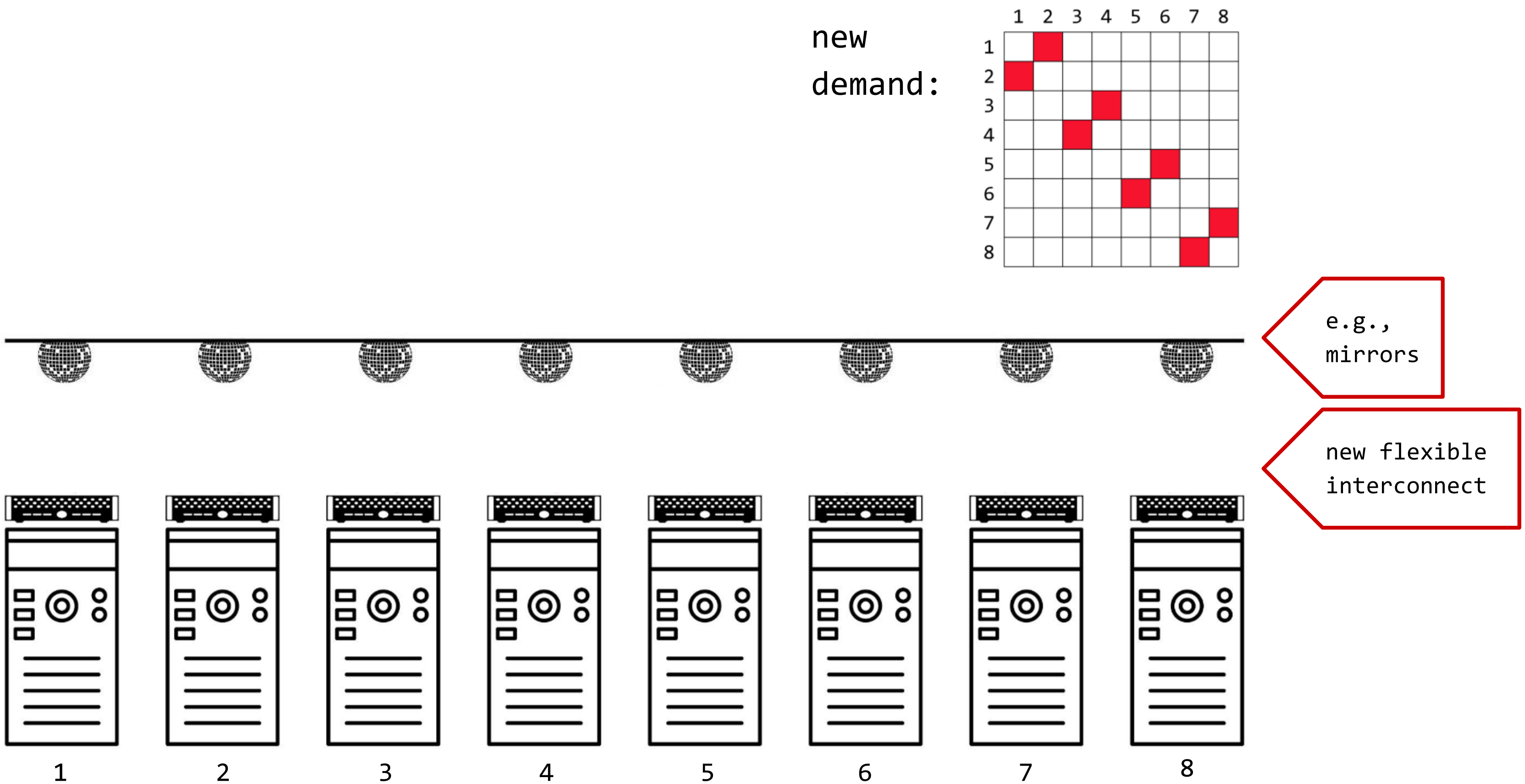
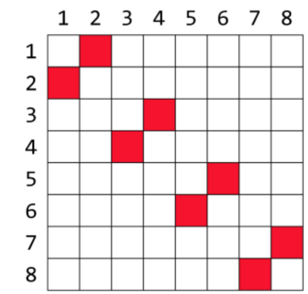


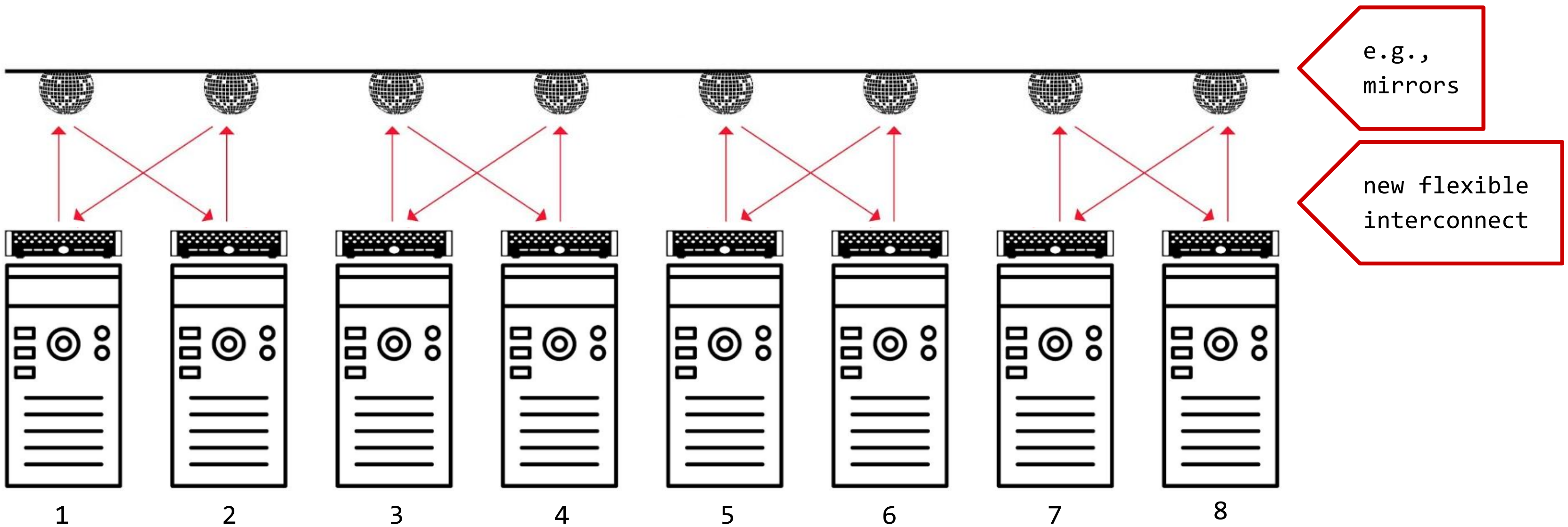
Trend:

Demand-Aware Graphs

Matches demand

new
demand:



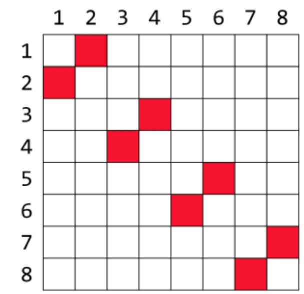


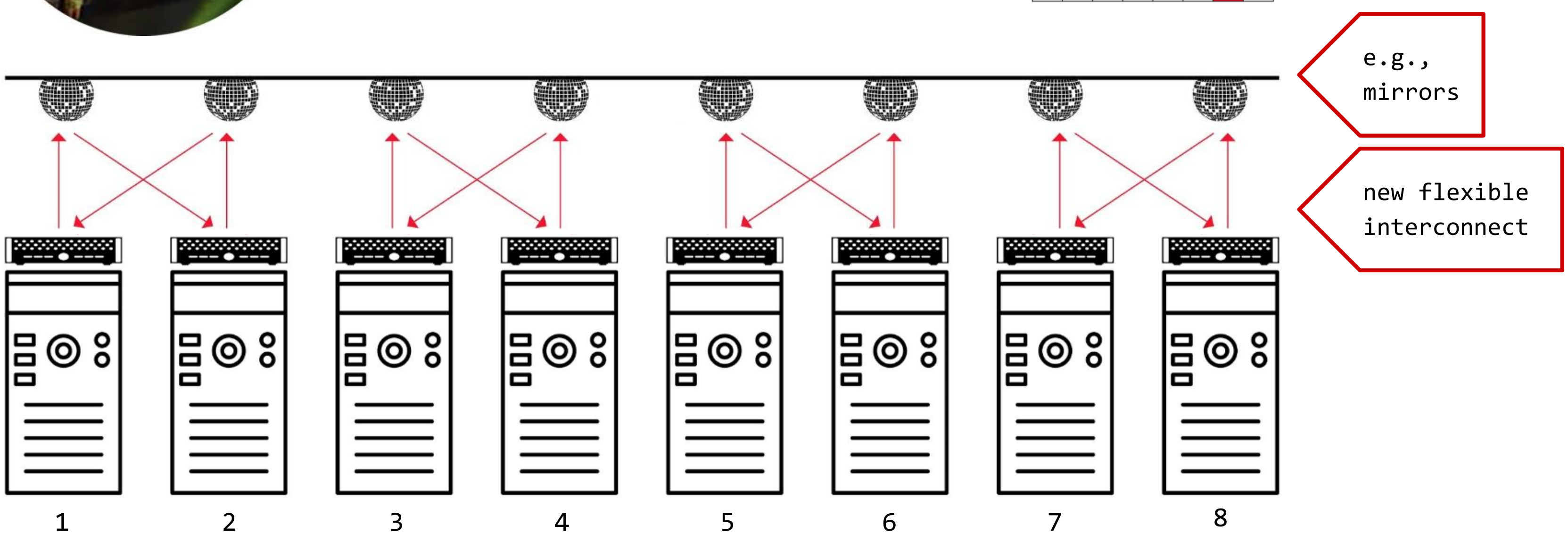
Trend:

Demand-Aware Graphs

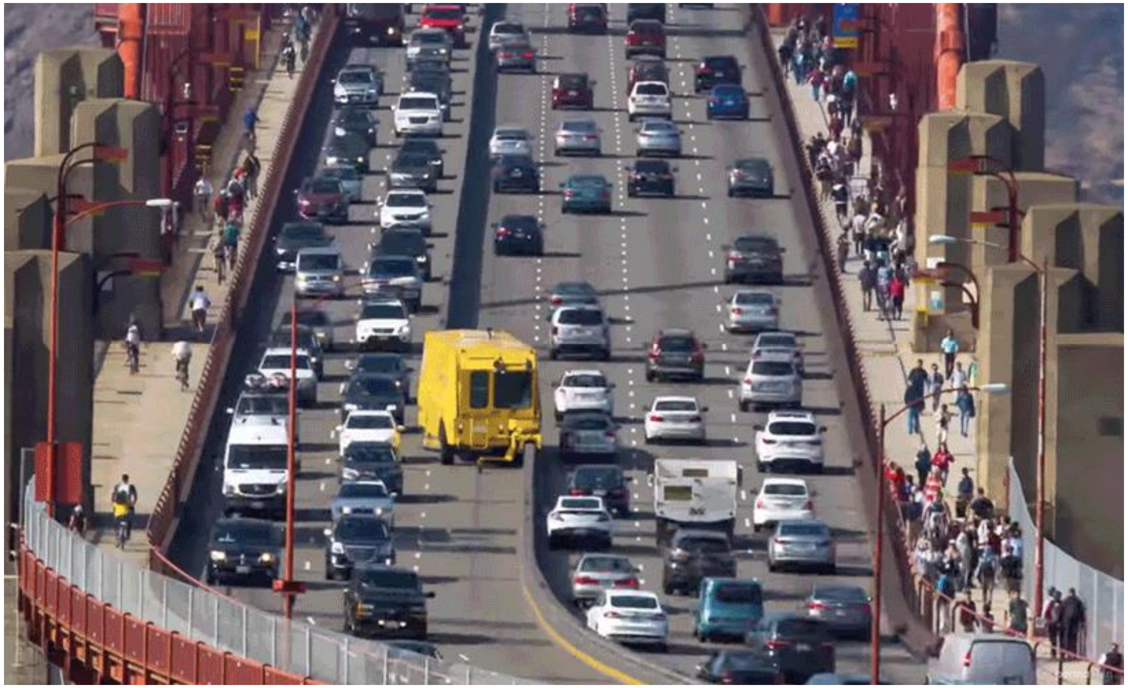
Demand-Aware Networks

new demand:





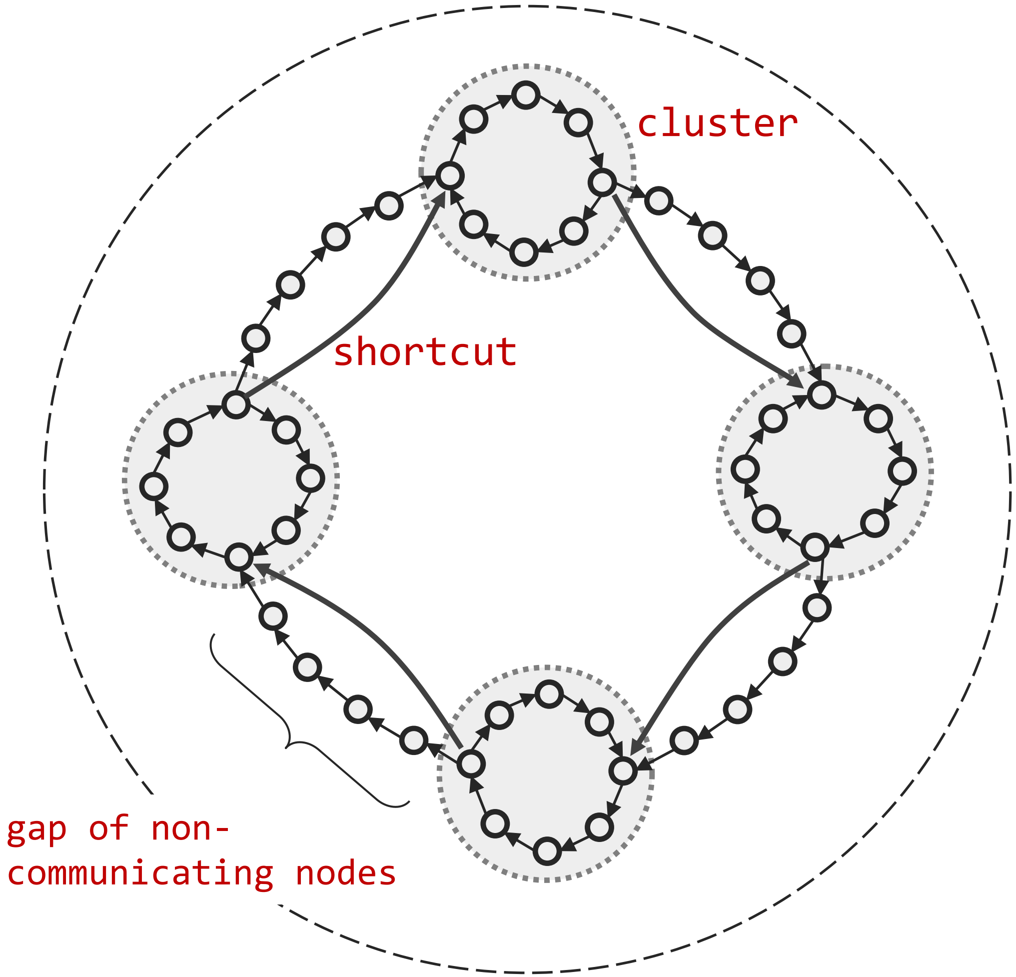
Analogy



Golden Gate Zipper

1-Dimensional Model:

Clusters in a Cycle



Nodes arranged in a cycle

Input: a $n \times n$ demand matrix D representing the traffic between any two nodes

⇢ y clusters

⇢ a cluster consists of x nodes

⇢ probability p: nodes communicate inside cluster

⇢ 1-p: to nodes in other clusters

⇢ Other nodes: $D_{u,v} = 0$ if u or v is not in a cluster

1-Dimensional Model:

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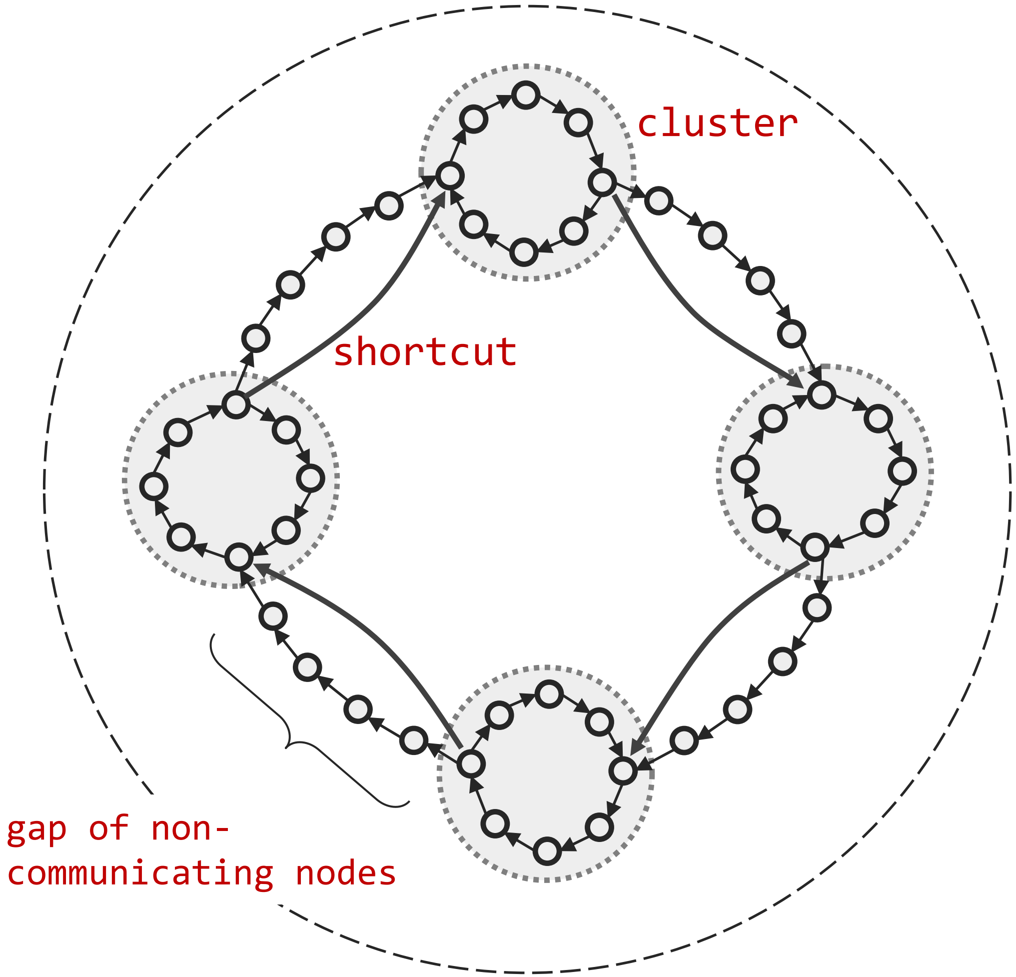

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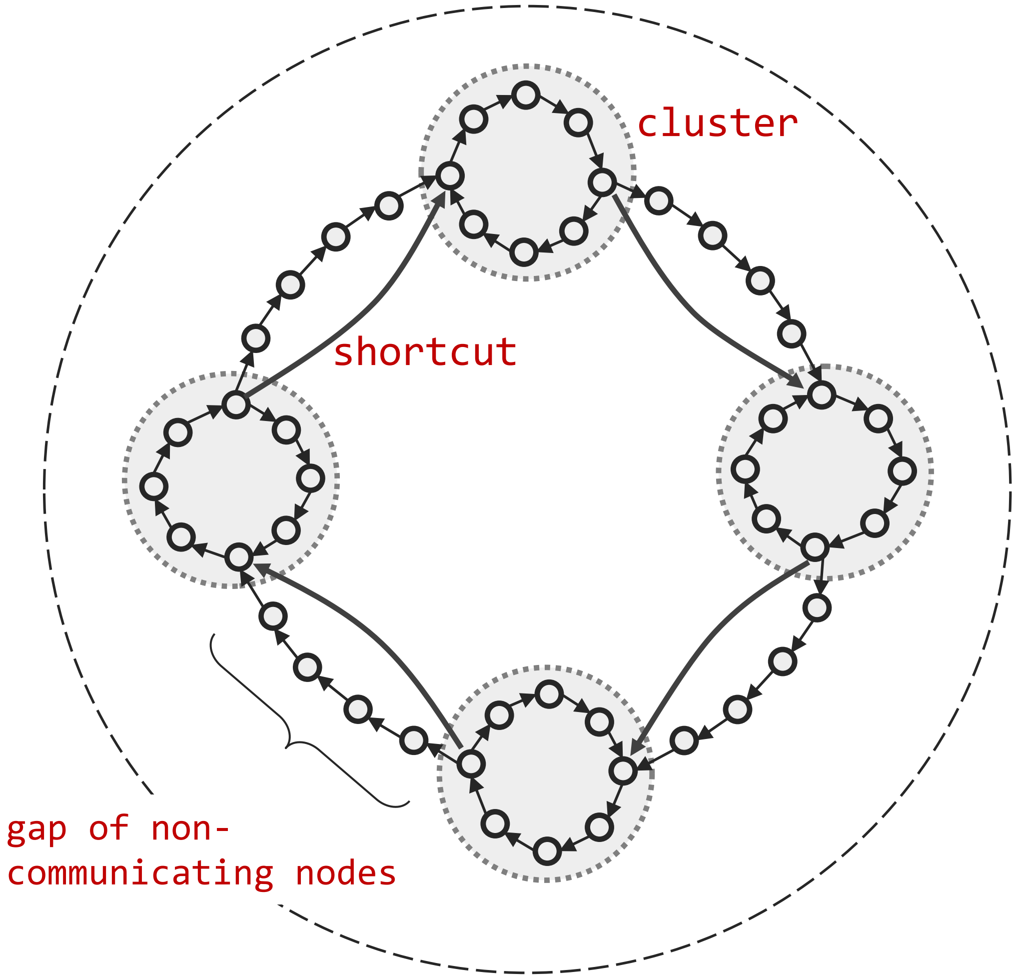
Question:
How to select augmenting edges: how to choose the distribution in a demand-aware manner?

1-Dimensional Model:

Clusters in a Cycle



Nodes arranged in a cycle

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⇢ probability p: nodes communicate inside cluster

⇢ q=1-p: to nodes in other clusters

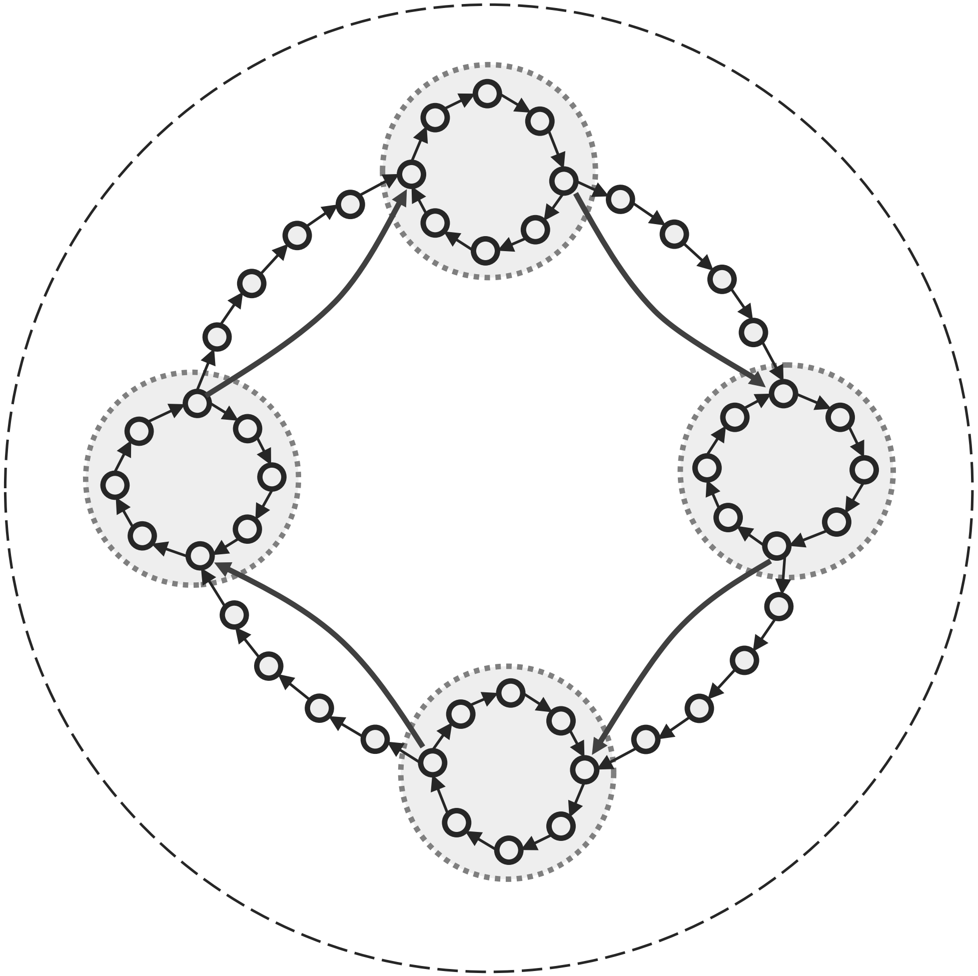
⇢ Other nodes: $D_{u,v} = 0$ if u or v is not in a cluster

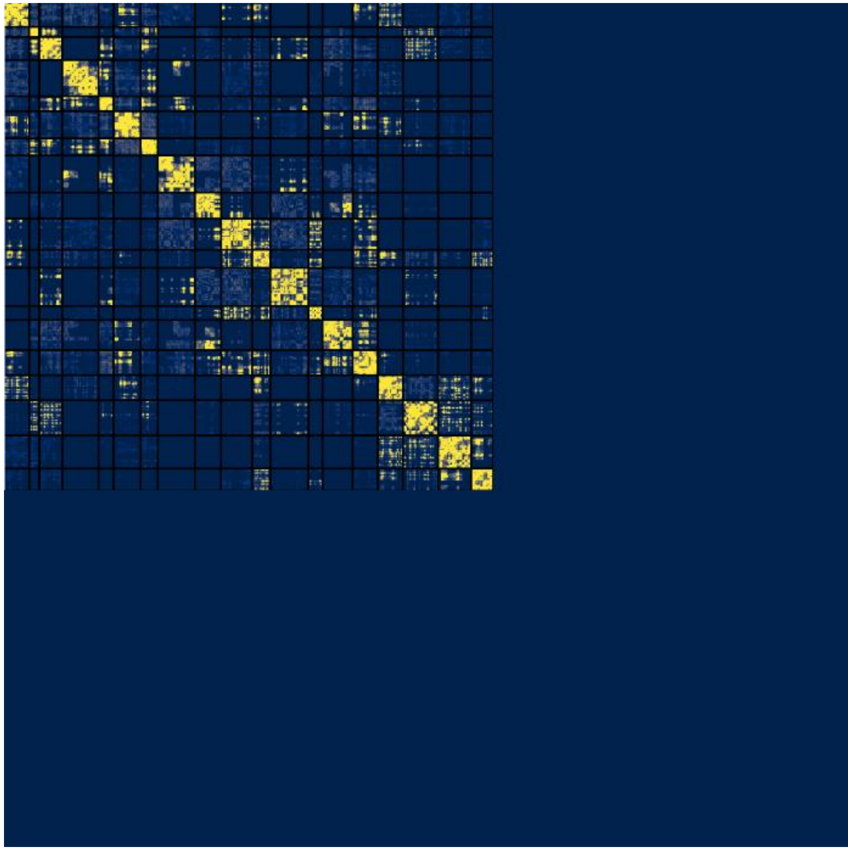
Question:

How to select augmenting edges: how to choose the distribution in a demand-aware manner?

We also study communication on a grid
(not the focus in this talk)

Empirical Correspondance





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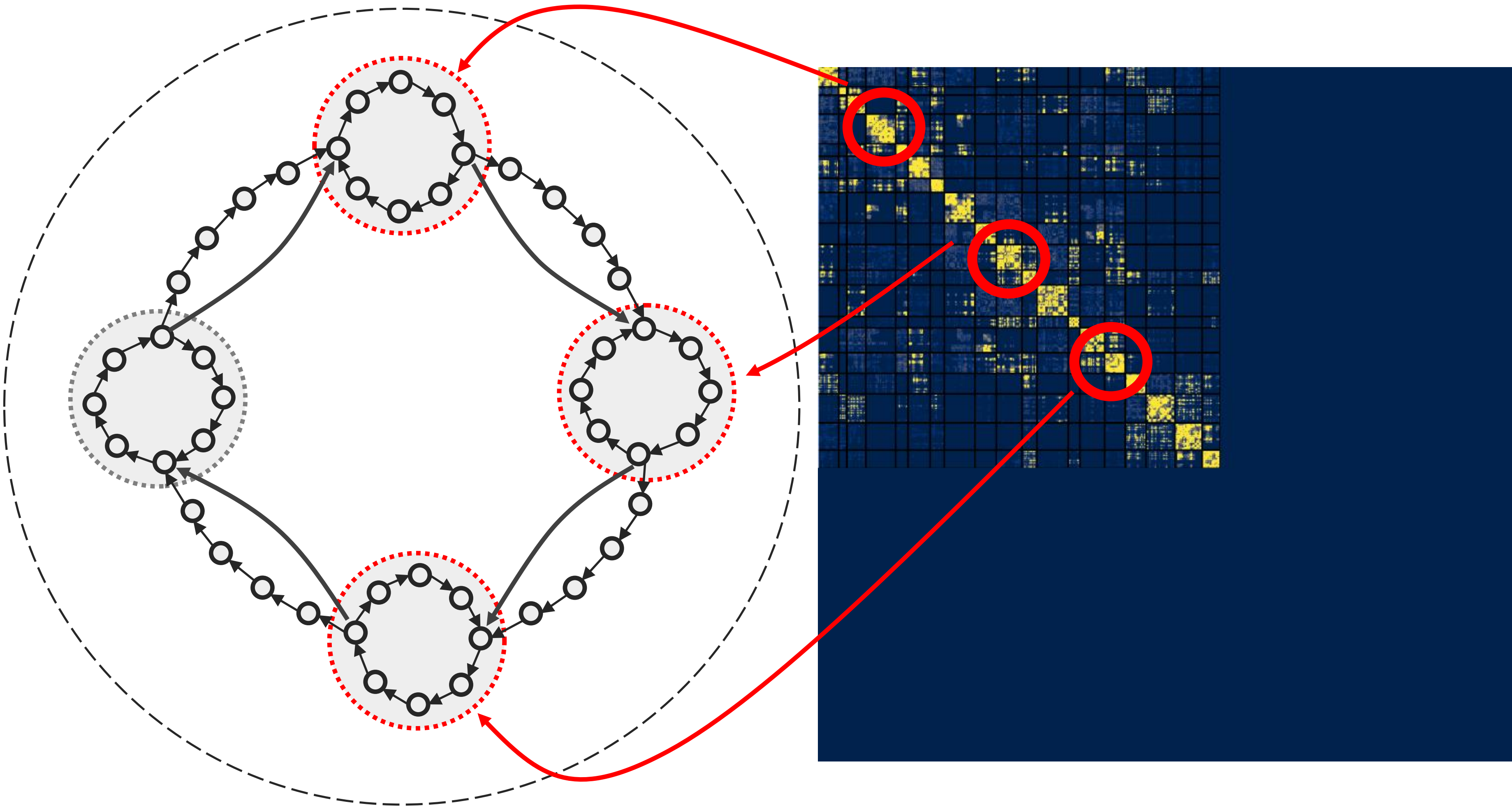

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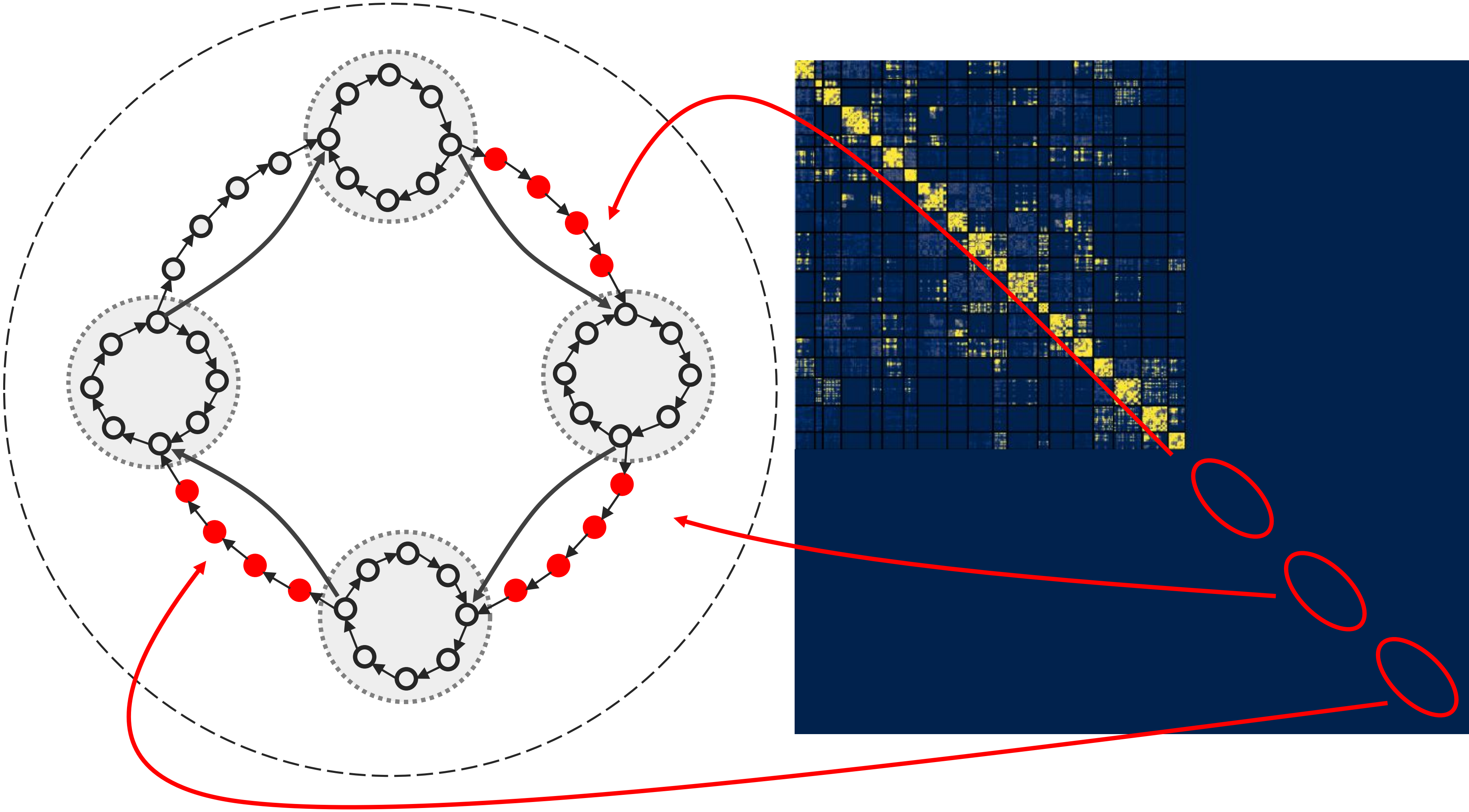
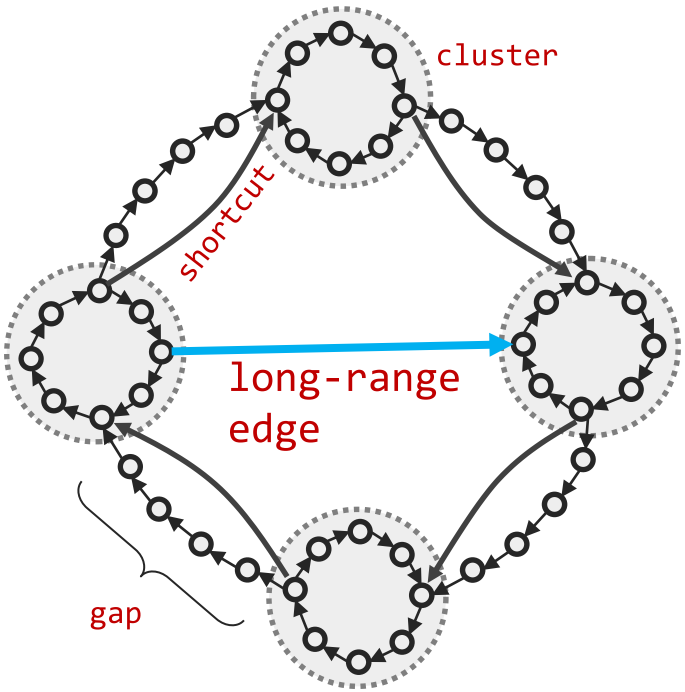


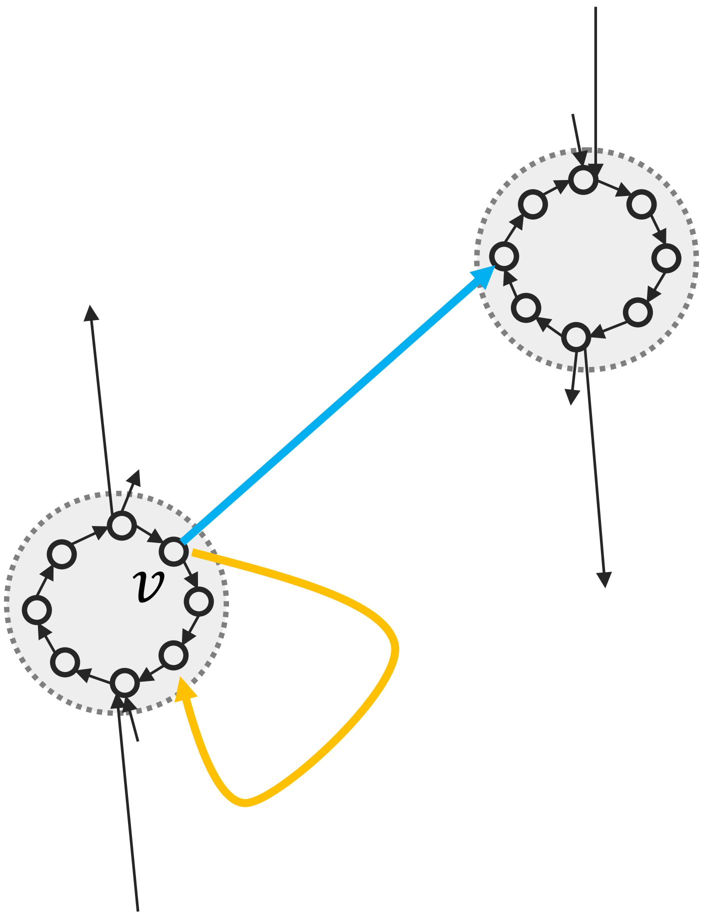
Objective



Goal:
Find a random distribution according to which the nodes add a directed edge (long-range edges) such that the weighted expected distance between all communicating nodes is minimized.

Nice to have:
Greedy routing from u to v:
⇢ take the edge to the cluster closest to the destination (between clusters)
⇢ take the edge to the node closest to the destination (inside clusters)

Our Approach



Let p_v be the probability that a packet originating from v is destined for a node belonging to the same cluster as v

Let $q_v = 1 - p_v$ be the probability that the packet is destined to a node outside v's cluster

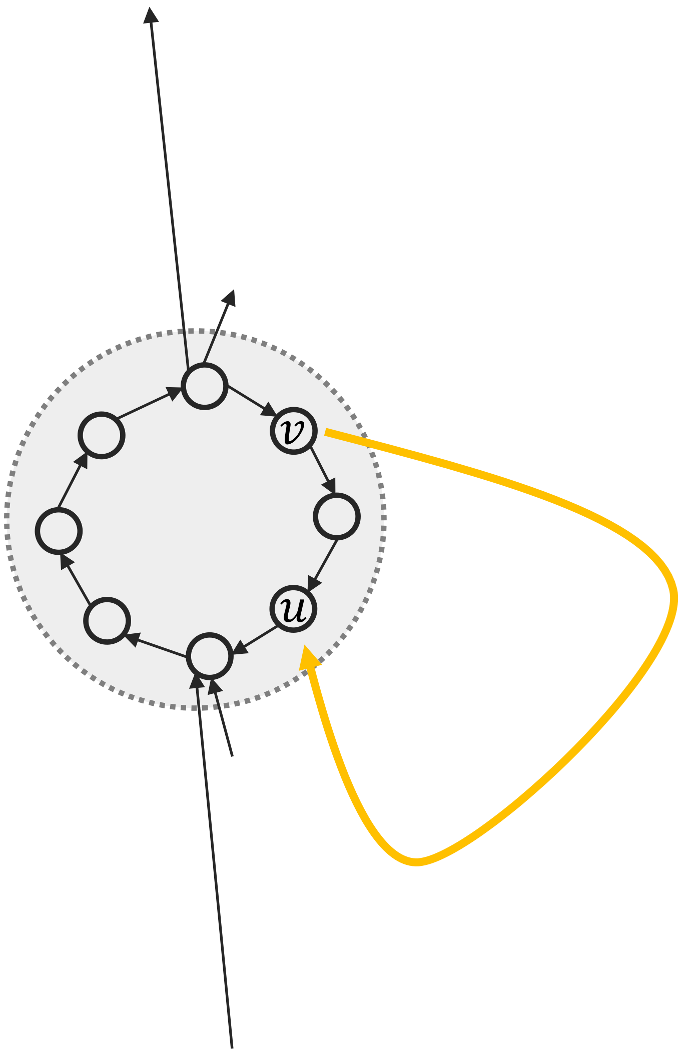
Idea:

⇢ Add a long-range edge originating from v with probability q_v proportional also to the inter-cluster distance

⇢ Otherwise add a mid-range edge originating from v, proportional to the ring-distance

Augmentation:

Mid-range edges



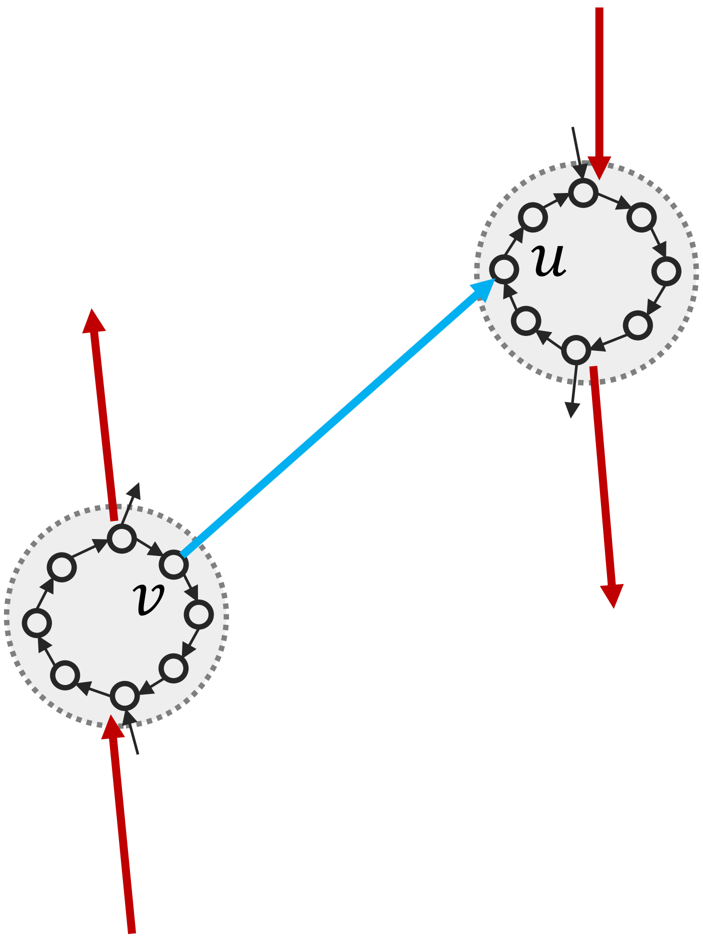
With probability q_v add a mid-range edge originating at v

Choose the endpoint u in the same cluster with probability proportional to:

$$\frac{1}{d(v,u)}$$

Augmentation:

Long-range edges



With probability q_v add a long-range edge originating at v

For every cluster C, $v \notin C$ choose an arbitrary $u \in C$.

Add the edge v, u with probability proportional to

$$\frac{1}{d_I(v,u)}$$

where $d_I(v,u)$ is the distance between v and u measured in the number of clusters between them (seeing clusters as "super-nodes" without internal distance)

Theoretical Analysis

For simplicity assume $p = p_v$ and $q = q_v$ for all v

Lemma: greedily routing a packet from v to u takes $O(\log(y) \cdot (p \cdot \log(x) + q \cdot \log(y))$ hops in expectation.

Analysis similar to that of Kleinberg.

Difficulty arises when cluster sizes are not fixed but vary according to some distribution X.

We consider:

- $\mathrm{X} \sim Pois(\lambda)$, i.e., $\Pr(\mathrm{X} = \mathrm{k}) = \frac{\lambda^k e^{-\lambda}}{k!}$, and
- $\mathrm{X} \sim PowerLaw(\alpha)$, i.e., $\Pr(\mathrm{X} = \mathrm{k}) \sim k^{-\alpha}$

Poisson Clusters

Lemma: Assume cluster sizes X are $Pois(\lambda)$ distributed. Then greedy routing takes this many steps in expectation:

1. $O(\log(y) \cdot (p \cdot \log(\lambda + C_1\sqrt{\lambda \log n}) + q \cdot \log(y)))$ if $\lambda > c \log n$
2. $O(\log(y) \cdot (p \cdot \log(C_2 \log n) + q \cdot \log(y)))$ if $\lambda < c \log n$
3. $O(\log(y) \cdot (p \cdot \log\left(C_3 \frac{\log n}{\log \log n}\right) + q \cdot \log(y)))$ if $\lambda = const$

Proof idea: Similar to before, but we need the average size of the clusters we visit within r hops. This requires us to show that the cluster sizes are concentrated. We consider the following cases:

1. $\lambda > c \log n$, then the cluster sizes are in $[\lambda - C_1\sqrt{\lambda \log n}, \lambda + C_1\sqrt{\lambda \log n}]$ w.h.p.
2. $\lambda < c \log n$, then the cluster sizes are at most $C_2 \log n$ w.h.p.
3. $\lambda = const$, then the cluster sizes are at most $\frac{\log n}{\log \log n}$ w.h.p.

w.h.p = probability at least $1 - {}^1/_{n^3}$

Each of the cases can solved using concentration bounds

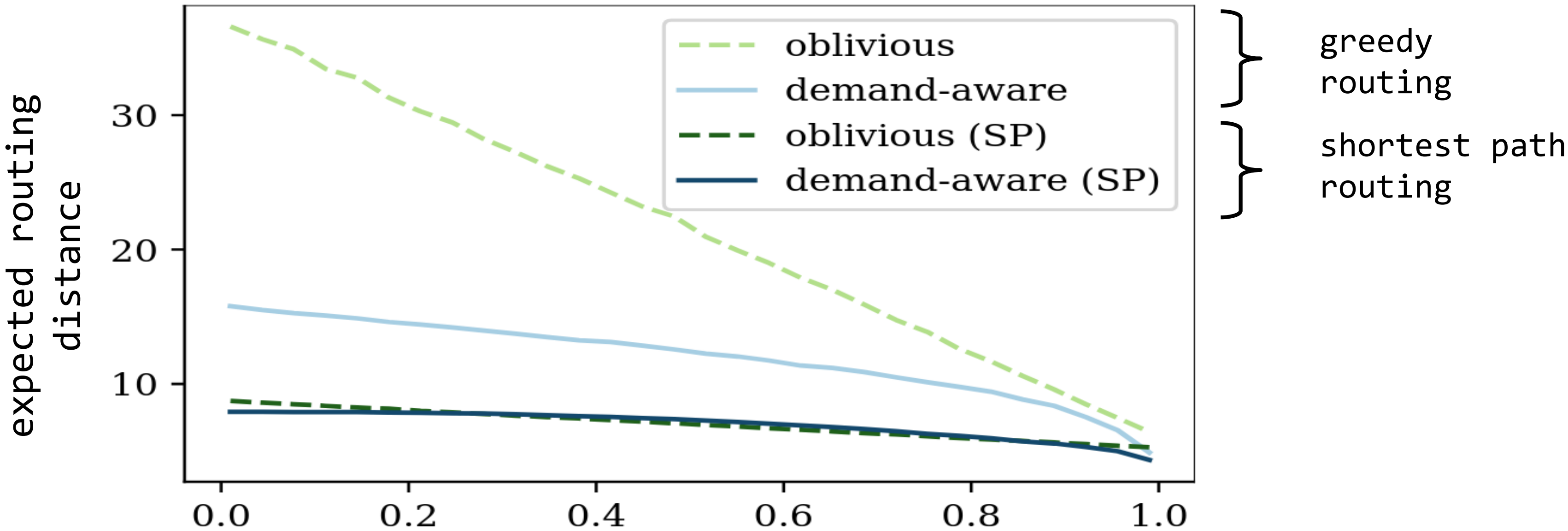
Power Law Clusters

Lemma: Assume cluster sizes X are $PowerLaw(\alpha)$ distributed. Then greedy routing takes in expectation: $O(\log(y) \cdot (4p \cdot \log\log n + q \cdot \log(y)))$

Proof idea: we cannot use concentration bounds to upper bound the average size of the clusters we visit within r steps.

Rather, we derive an upper bound with overestimating parts of the contribution (using bucketing and assuming parts of the Greedy routing "picks" cluster sizes uniformly at random).

Empirically...



in cluster communication probability p

fixed values: $x = 16, y = 32, n = 3xy$

Demand-aware significantly better the more structure we have and also especially for greedy routing : makes better shortcuts.

oblivious "Kleinberg"-like strategy: for a node v add an edge to a communicating node u with probability proportional to $d(v,u)^{-1}$

Thank you!