Demand-Aware Small-World Networks on Clustered Demands

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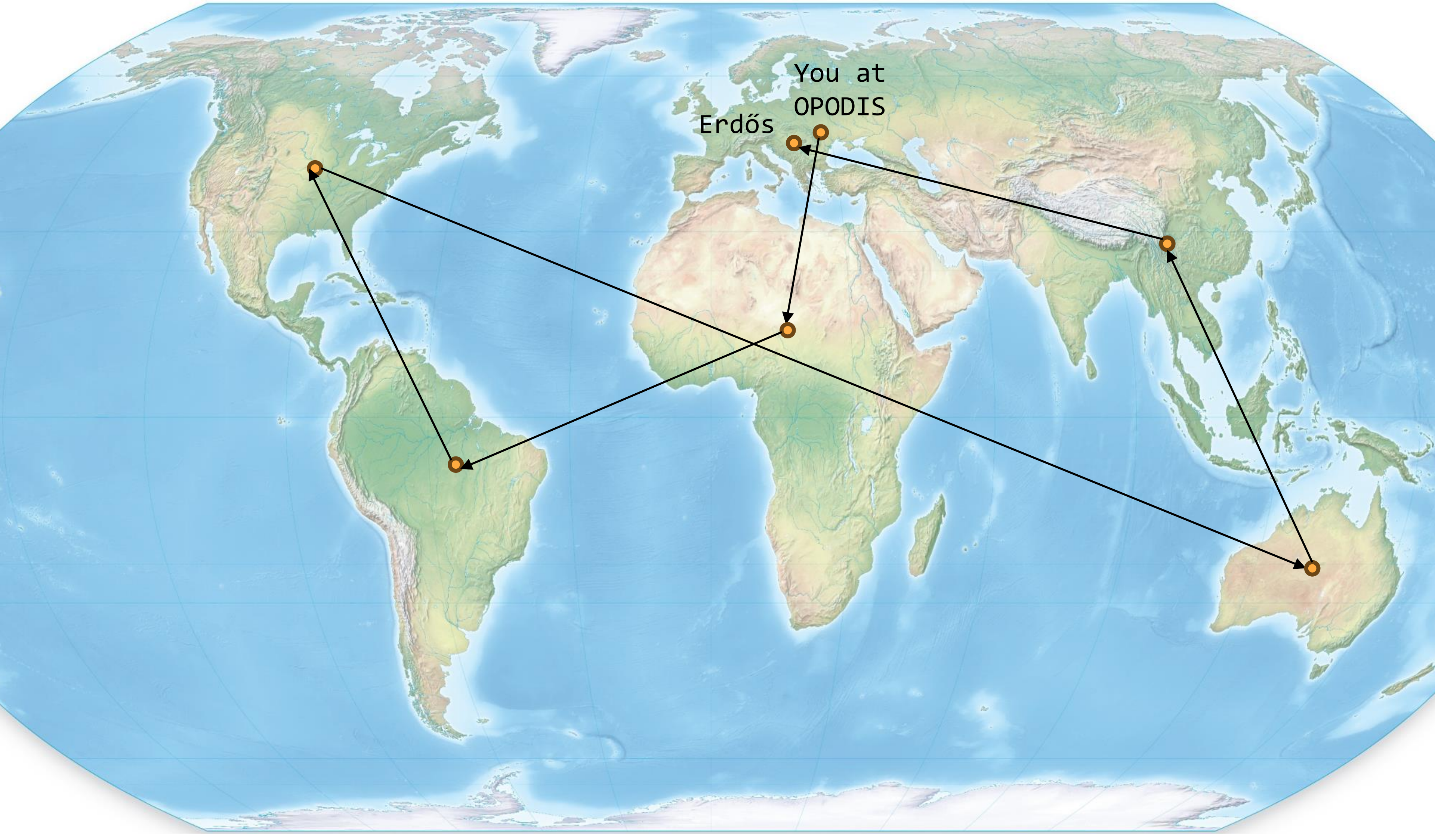
Stefan Schmid (TU Berlin)

Acknowledgements:

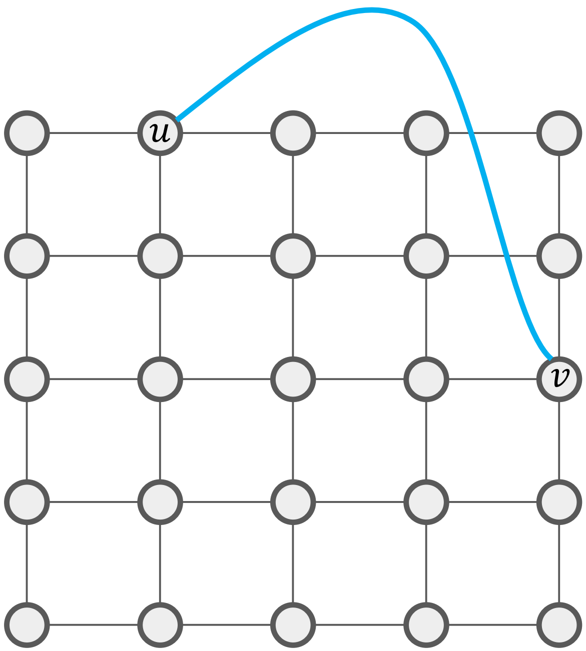
Context: “Small World Phenomenon”

Six Degrees of Separation



Popular Model

- Grid + long-range contact
- Kleinberg [STOC '00]: Certain powerlaw gives polylog routes which can even be *found*!
- Also inspired distributed system designs, e.g., p2p networks
- Works for "any demand" (all-to-all)



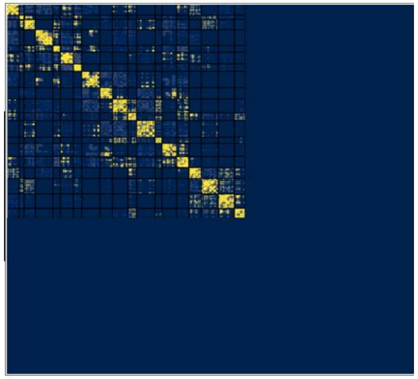
However, in practice:

Demand more structured

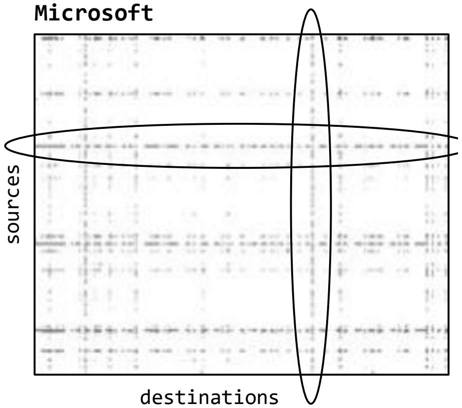
Empirical studies:

traffic matrices sparse and skewed

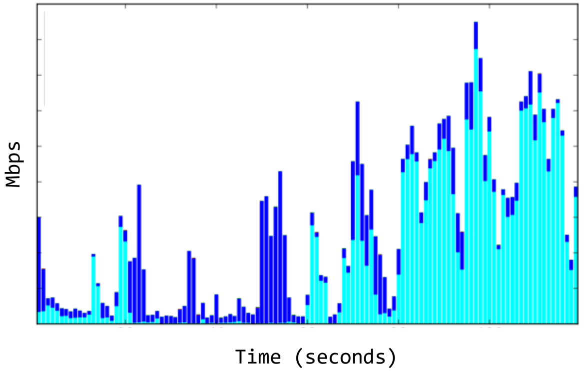
Facebook



Microsoft

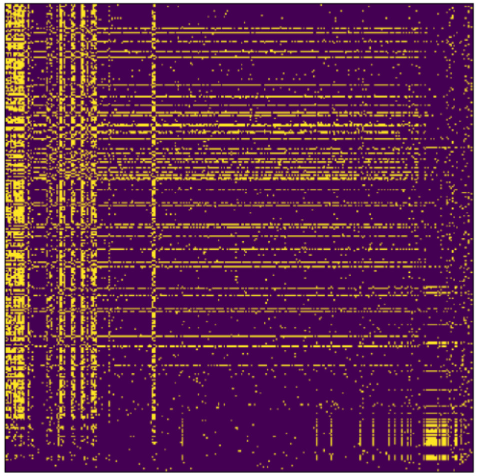


Facebook

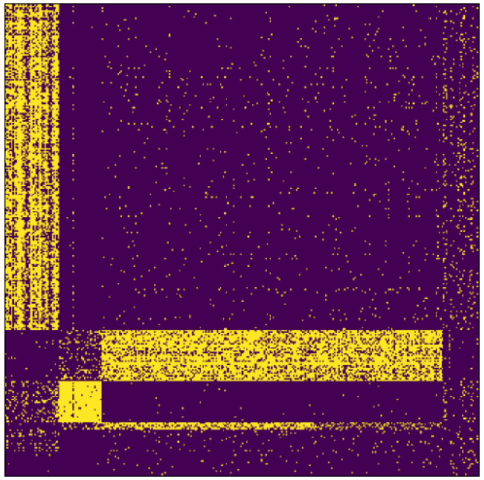


Traffic is also clustered: bi-clustering results

Small Stable Clusters



reordering based on
bicluster structure



Förster et al., Analyzing the Communication Clusters
in Datacenters. WWW 2023

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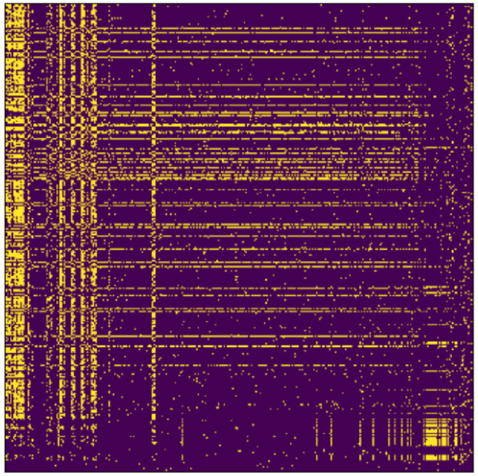
Small Stable Clusters



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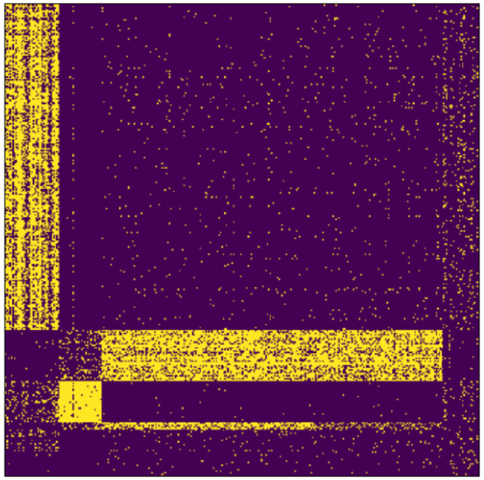
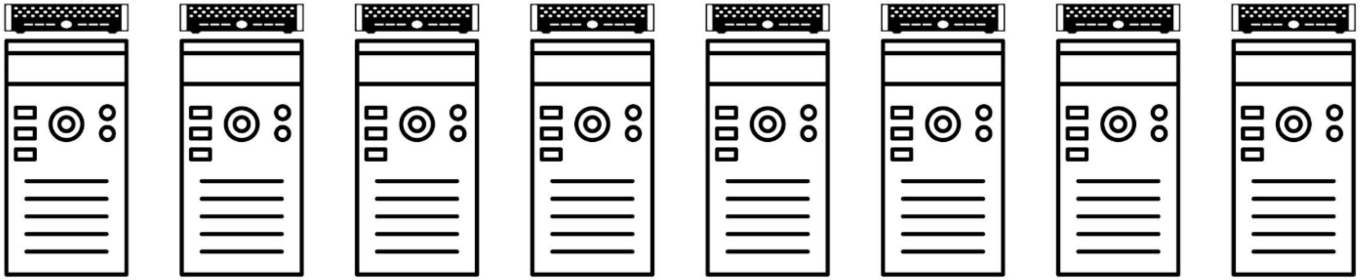


Can we exploit this?

Förster et al., Analyzing the Communication Clusters in Datacenters. WWW 2023

Trend:

Demand-Aware Graphs



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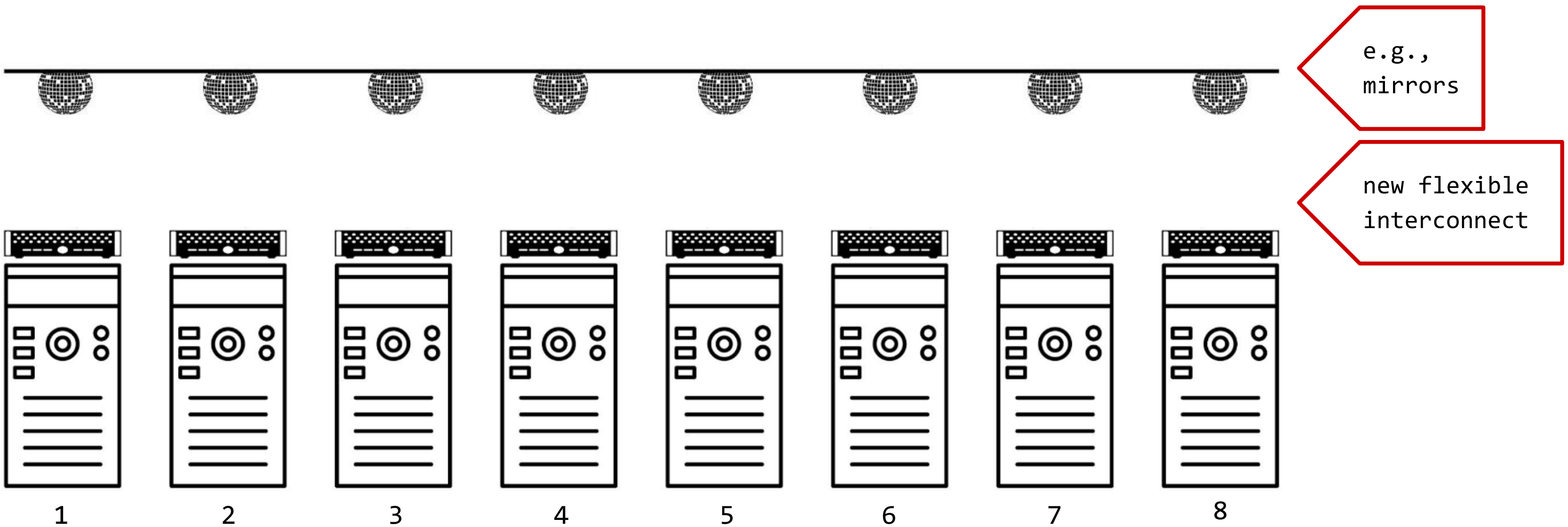

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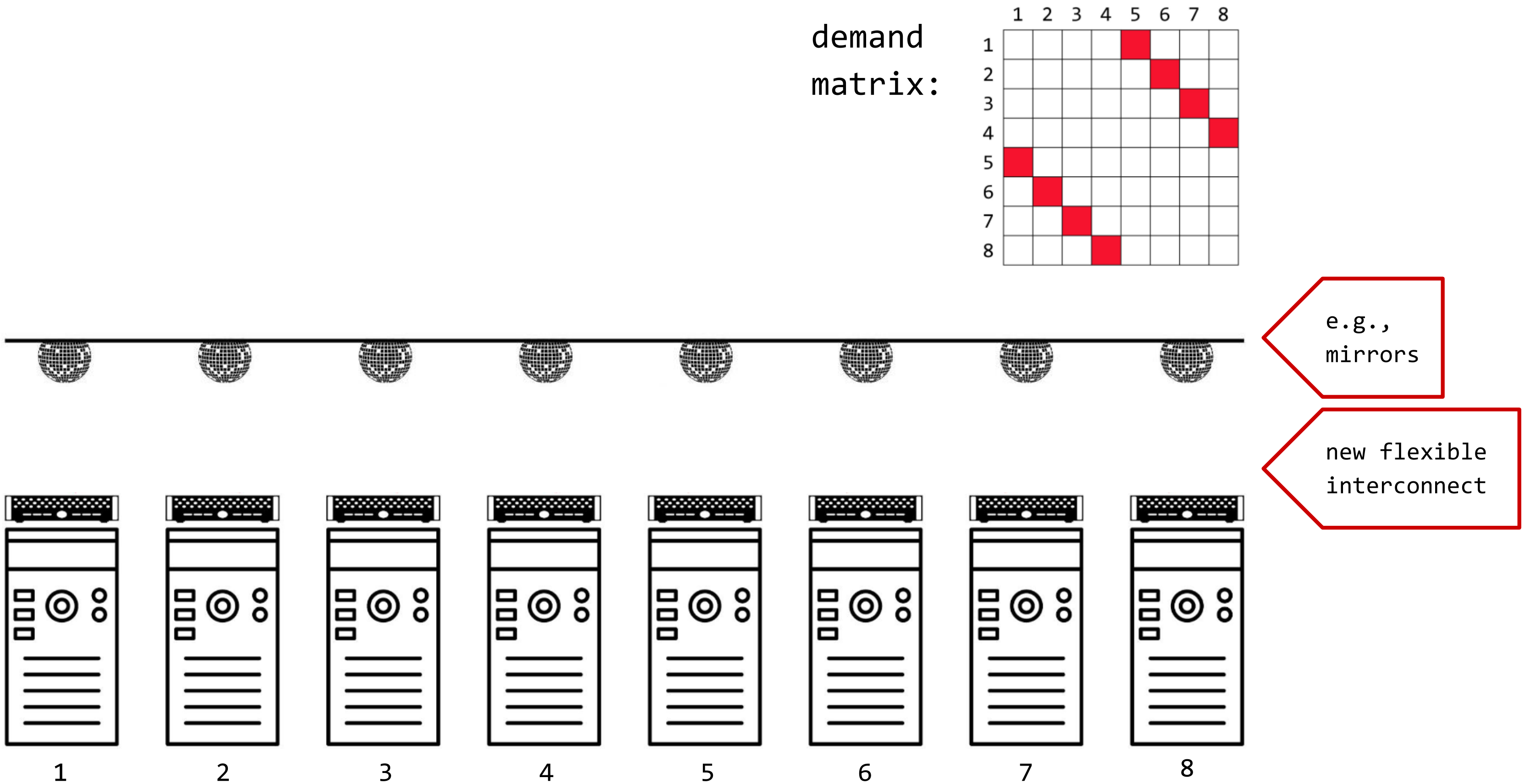
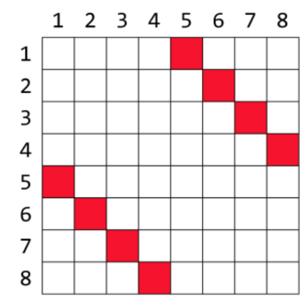


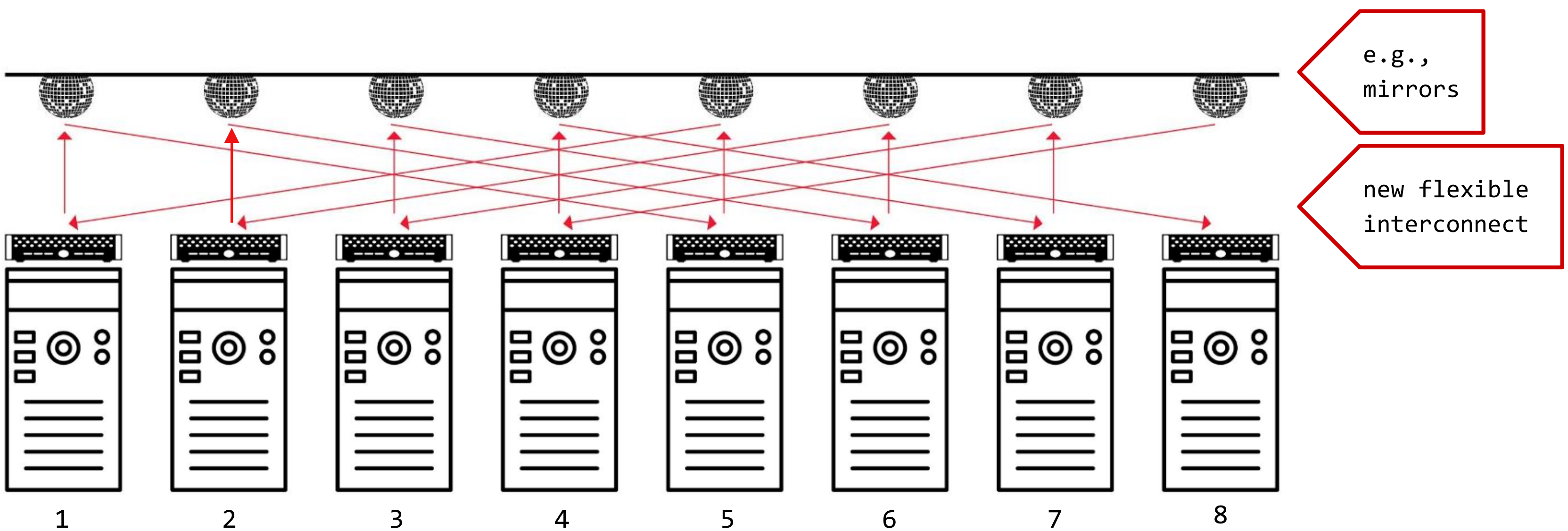
Trend:

Demand-Aware Graphs

Matches demand

demand matrix:

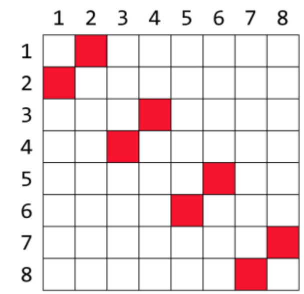


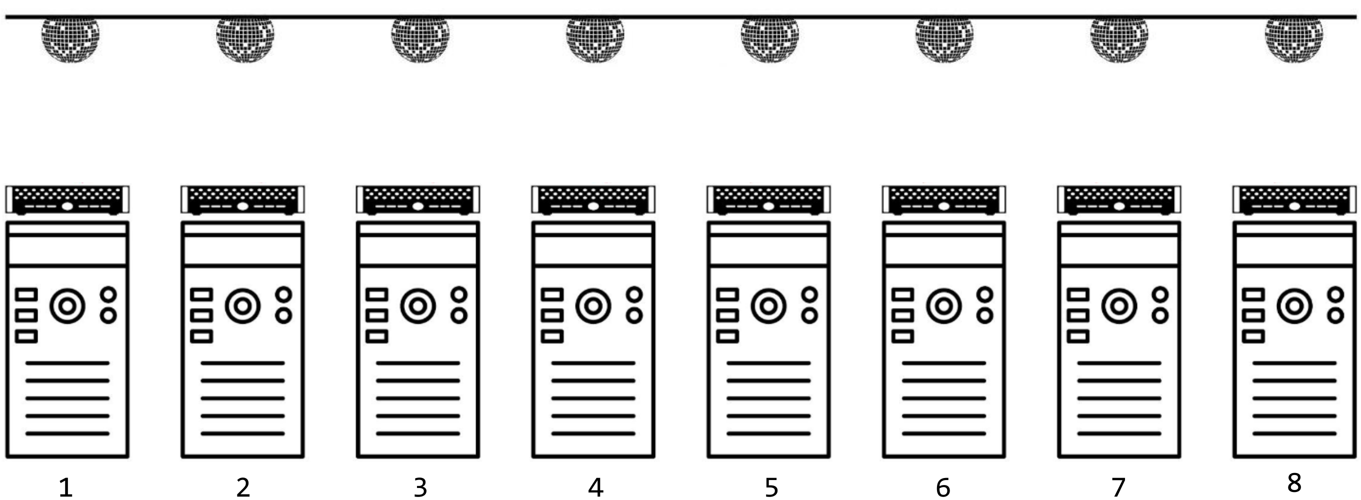


Trend:

Demand-Aware Graphs

new demand:





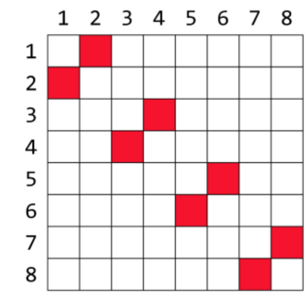
e.g., mirrors

new flexible interconnect

Trend:

Demand-Aware Graphs

new demand:



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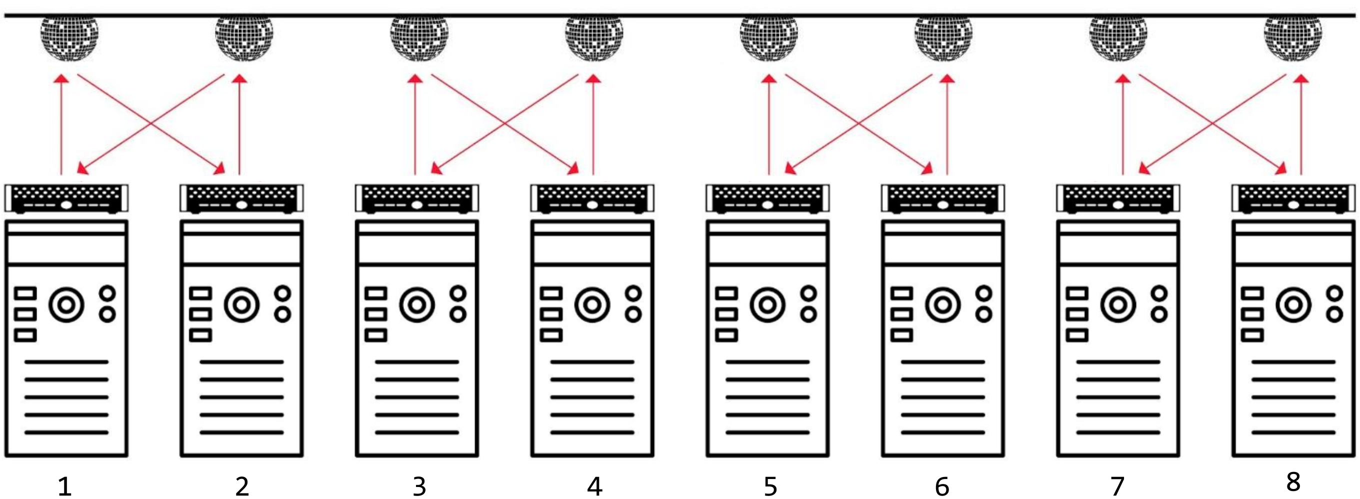
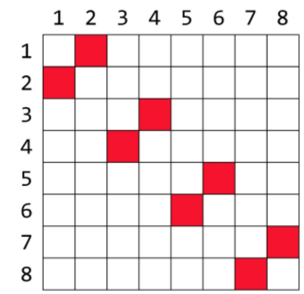


Trend:

Demand-Aware Graphs

Demand-Aware Networks

new demand:



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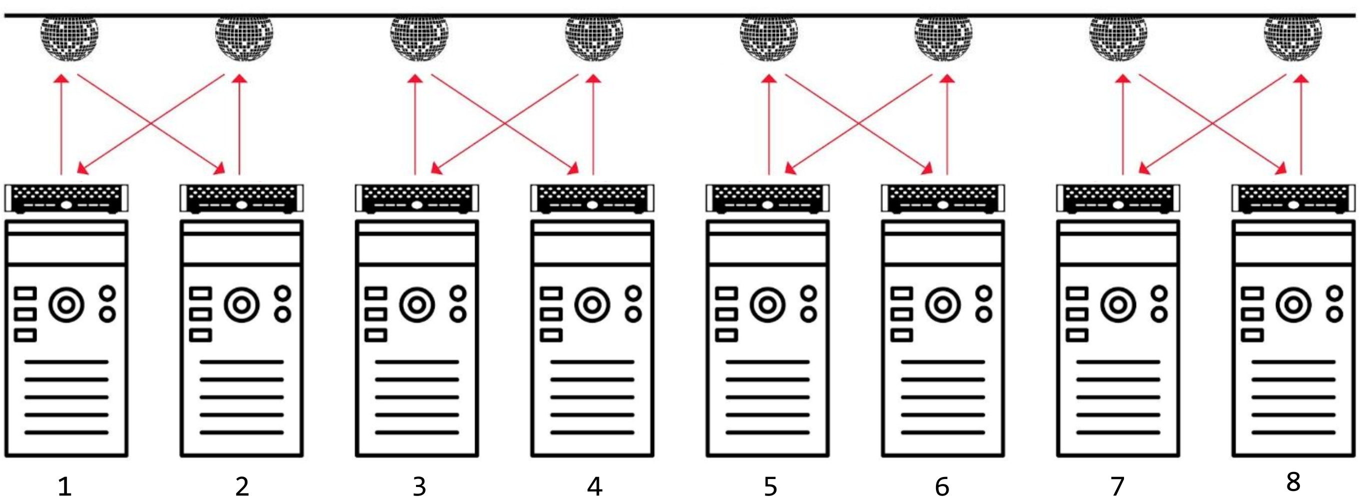
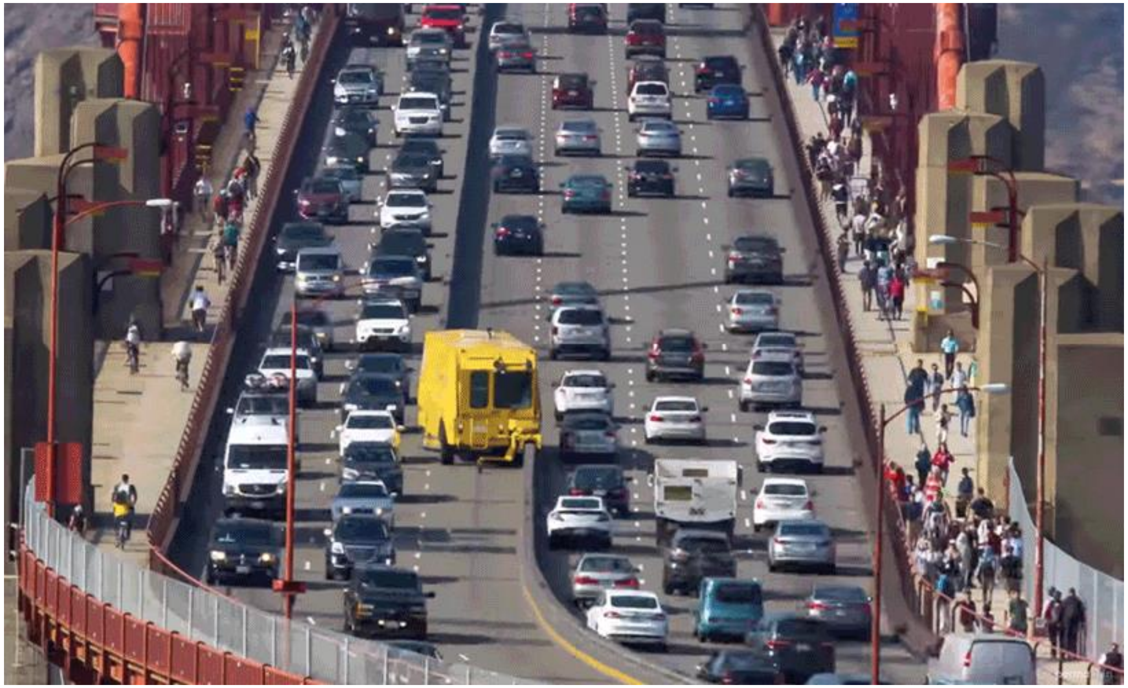


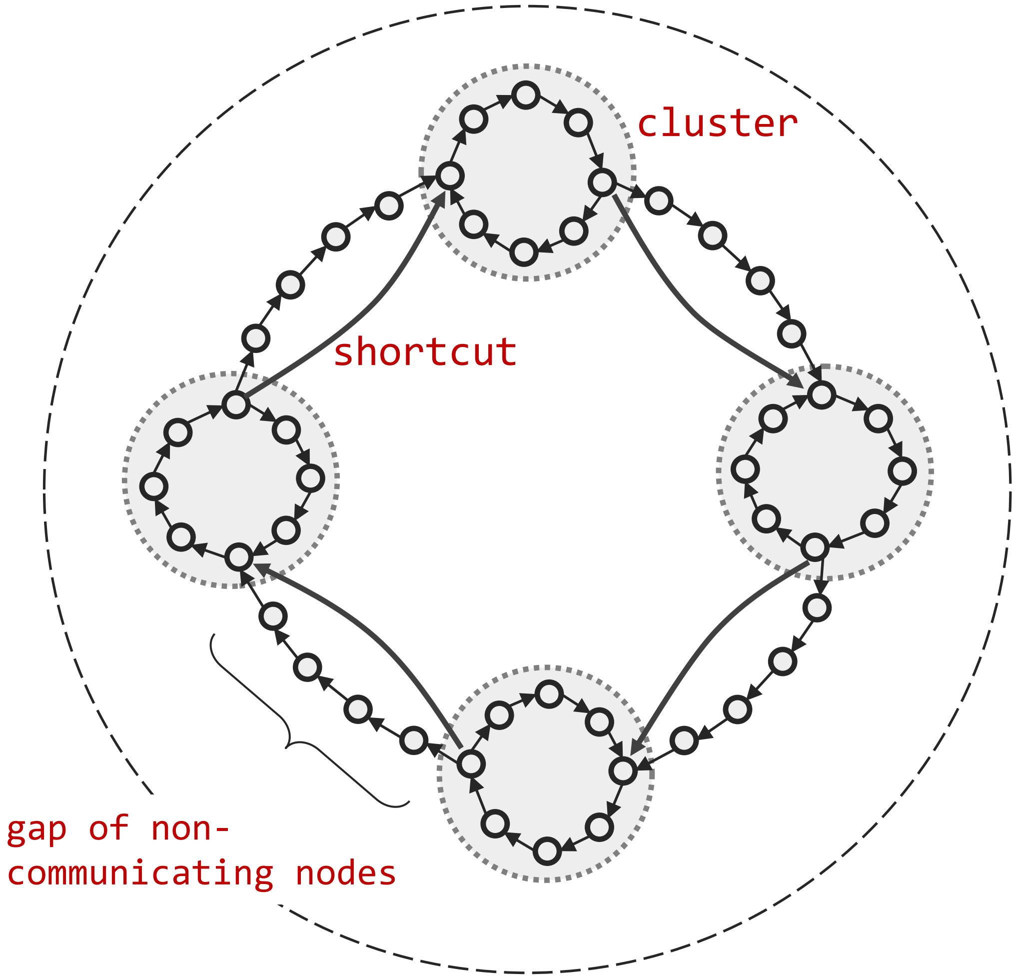
Analogy



Golden Gate Zipper

1-Dimensional Model:

Clusters in a Cycle



Nodes arranged in a cycle

Input: a $n \times n$ demand matrix D representing the traffic between any two nodes

⇢ y clusters

⇢ a cluster consists of x nodes

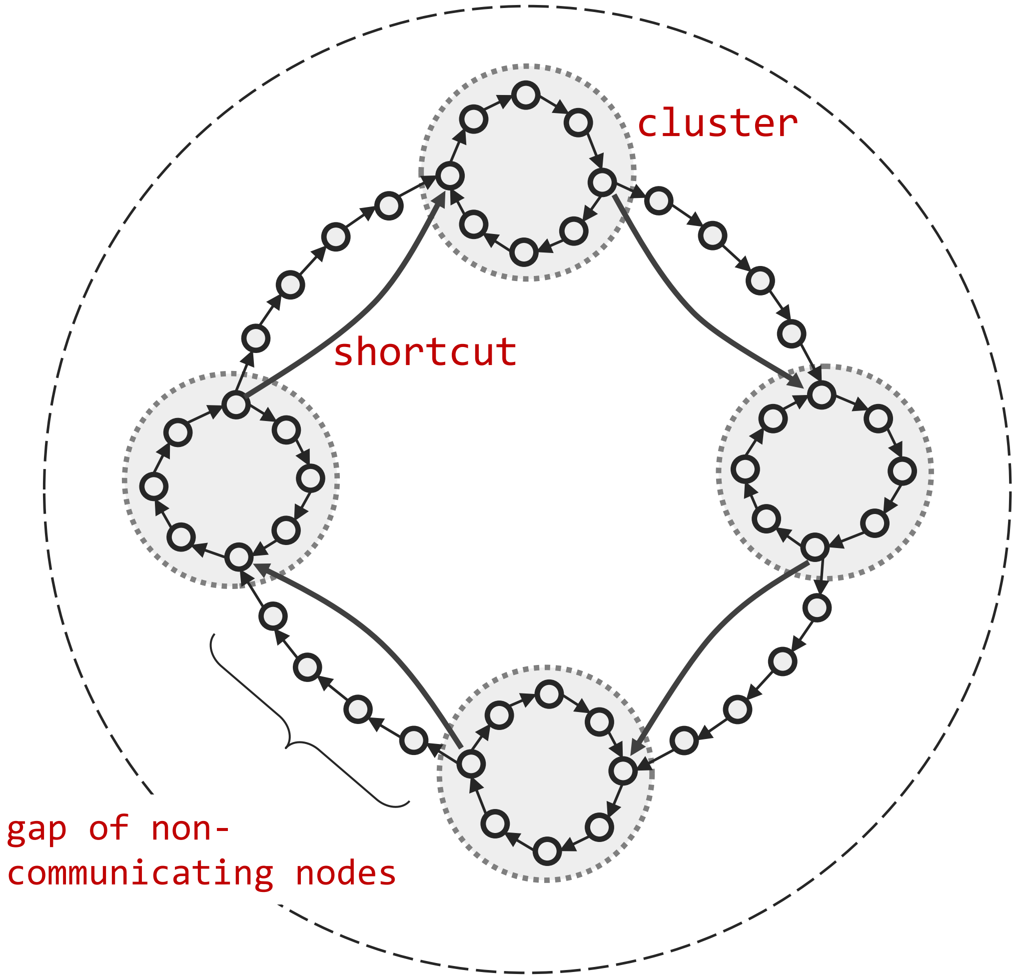
⇢ probability p: nodes communicate inside cluster

⇢ 1-p: to nodes in other clusters

⇢ Other nodes: $D_{u,v} = 0$ if u or v is not in a cluster

1-Dimensional Model:

Clusters in a Cycle



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Question:

How to select augmenting edges: how to choose the distribution in a demand-aware manner?

1-Dimensional Model:

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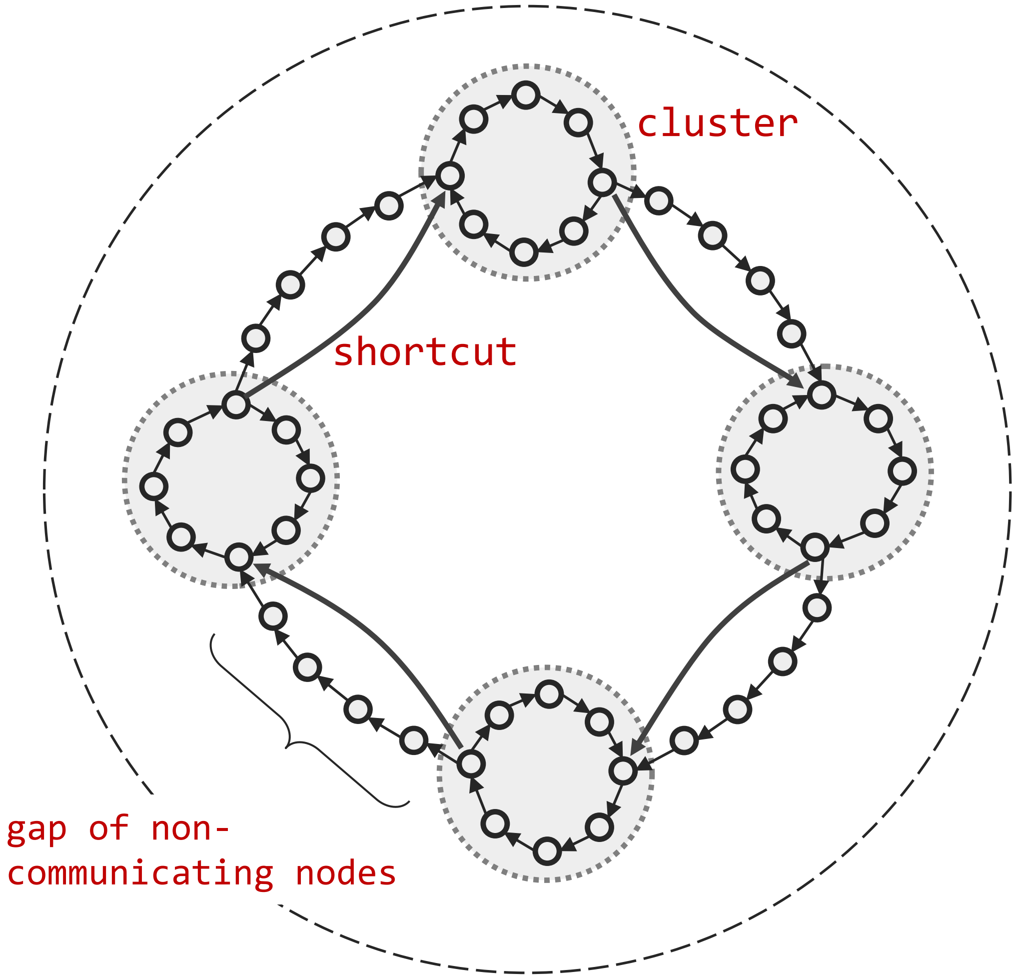

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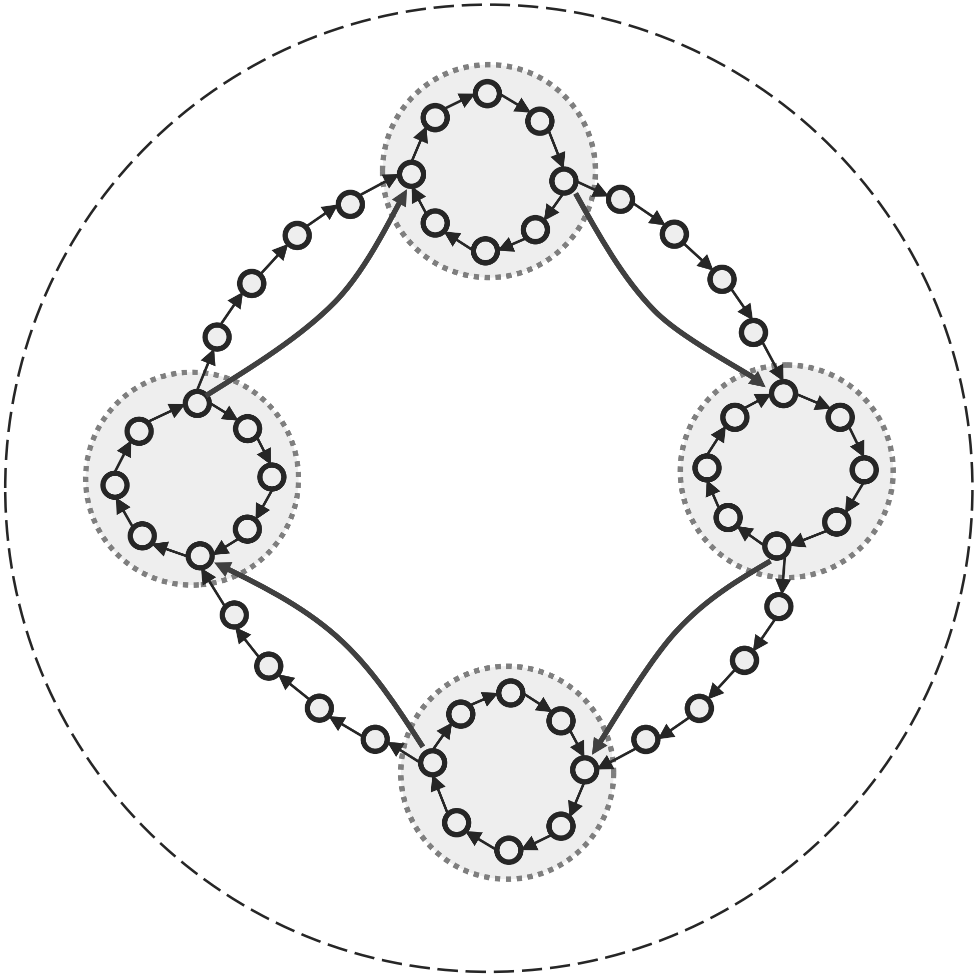
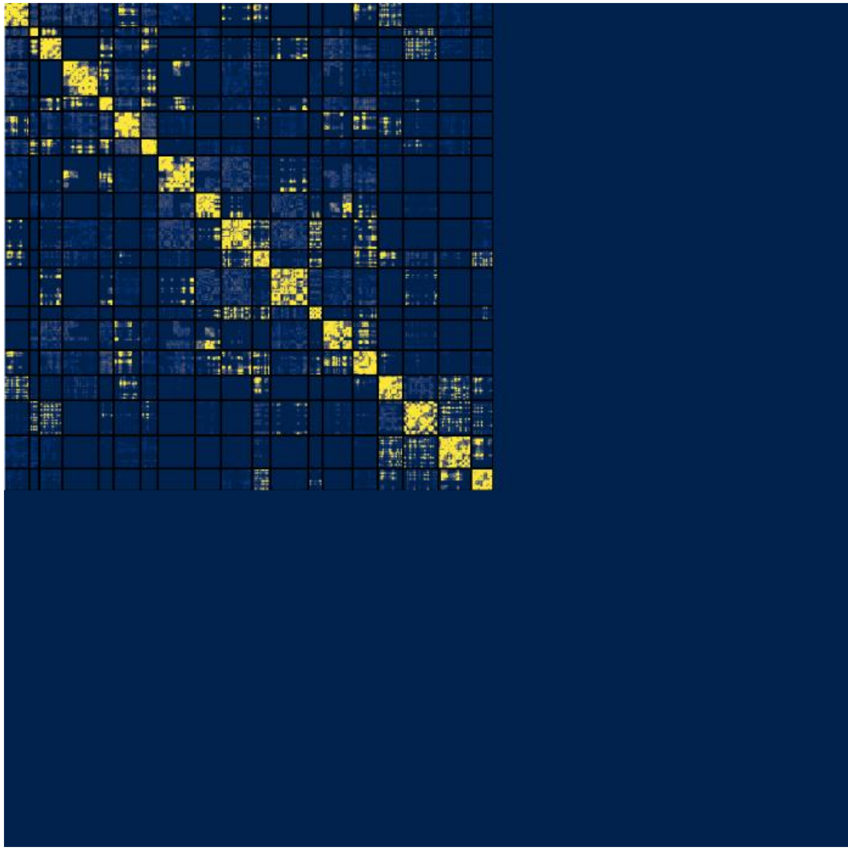
We also study communication on a grid
(not the focus in this talk)

Empirical Correspondance





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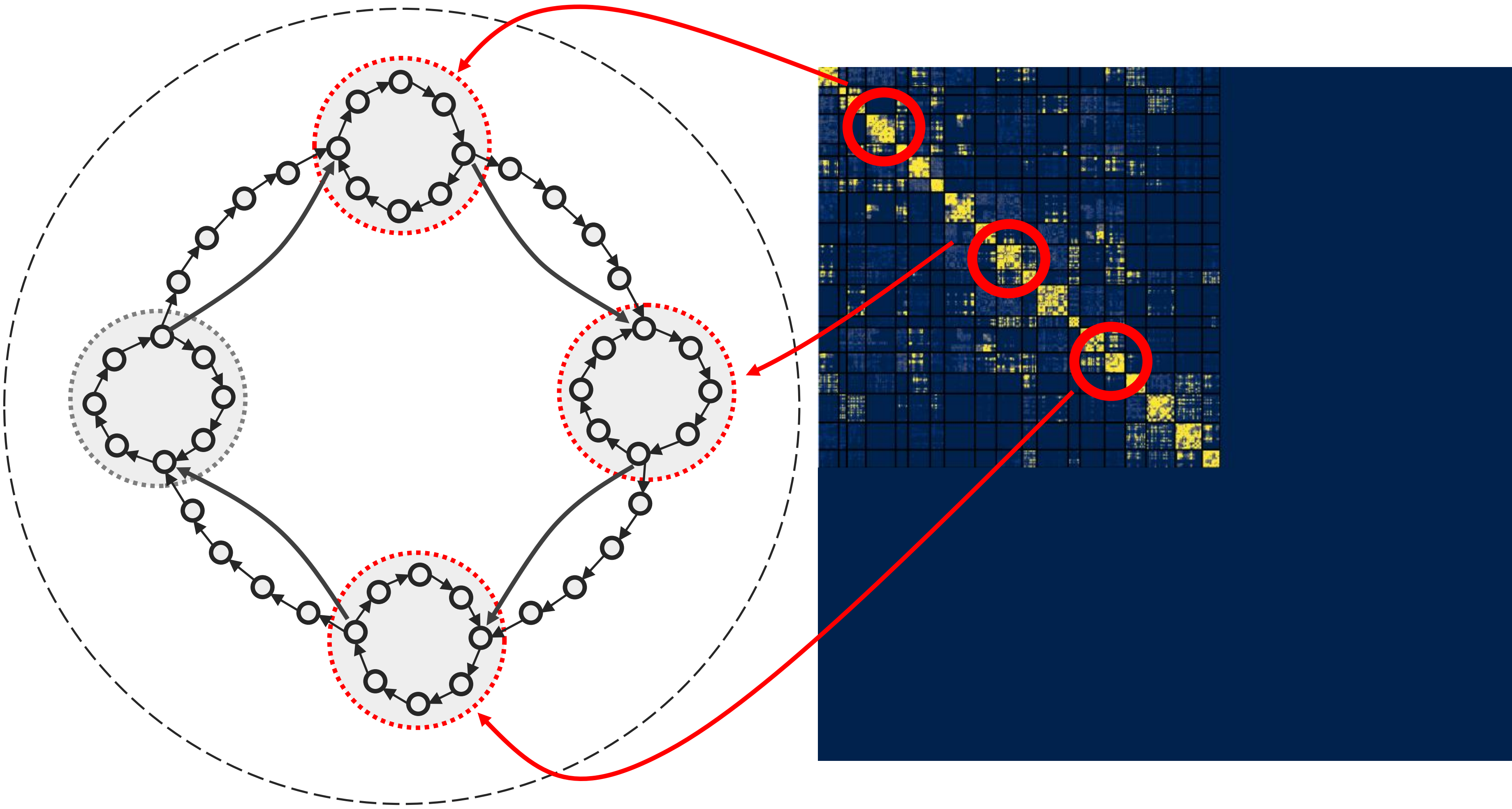

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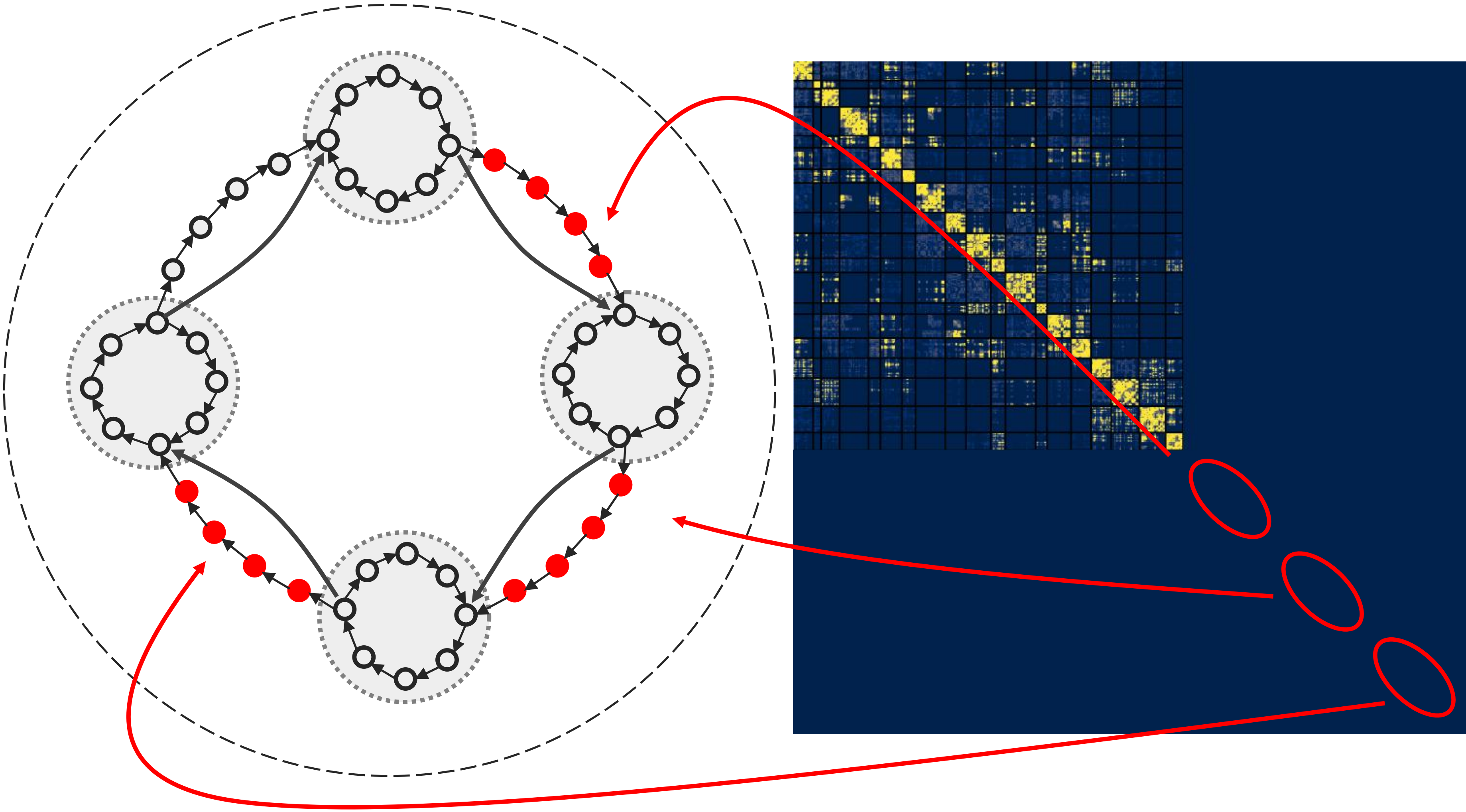
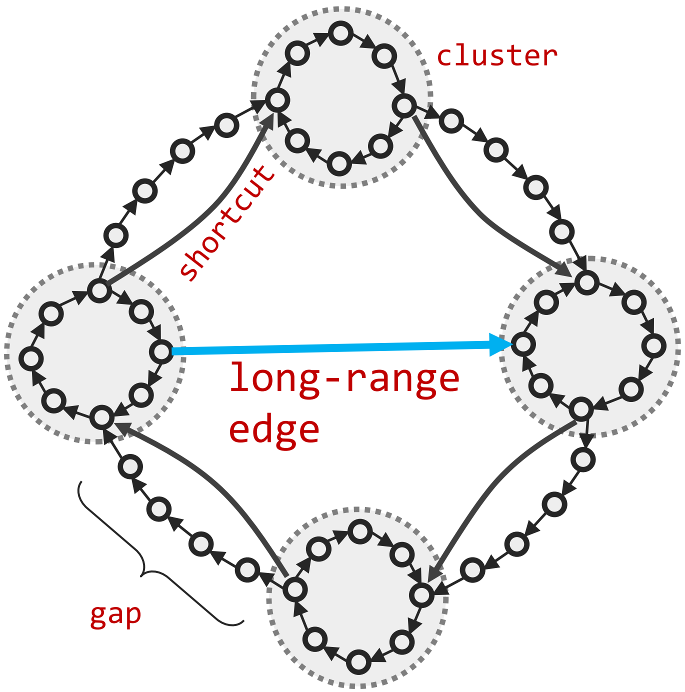


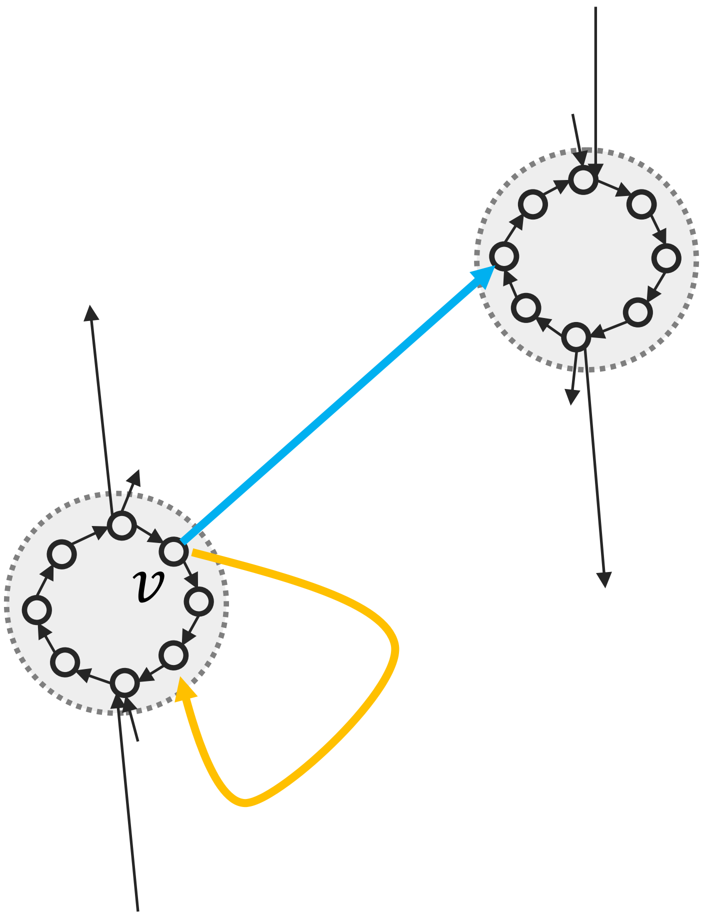
Objective



Goal:
Find a random distribution according to which the nodes add a directed edge (long-range edges) such that the weighted expected distance between all communicating nodes is minimized.few

Nice to have:
Greedy routing from u to v:
⇢ take the edge to the cluster closest to the destination (between clusters)
⇢ take the edge to the node closest to the destination (inside clusters)

Our Approach



Let p_v be the probability that a packet originating from v is destined for a node belonging to the same cluster as v

Let $q_v = 1 - p_v$ be the probability that the packet is destined to a node outside v's cluster

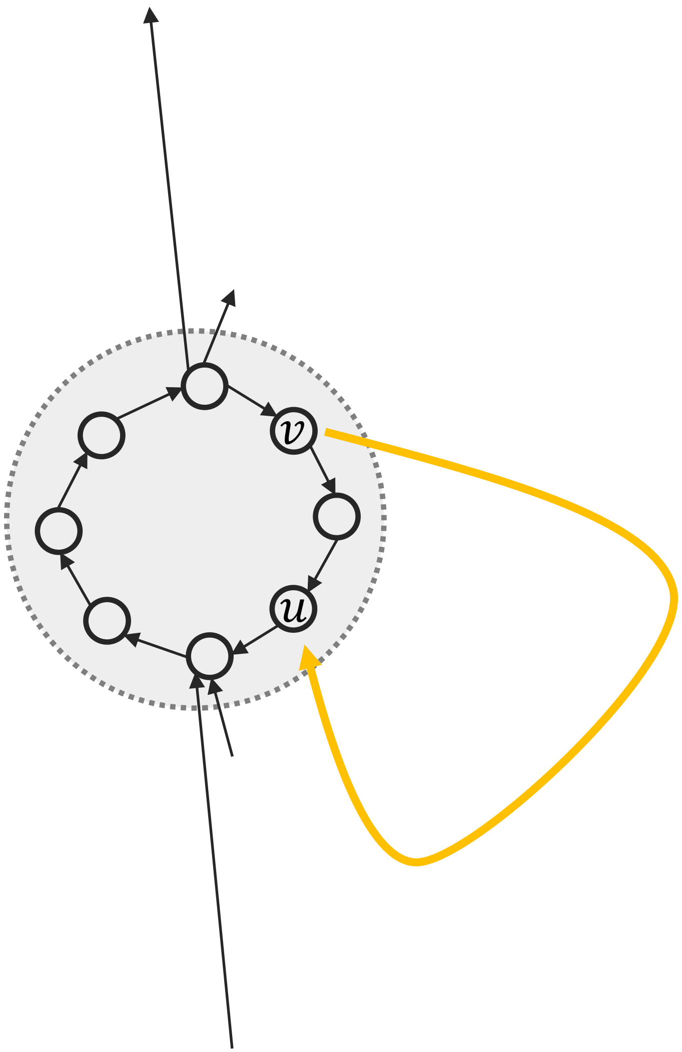
Idea:

⇢ Add a long-range edge originating from v with probability q_v proportional also to the inter-cluster distance

⇢ Otherwise add a mid-range edge originating from v, proportional to the ring-distance

Augmentation:

Mid-range edges



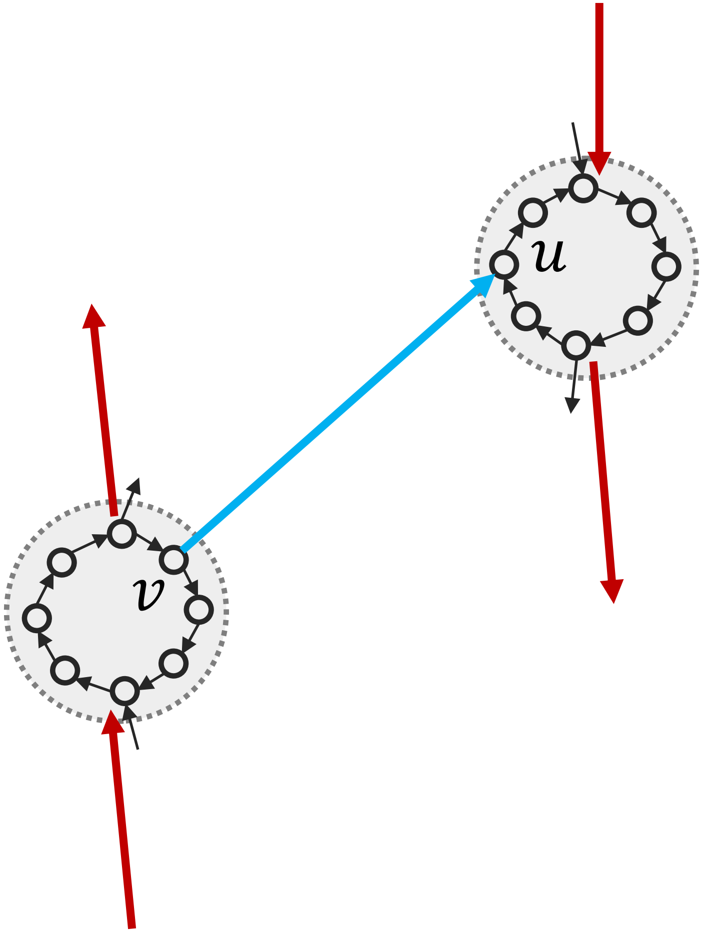
With probability q_v add a mid-range edge originating at v

Choose the endpoint u in the same cluster
with probability proportional to:

$$\frac{1}{d(v,u)}$$

Augmentation:

Long-range edges



With probability q_v add a long-range edge originating at v

For every cluster C, $v \notin C$ choose an arbitrary $u \in C$.

Add the edge v, u with probability proportional to

$$\frac{1}{d_I(v,u)}$$

where $d_I(v,u)$ is the distance between v and u measured in the number of clusters between them (seeing clusters as "super-nodes" without internal distance)

Theoretical Analysis

For simplicity assume $p = p_v$ and $q = q_v$ for all v

Lemma: greedily routing a packet from v to u takes $O(\log(y) \cdot (p \cdot \log(x) + q \cdot \log(y))$ hops in expectation.

Analysis similar to that of Kleinberg.

Difficulty arises when cluster sizes are not fixed but vary according to some distribution X.

We consider:

⇢ $\mathrm{X} \sim Pois(\lambda)$, i.e., $\Pr(\mathrm{X} = \mathrm{k}) = \frac{\lambda^k e^{-\lambda}}{k!}$, and

⇢ $\mathrm{X} \sim PowerLaw(\alpha)$, i.e., $\Pr(\mathrm{X} = \mathrm{k}) \sim k^{-\alpha}$

Poisson Clusters

Lemma: Assume cluster sizes X are $Pois(\lambda)$ distributed. Then greedy routing takes this many steps in expectation:

1. $O(\log(y) \cdot (p \cdot \log(\lambda + C_1\sqrt{\lambda \log n}) + q \cdot \log(y)))$ if $\lambda > c \log n$
2. $O(\log(y) \cdot (p \cdot \log(C_2 \log n) + q \cdot \log(y)))$ if $\lambda < c \log n$
3. $O\left(\log(y) \cdot \left(p \cdot \log\left(C_3 \frac{\log n}{\log\log n}\right) + q \cdot \log(y)\right)\right)$ if $\lambda = const$

Proof idea: Similar to before, but we need the average size of the clusters we visit within r hops. This requires us to show that the cluster sizes are concentrated. We consider the following cases:

1. $\lambda > c \log n$, then the cluster sizes are in $[\lambda - C_1\sqrt{\lambda \log n}, \lambda + C_1\sqrt{\lambda \log n}]$ w.h.p.
2. $\lambda < c \log n$, then the cluster sizes are at most $C_2 \log n$ w.h.p.
3. $\lambda = const$, then the cluster sizes are at most $\frac{\log n}{\log\log n}$ w.h.p.

w.h.p = probability at least $1 - {}^{1}/_{n^3}$

Each of the cases can solved using concentration bounds

Power Law Clusters

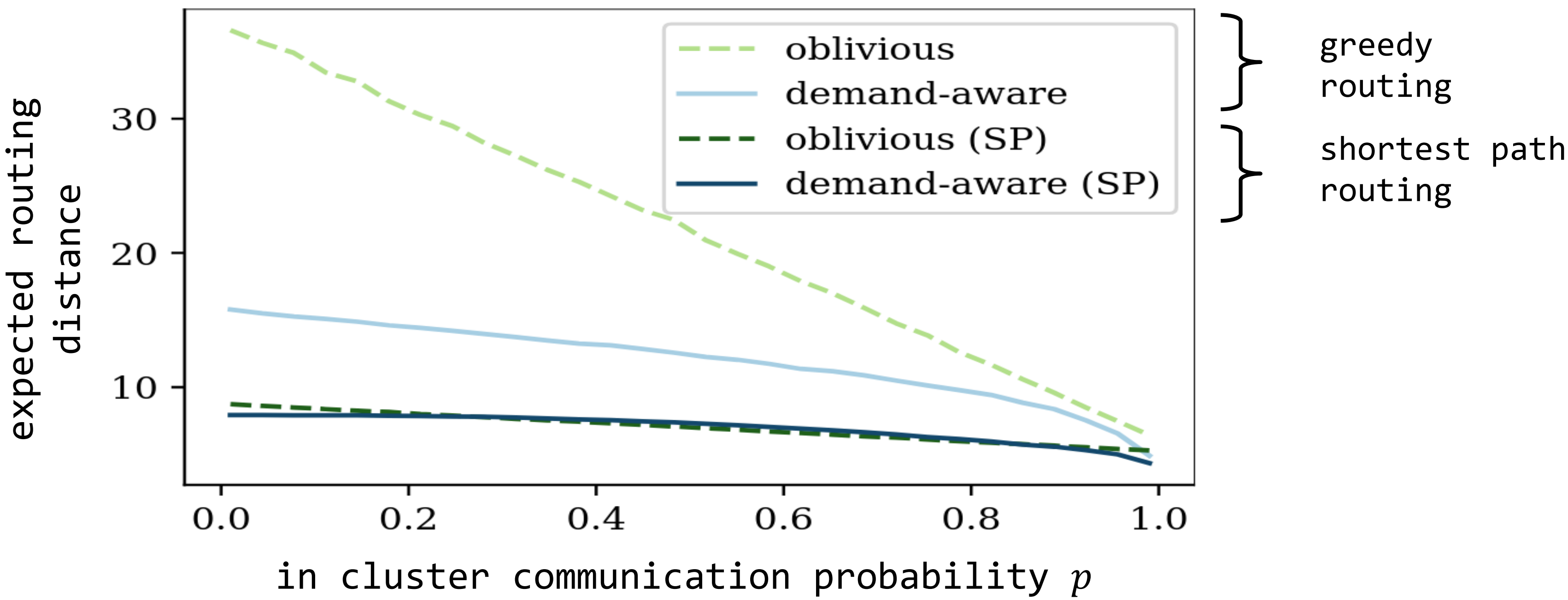
Lemma: Assume cluster sizes X are $PowerLaw(\alpha)$ distributed.
Then greedy routing takes in expectation:

$$O(\log(y) \cdot (4p \cdot \log\log n + q \cdot \log(y)))$$

Proof idea: we cannot use concentration bounds to upper bound the average size of the clusters we visit within r steps.

Rather, we upper bound it directly by dividing the cluster sizes into buckets. The average bucket size is upper bounded by assuming that the Greedy route "picks" cluster sizes uniformly at random.

Empirically...



fixed values: $x = 16, y = 32, n = 3xy$

Demand-aware significantly better for greedy routing and much inter-cluster communication: makes better shortcuts.

oblivious "Kleinberg"-like strategy: for a node v add an edge to a communicating node u with probability proportional to $d(v,u)^{-1}$

Thank you!