# Online algorithms with infused advice

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# The success of randomized algorithms

"We do not assume anything about the distribution of the instances of the problem to be solved. Instead we incorporate randomization into the algorithm itself... It may seem at first surprising that employing randomization leads to efficient algorithm."

- Michael Rabin, 1976

Randomized algorithms often perform exceptionally well in practice

Why?

This talk: an advice model for randomized algorithms

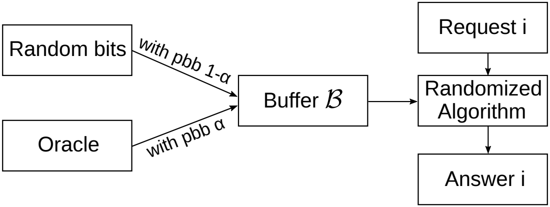
# Online algorithms and competitive analysis

- Stream of requests arrives one-by-one
- Algorithm answers without knowing the future
- Worst-case-oriented competitive analysis

$$CR(ALG) = \max_{\sigma} \frac{\mathbb{E}[ALG(\sigma)]}{OPT(\sigma)}$$

- The oblivious adversary (unaware of the algorithm's random choices)

# Our method: infused advice




(With probability $\alpha$ a clairvoyant oracle inserts "lucky" random bits)

# Infused advice: rationality

Randomized algorithms may enjoy good fortune (lucky random bits)

Infused advice measures impact of rounds with lucky random bits

# Our results

Analysis of three classic algorithms:

- **Caching**:
  RandomMark by Fiat, Karp, Luby, McGeoch, Sleator and Young
  (Journal of Algorithms 1991)
- **Uniform metrical task system**:
  algorithm by Borodin, Linial, Sacks
  (STOC 1987)
- **Online set cover**:
  primal-dual algorithm by Buchbinder and Naor
  (ESA 2005)

...and lower bounds for these problems

# Our results

| Problem | upper bound | lower bound |
|---|---|---|
| Caching | $\min\{2H_k, \frac{2}{\alpha}\}$ | $\min\{H_k, \frac{1}{\alpha}\}$ |
| Uniform MTS | $\min\{2H_n, \frac{2}{\alpha} + 2\}$ | $\min\{H_{n-1}, \frac{1}{\alpha}\}$ |
| Online set cover | $O(\min\{\log d \log n, \frac{\log n}{\alpha}\})$ | $\min\{\frac{1}{2} \log d, \frac{1}{2\alpha}\}$ |

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Lower bounds assumptions:

- Lazy algorithms
- Not storing random bits from the current round for future rounds

(Without assumptions about advice size)

# Reminder of this talk

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- Relaxed competitiveness: in expectation over random choices
- Limited power of the adversary: hiding the current state
- Increased power of the algorithm:
  opportunity to choose a cheaper execution path

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- Online list access: Move-To-Front vs Frequency Count (2-competitive vs $\Omega(n)$-competitive)
- Caching: First-In-First-Out vs Least Recently Used (k-competitive vs k-competitive)

# Beyond worst-case analysis (online algorithms)

1. Distributional analysis
2. Random arrival order
3. Smoothed analysis
4. Diffused adversaries
5. Parameterized analysis (e.g. access graph, locality of reference)
6. Resource augmentation
7. Loose competitiveness
8. Advice and predictions

...and many more; see a book "Beyond Worst Case Analysis", Tim Roughgarden

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2. Change the measure (resource augmentation, loose competitiveness)
3. Change the algorithm (advice, lookahead, predictions)

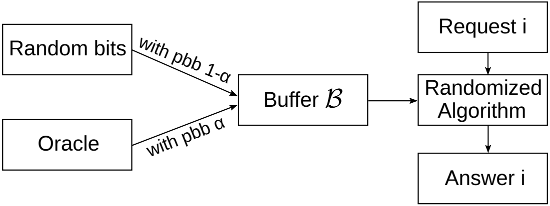
# Benefits of infused advice

- Can be applied to existing algorithms
- No assumptions about inputs
- Competitive ratio
- Lower bounds without bounding advice size

# Consequences

- Randomized algorithms design insights
- Beyond online algorithms: offline, distributed, streaming algorithms
- Augmenting existing randomized algorithms with predictions

# Summary

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