Appendix for: Quadratic Optimization Models for Balancing Preferential Access and Fairness: Formulations and Optimality Conditions

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There are three components in this appendix. Section A provides proofs of the lemmas and propositions that we omitted in the main text to preserve space. Section B details the estimation and sources of data that we summarize in Section 4. Section C provides additional figures and tables to supplement those in Section 5.

A. Supplemental propositions, lemmas, and examples

A.1. Three propositions

First, we provide proofs of the three propositions (restated below) that are used in the main text.

Proposition A.1. *The inequality*

$$\sum_{i\in I} U_i P_{i,j} x_{i,j} \le C_j y_j, \forall j \in J,$$

is a valid inequality for model (1).

Proof. See [34, Chapter 8]. □

Proposition A.2. *The model*

$$\begin{aligned}
\min_{x,y} \quad & \sum_{j\in J} C_j \big(1 - \frac{\sum_{i\in I} U_i P_{i,j} x_{i,j}}{C_j}\big)^2 \\
s.t. \quad & (1b) - (1f) \\
& 0 \le y_j \le 1 && \forall j \in J
\end{aligned}$$

provides an optimal solution for model (1).

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Proof. We prove that given a feasible solution for the x variables, the binary restrictions on the y variables can be reformulated with their continuous relaxations without loss of optimality. For all $j \in J$, we distinguish two cases in an optimal solution for the x variables: (i) $\exists i \in I : x_{i,j} > 0$, and (ii) $x_{i,j} = 0, \forall i \in I$.

(i) From equation (1f), we have $x_{i,j} = 1$ for this j. Then, it follows from equation (1e) that $y_j = 1$ in an optimal solution.

(ii) From equation (2b), we have $u_j = 0$ for this j. Then, $y_j = 0$ is feasible and achieves the same objective function value as any $y \in [0, 1]$. Hence, $y_j = 0$ is also optimal.

□

Proposition A.3. *The continuous relaxation of constraint* (1f) *is not a valid inequality for model* (1).

Proof. Consider an instance of model (1) with $|I| = |J| = B = 2$, $U_i = 10, P_{i,j} = 1, \forall i \in I, j \in J$, $C_1 = 10, C_2 = 15$. The optimal objective function of model (1) is $\frac{5}{3}$. Lower objective function values are obtained by allowing $x \in [0, 1]$ while maintaining y as binary; e.g., $x_{1,1} = 0.8, x_{1,2} = 0.2, x_{2,1} = 0, x_{2,2} = 1$ provides an objective function value of $1 < \frac{5}{3}$. □

A.2. A Lemma

Next, we provide a proof of Lemma 1 (restated below) that is used in the main text.

Lemma A.4. *In a feasible solution (x, y) to model (7), let $J_B = \{j \in J : \sum_{i\in I} U_i P_i x_{i,j} > 0\} \subseteq J$. Consider an $i^* \in I, j^* \in J$ such that $x_{i^*_,j^*} > 0$ (such a pair exists because of constraint (1d)). Then, there exists another feasible solution (x', y') that provides the same value of the objective function (7a) as (x, y), such that*

L.(i) $\sum_{i\in I} U_i P_i x'_{i,j} = \sum_{i\in I} U_i P_i x_{i,j},\ \forall j \in J$

L.(ii) $x'_{i^*_,j} > 0,\ \forall j \in J_B$.

Proof. Under the hypothesis, let

$$\tilde{J} = \{j \in J : \sum_{i\in I} U_i P_i x_{i,j} > 0, x_{i^*_,j} = 0\} \subseteq J_B. \tag{2}$$

By definition, $j^* \in J_B$ and $j^* \notin \tilde{J}$; thus, J_B is non-empty. If $\tilde{J} = \emptyset$, condition L.(ii) is already satisfied. Then, we define $x'_{i,j} \leftarrow x_{i,j}, \forall i \in I, j \in J$ and $y'_j \leftarrow y_j, \forall j \in J$. Then (x', y') trivially satisfies conditions L.(i) and L.(ii). In the rest of the proof we assume $\tilde{J}$ is non-empty.

From equations $\{(1e)\text{-}(1g)\} \cap (7b)$, we have (i) $x_{i^*_,j} = 0, y_j = 1, \forall j \in \tilde{J}$, (ii) $x_{i^*_,j} > 0, y_j = 1, \forall j \in J_B \backslash \tilde{J}$, and (iii) $x_{i^*_,j} = 0, y_j = \{0, 1\}, \forall j \in J \setminus J_B$.

Next, we define

$$i_j = \max\{i \in I \setminus \{i^*\} : x_{i,j} > 0\}, \quad \forall j \in \tilde{J} \tag{3a}$$

$$\tilde{I} = \cup_{j\in\tilde{J}} \{i_j\} \subseteq I \tag{3b}$$

$$J_i = \{j \in \tilde{J} : i_j = i\} \subseteq \tilde{J} \subseteq J_B, \quad \forall i \in \tilde{I}. \tag{3c}$$

Equation (3) defines a mapping between $\tilde{I}$ and $\tilde{J}$. Here, equation (3a) defines a mapping from $j \in \tilde{J}$ to $i \in I$, while equation (3c) defines the inverse set. We note that i_j is a single value $\forall j \in \tilde{J}$.

Under the hypothesis from equations (3a) and (3b), $i^* \neq i_j, \forall j \in \tilde{J}$, $i^* \notin \tilde{I}$, $j^* \notin J_i, \forall i \in \tilde{I}$ and $\sum_{i\in\tilde{I}} |J_i| = |\tilde{J}|$. We then have the following mutually exclusive and exhaustive partitions for both I and J:

$$I = \{i^*\} \sqcup \tilde{I} \sqcup (I \setminus (\tilde{I} \cup \{i^*\})) \tag{4a}$$

$$I = \{i^*\} \sqcup \{i_j\} \sqcup (I \setminus (\{i_j\} \cup \{i^*\})) \qquad \forall j \in \tilde{J} \tag{4b}$$

$$J = \{j^*\} \sqcup \tilde{J} \sqcup (J \setminus (\tilde{J} \cup \{j^*\})) \tag{4c}$$

$$J = \{j^*\} \sqcup J_i \sqcup (J \setminus (J_i \cup \{j^*\})) \qquad \forall i \in \tilde{I}, \tag{4d}$$

where $\sqcup$ denotes the disjoint union.

With this background, the idea of the proof is to modify x by reallocating some of the population assigned from i^* to j^* to those $j \in \tilde{J}$. To offset this change, we allocate the same amount of population from i_j to j^*. Formally, we construct this solution (x', y') via equations (4a), (4c) and (4d) for seven mutually exclusive and exhaustive subsets of $I \times J$:

$$
\begin{aligned}
& x'_{i^*j^*} \leftarrow x_{i^*j^*} - |\tilde{J}| \cdot \frac{\varepsilon}{U_{i^*}P_{i^*}} && && \text{(5a)} \\
& x'_{i^*j} \leftarrow \frac{\varepsilon}{U_{i^*}P_{i^*}}, && \forall j \in \tilde{J} && \text{(5b)} \\
& x'_{i^*j} \leftarrow x_{i^*j}, && \forall j \in J \setminus (\tilde{J} \cup \{j^*\}) && \text{(5c)} \\
& x'_{i,j^*} \leftarrow x_{i,j^*} + |J_i| \cdot \frac{\varepsilon}{U_i P_i}, && \forall i \in \tilde{I} && \text{(5d)} \\
& x'_{i,j} \leftarrow x_{i,j} - \frac{\varepsilon}{U_i P_i}, && \forall j \in J_i, i \in \tilde{I} && \text{(5e)} \\
& x'_{i,j} \leftarrow x_{i,j}, && \forall j \in J \setminus (J_i \cup \{j^*\}), i \in \tilde{I} && \text{(5f)} \\
& x'_{i,j} \leftarrow x_{i,j}, && \forall j \in J, i \in I \setminus (\tilde{I} \cup \{i^*\}) && \text{(5g)} \\
& y'_j \leftarrow y_j, && \forall j \in J. && \text{(5h)}
\end{aligned}
$$

We note that construction (5g) combines all three mutually exclusive and exhaustive subsets of J. In constructions (5a), (5b), (5d) and (5e), ε is defined as half the minimum of three positive quantities:

$$\varepsilon = \frac{1}{2} \min\{\varepsilon_1, \varepsilon_2, \varepsilon_3\} > 0, \tag{6}$$

where

$$
\begin{aligned}
\varepsilon_1 &= \frac{U_{i^*}P_{i^*}x_{i^*j^*}}{|\tilde{J}|} > 0, && \text{(7a)} \\
\varepsilon_2 &= \min_{i \in \tilde{I}} \left\{ \frac{U_i P_i \cdot (1 - x_{i,j^*})}{|\tilde{J}|} \right\} > 0, && \text{(7b)} \\
\varepsilon_3 &= \min_{j \in \tilde{J}} \{U_{i_j} P_{i_j} x_{i_j,j}\} > 0. && \text{(7c)}
\end{aligned}
$$

Here $\varepsilon_1 > 0$ since $x_{i^*j^*} > 0$ under the hypothesis, $\varepsilon_2 > 0$ follows from (1d) since $\tilde{I}$ is non-empty, and $\varepsilon_3 > 0$ follows from equation (3a).

Next, we show that (x', y') satisfies conditions L.(i) and L.(ii) and that it is a feasible solution to model (7) that provides the same value of the objective function (7a) as (x, y).

- We first show that (x', y') satisfies condition L.(i). We use the three subsets of J in equation (4c).

  (i) for $j = j^*$, we have $\sum_{i \in I} U_i P_i x'_{i,j^*} =$

$$
\begin{aligned}
& U_{i^*}P_{i^*}x'_{i^*j^*} + \sum_{i \in \tilde{I}} U_i P_i x'_{i,j^*} + \sum_{i \in I \setminus (\tilde{I} \cup \{i^*\})} U_i P_i x'_{i,j^*} && \text{(8a)} \\
=& U_{i^*}P_{i^*} \cdot (x_{i^*j^*} - |\tilde{J}| \cdot \frac{\varepsilon}{U_{i^*}P_{i^*}}) + \sum_{i \in \tilde{I}} U_i P_i \cdot (x_{i,j^*} + |J_i| \cdot \frac{\varepsilon}{U_i P_i}) + \sum_{i \in I \setminus (\tilde{I} \cup \{i^*\})} U_i P_i x_{i,j^*} && \text{(8b)} \\
=& U_{i^*}P_{i^*}x_{i^*j^*} - |\tilde{J}| \cdot \varepsilon + \sum_{i \in \tilde{I}} U_i P_i x_{i,j^*} + |\tilde{J}| \cdot \varepsilon + \sum_{i \in I \setminus (\tilde{I} \cup \{i^*\})} U_i P_i x_{i,j^*} && \text{(8c)}
\end{aligned}
$$

$$=\sum_{i\in I} U_i P_i x_{i,j^*} \tag{8d}$$

Here, equation (8b) holds from constructions (5a), (5d) and (5g), while equation (8c) uses that $\sum_{i\in\tilde{I}} |J_i| = |\tilde{J}|$.

(ii) for $j \in \tilde{J}$, we have $\sum_{i\in I} U_i P_i x'_{i,j} =$

$$U_{i^*} P_{i^*} x'_{i^*_,j} + U_{i_j} P_{i_j} x'_{i_j,j} + \sum_{i\in I\setminus(\{i_j\}\cup\{i^*\})} U_i P_i x'_{i,j} \tag{9a}$$

$$=U_{i^*} P_{i^*} \cdot \frac{\varepsilon}{U_{i^*} P_{i^*}} + U_{i_j} P_{i_j} \cdot (x_{i_j,j} - \frac{\varepsilon}{U_{i_j} P_{i_j}}) + \sum_{i\in I\setminus(\{i_j\}\cup\{i^*\})} U_i P_i x_{i,j} \tag{9b}$$

$$=\sum_{i\in I} U_i P_i x_{i,j}. \tag{9c}$$

Equation (9b) holds from constructions (5b), (5e) and (5g), while equation (9c) holds from the hypothesis that $x_{i^*_,j} = 0, \forall j \in \tilde{J}$.

(iii) for $j \in J \setminus (\tilde{J} \cup \{j^*\})$, we have $\sum_{i\in I} U_i P_i x'_{i,j} =$

$$U_{i^*} P_{i^*} x'_{i^*_,j} + \sum_{i\in\tilde{I}} U_i P_i x'_{i,j} + \sum_{i\in I\setminus(\tilde{I}\cup\{i^*\})} U_i P_i x'_{i,j} \tag{10a}$$

$$=U_{i^*} P_{i^*} x_{i^*_,j} + \sum_{i\in\tilde{I}} U_i P_i x_{i,j} + \sum_{i\in I\setminus(\tilde{I}\cup\{i^*\})} U_i P_i x_{i,j} \tag{10b}$$

$$=\sum_{i\in I} U_i P_i x_{i,j}. \tag{10c}$$

Equation (10b) holds from constructions (5c), (5f) and (5g).

- Next, we show that (x', y') satisfies condition L.(ii). We again use three subsets of $J_B \subseteq J$ as follows.

(i) for $j = j^*$, we have $x'_{i^*_,j^*} =$

$$x_{i^*_,j^*} - |\tilde{J}| \cdot \frac{\varepsilon}{U_{i^*} P_{i^*}} \geq x_{i^*_,j^*} - \frac{1}{2} x_{i^*_,j^*} = \frac{1}{2} x_{i^*_,j^*} > 0.$$

The first equality holds from construction (5a), while the first inequality follows from equations (6) and (7a). The last inequality holds from the hypothesis that $x_{i^*_,j^*} > 0$.

(ii) for $j \in \tilde{J}$, we have $x'_{i^*_,j} =$

$$\frac{\varepsilon}{U_{i^*} P_{i^*}} > 0.$$

The equality holds from construction (5b), while the inequality follows from equation (6).

(iii) for $j \in J_B \setminus (\tilde{J} \cup \{j^*\})$, we have $x'_{i^*_,j} =$

$$x_{i^*_,j} > 0.$$

The equality holds from construction (5c), while the inequality follows from the definitions of J_B and $\tilde{J}$.

Next, we show that (x', y') is a feasible solution to model (7) that provides the same value of the objective function (7a) as (x, y).

- (x', y') satisfy constraints (7b):
  This follows from condition L.(i).

- (x', y') satisfy constraints (1c):
  This follows from construction (5h).

- (x', y') satisfy constraints (1d):

  (i) for $i = i^*$, we have $x_{i^*j} =$

$$x'_{i^*j^*} + \sum_{j \in \tilde{J}} x'_{i^*j} + \sum_{j \in J \setminus (\tilde{J} \cup \{j^*\})} x'_{i^*j} \tag{11a}$$

$$= x_{i^*j^*} - |\tilde{J}| \cdot \frac{\varepsilon}{U_{i^*} P_{i^*}} + \sum_{j \in \tilde{J}} \frac{\varepsilon}{U_{i^*} P_{i^*}} + \sum_{j \in J \setminus (\tilde{J} \cup \{j^*\})} x_{i^*j} \tag{11b}$$

$$= x_{i^*j^*} + \sum_{j \in J \setminus (\tilde{J} \cup \{j^*\})} x_{i^*j} \tag{11c}$$

$$= \sum_{j \in J} x_{i^*j} \tag{11d}$$

$$= 1. \tag{11e}$$

  Equation (11b) holds from constructions (5a)-(5c). Equation (11d) follows since $x_{i^*j} = 0, \forall j \in \tilde{J}$, while equation (11e) follows from equation (1d).

  (ii) for $i \in \tilde{I}$, we have $\sum_{j \in J} x'_{i,j} =$

$$x'_{i,j^*} + \sum_{j \in J_i} x'_{i,j} + \sum_{j \in J \setminus (J_i \cup \{j^*\})} x'_{i,j} \tag{12a}$$

$$= x_{i,j^*} + |J_i| \cdot \frac{\varepsilon}{U_i P_i} + \sum_{j \in J_i} (x_{i,j} - \frac{\varepsilon}{U_i P_i}) + \sum_{j \in J \setminus (J_i \cup \{j^*\})} x_{i,j} \tag{12b}$$

$$= x_{i,j^*} + \sum_{j \in J_i} x_{i,j} + \sum_{j \in J \setminus (J_i \cup \{j^*\})} x_{i,j} \tag{12c}$$

$$= \sum_{j \in J} x_{i,j} \tag{12d}$$

$$= 1. \tag{12e}$$

  Equation (12b) holds from constructions (5d)-(5f), while equation (12e) follows from equation (1d).

  (iii) for $i \in I \setminus (\tilde{I} \cup \{i^*\})$, we have $\sum_{j \in J} x'_{i,j} =$

$$\sum_{j \in J} x_{i,j} = 1.$$

  The first equality holds from construction (5g), while the second equality follows from equation (1d).

- (x', y') satisfy constraints (1e). We use the seven subsets of $I \times J$ in constructions (5a)-(5g):

  (i) for $i = i^*, j = j^*$, we have $x'_{i^*_, j^*} =$

  $$x_{i^*_, j^*} - |\tilde{J}| \cdot \frac{\varepsilon}{U_{i^*} P_{i^*}} \leq x_{i^*_, j^*} \leq y_{j^*} = y'_{j^*}.$$

  The first equality holds from construction (5a), while the first inequality follows from $\varepsilon > 0$. The second inequality holds from equation (1e), while the second equality follows from construction (5h).

  (ii) for $i = i^*, j \in \tilde{J}$, we have $x'_{i^*_, j} =$

  $$\frac{\varepsilon}{U_{i^*} P_{i^*}} \leq \frac{1}{2} \frac{x_{i^*_, j^*}}{|\tilde{J}|} \leq y_j = y'_j.$$

  The first equality follows from construction (5b), while the first inequality follows from equations (6) and (7a). The second inequality holds from $x_{i^*_, j^*} \leq 1$—that is implied by equations (1d) and (1e)—and $|\tilde{J}| \geq 1$, while the last equality follows from construction (5h).

  (iii) for $i = i^*, j \in J \setminus (\tilde{J} \cup \{j^*\})$, we have $x'_{i^*_, j} =$

  $$x_{i^*_, j} \leq y_j = y'_j.$$

  The first equality holds from construction (5c), while the inequality follows from equation (1e). The last equality holds from construction (5h).

  (iv) for $i \in \tilde{I}, j = j^*$, we have x'_{i,j^*}=

  $$x_{i,j^*} + |J_i| \cdot \frac{\varepsilon}{U_i P_i} \leq x_{i,j^*} + |J_i| \cdot \frac{1 - x_{i,j^*}}{2|\tilde{J}|} \leq x_{i,j^*} + \frac{1 - x_{i,j^*}}{2} = \frac{1 + x_{i,j^*}}{2} \leq \frac{1 + y_{j^*}}{2} = y_{j^*} = y'_{j^*}.$$

  The first equality holds from construction (5d), while the first inequality follows from equations (6) and (7b). The second inequality holds since $|J_i| \leq |\tilde{J}|, \forall i \in \tilde{I}$. The third inequality holds from equation (1e), while the fourth equality follows from $y_{j^*} = 1$, which is implied by equation (1e) and the hypothesis that $x_{i^*_, j^*} > 0$. The last equality holds from construction (5h).

  (v) for $i \in \tilde{I}, j \in J_i$, we have $x'_{i,j} =$

  $$x_{i,j} - \frac{\varepsilon}{U_i P_i} \leq x_{i,j} \leq y_j = y'_j.$$

  The first equality holds from construction (5e), while the second inequality follows from equation (1e). The last equality holds from construction (5h).

  (vi) for $i \in \tilde{I}, j \in J \setminus (J_i \cup \{j^*\})$, we have $x'_{i,j} =$

  $$x_{i,j} \leq y_j = y'_j.$$

  The first equality holds from construction (5f), while the second inequality follows from equation (1e). The last equality holds from construction (5h).

(vii) for $i \in I \setminus (\tilde{I} \cup \{i^*\}), j \in J$, we have $x'_{i,j} =$

$$x_{i,j} \leq y_j = y'_j.$$

The first equality holds from construction (5g), while the second inequality follows from equation (1e). The last equality holds from construction (5h).

- (x', y') satisfy $x_{i,j} \geq 0, \forall i \in I, j \in J$:

(i) for $i = i^*, j = j^*$, we have $x'_{i^*_, j^*} =$

$$x_{i^*_, j^*} - |\tilde{J}| \cdot \frac{\varepsilon}{U_{i^*} P_{i^*}} \geq x_{i^*_, j^*} - \frac{1}{2} x_{i^*_, j^*} = \frac{1}{2} x_{i^*_, j^*} \geq 0.$$

The first equality holds from construction (5a), while the first inequality follows from equations (6) and (7a). The last inequality holds from equation (7b).

(ii) for $i = i^*, j \in \tilde{J}$, we have $x'_{i^*_, j} =$

$$\frac{\varepsilon}{U_{i^*} P_{i^*}} \geq 0.$$

The equality holds from construction (5b), while the inequality follows from $\varepsilon > 0$.

(iii) for $i = i^*, j \in J \setminus (\tilde{J} \cup \{j^*\})$, we have $x'_{i^*_, j} =$

$$x_{i^*_, j} \geq 0.$$

The equality holds from construction (5c), while the inequality follows from equation (7b).

(iv) for $i \in \tilde{I}, j = j^*$, we have $x'_{i,j^*}=$

$$x_{i,j^*} + |J_i| \cdot \frac{\varepsilon}{U_i P_i} \geq x_{i,j^*} \geq 0.$$

The equality holds from construction (5d), while the first inequality follows from $\varepsilon > 0$. The second inequality holds from equation (7b).

(v) for $i \in \tilde{I}, j \in J_i$, we have $x'_{i,j} =$

$$x_{i,j} - \frac{\varepsilon}{U_i P_i} \geq x_{i,j} - \frac{1}{2} x_{i,j} = \frac{1}{2} x_{i,j} \geq 0.$$

The first equality holds from construction (5e), while the first inequality follows from equations (6) and (7c). The second inequality holds from equation (7b).

(vi) for $i \in \tilde{I}, j \in J \setminus (J_i \cup \{j^*\})$, we have $x'_{i,j} =$

$$x_{i,j} \geq 0.$$

The equality holds from construction (5f), while the inequality follows from equation (7b).

(vii) for $i \in I \setminus (\tilde{I} \cup \{i^*\}), j \in J$, we have $x'_{i,j} =$

$$x_{i,j} \geq 0.$$

The equality holds from construction (5g), while the inequality follows from equation (7b).

- (x', y') satisfy constraints (1g):
  This follows from construction (5h).

Thus, the solution (x', y') is feasible. From condition L.(i), (x', y') provides the same value of the objective function (7a) as (x, y). $\square$

A.3. An Example

Example A.5. *Consider an instance of model (1) with $I = \{1,2\}, J = \{1,2\}, B = 2$, and let the populations U_i, capacities C_j and access $P_{i,j}, i \in I, j \in J$ be as illustrated in Fig. S1. Then, the only feasible solution is $y_1 = y_2 = 1$, $x_{1,2} = x_{2,1} = 1$ with utilization $u_1 = \frac{U_2 P_{2,1}}{C_1} = 0.75$ and $u_2 = \frac{U_1 P_{1,2}}{C_2} = 0.5$. Since $u_1 \neq u_2$, the allocation is not proportionally fair. The allocation is also not fair—consider $i = 2, j = 1, j' = 2$ in Definition 1; i.e., $i = 2$ is assigned to $j = 1$ even though $P_{2,1} < P_{2,2}$ and $u_1 \geq u_2$. This implies a DoF of less than 1; since the allocation has a warranted fairness for the other ordered pair of facilities $j = 1, j' = 2$, we have DoF $= 0.5$ and DoF$' = 0$.*

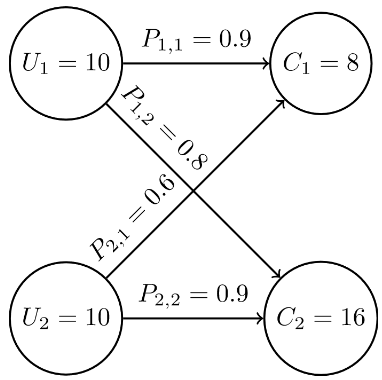


Figure S1: An instance of model (1) that has no solution with a fair or proportionally fair allocation. The two nodes on the left are users, and the two nodes on the right are facilities. For details, see Example A.5.

B. Estimation of data

B.1. Estimation of the set $i \in I$ and U_i

In this section, we provide our procedure to estimate the set of populations we use in Section 5 of the main text.

The smallest administrative unit in Germany for which data relevant to our study is available is the ZIP code. However, since ZIP codes are created by the federal post office [32], no official population statistics are available. We obtain ZIP code level population data for all ZIP codes, as well as the centroids of the ZIP codes, in Bavaria from suche-postleitzahl.org (Postleitzahlen Deutschland) [30]. Using grid-based population data of the 2011 census [28] and data on the boundaries of ZIP codes by OpenStreetMap available under the Open Database License [23], the Postleitzahlen Deutschland maps census data to ZIP codes. We do not include ZIP code 82475, which is reserved solely for Zugspitze—the highest mountain in Germany—and has no population; i.e., we consider all 2,060 inhabited ZIP codes in Bavaria. We multiply all populations with a population growth factor since 2011 (i.e., the most recent total Bavarian population divided by the 2011 Bavarian total population) to scale the data to current population numbers.

The Federal Ministry of Transport and Digital Infrastructure (BMVI) splits regions into two types—rural and urban [7]. This classification is based on the transportation requirements between communities near large cities and those further away by using the approach we summarize; for details, see [6]. First, the BMVI identifies large cities (primarily, cities with at least 100,000 inhabitants). Next, the catchment areas are determined—based on a commuting time of less than 30 minutes (by privately owned vehicles) or a commuter share of at least 25 percent to the nearest major city. These cities and municipalities are classified as urban, and all others as rural. Using this classification, the BMVI categorizes ZIP codes into rural and urban. We obtain a mapping between cities and ZIP codes from Postleitzahlen Deutschland [29]. Three ZIP codes contain several municipalities that are not uniformly classified as either urban or rural. We categorize these three based on where the majority of the population lives in. Thus, we obtain a classification of all the 2,060 ZIP codes as rural or urban.

B.2. Estimation of the set $j \in J$

In this section, we provide our procedure to estimate the set of recycling centers we use in Section 5 of the main text.

We were unable to obtain an official list of all the recycling centers in Bavaria. We create this list manually following the drop-down menu on the Abfallratgeber Bayern—a website jointly maintained by the Bavarian State Ministry of the Environment and Consumer Protection (StMUV) and the Bavarian State Office for the Environment (LfU) [4]; this website provides a listing of the authorities responsible for waste disposal in each of the Bavarian districts and district-free cities. The district Regen was missing from the drop-down menu; hence, we include its centers manually from [35]. The three districts of Kulmbach, Neustadt a.d. Waldnaab and Bayreuth do not follow the same concept of recycling centers as the other districts. For many types of waste these districts use collect-systems or have a multitude of containers set up; see, [16, 17, 18] for details. However, there still exist big facilities in these districts that, rather than accepting the whole spectrum of recyclable waste, specialize for certain types of recyclable waste. Instead of excluding these three districts, we opt to use the large facilities as proxies for recycling centers. For Kulmbach we include two landfill sites and one "waste transfer station", for Neustadt a.d. Waldnaab we include one landfill site, and for Bayreuth we include one "waste sorting facility" and two "collection points".

Following this process, we obtain a list of the exact latitudes and longitudes, as well as the ZIP codes, of all the recycling centers. We obtain 1,529 recycling centers, while 1,578 recycling centers are reported by the LfU in 2019 [2]. The discrepancy of the 49 recycling centers is in large part due to two districts, the districts of Aichach-Friedberg [19] and Neustadt a.d. Aisch [22], recently closing a substantial amount of their recycling centers in the context of structural reforms. The number of recycling centers we obtain in all the other districts and district-free cities is the same or nearly-same to the reported numbers [3], with the exception of the district of Neustadt a.d. Waldnaab where no information on recycling centers is available.

B.3. Estimation of C_j

As we mention in Section 4.1, we were unable to obtain explicit data on the capacity of the recycling centers, and we assume the capacities are proportionate to the amount of waste that is returned to the respective recycling centers. To approximate this, we use the notion of the "catchment population" that is pervasive in geographic information systems. Different methods for estimating catchment populations and/or the respective catchment areas exist. Some authors use empirical data on users' origin [15, 21, 31], including the BMVI (see B.1). Others define the catchment population of a facility based on proximity to the facility [1, 20]. We adopt the approach of [20] and define the catchment population of each recycling center as the number of people that are nearest, or most accessible, to this recycling center.

The 2011 census of Germany provides grid-based population data [28], and thus the resident-population of each 100m $\times$ 100m grid cell is available. Using open-source data from the Federal Agency for Cartography and Geodesy (BKG) we next link each grid cell to the corresponding states of Germany [5]; we filter the population of all the grid cells in Bavaria from this data. The grid cell data uses the ETRS89 Lambert Azimuthal Equal-Area (ETRS89 LAEA) projection coordinate reference system, while the manually collected coordinates of the recycling centers (see, Section B.2) use the World Geodetic System 1984 (WGS 84) coordinate system. We convert the data from WGS 84 to ETRS89 LAEA using an online coordinate converter available at `https://tool-online.com/en/` [33]. For an overview on coordinate reference systems, see, e.g [9, Chapter 2].

Next, we calculate the recycling center that is closest to each grid cell's centroid using the `geopy` Python client [14]; `geopy` uses an ellipsoidal model to compute the shortest distance on the surface of the earth. In order to significantly reduce the computationally expensive calls to `geopy`'s distance function, we obtain the catchment population by proceeding according to Algorithm S1. Instead of computing the distances from each of the more than 7 million grid cells to each of the 1,529 recycling centers, we now compute on average the distance to only 8.14 recycling centers from each grid cell.

Algorithm S1 goes through each grid cell to determine the closest recycling center in the following manner. In Step 4 of the algorithm, a "box" around the grid cell's centroid is created that contains only those recycling centers whose latitude and longitude differ by less than a tenth of a degree from the grid cell's centroid's latitude and longitude. If this box does not contain any recycling centers, we iteratively increase the size of the box by another tenth of a degree of latitude and longitude until is not empty anymore. In Step 10, we compute the closest recycling center to the grid cell's centroid and its corresponding distance d^* among those that are contained in the box. To ensure that no recycling center outside of the box can be closer to the grid cell's centroid, in Step 12 we compare d^* to the minimum distance d that a recycling center outside of the box could possibly have to the grid cell's centroid; i.e., the distance to the location that has the same latitude as the grid cell's centroid but whose longitude differs just enough to be outside of the box. If d is greater than or equal to d^*, the corresponding recycling center is indeed guaranteed to be the closest among all recycling centers. Otherwise, we stretch the box one last time along each axis by a factor of $\frac{d^*}{d} > 1$, which ensures that every location outside of the box is at least d^* away. Thus, in Step 18 we choose the closest recycling center as the recycling center with the minimum distance among the recycling centers inside the box. Next, we add the population of the grid cell to the catchment population of its closest recycling center.

For ZIP codes with more than one recycling center, we add up the catchment population of all the centers in the ZIP code. Finally, we scale the resulting catchment population with Germany's population growth factor since 2011. This factor is 1.5. A factor larger than 1 reflects the notion that more than sufficient capacity is available; i.e., not all recycling centers need to be opened, or a value of $B < |J|$ provides a feasible solution for model (1). This number provides our estimate of the capacity of facility j, or C_j.

B.4. Comparison of the amount of rural and urban waste production

Previous work considers the notion that the amount of waste produced in rural and urban regions are different. In a study from England, the authors find rural regions produce more waste than urban regions [12]. In contrast, in a study from Cyprus, the authors find urban regions dispose of more waste than rural regions [26]; similarly, the authors in [10] find regions with larger populations produce more per-capita waste.

We obtain data on the annual amounts of household waste produced for all district and so-called district-free cities ("Kreisfreie Städte") in Bavaria from the Regionaldatenbank Deutschland, a database maintained by the statistical offices of the federal government and the federal states [27]. For each district and district-free city in Bavaria, we sum up the populations of all of its urban and rural municipalities to classify whether it is predominantly urban or rural. We classify 28 districts and district-free cities (henceforth, "cities" in this section) as urban and 68 as rural. Thus, we have data for the amount of waste produced per person in each of the 28 urban and 68 rural cities.

In our sample, the average amount of waste produced per person per year is $\bar{U} = 474.0$ kg in urban regions and $\bar{R} = 502.8$ kg in rural regions. The standard deviation of the average amount of waste produced per person per year in the 68 rural cities is $s_R = 80.2$ kg while that in the 28 urban cities is $s_U = 40.4$ kg; i.e., the variance in the rural cities is about four times larger than that in the urban cities. We test the following hypothesis.

Algorithm S1 Estimating the catchment population of recycling centers

Input: WGS84-coordinates (lat_r, lon_r) of all recycling centers $r \in R$; WGS84-coordinates of the centroids (lat_z, lon_z) of all grid cells $z \in Z$ in Bavaria and their population U_z; a function *distance* that computes the distance between two pairs of WGS84-coordinates.

Output: the catchment population G_r of each recycling center $r \in R$, as defined in Section B.3.

$G_r \leftarrow 0, \forall r \in R$
for $z \in Z$
  $M \leftarrow \emptyset; \varepsilon \leftarrow 0; lat_l \leftarrow lat_z; lat_u \leftarrow lat_z; lon_l \leftarrow lon_z; lon_u \leftarrow lon_z.$
  while $M = \emptyset$
    $\varepsilon \leftarrow \varepsilon + 0.1$
    $lat_l \leftarrow lat_l - \varepsilon; lat_u \leftarrow lat_u + \varepsilon; lon_l \leftarrow lon_l - \varepsilon; lon_u \leftarrow lon_u + \varepsilon.$
    for $r \in R$
      if $lat_l < 1at_r < lat_u$ and $lon_l < lon_r < lon_u$
        $M \leftarrow M \cup \{r\}.$
  $d^* \leftarrow \min_{r \in M} distance((lat_z, lon_z), (lat_r, lon_r));$
  $r^* \leftarrow \arg\min_{r \in M} distance((lat_z, lon_z), (lat_r, lon_r)).$
  $d \leftarrow distance((lat_z, lon_z), (lat_z, lon_z + \varepsilon)).$
  if $d^* \geq d$
    $\varepsilon \leftarrow \varepsilon \cdot \frac{d^*}{d}$
    $lat_l \leftarrow lat_z - \varepsilon; lat_u \leftarrow lat_z + \varepsilon; lon_l \leftarrow lon_z - \varepsilon; lon_u \leftarrow lon_z + \varepsilon.$
    for $r \in R \setminus M$
      if $lat_l < 1at_r < lat_u$ and $lon_l < lon_r < lon_u$
        $M \leftarrow M \cup \{r\}.$
    $r^* \leftarrow \arg\min_{r \in M} distance((lat_z, lon_z), (lat_r, lon_r)).$
  $G_{r^*} \leftarrow G_{r^*} + U_z.$
Return: $G_r, \forall r \in R.$

$$H_0 : \mu_R = \mu_U$$
$$H_A : \mu_R \neq \mu_U,$$

where μ_R and μ_U denote the means of the waste produced per person per year in the rural and urban cities, respectively. We use the Welch t-test (see, e.g., [8]), with a test statistic of $t = \frac{\bar{R} - \bar{U}}{s_{\bar{\Delta}}}$ where $s_{\bar{\Delta}} = \sqrt{\frac{s_R^2}{68} + \frac{s_U^2}{28}}$. Then, we reject H_0 with a p-value of 0.022; we calculate a corresponding t-value of 2.33. Thus, there is statistically significant evidence that $\mu_R > \mu_U$. We inflate the capacities of the recycling centers in rural regions by the ratio of the two averages; i.e., by a factor of $\frac{\bar{U}}{\bar{R}} = 1.06$.

B.5. Estimation of $P_{i,j}$

In this section, we provide our procedure to estimate the preferences of populations in ZIP code i to recycling centers in ZIP code j that we use in Section 5 of the main text.

Our travel model resembles that developed in [24] for estimating peoples' willingness to travel in order to obtain antiviral drugs based on the travel distances. Similar travel models have also been employed in determining preferences to postal facilities [25]. We use data from Mobility in Germany (MiD) [11]—a nation-wide survey commissioned by the German Federal Ministry of Transport and Digital Infrastructure and carried out by the Institute for Applied Social Sciences; for details, see [13]. We use the dataset from 2017 that includes a total of 618,245 person-trips. For our estimation, we filter to only include trips for the purpose of running errands, and our data includes a total of 50,195 and 86,744 person-trips originating from rural and urban locations, respectively. We describe our model below.

Index set:

$k \in K$ set of distance bins from MiD database; $K = \{0, 1, \dots, 8\}$

Parameters:

$d_k \in \mathbb{R}_0^+$ left endpoint of distance in bin k [kilometers]

$T_k \in \mathbb{N}_0$ MiD person-trips in bin k

Model:

$\hat{P}(d) \in [0, 1]$ fraction of target population willing to travel at least d kilometers:

$$\hat{P}(d) = \begin{cases} a_1 \exp(-b_1 d^{c_1}) & \text{if } d < t \\ a_2 \exp(-b_2 d^{c_2}) & \text{if } d \geq t, \end{cases} \tag{14}$$

where all the parameters (a_1, a_2, b_1, b_2, c_1, c_2, and t) are positive. Model (14) reflects the notion that the fraction of people traveling a given distance decreases exponentially as the distance grows. Further, by including a threshold of t kilometers, we allow different travel behaviors for distances above and below this threshold. In other words, we allow the data to determine different models for preferences for locations that are closer and for locations that are farther away. We use $t = 5$ kilometers.

The MiD study uses the following distance intervals to record trips (in kilometers): $[0, 0.5)$, $[0.5, 1)$, $[1, 2)$, $[2, 5)$, $[5, 10)$, $[10, 20)$, $[20, 50)$, $[50, 100)$, and $[100, \infty)$; i.e., $d_0 = 0, d_1 = 0.5, d_2 = 1, d_3 = 2, d_4 = 5, d_5 = 10, d_6 = 20, d_7 = 50, d_8 = 100$. By summing up the number of person-trips of distances that are at least d_k and dividing the result by the total numbers of person-trips, we obtain an empirical value for the excess distribution of $P(d)$:

$$P(d_k) = \frac{\sum\limits_{l \geq k} T_l}{\sum\limits_{l \geq 0} T_l} \,. \tag{15}$$

Using a least-squares fit on our data, we obtain the following models:

$$\hat{P}_{urban}(d) = \begin{cases} \exp(-0.255d^{0.867}) & \text{if } d < 5\text{km} \\ 4.639\exp(-1.499d^{0.329}) & \text{if } d \geq 5\text{km}, \end{cases} \tag{16a}$$

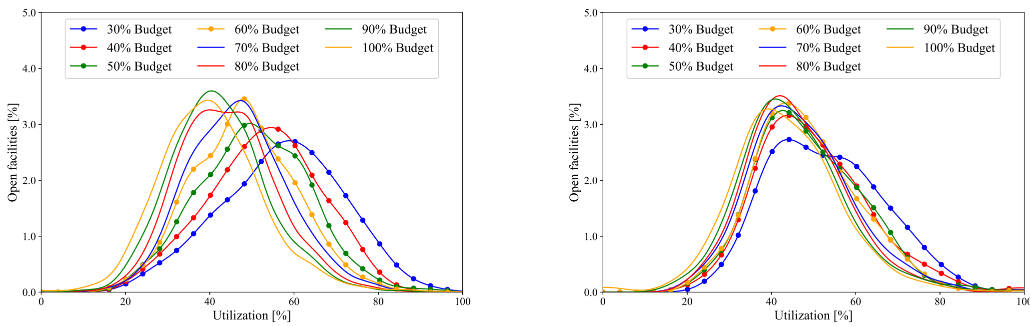
$$\hat{P}_{rural}(d) = \begin{cases} \exp(-0.250d^{0.820}) & \text{if } d < 5\text{km} \\ 1.611\exp(-0.689d^{0.437}) & \text{if } d \geq 5\text{km}. \end{cases} \tag{16b}$$

Models (16a) and (16b) fit the data well, as evidenced by R^2-values greater than 0.996 for both above and below the 5 kilometers threshold. We provide a plot of the two travel models, along with the MiD data, in Fig. 1 of the main text. Here the solid lines are given by equations (16a) and (16b) and the MiD data is given by circles. The willingness to travel with the purpose of running errands is slightly larger for person-trips originating in the rural regions.

Next, we use models (16a) and (16b) to determine the preferences, $P_{i,j}$. To this end, we first compute the geodesic distance between the centroids of all ZIP codes and the locations of the recycling centers. We have data for the latitude and longitude of the recycling centers, for details, see Appendix B.2; thus, their locations are given precisely. For ZIP codes that contain more than one recycling center, we use the centroid of those centers as the location of that ZIP code's recycling center. Since the locations are close to each other, the centroid's latitude and longitude is approximately the mean of the latitudes and longitudes, respectively. Then, the distances between each ZIP code's centroid and the recycling centers coupled with equations (16a) and (16b) provide us our estimates of the probability, $P_{i,j}$.

C. Supplemental figures and tables

In this section we present figures and tables to supplement those in Section 5. Figure S2 shows the distribution of the utilization of open facilities in rural (Figure S2a) and urban (Figure S2b) regions. Table S1 quantifies the effect of removing (i, j) combinations with a preference of less than 20% on the computational performance. Figure S3 illustrates that in an optimal solution the access of open facilities is roughly proportional to their capacity.



(a) rural **(b)** urban

Figure S2: Distribution of utilization for different budgets for open facilities in rural (Fig S2a) and urban (Fig S2b) regions. Here, the y-axis provides the relative frequency of the open facilities. A broad distribution suggests a large dispersion. For the analogous figure on the distribution of the utilization of facilities regardless of rural or urban, see Fig. 3b in Section 5.2. For details, see Section 5.2.

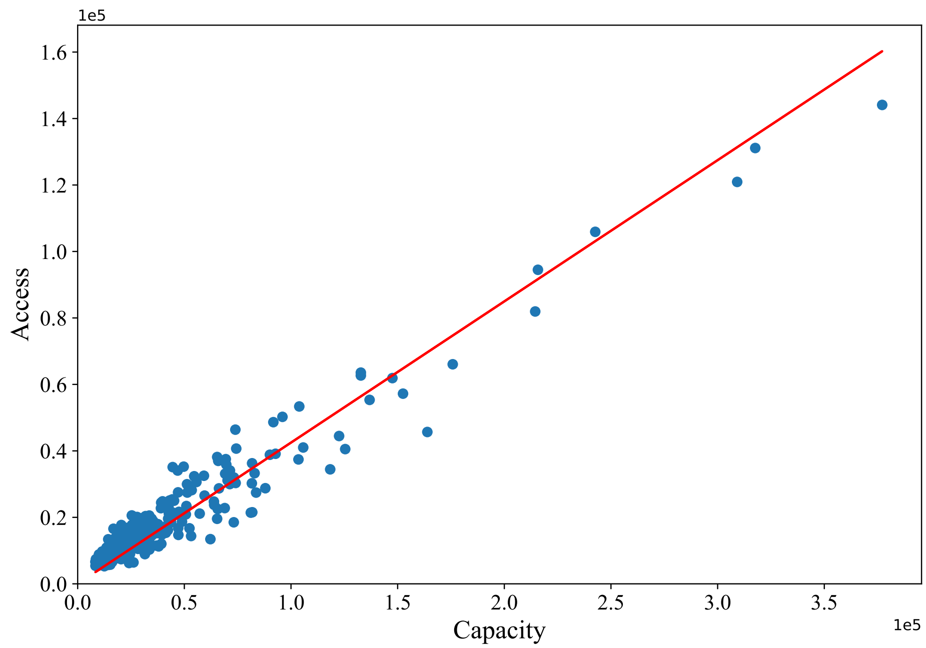


Figure S3: Comparison of the capacities (x-axis) and the access $\sum_{i \in I} U_i P_{i,j} x_{i,j}$ (y-axis) of the facilities that are open in an optimal solution of model (1) with a 30% budget. The blue dots show the 418 open facilities, while the red line fits a linear function, $y = 0.425x$, to these points. The plot shows a strong linear correlation reflected by a R^2-value of 0.909.

Instance		**Objective value**				**Time [s]**		
$\|I\|$	$\|J\|$	0% cutoff	20% cutoff	Δ	Guarantee	0% cutoff	20% cutoff	Δ
206	139	1,807,898.0	1,807,898.0	0.0%	0.5%	23.1	257.5	-1,016.8%
206	279	3,153,063.5	3,149,664.8	0.1%	0.3%	6.3	37.8	-497.5%
412	139	1,422,981.7	1,427,853.6	-0.3%	2.4%	T	427.8	96.0%
412	279	2,045,682.0	2,045,735.4	0.0%	2.1%	T	235.6	97.8%
412	418	3,517,002.0	3,516,904.6	0.0%	0.4%	114.8	73.6	35.9%
618	279	1,843,062.5	1,848,640.0	-0.3%	3.8%	T	6,357.1	41.1%
618	418	2,543,381.0	2,544,447.5	0.0%	3.1%	T	638.5	94.1%
618	558	4,186,160.3	4,186,269.9	0.0%	0.8%	T	44.9	99.6%
824	418	2,415,860.6	2,421,358.4	-0.2%	4.1%	T	1,150.7	89.3%
824	558	3,306,477.5	3,308,252.8	-0.1%	3.6%	T	512.3	95.3%
824	697	4,304,657.5	4,305,180.5	0.0%	1.7%	T	89.5	99.2%
1,030	558	3,234,110.6	3,236,474.9	-0.1%	4.3%	T	370.4	96.6%
1,030	697	3,815,682.8	3,816,522.3	0.0%	3.8%	T	272.4	97.5%

Table S1: Comparison of the computational performance of the implementation described in Section 5.1 where (i, j) combinations with a respective probability of less than 20% are removed ("20% cutoff") with the naive implementation where none of the (i, j) combinations are removed ("0% cutoff"). All instances are solved with a 70% budget, a maximum time limit of 3 hours, and an optimality gap of 0.5%. The "Δ" columns denote the relative difference between the two implementations; here, a positive Δ suggests that applying a 20% cutoff provides an improvement over the implementation without a cutoff. The "Guarantee" column denotes the conservative gap between the upper bound reported by the implementation with a 20% cutoff and the lower bound reported by the implementation without a cutoff. Entries marked with "T" in the "Time" columns denote the maximum time limit is reached. All entries are rounded to the first decimal place. For details, see Section 5.3.

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