

TAR²: Temporal-Agent Reward Redistribution for Optimal Policy Preservation in Multi-Agent Reinforcement Learning

Aditya Kapoor¹, Kale-ab Tessa², Mayank Baranwal³ and Harshad Khadilkar³

Stefano Albrecht²

Mingfei Sun¹

¹University of Manchester

²University of Edinburgh

³Indian Institute of Technology, Bombay

Abstract

In cooperative multi-agent reinforcement learning (MARL), learning effective policies is challenging when global rewards are sparse and delayed. This difficulty arises from the need to assign credit across both agents and time steps, a problem that existing methods often fail to address in episodic, long-horizon tasks. We propose *Temporal-Agent Reward Redistribution* (TAR²), a novel approach that decomposes sparse global rewards into agent-specific, time-step-specific components, thereby providing more frequent and accurate feedback for policy learning. Theoretically, we show that TAR² (i) aligns with potential-based reward shaping, preserving the same optimal policies as the original environment; and (ii) maintains policy gradient update directions identical to those under the original sparse reward, ensuring unbiased credit signals. Empirical results on two challenging benchmarks—SMACLite and Google Research Football—demonstrate that TAR² significantly stabilizes and accelerates convergence, outperforming strong baselines like AREL and STAS in both learning speed and final performance. These findings establish TAR² as a principled and practical solution for agent-temporal credit assignment in sparse-reward multi-agent systems.¹§

1 Introduction

Multi-agent reinforcement learning (MARL) enables autonomous agents to cooperate on complex tasks across a wide range of domains, including warehouse logistics [Krnjaic *et al.*, 2022], e-commerce [Shelke *et al.*, 2023; Baer *et al.*, 2019], robotics [Sartoretti *et al.*, 2019; Damani *et al.*, 2021], and routing [Zhang *et al.*, 2018; Vinitksy *et al.*, 2020; Zhang *et al.*, 2023]. MARL has also shown great promise in high-stakes coordination challenges in video games such as StarCraft II [Vinyals *et al.*, 2019], DOTA [Berner *et al.*, 2019], and Google Football [Kurach *et al.*, 2020], where teams of agents must align actions toward a shared goal.

¹Codebase: https://github.com/AdityaKapoor74/MARL-Agent_Temporal_Credit_Assignment

However, one of the most persistent bottlenecks in cooperative MARL is *credit assignment*: deciding how to allocate a global team reward among multiple agents in partial observability and decentralized execution. This bottleneck is particularly acute when rewards are *sparse or delayed*, which causes agents to struggle in correlating their intermediate actions with eventual outcomes [Arjona-Medina *et al.*, 2019; Ren *et al.*, 2021]. In such settings, there is a need to determine *when* along a multi-step trajectory the helpful actions occurred (*temporal credit assignment*) and *which* agent or subset of agents were responsible (*agent credit assignment*). Prior works often focus on only one of these aspects at a time—e.g., factorizing value functions to identify agent-specific credits [Sunehag *et al.*, 2017; Rashid *et al.*, 2020] or building dense temporal proxies to mitigate delayed rewards [Arjona-Medina *et al.*, 2019; Xiao *et al.*, 2022]. Yet, managing both agent and temporal credit assignment *simultaneously* remains challenging, especially in high-dimensional tasks where partial observability further complicates the learning problem [Papoudakis *et al.*, 2020].

In this paper, we propose *Temporal-Agent Reward Redistribution* (TAR²), a novel framework that jointly addresses *agent* and *temporal* credit assignment in cooperative MARL with sparse global rewards. Rather than relying solely on value-function factorization or temporal heuristics, TAR² **re-distributes** the final episodic reward across each time step and each agent, guided by a learned model (Figure 1). Concretely, TAR² maps sparse team returns into time-step-specific feedback signals and then allocates these among agents according to each agent’s (marginal) contribution, approximated via a dual-attention mechanism. Crucially, our approach is built upon *potential-based reward shaping* [Ng, 1999; Devlin and Kudenko, 2011], ensuring that the optimal policies of the original Markov Decision Process (MDP) are preserved in our reshaped rewards. This preserves *optimal policy invariance*, while providing finer-grained credit signals to accelerate learning.

We validate TAR² both through *theoretical guarantees* and *empirical results on challenging benchmarks*.

Contributions. Summarized below are our key contributions:

- **Unified Agent-Temporal Credit Assignment.** We develop TAR², a single framework that decomposes

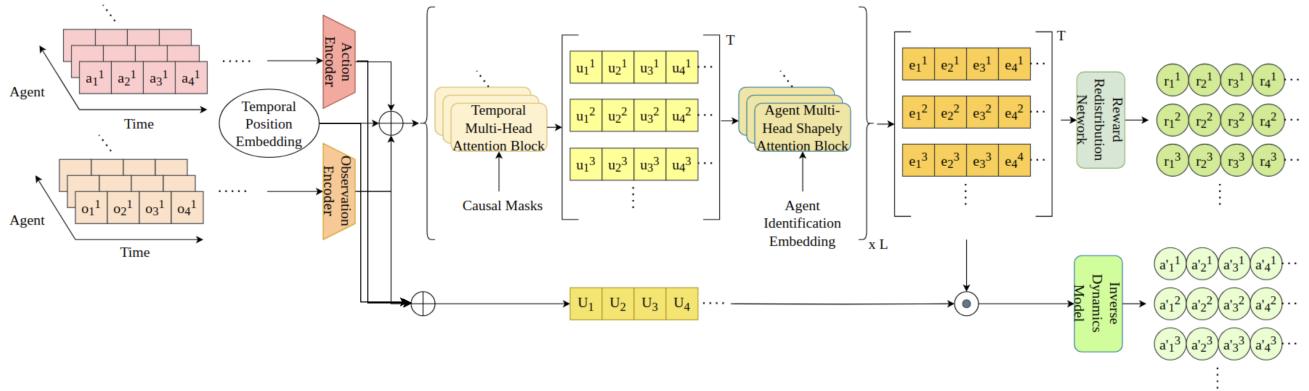


Figure 1: **Overview of TAR².** Our dual-attention mechanism uses a *temporal causal attention* module and an *agent Shapley attention* module to decompose rewards by time steps and agents. An inverse dynamics model further refines the temporal representation, and the final output yields per-agent, per-timestep redistributed rewards that enables effective reward attribution.

episodic rewards both across agents *and* over time, bridging the gap between factorization-based methods and purely temporal reward-shaping methods.

- **Policy Invariance via Potential-Based Shaping.** We formally prove that TAR²’s redistributed reward function preserves the original environment’s optimal policies and maintains equivalent gradient directions. To effectively capture agent contributions and temporal dependencies, we employ a dual-attention architecture that integrates Shapley attention for fair credit assignment, an inverse dynamics model to learn robust temporal representations, and final-state conditioning to link intermediate actions with final outcomes.

- **Robust Empirical Performance.** Through extensive experiments on challenging configurations of SMACLite and Google Research Football, TAR² surpasses strong baselines in convergence speed and final return under sparse rewards.

2 Related Works

In this section, we review and compare various methods addressing credit assignment in both single-agent and multi-agent reinforcement learning (MARL). While single-agent methods focus primarily on temporal credit assignment, multi-agent methods must manage the additional complexity of agent-specific credit assignment—particularly under sparse or delayed rewards. Our discussion emphasizes the unique challenge of combined agent-temporal credit assignment and highlights how existing approaches differ from our proposed solution, TAR².

2.1 Temporal Credit Assignment

Temporal credit assignment aims to decompose sparse or episodic rewards into informative, time-step-specific feedback, which is critical for learning in long-horizon tasks with delayed outcomes.

Early single-agent methods like RUDDER [Arjona-Medina *et al.*, 2019] redistribute rewards by analyzing return contributions at each step. While effective in single-agent settings, RUDDER depends on accurate return predictions and does not extend naturally to multi-agent scenarios where individual contributions must also be identified. Sequence modeling approaches [Liu *et al.*, 2019; Han *et al.*, 2022] use architectures like Transformers [Vaswani *et al.*, 2017] to capture long-term dependencies, but they similarly focus on temporal aspects without addressing agent-level credit.

Trajectory-based methods [Ren *et al.*, 2021; Zhu *et al.*, 2023] learn proxy rewards via smoothing and bi-level optimization; however, these approaches generally assume a single agent and do not account for multiple cooperating agents. Hindsight Policy Gradients (HPG) [Harutyunyan *et al.*, 2019] retrospectively assign rewards using future trajectory information but are again tailored to single-agent scenarios.

In multi-agent contexts, AREL [Xiao *et al.*, 2022] extends temporal credit assignment using attention mechanisms to redistribute rewards over time. Although AREL handles temporal dependencies among agents, it primarily focuses on the temporal domain and does not fully capture agent-specific contributions, particularly under sparse reward conditions. In contrast, our method, TAR², jointly addresses both temporal and agent-specific credit assignment, which is crucial for multi-agent systems with sparse rewards.

2.2 Agent Credit Assignment

Agent credit assignment seeks to allocate portions of a global reward to individual agents based on their contributions, a key aspect of learning cooperative policies.

Difference rewards and counterfactual methods, such as COMA [Foerster *et al.*, 2018; Devlin *et al.*, 2014], compute agent-specific advantages by considering counterfactual scenarios, but these approaches typically assume denser feedback and struggle when rewards are sparse or significantly delayed. Value function factorization methods like VDN [Sune-

hag *et al.*, 2017] and QMIX [Rashid *et al.*, 2020] decompose joint value functions into agent-specific components, improving scalability and coordination in environments with frequent rewards. However, their factorization assumptions may not hold in highly sparse settings, and they often do not explicitly handle the temporal aspect of credit assignment.

Shapley value-based approaches [Wang *et al.*, 2020] offer fair attribution of global rewards to agents based on marginal contributions, but exact computation is intractable in large systems, and approximations can miss subtle inter-agent dynamics—especially when rewards are sparse. Attention-based critics like PRD [Freed *et al.*, 2022; Kapoor *et al.*, 2024a,b] focus on identifying important agent interactions but still rely on assumptions about reward structure that may not hold in sparse-feedback scenarios.

TAR^2 distinguishes itself by directly addressing the challenges of sparse, delayed rewards through joint temporal and agent-specific redistribution, bypassing the limitations of TD-based bootstrapping methods. This dual focus makes TAR^2 more robust in sparse environments where previous agent credit assignment methods struggle.

2.3 Combined Agent-Temporal Credit Assignment

Simultaneously addressing agent and temporal credit assignment greatly increases complexity, as it requires reasoning over high-dimensional interactions across both agents and time.

Recent works like She *et al.* [2022] attempt combined credit assignment using attention-based methods, but they face scalability issues and do not offer theoretical guarantees. STAS [Chen *et al.*, 2023] specifically tackles joint agent-temporal credit assignment by employing a dual transformer structure with spatial-temporal attention mechanisms and Shapley value approximations. While STAS successfully decomposes rewards across time and agents, it lacks clear theoretical guarantees of policy optimality invariance and can be unstable in highly sparse environments.

In contrast, TAR^2 introduces several key enhancements over STAS:

- **Theoretical Guarantees:** Unlike STAS, TAR^2 is grounded in potential-based reward shaping, ensuring that the redistributed rewards preserve the optimal policy of the original environment.
- **Enhanced Architecture:** TAR^2 extends the dual-attention approach by incorporating an inverse dynamics model and conditioning rewards on final state embeddings. These additions improve the accuracy of temporal and agent-specific credit assignment, particularly in long-horizon tasks with sparse rewards.
- **Stability in Sparse Environments:** By normalizing rewards and leveraging theoretical insights, TAR^2 exhibits greater stability and improved performance compared to prior methods like STAS and AREL in sparse-reward settings.

While other methods attempt similar decompositions, TAR^2 's combination of theoretical guarantees, architectural innovations, and focus on sparse rewards sets it apart as a

novel and robust solution for joint agent-temporal credit assignment.

3 Background

In this section, we present the foundational concepts and problem setup that underpin our method. We begin by reviewing the standard framework of *decentralized partially observable Markov decision processes* (Dec-POMDPs) [Oliehoek and Amato, 2016; Amato, 2024], which formalizes many cooperative multi-agent reinforcement learning (MARL) tasks. We then focus on the *episodic* version of MARL with sparse or delayed rewards, highlighting why these settings pose unique credit-assignment challenges. Finally, we introduce *potential-based reward shaping* [Ng, 1999; Devlin and Kudenko, 2011], the key theoretical tool we build upon to preserve optimal policies while reshaping rewards.

3.1 Decentralized Partially Observable Markov Decision Processes (Dec-POMDPs)

A Dec-POMDP is defined by the tuple

$$\mathcal{M} = (\mathcal{S}, \{\mathcal{A}_i\}_{i=1}^N, \mathcal{P}, \{\mathcal{O}_i\}_{i=1}^N, \{\pi_i\}_{i=1}^N, \mathcal{R}_\zeta, \rho_0, \gamma),$$

where N agents interact in an environment with states $s \in \mathcal{S}$. Each agent $i \in \{1, \dots, N\}$ selects an action a_i from its action space \mathcal{A}_i , and receives an observation o_i from its observation space \mathcal{O}_i . The observation o_i is generated according to the observation function $\mathcal{T}(o_i \mid s, i)$, which defines the probability of agent i receiving observation o_i given the current state s . The transition function $\mathcal{P}(s_{t+1} \mid s_t, a_t)$ governs how states evolve, and ρ_0 is the initial state distribution. Agents operate according to a *joint policy* $\pi = \prod_{i=1}^N \pi_i$, where each π_i conditions on local observation histories $h_{i,t} = \{(o_{i,\tau}, a_{i,\tau})\}_{\tau=1}^t$, and $h_{|\tau|}$ along with $a_{|\tau|}$ represent the final approximate state and action of the multi-agent trajectory, respectively.

Unlike fully observable MDPs, partial observability means each agent sees only a slice of the global state, making coordination harder. A *global reward* function $\mathcal{R}_\zeta: \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ provides a team-wide signal, shared among all agents at each timestep (or, in our *episodic* case, at the end of the trajectory). The objective is to learn policies π that maximize the expected return:

$$\mathbb{E}_{s_0 \sim \rho_0, s \sim \mathcal{P}, a \sim \pi} \left[\sum_{t=0}^{|\tau|} \gamma^t r_{\text{global},t} \right].$$

3.2 Episodic Multi-Agent Reinforcement Learning

In many MARL domains, rewards are received only upon completing an *episode*, yielding a single *episodic* reward $r_{\text{global,episodic}}(\tau)$. This setting is common in tasks with sparse or delayed feedback—e.g., defeating all opponents in a battle environment or scoring a goal in a sports simulation. Although it accurately represents real-world scenarios, episodic (delayed) rewards significantly complicate learning, causing high variance and bias [Ng, 1999] in the policy gradient estimates. Agents must learn not only *which* actions lead to success, but also *when* those actions should occur within the trajectory—a problem known as *temporal credit assignment*.

3.3 Potential-Based Reward Shaping

A well-known approach to address delayed or sparse rewards is *potential-based reward shaping* [Ng, 1999]. In the single-agent case, one augments the reward function R with an extra shaping term F , derived from a potential function Φ . Formally, for any two consecutive states s and s' , the shaping reward is:

$$F(s, s') = \gamma \Phi(s') - \Phi(s).$$

Because this shaping term telescopes across a trajectory, it preserves the set of optimal policies. In multi-agent settings, the same principle applies if each agent’s shaping function is potential-based [Devlin and Kudenko, 2011; Xiaosong Lu, 2011]. In particular, adding

$$F_i(s, s') = \gamma \Phi_i(s') - \Phi_i(s)$$

to the reward function for agent i ensures that Nash equilibria remain unchanged. For more details, refer to Appendix 1.

Potential-based shaping motivates our **reward redistribution** strategy, as it allows us to create denser per-step feedback while provably preserving policy optimality. We leverage this shaping concept to break down a single episodic reward into agent- and time-step-specific components, ensuring that the reward augmentation does *not* distort the underlying solution set. In the next section, we detail how we design such a redistribution scheme—called TAR²—to tackle sparse multi-agent tasks effectively.

4 Approach

In this section, we present our *Temporal-Agent Reward Redistribution* (TAR²) algorithm, which addresses the dual challenge of *temporal* and *agent-specific* credit assignment in cooperative multi-agent reinforcement learning (MARL). We begin by introducing our reward redistribution mechanism (Sec. 4.1), then establish theoretical guarantees ensuring optimal policy preservation (Sec. 4.2). Next, we detail the architectural design of our model (Sec. 4.3) and outline the full training procedure (Sec. 4.4). Finally, we discuss interpretability considerations and reference ablation studies that provide further insight into each design choice.

4.1 Reward Redistribution Formulation

The central challenge in episodic MARL is to decompose a sparse global reward $r_{\text{global,episodic}}$ into more informative, time-step and agent-specific feedback, while preserving the original problem’s solutions. We achieve this by defining two weight functions:

$$w_t^{\text{temporal}}, \quad w_{i,t}^{\text{agent}},$$

which decompose the final return across time steps and agents, respectively. Formally, each agent i at time t receives

$$r_{i,t} = w_{i,t}^{\text{agent}} w_t^{\text{temporal}} r_{\text{global,episodic}}(\tau).$$

We impose normalization constraints such that $\sum_{i=1}^N w_{i,t}^{\text{agent}} = 1$ and $\sum_{t=1}^{|\tau|} w_t^{\text{temporal}} = 1$, ensuring the sum of all $r_{i,t}$ matches the original episodic reward as done in prior works [Xiao *et al.*, 2022; Chen *et al.*, 2023; Ren *et al.*, 2021; Efroni *et al.*, 2021]. Specifically, we make the following assumption about the reward redistribution:

Assumption 1. *The per timestep agent-specific reward can be approximated by a function conditioned on observation-action tuple and the final outcome of the multi-agent trajectory.*

$$r_{\text{global,episodic}}(\tau) = \sum_{i=1}^N \sum_{t=1}^{|\tau|} r_{i,t}(h_{i,t}, a_{i,t}, h_{|\tau|}, a_{|\tau|}), \quad (1)$$

By factoring the final reward into finer-grained terms, agents can receive meaningful feedback at each timestep (temporal credit), without conflating the contributions of other agents (agent credit). This alleviates the high-variance gradient updates typical in purely episodic settings, see Figure 4.

We propose a two-stage reward redistribution mechanism that decomposes the global episodic reward temporally and across agents (see Section 2 in the supplementary material for detailed formulation).

4.2 Theoretical Guarantees

Optimal Policy Invariance. We prove that the new reward function

$$\mathcal{R}_{\omega,\kappa}^i(s_t, a_t, s_{t+1}) = \mathcal{R}_\zeta(s_t, a_t, s_{t+1}) + r_{i,t}$$

is consistent with potential-based reward shaping (see Section 3.3). Specifically, if π_θ^* is optimal under $\mathcal{R}_{\omega,\kappa}$, it remains optimal under the original environment reward \mathcal{R}_ζ . This ensures that our reward redistribution does not alter the set of optimal policies. For a detailed proof of this invariance, please refer to Section 3 of the supplementary material.

Gradient Direction Preservation. Beyond preserving optimal solutions, we also show (in Section 6 or the supplemental material) that the *direction* of the policy gradient update under TAR² matches that under the original reward. Concretely, for an agent i ,

$$\nabla_{\theta_i} J_{\omega,\kappa}(\theta_i) \propto \nabla_{\theta_i} J_\zeta(\theta_i),$$

indicating that we do not bias the learning trajectory, only *densify* it. Proof details appear in Section 6 of the supplementary material.

4.3 Reward Model Architecture

Our reward redistribution model builds on the dual attention mechanisms of AREL [Xiao *et al.*, 2022] and STAS [Chen *et al.*, 2023], with key enhancements to better handle sparse, long-horizon multi-agent tasks.

Adaptations from Prior Work

We embed each agent’s observations, actions, and unique positional information at every timestep, then sum these embeddings to form a sequence that is input to a causal dual attention mechanism [Vaswani *et al.*, 2017]. This mechanism allows us to capture both temporal dependencies and inter-agent interactions. By using a Shapley attention network, similar to STAS, we approximate each agent’s marginal contribution via Monte Carlo rollouts over coalitions. The architecture comprises three dual attention blocks, each with four attention heads, selectively focusing on relevant spatial and temporal features.

Architectural Enhancements and Modifications

To improve credit assignment, we introduce three enhancements:

- **Final State Conditioning:** We condition the reward redistribution on the final observation-action embedding [Harutyunyan *et al.*, 2019; Amir *et al.*, 2023], linking intermediate actions to final outcomes. This contrasts with prior methods that ignore end-of-trajectory context, often averaging rewards irrespective of ultimate success. By incorporating the final outcome, our approach ensures that intermediate actions are accurately attributed based on their actual contribution to the trajectory’s success or failure, leading to more precise and meaningful credit assignment.
- **Inverse Dynamics Modeling:** We add an inverse dynamics module that predicts each agent’s action at every timestep from the learned embeddings. This regularizes temporal representations, capturing causality and stabilizing credit assignment.
- **Probabilistic Reward Redistribution:** At inference, we normalize predicted rewards across time and agents to form probability distributions. This ensures each per-timestep, per-agent reward is a positive fraction of the episodic reward, preserving the principles of potential-based shaping.

These modifications enable more precise credit assignment in sparse-reward environments. For detailed architectural specifications see Section 7 of the supplementary material.

4.4 Training Objective

We optimize the reward redistribution model parameters (ω, κ) using the objective:

$$L(\omega, \kappa) = \mathbb{E}_{\tau \sim B} \left[\left(r_{\text{global, episodic}}(\tau) - \sum_{t=1}^T \sum_{i=1}^N R(i, t; \omega, \kappa) \right)^2 - \lambda \sum_{t=1}^T \sum_{i=1}^N a_{i,t} \log(p_{i,t}) \right], \quad (2)$$

where $R(i, t; \omega, \kappa)$ denotes the predicted reward for agent i at time t , and $p_{i,t}$ is the inverse dynamics model’s predicted probability for action $a_{i,t}$. The first term minimizes the discrepancy between the sum of redistributed rewards and the true episodic return, while the second term trains the inverse dynamics component via cross-entropy loss. Training involves sampling trajectories from an experience buffer B and periodically updating (ω, κ) using this loss. For the complete training procedure, please refer to Algorithm in Section 8 of the supplementary material.

5 Experimental Setup

5.1 Baselines

We compare TAR^2 against several reward redistribution methods, all trained with Multi-Agent Proximal Policy Optimization (MAPPO) [Yu *et al.*, 2022]:

- **TAR^2 (Ours):** Our approach (Section 4) that redistributes rewards both across time and agents, leveraging an inverse dynamics model and final outcome conditioning.
- **Uniform (IRCR)** [Gangwani *et al.*, 2020]: Assigns the global episodic reward equally to each timestep and agent, i.e., $r_{\text{global},t} = r_{\text{episodic}}(\tau)/|\tau|$.
- **AREL-Temporal** [Xiao *et al.*, 2022]: Focuses on temporal credit assignment by predicting rewards for the entire multi-agent trajectory at each timestep.
- **AREL-Agent-Temporal:** We modified AREL-TEMPORAL version to assign rewards per agent at each timestep, rather than per joint observation.
- **STAS** [Chen *et al.*, 2023]: Employs a dual attention structure (temporal + Shapley-based agent attention) to decompose global rewards into agent-temporal components.

Additional hyperparameter details for each method can be found in Section 11 of the supplementary material.

5.2 Environments

We evaluate on two cooperative multi-agent benchmarks SMACLite [Michalski *et al.*, 2023] and Google Research Football [Kurach *et al.*, 2020], providing delayed (episodic) feedback by accumulating dense rewards during each trajectory and only returning them at the end. More details on the environment can be found in the supplementary material, Section 10.

5.3 Results and Discussion

Metrics. We report per-agent reward instead of win rate, as rewards provide continuous, granular feedback throughout training—particularly valuable in complex tasks where partial successes or incremental improvements occur long before an episode concludes.

Performance in GRF and SMAClite. Figures 2 and 3 show average agent rewards (with standard deviation) over three GRF tasks and three SMAClite scenarios. TAR^2 consistently outperforms baselines, converging to higher returns in all tasks.

Uniform Baseline (IRCR). The simplest baseline—assigning an equal portion of the global reward to each timestep and agent—plateaus early. By ignoring each agent’s varying contribution, it provides insufficient guidance for fine-grained strategy learning, leading to relatively stagnant performance.

AREL Variants & STAS. While STAS leverages Shapley-based decompositions and generally outperforms the AREL variants, it still trails TAR^2 . A key limitation is that both STAS and AREL produce unbounded per-agent, per-timestep predictions, which can destabilize training. Moreover, AREL (temporal or agent-temporal) struggles particularly in Google Football, leading to catastrophic drops in tasks like Pass and Shoot.

Overall, these results demonstrate that TAR^2 not only yields higher final performance but also converges more reliably across a range of sparse-reward tasks.

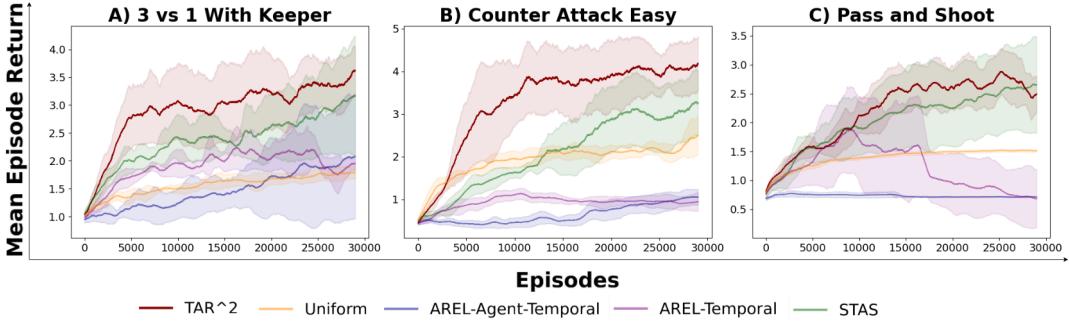


Figure 2: Performance comparison of different reward redistribution approaches using MAPPO across three Google Research Football scenarios. The graphs plot median episode returns versus episodes.

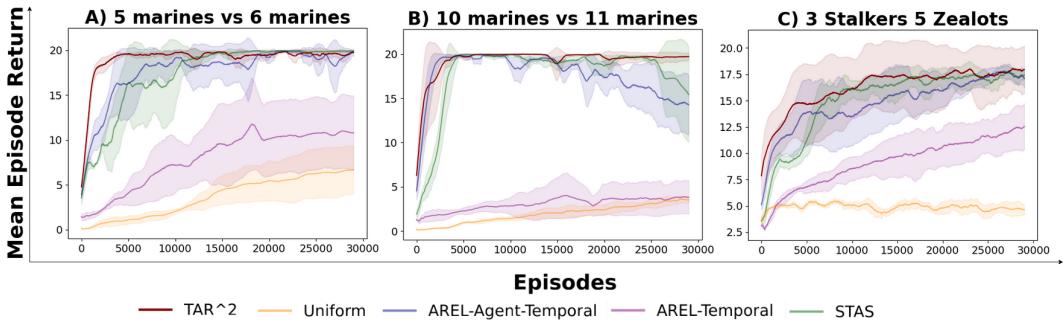


Figure 3: Performance comparison of different reward redistribution approaches using MAPPO across three SMACLite scenarios. The graphs plot median episode returns versus episodes.

While our primary focus in this work has been on achieving robust performance improvements and theoretical guarantees for reward redistribution, we also recognize the potential for interpreting TAR^2 's per-timestep, per-agent reward predictions. Although a detailed interpretability analysis is beyond the scope of this paper, preliminary insights and discussions are provided in the supplementary material, Section 9. We leave a comprehensive study of interpretability for future research, given its complexity in high-dimensional multi-agent environments.

5.4 Additional Analysis: Performance Bounds & Ablation Studies

Evaluation Baselines and Performance Bounds

To contextualize TAR^2 's performance, we evaluate it against several reward-assignment configurations in the environment, approximating both lower and upper performance bounds for MAPPO [Yu *et al.*, 2022]:

- **Episodic Team:** Provides a single global reward only at the episode's end. This is a minimal credit assignment scheme that generally yields a lower bound.
- **Episodic Agent:** Allocates total return to each agent based solely on its individual contribution. While more granular than Episodic Team, it can induce greediness and discourage cooperative behavior in tasks requiring teamwork.

- **Dense Temporal:** All agents receive dense feedback at every timestep based on global performance. This offers an approximate upper bound, since it supplies immediate signals for each action.
- **Dense Agent-Temporal:** Each agent obtains a dense reward for its individual contributions at every timestep. Similar to Episodic Agent, but with dense signals for each action, making it another upper-bound scenario.

Figure 4 illustrates these bounds alongside TAR^2 in three Google Research Football tasks. TAR^2 outperforms or matches the best dense baselines, demonstrating that its structured redistribution approach rivals the benefits of dense rewards while preserving policy optimality invariance.

Ablation Studies

We also conduct ablations to isolate the impact of three key components in TAR^2 : (1) the *inverse dynamics* model, (2) *final outcome conditioning*, and (3) *normalization* of predicted rewards (Fig. 5).

Inverse Dynamics Model. Removing the inverse dynamics task results in notably slower convergence and higher variance, indicating that predicting each agent's actions helps the model capture causal structure in multi-agent trajectories. This aligns with prior findings [Pathak *et al.*, 2017; Agrawal *et al.*, 2015] that inverse dynamics objectives can improve temporal representations.

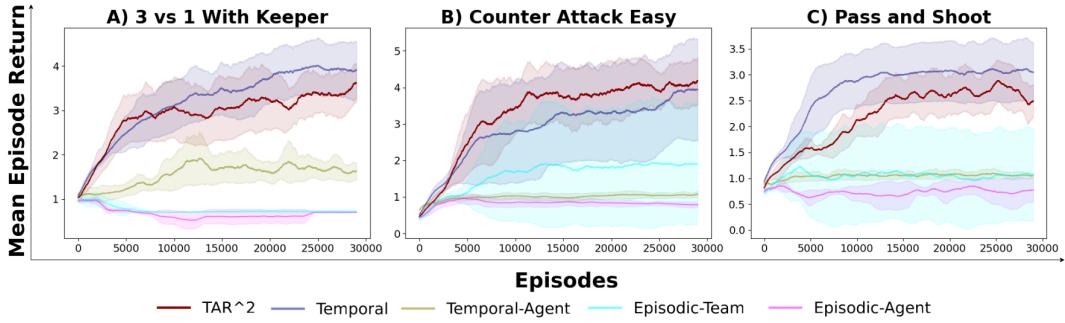


Figure 4: Performance comparison of TAR^2 reward redistribution using MAPPO across three Google Research Football scenarios. The graphs plot median episode returns versus episodes, bounding TAR^2 's performance with heuristically designed reward functions for the environment.

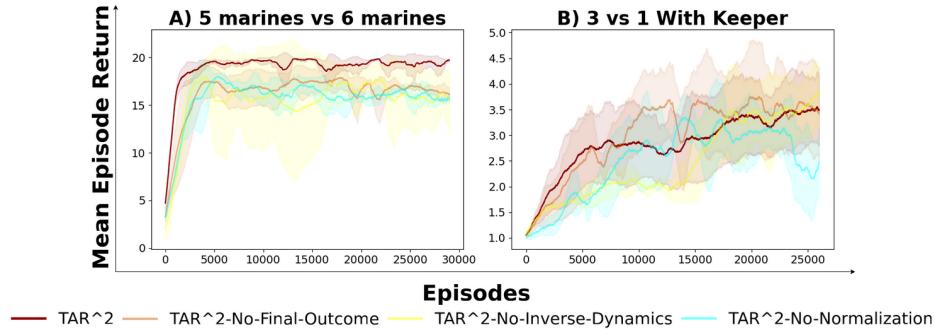


Figure 5: Ablation study of TAR^2 components across two environments. The graphs plot median episode returns versus episodes, analyzing the impact of key design choices by comparing TAR^2 with variations that exclude specific components.

Final Outcome Conditioning. When the reward predictor ignores the final state-action context, performance significantly degrades and fluctuates. Conditioning on the ultimate outcome helps the model attribute credit more accurately, mapping intermediate actions to actual successes or failures instead of an average or hypothetical return [Harutyunyan *et al.*, 2019].

Reward Normalization. Without normalizing the predicted rewards to sum to the true episodic return, learning becomes unstable leading to suboptimal policies, with wide performance fluctuations. Enforcing this normalization ensures each per-agent, per-timestep reward aligns with the global trajectory outcome, reducing volatility and preserving potential-based shaping requirements.

Summary of Ablations. Each component contributes to TAR^2 's performance and stability. Removing any of them leads to slower learning or higher fluctuations, underscoring their collective importance for effective agent-temporal credit assignment.

6 Conclusion and Future Work

In this paper, we tackled the dual challenges of temporal and agent-specific credit assignment in multi-agent reinforcement learning with delayed episodic rewards. We introduced TAR^2 , a novel reward redistribution method that decomposes

a global episodic reward into agent-specific, per-timestep rewards while preserving policy optimality via potential-based reward shaping. TAR^2 effectively aligns intermediate actions with final outcomes by leveraging final state conditioning, inverse dynamics modeling, and probabilistic normalization—innovations that substantially improve learning in sparse-reward settings. Our extensive evaluations on challenging benchmarks like SMACLite and Google Research Football demonstrate that TAR^2 outperforms state-of-the-art baselines in terms of sample efficiency and final performance. These results confirm that our approach provides more precise credit assignment, leading to faster and more stable policy convergence.

Looking ahead, we envision several exciting avenues for expanding this work. For instance, integrating Hindsight Credit Assignment [Harutyunyan *et al.*, 2019] into the TAR^2 framework could further refine reward signals based on alternate goals or outcomes. Additionally, exploring transfer-learning capabilities—such as applying a model trained with a certain number of agents to scenarios with more agents or transferring knowledge across similar environments—could reveal the academic offspring of TAR^2 , broadening its applicability and impact.

References

- Pulkit Agrawal, Joao Carreira, and Jitendra Malik. Learning to see by moving, 2015.
- Christopher Amato. (a partial survey of) decentralized, cooperative multi-agent reinforcement learning. *arXiv preprint arXiv:2405.06161*, 2024.
- Nadav Amir, Yael Niv, and Angela Langdon. States as goal-directed concepts: an epistemic approach to state-representation learning. *ArXiv*, abs/2312.02367, 2023.
- Jose A Arjona-Medina, Michael Gillhofer, Michael Widrich, Thomas Unterthiner, Johannes Brandstetter, and Sepp Hochreiter. Rudder: Return decomposition for delayed rewards. *Advances in Neural Information Processing Systems*, 32, 2019.
- Schirin Baer, Jupiter Bakakeu, Richard Meyes, and Tobias Meisen. Multi-agent reinforcement learning for job shop scheduling in flexible manufacturing systems. In *2019 Second International Conference on Artificial Intelligence for Industries (AI4I)*, pages 22–25, 2019.
- Christopher Berner, Greg Brockman, Brooke Chan, Vicki Cheung, Przemysław Dębiak, Christy Dennison, David Farhi, Quirin Fischer, Shariq Hashme, Chris Hesse, et al. Dota 2 with large scale deep reinforcement learning. *arXiv preprint arXiv:1912.06680*, 2019.
- Sirui Chen, Zhaowei Zhang, Yali Du, and Yaodong Yang. Stas: Spatial-temporal return decomposition for multi-agent reinforcement learning. *ArXiv*, abs/2304.07520, 2023.
- Mehul Damani, Zhiyao Luo, Emerson Wenzel, and Guillaume Sartoretti. Primal _2: Pathfinding via reinforcement and imitation multi-agent learning-lifelong. *IEEE Robotics and Automation Letters*, 6(2):2666–2673, 2021.
- Sam Devlin and Daniel Kudenko. Theoretical considerations of potential-based reward shaping for multi-agent systems. In *Adaptive Agents and Multi-Agent Systems*, 2011.
- Sam Devlin, Logan Yliniemi, Daniel Kudenko, and Kagan Tumer. Potential-based difference rewards for multiagent reinforcement learning. In *Proceedings of the 2014 international conference on Autonomous agents and multi-agent systems*, pages 165–172, 2014.
- Yonathan Efroni, Nadav Merlis, and Shie Mannor. Reinforcement learning with trajectory feedback. In *Proceedings of the AAAI conference on artificial intelligence*, volume 35, pages 7288–7295, 2021.
- Jakob Foerster, Gregory Farquhar, Triantafyllos Afouras, Nantas Nardelli, and Shimon Whiteson. Counterfactual multi-agent policy gradients. In *Proceedings of the AAAI conference on artificial intelligence*, volume 32, 2018.
- Benjamin Freed, Aditya Kapoor, Ian Abraham, Jeff Schneider, and Howie Choset. Learning cooperative multi-agent policies with partial reward decoupling. *IEEE Robotics and Automation Letters*, 7(2):890–897, April 2022.
- Tanmay Gangwani, Yuan Zhou, and Jian Peng. Learning guidance rewards with trajectory-space smoothing. *Advances in Neural Information Processing Systems*, 33:822–832, 2020.
- Beining Han, Zhizhou Ren, Zuofan Wu, Yuan Zhou, and Jian Peng. Off-policy reinforcement learning with delayed rewards. In *International Conference on Machine Learning*, pages 8280–8303. PMLR, 2022.
- Anna Harutyunyan, Will Dabney, Thomas Mesnard, Mohammad Gheshlaghi Azar, Bilal Piot, Nicolas Heess, Hado P van Hasselt, Gregory Wayne, Satinder Singh, Doina Precup, et al. Hindsight credit assignment. *Advances in neural information processing systems*, 32, 2019.
- Aditya Kapoor, Benjamin Freed, Howie Choset, and Jeff Schneider. Assigning credit with partial reward decoupling in multi-agent proximal policy optimization. *arXiv preprint arXiv:2408.04295*, 2024.
- Aditya Kapoor, Sushant Swamy, Kale ab Tessera, Mayank Baranwal, Mingfei Sun, Harshad Khadilkar, and Stefano V. Albrecht. Agent-temporal credit assignment for optimal policy preservation in sparse multi-agent reinforcement learning, 2024.
- Aleksandar Krnjaic, Raul D Steleac, Jonathan D Thomas, Georgios Papoudakis, Lukas Schäfer, Andrew Wing Keung To, Kuan-Ho Lao, Murat Cubuktepe, Matthew Haley, Peter Börsting, et al. Scalable multi-agent reinforcement learning for warehouse logistics with robotic and human co-workers. *arXiv preprint arXiv:2212.11498*, 2022.
- Karol Kurach, Anton Raichuk, Piotr Stańczyk, Michał Zając, Olivier Bachem, Lasse Espeholt, Carlos Riquelme, Damien Vincent, Marcin Michalski, Olivier Bousquet, et al. Google research football: A novel reinforcement learning environment. In *Proceedings of the AAAI conference on artificial intelligence*, volume 34, pages 4501–4510, 2020.
- Yang Liu, Yunan Luo, Yuanyi Zhong, Xi Chen, Qiang Liu, and Jian Peng. Sequence modeling of temporal credit assignment for episodic reinforcement learning. *arXiv preprint arXiv:1905.13420*, 2019.
- Ryan Lowe, Aviv Tamar, Jean Harb, OpenAI Pieter Abbeel, and Igor Mordatch. Multi-agent actor-critic for mixed cooperative-competitive environments. *Advances in neural information processing systems*, 30, 2017.
- Adam Michalski, Filippos Christianos, and Stefano V Albrecht. Smaclite: A lightweight environment for multi-agent reinforcement learning. *arXiv preprint arXiv:2305.05566*, 2023.
- AY Ng. Policy invariance under reward transformations: Theory and application to reward shaping. In *Proceedings of the 16th International Conference on Machine Learning*, page 278, 1999.
- Frans A. Oliehoek and Chris Amato. A concise introduction to decentralized pomdps. In *SpringerBriefs in Intelligent Systems*, 2016.

- Georgios Papoudakis, Filippos Christianos, Lukas Schäfer, and Stefano V. Albrecht. Benchmarking multi-agent deep reinforcement learning algorithms in cooperative tasks. In *NeurIPS Datasets and Benchmarks*, 2020.
- Deepak Pathak, Pulkit Agrawal, Alexei A. Efros, and Trevor Darrell. Curiosity-driven exploration by self-supervised prediction, 2017.
- Tabish Rashid, Mikayel Samvelyan, Christian Schroeder De Witt, Gregory Farquhar, Jakob Foerster, and Shimon Whiteson. Monotonic value function factorisation for deep multi-agent reinforcement learning. *Journal of Machine Learning Research*, 21(178):1–51, 2020.
- Zhizhou Ren, Ruihan Guo, Yuan Zhou, and Jian Peng. Learning long-term reward redistribution via randomized return decomposition. *arXiv preprint arXiv:2111.13485*, 2021.
- Mikayel Samvelyan, Tabish Rashid, Christian Schroeder De Witt, Gregory Farquhar, Nantas Nardelli, Tim GJ Rudner, Chia-Man Hung, Philip HS Torr, Jakob Foerster, and Shimon Whiteson. The starcraft multi-agent challenge. *arXiv preprint arXiv:1902.04043*, 2019.
- Guillaume Sartoretti, Justin Kerr, Yunfei Shi, Glenn Wagner, TK Satish Kumar, Sven Koenig, and Howie Choset. Primal: Pathfinding via reinforcement and imitation multi-agent learning. *IEEE Robotics and Automation Letters*, 4(3):2378–2385, 2019.
- John Schulman, Philipp Moritz, Sergey Levine, Michael Jordan, and Pieter Abbeel. High-dimensional continuous control using generalized advantage estimation. *arXiv preprint arXiv:1506.02438*, 2015.
- Jennifer She, Jayesh K Gupta, and Mykel J Kochenderfer. Agent-time attention for sparse rewards multi-agent reinforcement learning. *arXiv preprint arXiv:2210.17540*, 2022.
- Omkar Shelke, Pranavi Pathakota, Anandsingh Chauhan, Harshad Khadilkar, Hardik Meisher, and Balaraman Ravindran. Multi-agent learning of efficient fulfilment and routing strategies in e-commerce. *arXiv preprint arXiv:2311.16171*, 2023.
- Peter Sunehag, Guy Lever, Audrunas Gruslys, Wojciech Marian Czarnecki, Vinicius Zambaldi, Max Jaderberg, Marc Lanctot, Nicolas Sonnerat, Joel Z Leibo, Karl Tuyls, et al. Value-decomposition networks for cooperative multi-agent learning. *arXiv preprint arXiv:1706.05296*, 2017.
- Richard S. Sutton and Andrew G. Barto. *Reinforcement Learning: An Introduction*. The MIT Press, Cambridge, MA, 1998.
- Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz Kaiser, and Illia Polosukhin. Attention is all you need. *Advances in neural information processing systems*, 30, 2017.
- Eugene Vinitsky, Nathan Lichte, Kanaad Parvate, and Alexandre Bayen. Optimizing mixed autonomy traffic flow with decentralized autonomous vehicles and multi-agent rl. *arXiv preprint arXiv:2011.00120*, 2020.
- Oriol Vinyals, Igor Babuschkin, Wojciech M. Czarnecki, Michaël Mathieu, Andrew Dudzik, Junyoung Chung, David Choi, Richard Powell, Timo Ewalds, Petko Georgiev, Junhyuk Oh, Dan Horgan, Manuel Kroiss, Ivo Danihelka, Aja Huang, L. Sifre, Trevor Cai, John P. Agapiou, Max Jaderberg, Alexander Sasha Vezhnevets, Rémi Leblond, Tobias Pohlen, Valentin Dalibard, David Buden, Yury Sulsky, James Molloy, Tom Le Paine, Caglar Gulcehre, Ziyun Wang, Tobias Pfaff, Yuhuai Wu, Roman Ring, Dani Yogatama, Dario Wünsch, Katrina McKinney, Oliver Smith, Tom Schaul, Timothy P. Lillicrap, Koray Kavukcuoglu, Demis Hassabis, Chris Apps, and David Silver. Grandmaster level in starcraft ii using multi-agent reinforcement learning. *Nature*, 575:350 – 354, 2019.
- Jianhong Wang, Yuan Zhang, Tae-Kyun Kim, and Yunjie Gu. Shapley q-value: A local reward approach to solve global reward games. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 34, pages 7285–7292, 2020.
- Baichen Xiao, Bhaskar Ramasubramanian, and Radha Poovendran. Agent-temporal attention for reward redistribution in episodic multi-agent reinforcement learning. *arXiv preprint arXiv:2201.04612*, 2022.
- Sidney N. Givigi Jr Xiaosong Lu, Howard M. Schwartz. Policy invariance under reward transformations for general-sum stochastic games. *Journal of Artificial Intelligence Research*, 41:397–406, 2011.
- Chao Yu, Akash Velu, Eugene Vinitsky, Jiaxuan Gao, Yu Wang, Alexandre Bayen, and Yi Wu. The surprising effectiveness of ppo in cooperative multi-agent games. *Advances in Neural Information Processing Systems*, 35:24611–24624, 2022.
- Kaiqing Zhang, Zhuoran Yang, Han Liu, Tong Zhang, and Tamer Basar. Fully decentralized multi-agent reinforcement learning with networked agents. In *International Conference on Machine Learning*, pages 5872–5881. PMLR, 2018.
- Yulin Zhang, William Macke, Jiaxun Cui, Sharon Hornstein, Daniel Urieli, and Peter Stone. Learning a robust multi-agent driving policy for traffic congestion reduction. *Neural Computing and Applications*, pages 1–14, 2023.
- Tianchen Zhu, Yue Qiu, Haoyi Zhou, and Jianxin Li. Towards long-delayed sparsity: Learning a better transformer through reward redistribution. In Edith Elkind, editor, *Proceedings of the Thirty-Second International Joint Conference on Artificial Intelligence, IJCAI-23*, pages 4693–4701. International Joint Conferences on Artificial Intelligence Organization, 8 2023. Main Track.

Supplementary Material

A Detailed Discussion on Potential-Based Reward Shaping

Ng [1999] presented a single-agent reward shaping method to address the credit assignment problem by introducing a potential-based shaping reward to the environment. The combination of the shaping reward with the original reward can enhance the learning performance of a reinforcement learning algorithm and accelerate the convergence to the optimal policy. Devlin and Kudenko [2011] and Xiaosong Lu [2011] extended potential-based reward shaping to multi-agent systems as follows:

Theorem 1. *Given an n -player discounted stochastic game $M = (S, A_1, \dots, A_n, T, \gamma, R_1, \dots, R_n)$, we define a transformed n -player discounted stochastic game $M' = (S, A_1, \dots, A_n, T, \gamma, R_1 + F_1, \dots, R_n + F_n)$, where $F_i \in S \times S$ is a shaping reward function for player i . We call F_i a potential-based shaping function if F_i has the form:*

$$F_i(s, s') = \gamma\Phi_i(s') - \Phi_i(s),$$

where $\Phi_i : S \rightarrow \mathbb{R}$ is a potential function. Then, the potential-based shaping function F_i is a necessary and sufficient condition to guarantee the Nash equilibrium policy invariance such that:

- (**Sufficiency**) If F_i ($i = 1, \dots, n$) is a potential-based shaping function, then every Nash equilibrium policy in M' will also be a Nash equilibrium policy in M (and vice versa).
- (**Necessity**) If F_i ($i = 1, \dots, n$) is not a potential-based shaping function, then there may exist a transition function T and reward function R such that the Nash equilibrium policy in M' will not be the Nash equilibrium policy in M .

In summary, potential-based reward shaping ensures that Nash equilibrium policies are preserved, enhancing learning without altering the strategic dynamics. This principle underpins our proposed reward redistribution method, which we will validate in the following sections, demonstrating its effectiveness in multi-agent reinforcement learning.

Relevance to TAR². Potential-based shaping provides the theoretical underpinning for our reward redistribution strategy. By ensuring that our redistribution function adheres to a potential-based form, we guarantee that:

- The reshaped rewards do not alter the set of optimal policies (policy optimality invariance).
- The convergence and strategic dynamics are preserved, even as we provide denser, agent- and time-specific rewards.

This detailed understanding justifies the design choices in TAR² and underlines its robustness in multi-agent scenarios.

B Formulating the Reward Redistribution Mechanism

To effectively address both temporal and agent-specific credit assignment in episodic MARL, we introduce a two-stage reward redistribution mechanism. This mechanism decomposes the global episodic reward into more informative, granular components that facilitate better learning and coordination among agents.

Temporal Redistribution We first redistribute the global episodic reward across the trajectory's time steps using a temporal weighting function

$$w_t^{\text{temporal}} \sim \mathcal{W}_\omega(h_t, a_t, h_{|\tau|}, a_{|\tau|}),$$

parameterized by ω . This function takes as input the history h_t , the joint action a_t at time t , and the final state-action pair $(h_{|\tau|}, a_{|\tau|})$, assigning a portion of the global reward to each timestep t :

$$r_{\text{global},t} = w_t^{\text{temporal}} \cdot r_{\text{global,episodic}}(\tau). \quad (3)$$

We enforce the normalization condition:

$$\sum_{t=1}^{|\tau|} w_t^{\text{temporal}} = 1. \quad (4)$$

Agent-wise Redistribution Next, each temporally redistributed reward $r_{\text{global},t}$ is allocated to individual agents using agent-specific weighting functions

$$w_{i,t}^{\text{agent}} \sim \mathcal{W}_\kappa(h_{i,t}, a_{i,t}, h_{|\tau|}, a_{|\tau|}),$$

parameterized by κ . Each $w_{i,t}^{\text{agent}}$ takes as input agent i 's history $h_{i,t}$, action $a_{i,t}$ at time t , and the final state-action pair $(h_{|\tau|}, a_{|\tau|})$, and redistributes the temporal reward among agents:

$$r_{i,t} = w_{i,t}^{\text{agent}} \cdot r_{\text{global},t}. \quad (5)$$

The agent weights are normalized at each timestep:

$$\sum_{i=1}^N w_{i,t}^{\text{agent}} = 1 \quad \forall t. \quad (6)$$

Overall Reward Redistribution Combining temporal and agent-wise redistributions, the per-agent reward at timestep t is given by:

$$r_{i,t} = w_{i,t}^{\text{agent}} \cdot w_t^{\text{temporal}} \cdot r_{\text{global,episodic}}(\tau). \quad (7)$$

The normalization constraints (Eqs. 4 and 6) ensure that the sum of all $r_{i,t}$ equals the original global episodic reward (refer Assumption 1):

$$\sum_{i=1}^N \sum_{t=1}^{|\tau|} r_{i,t} = r_{\text{global,episodic}}(\tau). \quad (8)$$

Constructing the New Reward Function Using the redistributed rewards, we define a new reward function for each agent i at timestep t :

$$\begin{aligned} \mathcal{R}_{\omega,\kappa}^i(s_t, a_t, s_{t+1}) &= \mathcal{R}_\zeta(s_t, a_t, s_{t+1}) + r_{i,t} \\ &= \mathcal{R}_\zeta(s_t, a_t, s_{t+1}) + w_{i,t}^{\text{agent}} \cdot w_t^{\text{temporal}} \cdot r_{\text{global,episodic}}(\tau). \end{aligned} \quad (9)$$

In Dec-POMDPs, where the true state s_t is not directly observable, we approximate this function using agent histories $h_{i,t}$ and actions $a_{i,t}$.

Summary Our mechanism parameterizes weighting functions using ω and κ , which condition on the current histories, actions, and the final outcome of the trajectory. By breaking down the episodic reward into time-step-specific and agent-specific components while enforcing normalization constraints, we ensure that the redistributed rewards provide detailed feedback to each agent without altering the underlying global objective.

C Optimal Policy Preservation

We establish that the optimal policy learned under the densified reward function $\mathcal{R}_{\omega,\kappa}$ remains optimal for the original reward function \mathcal{R}_ζ .

Theorem 2 (Optimal Policy Preservation). *Consider two Dec-POMDPs:*

$$\begin{aligned} \mathcal{M}_{\text{env}} &= (\mathcal{S}, \mathcal{A}, \mathcal{P}, T, O, N, \mathcal{R}_\zeta, \rho_0, \gamma), \\ \mathcal{M}_{\text{rrf}} &= (\mathcal{S}, \mathcal{A}, \mathcal{P}, T, O, N, \mathcal{R}_{\omega,\kappa}, \rho_0, \gamma), \end{aligned}$$

where $\mathcal{R}_{\omega,\kappa}^i(s_t, a_t, s_{t+1}) = \mathcal{R}_\zeta(s_t, a_t, s_{t+1}) + w_{i,t}^{\text{agent}} w_t^{\text{temporal}} r_{\text{global,episodic}}(\tau)$ for each agent i . If π_θ^* is optimal in \mathcal{M}_{rrf} , then it is also optimal in \mathcal{M}_{env} .

Proof Sketch. To prove optimality preservation, we show that $\mathcal{R}_{\omega,\kappa}$ can be expressed as

$$\mathcal{R}_{\omega,\kappa}^i(s_t, a_t, s_{t+1}) = \mathcal{R}_\zeta(s_t, a_t, s_{t+1}) + F_i(s_t, a_t, s_{t+1}),$$

where F_i is a potential-based shaping function. For simplicity, assume $\gamma = 1$.

Given Eq. 9, we seek functions $\phi^i: S \rightarrow \mathbb{R}$ such that

$$w_{i,t}^{\text{agent}} w_t^{\text{temporal}} r_{\text{global,episodic}}(\tau) = \phi^i(s_{t+1}) - \phi^i(s_t).$$

This relation holds by defining the potential function for each agent as

$$\phi^i(s_t) = r_{\text{global,episodic}}(\tau) \cdot \sum_{t'=0}^t w_{i,t'}^{\text{agent}} w_{t'}^{\text{temporal}}.$$

With this definition, the shaping function $F_i(s_t, a_t, s_{t+1}) = \phi^i(s_{t+1}) - \phi^i(s_t)$ matches the additional term in $\mathcal{R}_{\omega,\kappa}^i$. By the potential-based shaping theorem [Ng, 1999; Devlin and Kudenko, 2011], such an augmentation preserves the optimal policy. \square

This theorem guarantees that learning with our redistributed rewards does not alter the set of optimal policies.

D Impact of Faulty Credit Assignment on Policy Gradient Variance

To understand the impact of imperfect credit assignment, we examine the influence of other agents on the policy gradient update for agent i in a Dec-POMDP setting. Without assuming any specific policy parameters, the policy gradient update for agent i is computed as:

$$\hat{\nabla}_{\theta_i} J(\theta, h) = \nabla_{\theta_i} \log \pi_i(a_i|h_i) \mathbb{E}_{\neg h_i, \neg a_i} [A(h, a)] \quad (10)$$

Calculating $A_i = \mathbb{E}_{\neg h_i, \neg a_i} [A(h, a)]$ is complex due to the high dimensionality and inter-agent dependencies. In practice, multi-agent policy gradient methods like MAPPO [Yu *et al.*, 2022] and MADDPG [Lowe *et al.*, 2017] use $A(h, a)$ for the policy update thus,

$$\hat{\nabla}_{\theta_i} J(\theta, h) = \nabla_{\theta_i} \log \pi_i(a_i|h_i) A(h, a) \quad (11)$$

This often leads to high variance in advantage estimates, slowing down learning due to noisier gradient updates.

Multi-agent policy gradient methods estimate the true *advantage* by calculating \hat{A} , a stochastic approximation of the advantage function based on joint actions a , joint agent histories h , and the joint policy π in Dec-POMDPs. The advantage function is defined as $A^\pi(s, a) = Q^\pi(s, a) - V^\pi(s)$, where $Q^\pi(s, a)$ and $V^\pi(s)$ are the state-action value and state-value functions, respectively [Sutton and Barto, 1998]. In Dec-POMDPs, these are approximated as $\hat{Q}^\pi(h, a)$ and $\hat{V}^\pi(h)$. Since the true value functions are unknown, various methods are used to compute \hat{A} , introducing errors [Sutton and Barto, 1998; Schulman *et al.*, 2015]. The advantage function reflects how much better it is to take a joint action a versus a random action from π , while in state s . For agent i , the goal is to compute the advantage of its action a_i within the multi-agent context, where perfect credit assignment would require perfectly calculating the agent-specific advantage based on its contribution to the overall reward of the group.

Theorem 3. Given that agent i 's reward contribution at an arbitrary time-step t is $r_{i,t}(h, a)$ and the episodic reward $r_{global, episodic}(\tau) = \sum_{t=1}^{|\tau|} \sum_{i=1}^N r_{i,t}(h, a)$, the conditional variance of $(\hat{\nabla}_{\theta_i} J|h, a)$ is proportional to the conditional variance of the advantage estimate $\hat{A}_{\neg i}(h, a)$

Proof. The variance of the policy gradient update in eq 11 for agent i is:

$$\text{Var}(\hat{\nabla}_{\theta_i} J|h, a) = (\nabla_{\theta_i} \log \pi(a_i|h_i)) (\nabla_{\theta_i} \log \pi(a_i|h_i))^T \text{Var}(\hat{A}|h, a).$$

This expression shows that the conditional variance of $(\hat{\nabla}_{\theta_i} J|h, a)$ is proportional to the conditional variance of the joint-advantage estimate $\hat{A}|h, a$. While estimating variance typically requires multiple samples, we can initially analyze a single sample to isolate the variance induced by the contributions of other agents. We can express the state-action and state-value function as:

$$\begin{aligned} \mathcal{Q}(h, a) &= \mathbb{E}_{s_0 \sim \rho_0, s \sim \mathcal{P}, a_i \sim \pi_i} \left[\sum_{t=1}^{|\tau|} \sum_{i=1}^N r_{i,t}(h, a) \right] \\ \mathcal{V}(h) &= \mathbb{E}_\pi [\mathcal{Q}(h, a)] \end{aligned}$$

$$\begin{aligned} \mathcal{A}(h, a) &= \mathcal{Q}(h, a) - \mathcal{V}(h) \\ \mathcal{A}(h, a) &= \mathbb{E}_{s_0 \sim \rho_0, s \sim \mathcal{P}, a_i \sim \pi_i} \left[\sum_{t=1}^{|\tau|} \sum_{i=1}^N r_{i,t}(h, a) \right] \\ &\quad - \mathbb{E}_\pi \left[\mathbb{E}_{s_0 \sim \rho_0, s \sim \mathcal{P}, a_i \sim \pi_i} \left[\sum_{t=1}^{|\tau|} \sum_{i=1}^N r_{i,t}(h, a) \right] \right] \end{aligned}$$

Based on the linearity of expectations on $\sum_{t=1}^{|\tau|} \sum_{i=1}^N r_{j,t} = \sum_{t=1}^{|\tau|} r_{i,t} + \sum_{t=1}^{|\tau|} \sum_{j \neq i} r_{j,t}$ and by rearranging the terms we get:

$$\begin{aligned}
\mathcal{A}(h, a) &= \mathbb{E}_{s_0 \sim \rho_0, s \sim \mathcal{P}, a_i \sim \pi_i} \left[\sum_{t=1}^{|\tau|} r_{i,t}(h, a) \right] \\
&\quad - \mathbb{E}_\pi \left[\mathbb{E}_{s_0 \sim \rho_0, s \sim \mathcal{P}, a_i \sim \pi_i} \left[\sum_{t=1}^{|\tau|} r_{i,t}(h, a) \right] \right] \\
&\quad + \mathbb{E}_{s_0 \sim \rho_0, s \sim \mathcal{P}, a_i \sim \pi_i} \left[\sum_{t=1}^{|\tau|} \sum_{j \neq i}^N r_{j,t}(h, a) \right] \\
&\quad - \mathbb{E}_\pi \left[\mathbb{E}_{s_0 \sim \rho_0, s \sim \mathcal{P}, a_i \sim \pi_i} \left[\sum_{t=1}^{|\tau|} \sum_{j \neq i}^N r_{j,t}(h) \right] \right]
\end{aligned}$$

The advantage estimate considering only the contribution of agent i is the only advantage term that should be considered while calculating the policy gradient update for agent i as shown in eq 10

$\mathcal{A}_i = \mathbb{E}_{s_0 \sim \rho_0, s \sim \mathcal{P}, a_i \sim \pi_i} [\sum_{t=1}^{|\tau|} r_{i,t}(h, a)] - \mathbb{E}_\pi [\mathbb{E}_{s_0 \sim \rho_0, s \sim \mathcal{P}, a_i \sim \pi_i} [\sum_{t=1}^{|\tau|} r_{i,t}(h, a)]]$ whereas the advantage estimate due to other agents given by

$$\begin{aligned}
\mathcal{A}_{\neg i} &= \mathbb{E}_{s_0 \sim \rho_0, s \sim \mathcal{P}, a_i \sim \pi_i} [\sum_{t=1}^{|\tau|} \sum_{j \neq i}^N r_{j,t}(h, a)] \\
&\quad - \mathbb{E}_\pi [\mathbb{E}_{s_0 \sim \rho_0, s \sim \mathcal{P}, a_i \sim \pi_i} [\sum_{t=1}^{|\tau|} \sum_{j \neq i}^N r_{j,t}(h)]]
\end{aligned}$$

induces noise into the policy gradient update for agent i . Thus,

$$\mathcal{A}(h, a) = \mathcal{A}_i(h, a) + \mathcal{A}_{\neg i}(h, a)$$

Using variance of the sum of random variables

$$\begin{aligned}
\text{Var}(\mathcal{A}(h, a)) &= \text{Var}(\mathcal{A}_i(h, a)) + \text{Var}(\mathcal{A}_{\neg i}(h, a)) \\
&\quad + 2\text{Cov}(\mathcal{A}_i(h, a), \mathcal{A}_{\neg i}(h, a))
\end{aligned}$$

To express the equation in terms of variance, we use the Cauchy-Schwarz inequality, which states that for any two random variables \mathcal{A}_i and $\mathcal{A}_{\neg i}$:

$$\text{Cov}(\mathcal{A}_i(h, a), \mathcal{A}_{\neg i}(h, a)) \leq \sqrt{\text{Var}(\mathcal{A}_i(h, a))\text{Var}(\mathcal{A}_{\neg i}(h, a))}$$

By substituting this inequality, we get an upper bound on our equation,

$$\begin{aligned}
\text{Var}(\mathcal{A}(h, a)) &\leq \text{Var}(\mathcal{A}_i(h, a)) + \text{Var}(\mathcal{A}_{\neg i}(h, a)) \\
&\quad + 2\sqrt{\text{Var}(\mathcal{A}_i(h, a))\text{Var}(\mathcal{A}_{\neg i}(h, a))} \\
\text{Var}(\mathcal{A}(h, a)) &\leq (\sqrt{\text{Var}(\mathcal{A}_i(h, a))} + \sqrt{\text{Var}(\mathcal{A}_{\neg i}(h, a))})^2
\end{aligned}$$

Thus, we get $\text{Var}(\hat{\nabla}\theta_i J|h, a) \propto \text{Var}(\hat{A}_{\neg i}|h, a)$

□

The above equation shows that the variance of the policy gradient update grows approximately linearly with the number of agents in the multi-agent system. This increase in variance reduces the signal-to-noise ratio of the policy gradient, necessitating more updates for effective learning. Proper credit assignment can mitigate this issue by enhancing the signal-to-noise ratio, thereby facilitating more sample-efficient learning.

E Policy Gradient Update Equivalence with Reward Redistribution

In this subsection, we establish that the policy gradient update for an arbitrary agent k , derived from the reward redistribution function $R_{\omega, \kappa}$, shares the same direction as the policy gradient update under the environment's original reward function R_ζ , though potentially with a different magnitude. This ensures that the policy update trajectory towards the optimal policy is preserved for every agent.

Proposition 1. Let π_θ be the joint policy in a decentralized execution paradigm, where the joint policy is the product of individual agent policies: $\pi_\theta = \prod_{k=1}^N \pi_{\theta_k}$. The policy gradient update for an arbitrary agent k under the reward redistribution function $R_{\omega, \kappa}$ is proportional to the policy gradient update under the environment's original reward function R_ζ , preserving the direction of the policy gradient and hence the joint policy update trajectory towards the optimal joint policy.

$$\nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{t=1}^{|\tau|} r_{k,t} \right] = \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} [\delta(\tau) r_{\text{global,episodic}}(\tau)] \quad (12)$$

where τ is the multi-agent trajectory attained from the joint policy π_θ and $\delta : \mathcal{H} \times \mathcal{A} \times \mathcal{H}_{|\tau|} \times \mathcal{A}_{|\tau|} \rightarrow \mathbb{R} \in [0, 1]$ is a function conditioned on the trajectory.

Proof. Consider the policy gradient update for agent k under the reward redistribution function (For brevity, we drop the detailed notation of the variables.):

$$\nabla_{\theta_k} J(\theta_k) = \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} [r_{\text{global,episodic}}(\tau)] \quad (13)$$

From the definition of the reward redistribution function in Assumption 1 of the main text , we have:

$$\begin{aligned} \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} [r_{\text{global,episodic}}(\tau)] &= \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{i=1}^N \sum_{t=1}^T r_{i,t} \right] \\ &= \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{t=1}^T r_{k,t} + \sum_{i \neq k} \sum_{t=1}^T r_{i,t} \right] \\ &= \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{t=1}^T r_{k,t} \right] + \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{i \neq k} \sum_{t=1}^T r_{i,t} \right]. \end{aligned}$$

Given the definitions $\sum_{i=1}^N w_{t,i}^{\text{agent}} = 1$, equation 7 in the main text , and $\sum_{t=1}^{|\tau|} w_t^{\text{temporal}} = 1$, equation 8 in the main text , we rewrite the above equation as:

$$\begin{aligned} \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} [r_{\text{global,episodic}}(\tau)] &= \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{t=1}^{|\tau|} r_{k,t} \right] \\ &\quad + \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\left(\sum_{t=1}^{|\tau|} w_t^{\text{temporal}} (1 - w_{k,t}^{\text{agent}}) \right) r_{\text{global,episodic}}(\tau) \right] \\ &\quad \text{Let } (w_t^{\text{temporal}} (1 - w_{k,t}^{\text{agent}})) = M_t, \\ \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} [r_{\text{global,episodic}}(\tau)] &= \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{t=1}^{|\tau|} r_{k,t} \right] \\ &\quad + \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\left(\sum_{t=1}^{|\tau|} M_t \right) r_{\text{global,episodic}}(\tau) \right] \\ \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\left(1 - \sum_{t=1}^{|\tau|} M_t \right) r_{\text{global,episodic}}(\tau) \right] &= \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{t=1}^{|\tau|} r_{k,t} \right] \end{aligned}$$

Comparing with equation 12 we get:

$$\delta(\tau) = 1 - \sum_{t=1}^{|\tau|} M_t.$$

Since $1 \geq (1 - w_{k,t}^{\text{agent}}) \geq 0$, $1 \geq w_t^{\text{temporal}} \geq 0$, and $\sum_{t=1}^{|\tau|} w_t^{\text{temporal}} = 1$, the term $\sum_{t=1}^{|\tau|} w_t^{\text{temporal}} \times (1 - w_{k,t}^{\text{agent}})$ represents a weighted sum, ensuring that:

$$1 \geq \delta(\tau) = 1 - \sum_{t=1}^{|\tau|} M_t \geq 0. \quad (14)$$

Now we show that the individual policy updates of each agent push the joint policy update towards the optimal joint policy, given that the update direction of each agent's individual policy under the reward redistribution function is the same as the joint policy update under the original reward function.

The joint policy is the product of the individual agents' policies $\pi_\theta = \prod_{k=1}^N \pi_{\theta_k}$ where $\theta = (\theta_1, \theta_2, \dots, \theta_N)$ represents the parameters of the joint policy. In multi-agent systems, the joint policy gradient can be decomposed into the sum of individual agents' policy gradients:

$$\nabla_\theta J(\theta) = \sum_{k=1}^N \nabla_{\theta_k} J(\theta_k)$$

For agent k , the policy gradient update under the reward redistribution function $\mathcal{R}_{\omega,\kappa}$ is:

$$\nabla_{\theta_k} J_{\omega,\kappa}(\theta_k) = \mathbb{E}_{\pi_{\theta_k}} [\nabla_{\theta_k} \log \pi_{\theta_k}(a_k | h_k) A_k^{\omega,\kappa}(h_k, a_k, h_{|\tau|}, a_{|\tau|})]$$

where $A_k^{\omega,\kappa}(h_k, a_k, h_{|\tau|}, a_{|\tau|})$ is the advantage function for agent k under the redistributed reward function $\mathcal{R}_{\omega,\kappa}$. Similarly, under the environment's original reward function \mathcal{R}_ζ , the policy gradient for agent k is:

$$\nabla_{\theta_k} J_\zeta(\theta_k) = \mathbb{E}_{\pi_{\theta_k}} [\nabla_{\theta_k} \log \pi_{\theta_k}(a_k | h_k) A_k^\zeta(h_k, a_k)]$$

Given that the update direction is preserved under the reward redistribution, the advantage function $A_k^{\omega,\kappa}(h_k, a_k, h_{|\tau|}, a_{|\tau|})$ and $A_k^\zeta(h_k, a_k)$ share the same sign for the same (h_k, a_k) , meaning that both updates point in the same direction.

Now, for the joint policy gradient under the original reward function, we have:

$$\nabla_\theta J_\zeta(\theta) = \sum_{k=1}^N \nabla_{\theta_k} J_\zeta(\theta_k)$$

Similarly, for the joint policy gradient under the redistributed reward function:

$$\nabla_\theta J_{\omega,\kappa}(\theta) = \sum_{k=1}^N \nabla_{\theta_k} J_{\omega,\kappa}(\theta_k)$$

From equation 12 we know that,

$$\nabla_{\theta_k} J_{\omega,\kappa}(\theta_k) = \delta_k \nabla_{\theta_k} J_\zeta(\theta_k)$$

Since the individual policy gradient updates point in the same direction under both reward functions, it follows that the joint policy update direction is also preserved. More formally, we can express the policy gradient under the redistributed reward function as:

$$\nabla_\theta J_{\omega,\kappa}(\theta) = \sum_{k=1}^N \delta_k \nabla_{\theta_k} J_\zeta(\theta_k)$$

where $\delta_k \in [0, 1]$ represents the magnitude scaling based on the contribution of agent k . The scaling factor δ_k is less than or equal to 1 due to the redistribution, but crucially, the direction remains the same. This means that each agent's policy update, while potentially smaller in magnitude, still pushes the joint policy in the direction of the optimal joint policy.

The reward redistribution function improves learning by reducing the variance in the gradient estimates and improves the accuracy of the advantage function approximation. This variance reduction improves the overall signal-to-noise ratio that leads to more reliable updates for each agent, which in turn ensures that the joint policy improves more consistently towards optimality, refer Appendix D. Since the direction of the updates remains the same, the overall policy update trajectory is preserved, and the agents collectively converge to the optimal joint policy.

Thus, training a policy with the reward redistribution function is equivalent to training with the environment's original reward function, as it preserves the direction of each agent's policy gradient update. This ensures that the policy evolves similarly in both settings and that the policy update trajectory for an arbitrary initial policy is preserved. \square

F Preserving Gradient Direction Under Reward Redistribution

We now show that the policy gradient update for any individual agent under our redistributed reward function shares the same *direction* as it would under the environment's original reward function. This guarantee ensures that learning trajectories remain consistent, despite the introduction of denser rewards.

Proposition 2 (Gradient Direction Preservation). *Let $\pi_\theta = \prod_{k=1}^N \pi_{\theta_k}$ be the joint policy in a decentralized MARL setting. Denote the reward functions by \mathcal{R}_ζ (original) and $\mathcal{R}_{\omega,\kappa}$ (redistributed) as introduced in Assumption 1 of the main text. Then, for any agent k , the gradient update under $\mathcal{R}_{\omega,\kappa}$ is proportional to the gradient update under \mathcal{R}_ζ , ensuring the same direction of policy improvement:*

$$\nabla_{\theta_k} J_{\omega,\kappa}(\theta_k) = \delta_k \nabla_{\theta_k} J_\zeta(\theta_k),$$

where $\delta_k \in [0, 1]$ is a scalar that may scale the gradient magnitude but not its direction.

Proof. **Step 1: Expressing the Joint and Individual Rewards**

Under the redistributed reward $\mathcal{R}_{\omega,\kappa}$, the global episodic reward $r_{\text{global,episodic}}(\tau)$ is decomposed as

$$r_{\text{global,episodic}}(\tau) = \sum_{i=1}^N \sum_{t=1}^{|\tau|} r_{i,t},$$

with $r_{i,t}$ being agent i 's assigned reward at timestep t . By definition, each $r_{i,t}$ is scaled by the temporal and agent-specific weights w_t^{temporal} and $w_{i,t}^{\text{agent}}$ satisfying

$$\sum_{t=1}^{|\tau|} w_t^{\text{temporal}} = 1 \quad \text{and} \quad \sum_{i=1}^N w_{i,t}^{\text{agent}} = 1 \quad \forall t.$$

Step 2: Gradient Update Under the Redistributed Reward

Let $J_{\omega,\kappa}(\theta_k) = \mathbb{E}_{\pi_{\theta_k}}[r_{\text{global,episodic}}(\tau)]$ be the expected return for agent k under $\mathcal{R}_{\omega,\kappa}$. Then:

$$\nabla_{\theta_k} J_{\omega,\kappa}(\theta_k) = \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{i=1}^N \sum_{t=1}^{|\tau|} r_{i,t} \right].$$

Since $r_{i,t} = r_{k,t}$ when $i = k$ and $r_{i,t}$ otherwise, we split the sum into

$$\sum_{t=1}^{|\tau|} r_{k,t} + \sum_{i \neq k} \sum_{t=1}^{|\tau|} r_{i,t}.$$

Thus,

$$\nabla_{\theta_k} J_{\omega,\kappa}(\theta_k) = \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{t=1}^{|\tau|} r_{k,t} \right] + \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{i \neq k} \sum_{t=1}^{|\tau|} r_{i,t} \right].$$

Using the normalization properties of w_t^{temporal} and $w_{i,t}^{\text{agent}}$ (Assumption 1), we rewrite the second term in terms of $r_{\text{global,episodic}}(\tau)$ and define $M_t = w_t^{\text{temporal}}(1 - w_{k,t}^{\text{agent}})$. The expression becomes:

$$\nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} [r_{\text{global,episodic}}(\tau)] = \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{t=1}^{|\tau|} r_{k,t} \right] + \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\left(\sum_{t=1}^{|\tau|} M_t \right) r_{\text{global,episodic}}(\tau) \right].$$

Collecting terms gives:

$$\nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\left(1 - \sum_{t=1}^{|\tau|} M_t \right) r_{\text{global,episodic}}(\tau) \right] = \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{t=1}^{|\tau|} r_{k,t} \right].$$

Defining $\delta(\tau) = 1 - \sum_{t=1}^{|\tau|} M_t$ where

$$M_t = w_t^{\text{temporal}} \cdot (1 - w_{k,t}^{\text{agent}}).$$

We rely on the following properties of the weighting functions: Firstly, for each t , $0 \leq w_t^{\text{temporal}} \leq 1$ and $0 \leq w_{k,t}^{\text{agent}} \leq 1$. Secondly, the agent-specific weights satisfy $\sum_{i=1}^N w_{i,t}^{\text{agent}} = 1$, which implies $0 \leq 1 - w_{k,t}^{\text{agent}} \leq 1$. Finally, the temporal weights sum to one: $\sum_{t=1}^{|\tau|} w_t^{\text{temporal}} = 1$.

Given these:

1. Since $0 \leq 1 - w_{k,t}^{\text{agent}} \leq 1$ and $w_t^{\text{temporal}} \geq 0$, it follows that

$$0 \leq M_t = w_t^{\text{temporal}} \cdot (1 - w_{k,t}^{\text{agent}}) \leq w_t^{\text{temporal}}.$$

2. Summing over all t ,

$$0 \leq \sum_{t=1}^{|\tau|} M_t \leq \sum_{t=1}^{|\tau|} w_t^{\text{temporal}} = 1.$$

3. Therefore,

$$0 \leq \sum_{t=1}^{|\tau|} M_t \leq 1.$$

4. Substituting this range into the definition of $\delta(\tau)$,

$$\delta(\tau) = 1 - \sum_{t=1}^{|\tau|} M_t,$$

implies

$$1 - 1 \leq \delta(\tau) \leq 1 - 0,$$

which simplifies to

$$0 \leq \delta(\tau) \leq 1.$$

Thus, $\delta(\tau)$ is guaranteed to lie within the interval $[0, 1]$.

Therefore,

$$\nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} [\delta(\tau) r_{\text{global,episodic}}(\tau)] = \nabla_{\theta_k} \mathbb{E}_{\pi_{\theta_k}} \left[\sum_{t=1}^{|\tau|} r_{k,t} \right].$$

implies $\delta(\tau)$ scales the original reward but does not change its sign, indicating equivalence in *direction* (though not necessarily magnitude).

Step 3: Equivalence of Gradient Directions

Consider the policy gradient for agent k under \mathcal{R}_ζ :

$$\nabla_{\theta_k} J_\zeta(\theta_k) = \mathbb{E}_{\pi_{\theta_k}} \left[\nabla_{\theta_k} \log \pi_{\theta_k}(a_k | h_k) A_k^\zeta(h_k, a_k) \right],$$

and under $\mathcal{R}_{\omega,\kappa}$:

$$\nabla_{\theta_k} J_{\omega,\kappa}(\theta_k) = \mathbb{E}_{\pi_{\theta_k}} \left[\nabla_{\theta_k} \log \pi_{\theta_k}(a_k | h_k) A_k^{\omega,\kappa}(h_k, a_k, h_{|\tau|}, a_{|\tau|}) \right].$$

From the above argument, each advantage $A_k^{\omega,\kappa}$ is effectively a scaled version of A_k^ζ (or has the same sign for every (h_k, a_k)). Thus there exists a $\delta_k \in [0, 1]$ such that

$$\nabla_{\theta_k} J_{\omega,\kappa}(\theta_k) = \delta_k \nabla_{\theta_k} J_\zeta(\theta_k).$$

Summing over all agents k implies the *joint* policy gradient under $\mathcal{R}_{\omega,\kappa}$ is a scaled version of that under \mathcal{R}_ζ , preserving the direction of policy improvement across agents.

Step 4: Implications for Learning Trajectories

Since scaling a gradient vector by a positive factor does not affect its direction, each agent's update drives policy parameters toward the same attractors as in the original reward setting. Hence, the learning trajectory is preserved, ensuring that:

- No spurious local optima are introduced by the redistribution.
- Convergence to optimal joint policies remains unchanged.
- Variance reductions from denser rewards improve stability without biasing the solution.

Therefore, training under the redistributed reward function $\mathcal{R}_{\omega,\kappa}$ is *directionally equivalent* to training under \mathcal{R}_ζ , ensuring that agents converge to the same optimal policies, only faster and more stably. \square

Implications for Joint Policy Updates and Convergence. While Proposition 2 guarantees that each individual agent's gradient under the redistributed rewards is aligned with its counterpart under the original rewards (up to a non-negative scaling factor), the aggregate joint policy gradient—being the sum of these scaled vectors—may not be perfectly parallel to the original joint gradient. However, because the scaling factors δ_k are non-negative and each agent's update direction is preserved, no agent receives a misleading gradient signal. This alignment reduces the variance of gradient estimates by providing more immediate, fine-grained feedback, which typically leads to more stable updates. Although potential-based reward shaping does not universally guarantee faster convergence from a purely theoretical standpoint, our empirical results (see Section X) demonstrate that TAR² often accelerates learning and improves sample efficiency in practice, likely due to the reduced variance and more informative credit assignment.

G Reward Modeling Details

Below is an expanded discussion for each of the three architectural design choices.

Final State Conditioning

When agents are not conditioned on the final outcome of the episode, each intermediate state or action is evaluated against an *average* future return (i.e., the expected reward given only the current local context). Such a perspective fails to distinguish sequences of actions that may appear similar locally but differ substantially in terms of eventual success or failure. For instance, two nearly identical trajectories might diverge in a crucial final step—one results in a high payoff (e.g., defeating an opponent, scoring a goal), while the other yields no payoff.

By explicitly incorporating the *final* state and action into the reward prediction, we “tell” the model how the trajectory actually ended. This makes each intermediate state-action pair consequential on the actual *final* outcome, rather than some local or average guess. In essence:

1. *Outcome Awareness*: Agents learn that an action *now* might be pivotal in leading to a successful (or failed) final outcome.
2. *Reduced Ambiguity*: Instead of returning an average future reward for a partially observed path, the model learns a more grounded mapping, since it knows exactly whether the trajectory eventually succeeded or failed.
3. *Sharper Credit Signals*: Conditioning on the final outcome helps separate important from unimportant steps (and which agent is responsible), making credit assignment clearer and potentially speeding up learning.

In single-agent RL, similar outcome- or goal-conditioning methods have shown improvements in credit assignment under delayed rewards [Harutyunyan *et al.*, 2019; Ren *et al.*, 2021]. By extending this approach to multi-agent settings, we ensure that the reward redistribution mechanism can focus on how each agent’s intermediate action contributed to the eventual outcome.

Inverse Dynamics Modeling

An inverse dynamics model predicts $a_{i,t}$ given the embeddings of $(h_{i,t}, h_{i,t+1})$ or some representation of the consecutive states. Including such a module has two benefits:

1. *Causality in Latent Space*: If the network can accurately predict which action was taken to go from one latent representation to another, it necessarily encodes features that distinguish different agent behaviors. This encourages the latent space to reflect transitions that are *functionally* relevant, rather than arbitrary correlations. In turn, reward predictions become grounded in actual causal relationships between states, actions, and outcomes.
2. *Stabilized Credit Assignment*: By having to reconstruct or identify each agent’s action, the network implicitly learns a more structured notion of time, ordering, and agent identity. This can reduce confusion about “who did what” when multiple agents act in parallel.

In single-agent RL, inverse dynamics objectives have been used to improve state representations, encourage meaningful features, and stabilize training [Pathak *et al.*, 2017; Agrawal *et al.*, 2015]. While these works focus primarily on single-agent or self-supervised exploration, the central insight remains: inverse dynamics tasks force the model to learn more discriminative, causally grounded embeddings. Although less common in multi-agent literature, these principles still apply—each agent’s identity and action path is clearer when the model learns to predict actions from state embeddings. This clarity can, in turn, facilitate more coherent reward redistribution.

Probabilistic Reward Redistribution

Without explicit normalization, the model may produce reward predictions $\{\hat{r}_{i,t}\}$ that do not sum to the true global return $r_{\text{global,episodic}}$. This can introduce inconsistencies and may violate the assumptions of potential-based reward shaping (where we require the sum of shaped rewards to match the original environmental return). Normalizing across time and agents solves this mismatch in multiple ways:

1. *Ensuring Consistency with the Environment*: By forcing $\sum_{i=1}^N \sum_{t=1}^{|\tau|} r_{i,t} = r_{\text{global,episodic}}$, we guarantee that each agent’s shaped reward remains faithful to the actual outcome, maintaining optimal policy invariance.
2. *Controlling Scale and Sign*: Rewards exceeding the actual global return or dropping below zero may lead to distorted policy updates. Normalization keeps each predicted reward in a sensible range, often $[0, 1]$ when scaled by the total return. Ensuring nonnegative rewards can simplify policy learning and avoid contradictory signals (where some agent “loses” reward that doesn’t exist in the environment).
3. *Variance Reduction and Interpretability*: By bounding each per-agent reward, the magnitude of the gradients may be more stable, reducing learning variance. Probability-like distributions also yield an intuitive interpretation: how the global reward is “allocated” among agents and time steps.
4. *Preventing Drift in Unconstrained Predictions*: In unconstrained models (e.g., a simple regressor for each time step), predictions can drift or accumulate error. This normalization step ensures that even if the model’s unconstrained outputs $\{\hat{r}_{i,t}\}$ become large or negative, the final distributed rewards remain consistent with the episode’s actual return.

Hence, probabilistic reward redistribution enforces a strict accounting of the environment’s episodic reward, preventing the model from misallocating or “inventing” reward signals, while maintaining alignment with potential-based shaping principles.

H Pseudocode

Below is the pseudocode to train TAR^2 and $MAPPO$:

Algorithm 1 Temporal-Agent Reward Redistribution with Multi-Agent Proximal Policy Optimization

- 1: Number of agents M , Initialize θ , the parameters for policy π , μ , the parameters for state value critic V and $\phi = (\omega, \kappa)$, the parameters for reward redistribution function R , using orthogonal initialization (Hu et al., 2020)
- 2: Set experience buffer $B \leftarrow \emptyset$; Reward redistribution model training frequency ϵ
- 3: Set learning rate $\alpha_\pi, \alpha_Q, \alpha_R$ for AdamW optimizer
- 4: **while** $step \leq step_{max}$ **do**
- 5: set data buffer $D = \{\}$
- 6: **for** $i = 1$ to $batch_size$ **do**
- 7: $\tau = []$ – empty list
- 8: initialize $h_{0,\pi}^{(1)}, \dots, h_{0,\pi}^{(M)}$ actor RNN states
- 9: initialize $h_{0,V}^{(1)}, \dots, h_{0,V}^{(M)}$ state value RNN states
- 10: **for** $t = 1$ to T **do**
- 11: **for all agents** a **do**
- 12: $u_t^{(a)}, h_{t,\pi}^{(a)} = \pi(o_t^{(a)}, h_{t-1,\pi}^{(a)}; \theta)$
- 13: **end for**
- 14: $(v_t^{(1)}, \dots, v_t^{(M)}), (h_{t,V}^{(1)}, \dots, h_{t,V}^{(M)}) = V(s_t^{(1)}, \dots, s_t^{(M)}, u_t^{(1)}, \dots, u_t^{(M)}, h_{t-1,V}^{(1)}, \dots, h_{t-1,V}^{(M)}; \mu)$ – we mask out the actions of agent a while calculating its state value $v^{(a)}$
- 15: Execute actions u_t , observe r_t, s_{t+1}, o_{t+1}
- 16: $\tau += [s_t, o_t, h_{t,\pi}, h_{t,V}, u_t, r_t, s_{t+1}, o_{t+1}]$
- 17: **end for**
- 18: Collect episodic return $r_{\text{global,episodic}}(\tau)$ and the trajectory τ , store $(r_{\text{global,episodic}}(\tau), \tau)$ in the buffer B
- 19: Compute the redistributed reward for trajectory τ by normalizing $R(i, t; \phi)$ across i and t axes to generate $w_{i,t}^{\text{agent}}$ and w_t^{temporal} and then multiply the normalized weights with the actual episodic reward $r_{\text{global,episodic}}$
- 20: Compute return G_i for each agent $i = 1, \dots, M$ using $R(i, t; \omega, \kappa)$, to learn the V function on τ and normalize with PopArt
- 21: Compute advantage estimate and target values $\hat{A}^1, \dots, \hat{A}^M$ and $\hat{V}^1, \dots, \hat{V}^M$ via GAE using state value estimates on τ , after PopArt denormalization
- 22: Split trajectory τ into chunks of length L
- 23: **for** $l = 0, 1, \dots, T//L$ **do**
- 24: $D = D \cup (\tau[l : l + T], \hat{A}[l : l + L], G[l : l + L], \bar{G}[l : l + L])$
- 25: **end for**
- 26: **end for**
- 27: **for** mini-batch $k = 1, \dots, K$ **do**
- 28: $b \leftarrow$ random mini-batch from D with all agent data
- 29: **for** each data chunk c in the mini-batch b **do**
- 30: update RNN hidden states for π, Q and V from first hidden state in data chunk
- 31: **end for**
- 32: **end for**
- 33: Adam update θ on $L(\theta)$ with data b
- 34: Adam update μ on $L(\mu)$ with data b
- 35: **if** step % ϵ is 0 **then** **then**
- 36: **for** $i = 1$ to $num_updates$ **do**
- 37: $\tau \leftarrow$ randomly sample trajectories from B
- 38: Compute reward redistribution loss using Eq 2 in Section 4.4 of the main paper
- 39: Adam update (ω, κ) on $L(\omega, \kappa)$ with loss calculated using τ
- 40: **end for**
- 41: **end if**
- 42: **end while**

I Interpretability and Insights

Although our primary focus is on performance and theoretical properties, TAR²'s per-timestep, per-agent reward predictions lend themselves to partial interpretability:

- Agents' importance at specific timesteps can be visualized by examining $w_t^{\text{temporal}} w_{i,t}^{\text{agent}}$.
- Comparing predicted reward distributions across different episodes can hint at consistent agent roles or strategic pivot points in the trajectory.

However, direct interpretability is challenging in high-dimensional multi-agent environments like SMACLite and Google Football, where the intricate interactions and vast state-action spaces complicate simple visualizations. Additionally, developing a systematic interpretability study would require significant additional methodologies and resources, extending beyond the scope of the current work. While we recognize the importance of interpretability and plan to explore it in future research, our current focus remains on establishing robust performance improvements and theoretical guarantees for reward redistribution.

J Detail Task Descriptions

SMACLite [Michalski *et al.*, 2023] A computationally efficient variant of StarCraft II [Samvelyan *et al.*, 2019], We experiment on SMACLite's three battle scenarios with varying levels of complexity: *(i)* 5m_vs_6m, *(ii)* 10m_vs_11m, *(iii)* 3s5z. Each agent's local observation includes the relative positions, unit types, health, and shield strength of both the agent's allies and enemies within its field of view, as well as the agent's own health and shield status. Agents can move in any of the four cardinal directions, stop, perform no operation, or attack any enemy agent within their field of view. The environment includes action masks that highlight valid actions based on the agent's current state. If an agent dies, the default action becomes 'no-op'. Each combat scenario lasts for 100 timesteps (agents can be eliminated sooner). The environment's reward function combines partial rewards for damaging or eliminating enemies, with a maximum possible team return normalized to 20. Link to repository: <https://github.com/ueo-agents/smaclite> (MIT License)

Google Research Football (GRF) [Kurach *et al.*, 2020] A high-fidelity multi-agent environment simulating football (soccer), evaluated on: *(i)* academy 3 vs 1 with keeper, *(ii)* academy counterattack easy, *(iii)* academy pass and shoot with keeper. GRF provides a comprehensive observation space, including features like player positions, ball coordinates, velocity vectors, stamina levels, and proximity to opponents and teammates. The environment's action space includes various football maneuvers such as passing, shooting, dribbling, and tackling, as well as movement in the four cardinal directions and the option to sprint or remain stationary. The input representation type used is 'simple11v2', which encodes critical information about player and ball positioning. Each episode is designed to end when a goal is scored or when a predefined number of timesteps, 100, is reached. GRF environments include various game scenarios like 'academy_3_vs_1_with_keeper', 'academy_counterattack_easy', and 'academy_pass_and_shoot_with_keeper', each offering different challenges for coordination and team play. Reward signals ('scoring,checkpoints') are sparse and tied to events such as goals scored or distance traveled to opponent team's goal post, with an emphasis on long-term planning. The repository for GRF is available at: <https://github.com/google-research/football> (Apache License 2.0).

K Implementation Details and Hyperparameters

The code was run on Lambda Labs deep learning workstation with 2-4 Nvidia RTX 2080 Ti graphics cards. Each training run was run on one single GPU, and required approximately 8 hours.

Hyperparameters used for TAR², STAS, AREL-Temporal, AREL-Agent-Temporal, Uniform and various environment reward configurations that are common to all tasks are shown in Tables 1. The task-specific hyperparameters considered in our grid search for TAR², STAS, AREL-variants in Tables 2, 3 and 4 respectively. Bold values indicate the optimal hyperparameters.

Table 1: Common Hyperparameters for MAPPO algorithms, including PRD variants.

common hyperparameters	value
ppo_epochs	15
ppo_batch_size	30
gamma	0.99
max_episodes	30000
max_time_steps	100
rnn_num_layers_v	1
rnn_hidden_v	64
v_value_lr	5e-4
v_weight_decay	0.0
v_hidden_shape	64
grad_clip_critic_v	0.5
value_clip	0.2
data_chunk_length	10
rnn_num_layers_actor	1
rnn_hidden_actor	64
policy_lr	5e-4
policy_weight_decay	0.0
policy_hidden_shape	64
grad_clip_actor	0.5
policy_clip	0.2
entropy_pen	1e-2
gae_lambda	0.95

Table 2: TAR² hyperparameters.

Env. Name	num heads	depth	dropout	comp. dim	batch size	lr	weight decay	inv. dyn. loss coef.	grad clip val.	model upd. freq.	model upd. epochs	policy lr	entropy coef
Google Football	[3, 4]	[3, 4]	[0.0, 0.1, 0.2]	[16, 64, 128]	[32, 64, 128]	[1e-4, 5e-4, 1e-3]	[0.0, 1e-5, 1e-4]	[1e-3, 1e-2, 5e-2]	[0.5, 5.0, 10.0]	[50, 100, 200]	[100, 200, 400]	[5e-4, 1e-3]	[5e-3, 8e-3, 1e-2]
SMACLite	[3, 4]	[3, 4]	[0.0, 0.1, 0.2]	[16, 64, 128]	[32, 64, 128]	[1e-4, 5e-4, 1e-3]	[0.0, 1e-5, 1e-4]	[1e-3, 1e-2, 5e-2]	[0.5, 5.0, 10.0]	[50, 100, 200]	[100, 200, 400]	[5e-4, 1e-3]	[5e-3, 8e-3, 1e-2]

Table 3: STAS hyperparameters.

Env. Name	num heads	depth	dropout	comp. dim	batch size	lr	weight decay	grad clip val.	model upd. freq.	model upd. epochs
Google Football	[3, 4]	[3, 4]	[0.0, 0.1, 0.2]	[16, 64, 128]	[32, 64, 128]	[1e-4, 5e-4, 1e-3]	[0.0, 1e-5, 1e-4]	[0.5, 5.0, 10.0]	[50, 100, 200]	[100, 200, 400]
SMACLite	[3, 4]	[3, 4]	[0.0, 0.1, 0.2]	[16, 64, 128]	[32, 64, 128]	[1e-4, 5e-4, 1e-3]	[0.0, 1e-5, 1e-4]	[0.5, 5.0, 10.0]	[50, 100, 200]	[100, 200, 400]

Table 4: AREL hyperparameters.

Env. Name	num heads	depth	dropout	comp. dim	batch size	lr	weight decay	grad clip val.	model upd. freq.	model upd. epochs
Google Football	[3, 4]	[3, 4]	[0.0, 0.1, 0.2]	[16, 64 , 128]	[32, 64, 128]	[1e-4, 5e-4 , 1e-3]	[0.0, 1e-5, 1e-4]	[0.5, 5.0, 10.0]	[50, 100, 200]	[100, 200 , 400]
SMACLite	[3, 4]	[3, 4]	[0.0, 0.1, 0.2]	[16, 64 , 128]	[32, 64, 128]	[1e-4, 5e-4 , 1e-3]	[0.0, 1e-5, 1e-4]	[0.5, 5.0, 10.0]	[50, 100, 200]	[100, 200 , 400]