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RESEARCH PAPER

Large neighbourhood search with adaptive guided ejection search for the pickup and delivery problem with time windows

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Abstract An effective and fast hybrid metaheuristic is proposed for solving the pickup and delivery problem with time windows. The proposed approach combines local search, large neighbourhood search and guided ejection search in a novel way to exploit the benefits of each method. The local search component uses a novel neighbourhood operator. A streamlined implementation of large neighbourhood search is used to achieve an effective balance between intensification and diversification. The adaptive ejection chain component perturbs the solution and uses increased or decreased computation time according to the progress of the search. While the local search and large neighbourhood search focus on minimising travel distance, the adaptive ejection chain seeks to reduce the number of routes. The proposed algorithm design results in an effective and fast solution method that finds a large number of new best-known solutions on a well-known benchmark dataset. Experiments are also performed to analyse the benefits of the components and

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heuristics and their combined use to achieve a better understanding of how to better tackle the subject problem.

Keywords Large neighbourhood · Guided ejection · Vehicle routing

1 Introduction

The pickup and delivery problem (PDP) is a vehicle routing problem in which customers are paired together and a pair must be serviced by the same vehicle (Savelsbergh and Sol 1995). In other words, a load must be collected from one location and delivered to another location by a single vehicle. Clearly there are also ordering or precedence constraints to ensure that the collection site is visited before the delivery site. If there are time windows during which the customers must be visited then the problem is known as pickup and delivery problem with time windows (PDPTW) (Dumas et al. 1991). The problem commonly arises in real-world logistics and solution methodologies have significant practical application. As such, a large number of techniques have been developed for PDPTW. These include approaches based on exact methods as well as heuristics. The exact based methods have advantages such as solving to provable optimality or providing bounds. They also tend to perform very well on smaller and medium sized instances. The significant disadvantage with these methods though is that they sometimes perform poorly on larger difficult instances. Heuristic methods on the other hand tend to scale well and are more robust for larger instances but are easily outperformed on smaller instances. It could also be argued that some heuristic methods are easier to develop and maintain and to adapt to new problem requirements. These could be the reasons that the majority of commercial vehicle routing software packages use heuristic-based methods (Hall and Partyka 2016).

Most of the exact methods proposed for the PDPTW are versions of branch and cut and/or branch and price (Berbeglia et al. 2007; Parragh et al. 2008). Branch and price is a branch and bound approach where each node in the branch and bound tree is solved using column generation. For PDPTW, the nodes are linear programming, set partitioning formulations of the problem and the columns (variables) represent possible routes. Generating the new columns, known as the pricing problem, could be solved by a variety of exact and heuristic methods. Most approaches use a dynamic programming method applied to a shortest path type formulation. Solving the pricing problem efficiently is the key to a successful approach because this is where most of the computation time is used. Very efficient pricing problem solving can be achieved though through the use of heuristics and problem structure exploitation. Other significant speed ups and algorithm improvements can often be achieved through other ideas such as branching strategies, stabilisation, column management, approximations and other heuristics.

One of the earliest applications of branch and price to PDPTW is given by Dumas et al. (1991) although it was clearly limited by the computing hardware available at the time. A branch and price method published later in the decade by Savelsbergh and Sol (1998) was already able to solve larger instances of the problem in practical computation times. One of the most recent examples of branch and price applied to PDP is from Venkateshan and Mathur (2011) and an example of a column generation based heuristic applied to PDPTW is given by Xu et al. (2003).

An alternative exact method is branch and cut. Branch and cut differs from branch and price in that a different formulation is used in which all the variables are present at the start rather than being dynamically generated. New constraints are generated at the nodes of the branch and bound tree. These cuts aim to accelerate the discovery of the optimal integer solution. For PDPTW and other vehicle routing problems different families of cuts have been proposed. Examples of branch and cut methods applied to other pickup and delivery problems include (Lu and Dessouky 2004; Ruland and Rodin 1997). A recent paper by Ropke and Cordeau (2009) presents a branch and cut and price method and test it on one of the most commonly used sets of benchmark instances by Li and Lim (2003). The results show that although the smaller instances can be solved to optimality very efficiently, the larger instances are still out of reach for exact methods.

Although there are several examples of exact methods for PDP, the majority of publications are on heuristic methods and in particular metaheuristics. Many of these approaches use a variant of neighbourhood search. Due to the pairing and precedence constraints present in PDP but not present in other variants of vehicle routing problems, there is less choice of neighbourhood operators available for the problem. There are, however, three operators which were employed in the earlier metaheuristics. The first is to simply move a pickup and delivery pair from one route to another. The second is to swap a pair of pickups and deliveries between two routes. The third is to move the pickup and delivery within a route. Li and Lim (2003) and Nanry and Barnes (2000) both present metaheuristics built around these local search operators. These simpler metaheuristics were later outperformed though by methods based on large neighbourhood search (LNS). As the name implies, LNS uses much larger neighbourhoods and searches them using heuristic and/or exact methods. The motivation is that larger neighbourhoods should provide better local optima. The disadvantage is that they can be slower to search due to their increased size. One of the first examples of LNS applied to PDP is from Bent and Van Hentenryck (2006). They explore large neighbourhoods using a branch and bound method. Ropke and Pisinger (2006) then published an alternative LNS which uses a simpler heuristic method for creating and searching large neighbourhoods. Their heuristic is based on the “disrupt and repair” or “ruin and recreate” heuristics. Customers are iteratively removed from solution routes using heuristics such as similarity measures and then re-inserted using heuristics such as regret assignment or greedy assignment. The parameters for these heuristics are dynamically adapted based on their success rate. Their LNS produced many new best-known solutions on the Li and Lim benchmark instances. Another paper that has achieved notable results on these benchmark instances is the Guided Ejection Search of Nagata and Kobayashi (2010a). The algorithm only aims to optimise the single objective of reducing the number of vehicles used but does so very effectively. It is a relatively simple but effective iterative heuristic that randomly adjusts the solution using customer swaps and moves. At each iteration, attempts are made to insert heuristically selected customers from removed routes. They later adapted this method into an evolutionary approach to again further improve their results with respect to the full objective function (Nagata and Kobayashi 2010b). There are also several publications detailing metaheuristics applied to specific variants of PDP. For example, PDP with transfers (Masson et al. 2012), PDP with LIFO loading (Cherkesly et al. 2015), single vehicle PDP (Gendreau et al. 1999; Kammarti et al.2004) and dynamic PDP (Gendreau et al. 2006). For further reading on PDP there are also several survey and overview papers (Battarra et al. 2014; Berbeglia et al. 2007; Parragh et al. 2008; Savelsbergh and Sol 1995).

From the above review of related literature, it is clear that PDPTW is a well-studied variant of the vehicle routing problem. A variety of exact and heuristic methods have been proposed and this has helped to advance our knowledge into how to tackle this difficult combinatorial optimization problem. Nevertheless, it is also clear that large instances of PDPTW remain a difficult challenge to state-of-the-art techniques. In this paper, an effective and efficient heuristic method is proposed which uses several tailored neighbourhood moves within a streamlined version of adaptive large neighbourhood search (Ropke and Pisinger 2006) also incorporating guided ejection search (Nagata and Kobayashi 2010a). The proposed technique is not only competitive with state-of-the-art methods but it produces a number of new best-known solutions for the well-known Li and Lim benchmark instances (2003). Moreover, this paper also makes a contribution to increase our understanding of which combination of search mechanisms can result in a highly effective and fast hybrid metaheuristic algorithm capable of solving large instances of the PDPTW. Experimental results also show that each of the components plays an essential role to make the overall algorithm perform very well over a wide range of problem sizes.

In the next section, we provide the problem definition for the benchmark PDPTW instances tested on. Section 3 introduces the algorithm and Sect. 4 details the computational results. Finally we present some conclusions and suggested future research in Sect. 5.

2 Problem definition

A solution to the pickup and delivery routing problem with time windows or PDPTW is a set of routes for a fleet of vehicles. Each route is executed by one vehicle and consists of a sequence of pickups and deliveries at customers' locations. Each transportation request is a pickup and delivery pair which must be executed in that order by the same vehicle while satisfying the vehicle capacity and the given time windows. The goal is to minimise the number of routes, hence the number of vehicles needed and to minimise the total travelled distance.

2.1 Parameters

M Set of customers $1 \ldots m$

L Set of locations $0, 1 \ldots m$ where 0 is the depot and $1 \ldots m$ are the customers

P Set of pickup customers

D Set of delivery customers

The intersection of P and D is the empty set $(P \cap D = \emptyset)$ and the union of P and D is M $(P \cup D = M)$. Each pickup $p_i \in P$ is associated with he corresponding delivery $d_i \in D$. Let:

- t_{ij} The travel time between locations i and j
- d_{ij} The distance between locations i and j
- s_i The service duration for location i
- e_i The earliest time at which the service at location i can start
- l_i The latest time at which the service at location i it must start

A service duration and service window have been included for the depot to make the model tidier but in the test instances the service duration for the depot is zero and the service window for the depot is unbounded. This is done in previously published models also, for example: (Nagata and Kobayashi 2010a).

2.2 Constraints

A route is a sequence of locations visited by a vehicle. A vehicle must start at a depot, visit at least two customers (corresponding to a pickup and a delivery) and return to the depot. A route of length n is therefore denoted by $v_0, v_1 \ldots v_n, v_{n+1}$ where v_0 and v_{n+1} are the depot and visits $v_1 \ldots v_n$ are customers. If a route contains a pickup p_i then it must also contain its corresponding delivery d_i (and vice versa) and p_i must precede d_i in the sequence. These are the pairing and precedence constraints, respectively. Each pickup p_i has a nonnegative demand q_i and the corresponding delivery d_i has the demand $-q_i$.

The current total load c_{v_i} carried by a vehicle v at a visit i where $i \geq 1$ is

$$c_{v_i} = \sum_{j=1}^{i} q_{v_j}. \tag{1}$$

All vehicles have an identical capacity Q and at all visits in a route the total carried load must not exceed the vehicle capacity,

$$c_{v_i} \leq Q \quad \forall v_i \in \{1 \ldots n\}. \tag{2}$$

The begin time b_v for each visit's service in the route is calculated as

$$b_{v_0} = 0,$$

$$b_{v_i} = \max\{b_{v_{i-1}} + s_{v_{i-1}} + t_{v_{i-1}v_i}, e_{v_i}\} \quad \forall i \in \{1 \ldots n\}. \tag{3}$$

In a route the services must begin before or at a location's latest service start time

$$b_{v_i} \leq l_{v_i} \quad \forall v_i \in \{0 \ldots n\}. \tag{4}$$

A solution to this PDPTW described above is set of feasible routes which together service all customers exactly once according to the conditions established in the formulation.

2.3 Objectives

The primary objective is to minimise the number of routes, hence the number of vehicles, in the solution. To compare solutions which have the same number of routes, a secondary objective is commonly used. This secondary objective is to minimise the total distance of all routes where distance of a route of length n is

$$\sum_{i=0}^{n} d_{v_i,v_{i+1}}. \tag{5}$$

3 Hybrid large neighbourhood search and guided ejection search

As discussed in the introduction, a number of methods have been proposed in the literature to tackle the PDPTW. The algorithmic design proposed here incorporates specialised neighbourhood operators to enhance the effectiveness of the local search, adaptive ejection search to reduce the number of routes and streamlined large neighbourhood search to enhance the efficiency of the search. The motivation behind the proposed algorithm design was to identify the essential mechanisms to reduce the number of routes and the total travelled distance and combine them into a streamlined yet effective and fast method.

The algorithm proposed here is a combination of three separate methods:

(1) A local search which uses four tailored neighbourhood operators.
(2) A simplified version of the adaptive large neighbourhood search (ALNS) of Ropke and Pisinger (2006). One of the simplifications is to remove the adaptive feature so this sub-routine will be referred to as LNS only.
3) A version of the guided ejection search (GES) by Nagata and Kobayashi (2010a).

An outline of the overall algorithm approach is given in Fig. 1 and each of the steps is described in detail in the following subsections. The overall strategy is to perform an effective large neighbourhood search on a solution while exploiting guided ejection chain to reduce the number of routes or perturbing the current solution if reducing the number of routes is not possible. This balance between intensification and diversification results in an effective algorithm as shown by the experimental results presented later in the paper.

```
LocalSearch

while (time remaining)
{
       try and reduce number of vehicles in best solution so far using GES

       if vehicles not reduced then perturb best solution so far

       LocalSearch

       LNS
}
```

Fig. 1 Overall algorithm outline

3.1 Local search

The main purpose of the local search is to construct good-quality initial solutions quickly. To do this it uses four neighbourhood structures and the corresponding operators perform an exhaustive search until no further improvements can be made with respect to all neighbourhoods. The first three neighbourhood moves are used in (Li and Lim 2003; Nanry and Barnes 2000). The neighbourhood moves are:

M1: Insert an un-assigned pickup and delivery (PD) pair into an existing route or create a new route for the PD pair
M2: Un-assign an assigned PD pair and try and insert it into a different route or create a new route for the PD pair
M3: Un-assign a PD pair (*pd1*) from a route (*r1*), un-assign a PD pair (*pd2*) from a route (*r2*) and then try and insert *pd1* into route *r2* and *pd2* into route *r1*
M4: Un-assign a PD pair (*pd1*) from a route (*r1*), un-assign a PD pair (*pd2*) from a route (*r2*) and then try and insert *pd1* into route *r2* and *pd2* into a third route *r3*

Note that in all of the neighbourhoods above, when trying to insert a PD pair into a route the local search tries every possible position for the pickup and for each feasible position for the pickup, also tries every possible feasible position for the drop (position refers to order position in the route). This means that each neighbourhood is explored exhaustively and the best of all neighbour solutions is selected. Hence, this is part of the intensification mechanism in the proposed approach. We are not aware of the move M4 being used for PDPTW previously. The rationale behind this move is to have another mechanism for transferring PD pairs between routes. M3 does this while maintaining the same number of PD pairs in each of the two routes involved. In M4 the transfer ends up with two routes having a different number of PD pairs after the move.

A single neighbour operator is applied exhaustively until no more improving moves can be made using that operator. The next operator is then similarly applied exhaustively until no more improving moves are available, and then the next operator and so on. The order the operators are applied is M1 to M4 as in the list above. When operator M4 has been exhausted then the local search returns to operator M1. This process is repeated until there are no available improvements using any of the operators. For the smaller neighbourhoods defined by operators M1 and M2, when testing a possible insertion, a best improvement strategy is used, meaning that the insertion is tested on all available routes and the best improvement move is used. For the larger operators M3 and M4, a first found improvement strategy is used, meaning that as soon as an improving move is found then it is accepted. The local search uses the hierarchical objective because it is possible to reduce the number of routes in the solution using operators M2 and M4.

The intensified local search described above can be completed quickly but the solutions can often still be significantly improved with respect to the objectives of minimising the number of routes and minimising total distance. The next step in the algorithm is to focus on minimising the number of routes used.

3.2 Guided ejection search

Guided ejection search was originally proposed by Nagata and Braysy (2009) for the vehicle routing problem with time windows (VRPTW). Nagata and Kobayashi (2010a) then developed a version for PDPTW. It only focuses on the objective of minimising the number of routes and their analysis showed that it was very effective on this single objective. An overview of the procedure is given in Fig. 2.

The method starts by randomly selecting a route and un-assigning all the PD pairs in it. It then proceeds to try and re-insert the un-assigned pairs over the remaining routes. When it cannot insert a pair, it un-assigns (ejects) another pair(s) to allow it to insert it. It then perturbs the partial solution and tries again to insert an un-assigned pair. This is repeated until either there are no un-assigned pairs, in which case a route has been successfully removed, or a maximum number of iterations have been reached. If a route is removed then the procedure is repeated by selecting another route and un-assigning the pairs within it and then trying to insert them again over the remaining routes and so on.

The next pair selected for insertion is selected from an un-assigned pairs list on a last in first out (LIFO) basis. LIFO was also used in (Nagata and Kobayashi 2010a), possibly because it improves the efficacy of the ejection heuristic which will be described later. When trying to insert the pair each route is tested in a random order and every possible position for the pair in the route is tested. If a feasible position for the pair is found then it is inserted. If more than one feasible position is found then the position for insertion is selected randomly. As with the local search, testing each possible position means trying each possible position for the pickup and for each possible position for the pickup also trying each possible position for the drop.

If the pair cannot be inserted then an attempt is made to insert it by ejecting one or two pairs from another route. First an attempt is made to insert it by ejecting a single pair and if this fails then every set of two pairs is tested to see if their ejection

```
randomly select a route and un-assign all PD-pairs from it

for a fixed number of iterations
{
        select an un-assigned PD-pair and try and insert it into an existing route

        if PD-pair inserted
        {
                if no more PD-pairs to assign
                        go to 1.
        }
        else
        {
                try and insert the PD-pair by un-assigning one or more other pairs (the
                ejection is heuristically selected by trying to avoid un-assigning
                PD-pairs that were difficult to insert before)

                perturb the solution by randomly moving or swapping PD-pairs between
                routes
        }
}
```

Fig. 2 GES outline

would allow the insertion. A maximum of two pairs was used for increased speed. If more than one set of pairs can be ejected to allow the insertion of the pair, then the set to eject is selected heuristically. Every time an attempt is made to insert a pair, a counter for that pair is increased by one. The heuristic for choosing which pair to eject is the pair with the lowest sum of the counter values (i.e. the set that has been previously attempted to be inserted the least number of times). The motivation behind the heuristic is that if a pair was previously difficult to insert (i.e. the counter value is high) then try not to eject it because it may be difficult to insert again.

The perturbation procedure at line 16 of Fig. 2 not only creates the possibility of later being able to insert pairs but it also reduces the risks of cycling. The perturbation randomly selects one of two possible move operators (each with 0.5 probability) and then executes the move on the current partial solution. The first move (*PairMove*) randomly selects a route and a PD pair within it, then randomly selects a second route and attempts to move the pair to a feasible position in the second route. If there is more than one feasible position in the second route then one is randomly selected. The second move (*SwapMove*) randomly selects two routes and a pair within each route. It then un-assigns the pairs and attempts to insert them into feasible positions in the opposite route. This time it selects the best possible positions (according to the secondary objective function—minimise total distance) rather than a random position. The perturbation finishes when ten moves have been executed.

The implementation in the present work is similar to the original version by Nagata and Braysy except for two changes. The first difference is at line 14. The original algorithm examines all sets of pairs for ejection up to a fixed size. The larger the fixed size, the more sets there are to examine and the longer the algorithm takes. The approach in this paper only examines sets of length one first. That is, it tries ejecting a single pair first and then if this fails in allowing the insertion, then it tries ejecting two pairs. Again the two pairs are selected by minimising the sum of their previous insertion attempt counters.

The second main difference is the stopping condition. Instead of finishing after a certain number of iterations or a fixed time limit, the number of iterations is extended based on the progress of reducing the number of un-assigned pairs. Every time a new partial solution with a new smallest number of un-assigned pairs is found then a counter is reset to zero. The counter is increased by one each time an attempt is made to perturb the solution by doing either *PairMove* or *SwapMove*. The procedure terminates if the counter reaches a predefined value (one million in our implementation). The motivation behind this heuristic is to terminate quickly if the progress suggests that the route will not be removed but to provide more time when the number of un-assigned pairs is being reduced but more slowly. This modified guided ejection chain mechanism maintains the intensification ability of the original approach but it also incorporates an adaptive ability to push the intensification or not according to the current solution.

3.3 Large neighbourhood search

After the modified GES, the local search is applied again followed by a large neighbourhood search. An overview of the LNS is shown in Fig. 3.

The LNS can be described as a "disrupt and repair" heuristic. It repeatedly un-assigns some PD pairs from a solution and then attempts to heuristically re-assign them but creating an improved solution. The method is based on the ALNS of Ropke and Pisinger but with several changes. One of the main changes was to replace a simulated annealing + noise acceptance criterion with late acceptance hill climbing (LAHC) (Burke and Bykov 2016). The main reason for this was to have a streamlined version by simplifying parameter setting because LAHC has only one parameter to set. LAHC is very similar to SA in that it accepts non-improving solutions but it replaces the probability-based acceptance criterion by a time-based deterministic one. At the start of the algorithm, LAHC may accept many non-improving solutions and so provide more search diversification whereas at the end the search intensifies as less and less non-improving solutions are accepted. LAHC is described by Fig. 4. In the figure, the initial solution is the solution created by the GES phase followed by the local search and the candidate solutions are the solutions generated by the removal and re-assignment heuristics. The *LHC_LEN* parameter was set as 2000 in all the experiments. A small amount of testing was performed in selecting this parameter but these initial tests suggested that this parameter did not have a large impact on the overall performance of the entire algorithm. It is possible that some additional performance gains could be achieved by tuning this parameter or more advanced sensitivity analysis (or even dynamically adapting it).

In the LNS, two removal heuristics are used: Shaw removal (1998) and random removal (Ropke and Pisinger 2006; Shaw 1998). At each iteration, one of the removal heuristics is randomly selected and applied. The Shaw removal heuristic aims to select a set of PD pairs that are similar. The idea is that if the pairs are similar then there is more possibility of re-arranging them in a new and possibly better way. If the pairs are all very different then they will probably be replaced exactly where they were originally assigned. The pair characteristics that are used to measure their similarity are: distance from each other, arrival times and demand. The formula for calculating the similarity is the same as given in Ropke and Pisinger (2006). The second heuristic is to simply randomly select a set of pairs. The probability of selecting the Shaw heuristic is set at 0.6, else the random selection

```
for a minimum number of iterations and maximum time limit
{
        select a removal heuristic

        un-assign heuristically selected PD-pairs in the solution using the removal
        heuristic

        select an assignment heuristic

        re-assign un-assigned PD-pairs using the assignment heuristic

        accept or reject the new solution as the current solution using LAHC
}
```

Fig. 3 LNS outline

```
Input parameters:
 Input solution s
 The length of the costs array LHC_LEN
Calculate initial cost function C(s)
Create a new array (costs) of length LHC_LEN
FOR x{0..LHC_LEN-1} SET costs[x] := C(s)
SET iter := 0
UNTIL stopping condition
   Construct a candidate solution s* from s
   SET x := iter mod LHC_LEN
   IF C(s*) ≤ costs[x] or C(s*) ≤ C(s)
      then accept the candidate (SET s := s*)
   ELSE
      reject the candidate (SET s := s)
   SET costs[x] := C(s)
   SET iter := iter+1
END UNTIL
```

Fig. 4 LAHC outline

heuristic is used. This creates a slight bias towards using the intelligent Shaw heuristic over the un-intelligent random heuristic. The number of pairs to remove by each heuristic is a number randomly selected from the range 4–80. These values were selected based on the results and guidance from (Ropke and Pisinger 2006).

To re-assign the pairs, the regret assignment heuristic only is used (Ropke and Pisinger 2006). The regret heuristic tries to improve upon greedy assignment by incorporating look-ahead. It does so by not only considering the best possible route for a pair insertion but by also the second, third, fourth… *k*th best routes. When selecting which pair to insert next it selects pairs that have less possible positions for insertion that are low cost relative to their other possible positions. The motivation is that if that pair is not inserted now there may be regret later if that position is no longer available due to a previous insertion in the route. The parameter k is randomly selected from 2, 3, 4, 5, *#R,* where *#R* is the number of routes in the current solution.

Although the GES only uses the objective of minimising the number of routes and ignores the objective of minimising distance, the LNS uses the full hierarchical objective. It is possible for the LNS to remove routes if a removal heuristic selects a set of pairs which includes all the pairs for an entire route and then the assignment heuristic re-assigns them over other routes. During the testing we did observe the LNS reducing the number of routes in a solution occasionally but as will be shown, the GES is far more effective for minimising the total number of routes.

The LNS stops when a minimum number of iterations (800 in this paper) without improvement have been done or both of the following are satisfied:

(1) There was no improvement in the last 400 iterations.
(2) And a minimum time limit has been reached, set as twice the time taken to complete the GES phase.

This streamlined LNS maintains the diversification and intensification ability but at the same time it excludes the adaptive mechanism which was shown to provide only a few extra percent benefit in performance (Ropke and Pisinger 2006).

3.4 Restarts

After the LNS is completed, the overall algorithm goes to step 3 in Fig. 1 to try reducing the number of vehicles again in the best solution found so far, using GES. After, if the number of vehicles was not reduced then the best solution so far is perturbed using the same perturbation function as in GES. The number of perturbation moves is set as the instance's number of PD pairs multiplied by 0.2. This is larger than the perturbation used within the GES phase where only ten moves are made. A larger perturbation is performed here to increase the search diversification, whereas within the GES phase the perturbation is to try and allow a single PD pair to be inserted. The algorithm then continues by applying the local search followed by LNS again and so on. The algorithm terminates when a maximum time limit is reached. Hence, the heuristic approach proposed in this paper alternates between a random (when no route is removed) and a greedy (when a route is removed) perturbation to the current solution to then perform an effective LNS that balances intensification and diversification. All algorithm parameters are summarised in Table 1.

4 Results

To test the algorithm, the benchmark instances of Li and Lim (2003) are used.[1] There are approximately 360 instances categorised into six groups of different sizes ranging from approximately 50 PD pairs up to 500 PD pairs. Each group is also subdivided into instances with clustered locations, randomly distributed locations and randomly clustered locations. Each subgroup is then further split by instances with short planning horizons and instances with long planning horizons.

Three sets of experiments were performed. The first was to investigate the benefit of the adaptive heuristic added to the GES. As described earlier, this heuristic terminates the GES phase more quickly when the progress suggests that an extra route will not be eliminated but allows more time when the progress suggests it is getting closer to removing a route. The second set of experiments was to investigate different configurations of the individual components and their combined benefit. The third analysis was to simply compare solutions generated with the current best knowns.

[1] Available at http://www.sintef.no/projectweb/top/pdptw/li-lim-benchmark/.

Table 1 Parameters summary

GES	
Maximum total perturbs	1,000,000
Perturbation	
Max perturbs within GES	10
Max perturbs during Restarts	Max [20, m * 0.2]
Probability of selecting *PairMove*	0.5
Probability of selecting *SwapMove*	0.5
LNS	
Stopping conditions	Min 800 consecutive iterations without improvement OR (Min 400 without improvement AND Min 2× CPU time used by GES)
LAHC array length (LHC_LEN)	2000
Min PD pairs removed by removal heuristic	4
Max PD pairs removed by removal heuristic	80
Probability of selecting Shaw heuristic	0.6
Probability of selecting removal heuristic	0.4

For the first two sets of experiments, five different algorithms were applied to all the test instances. The algorithms tested are the full algorithm and then four other versions, each with different components removed. The aim was to investigate the impact of the individual components or whether there was not any benefit in combining components when given the same computation time, or if combining the algorithms produces a more effective overall algorithm. The configurations were as follows:

1. LS + AGES + LNS (Algo1): The full algorithm as described and using the adaptive heuristic for the GES (labelled **A**daptive GES).
2. LS + GES + LNS (Algo2): The same as 1 but without the adaptive heuristic in GES.
3. LS + AGES (Algo3): The same as 1 but without the LNS phase, to see if the LS alone is sufficient at minimising the distance objective.
4. LS + LNS (Algo4): The same as 1 but without the AGES phase which aims at minimising total routes. LS and LNS are both also able to minimise total routes on their own but this test was to investigate whether they are sufficient on their own if given the extra time not used by the removed AGES phase.
5. AGES + LNS (Algo5): The same as 1 but with the LS phase removed. Previous papers indicated that LNS is much more effective than LS so there

may be no benefit in including the LS phase, and instead just giving more time to the LNS phase.

Note that GES will exit sooner than AGES and so LNS in Algo2 will also have less time than LNS in Algo1 per iteration. However, because all algorithms are being run for the same fixed time Algo2 will complete more iterations than Algo1 and so the overall CPU time distributed between the different phases will be similar overall.

On the '100' group of instances, 5 min of computation time was allowed. On the '200' and '400' groups, 15 min. On the '600' group, 30 min and on the '800' and '1000' groups, 60 min. These values were chosen based on similar run times in other papers (Ropke and Pisinger 2006; Nagata and Kobayashi 2010a). All runs were performed on an Intel Xeon CPU E5-1620 @ 3.5 GHz utilising a single core per run. 32 GB RAM was available (although testing showed the algorithm requires a maximum of 70 MB on the largest instances). The code was written in C#.

Table 2 lists the total number of vehicles used and the total distance for all the solutions for each group of instances, for each algorithm. These results are further broken down in Table 3 in which we rank the algorithms by how they performed against each other. For each group of instances, we record the total number of times that each algorithm found the best solution, the second best solution, the third best, fourth best and fifth best out of the five algorithms. Kendall's non-parametric test is applied to the rankings to determine if the pairwise comparisons between two algorithms are statistically significant. The mean values and P values used in the statistical test are given in Table 4. Pairwise comparisons are made to see if the differences are statistically significant at the 0.05 level.

For the pairwise comparisons we define $A < B$ as meaning A has lower rank than B but the pairwise comparison is not significant. We define $A \ll B$ as meaning A has lower rank than B and the pairwise comparison is statistically significant. The rankings are as follows:

Over all instances we may rank the five algorithms as Algo1 < Algo2 ≪ Algo5 ≪ Algo4 < Algo3.

On the '100' instances the rank result is Algo2 < Algo1 < Algo4 < Algo3 < Algo5.
On the '200' instances the rank result is Algo2 < Algo1 ≪ Algo4 < Algo5 < Algo3.
On the '400' instances the rank result is Algo2 < Algo1 ≪ Algo5 < Algo3 < Algo4.
On the '600' instances the rank result is Algo1 < Algo2 < Algo5 ≪ Algo3 < Algo4. (Algo1 ≪ Algo5).
On the '800' instances the rank result is Algo1 < Algo2 < Algo5 ≪ Algo3 < Algo4.
On the '1000' instances the rank result is Algo1 < Algo2 < Algo5 ≪ Algo3 < Algo4.

Table 2 Algorithm comparisons on all instances

Inst.	*t* (m)	LS + AGES + LNS		LS + GES + LNS		LS + AGES		LS + LNS		AGES + LNS	
		Veh.	Dist.	Veh.	Dist.	Veh.	Dist.	Veh.	Dist.	Veh.	Dist.
100	5	402	58,163.22	**402**	**58,162.29**	402	59,145.32	404	58,175.81	402	58,604.47
200	15	601	186,158.57	601	185,610.99	**600**	**197,910.36**	619	176,257.69	601	187,890.12
400	15	1142	447,627.39	1145	441,007.53	**1140**	**485,337.69**	1183	408,446.49	1144	452,306.60
600	30	**1643**	**935,948.36**	1653	915,686.48	1644	998,440.66	1710	824,991.45	1643	947,724.82
800	60	2146	1,551,495.35	2153	1,550,252.37	2146	1,664,710.86	2223	1,383,687.84	**2136**	**1,545,456.09**
1000	60	2634	2,310,830.27	2645	2,282,149.28	2636	2,443,110.48	2733	2,047,589.35	**2605**	**2,259,241.06**

The best results are indicated in bold

Table 3 Algorithm rankings

Size	Rank	LS + AGES + LNS	LS + GES + LNS	LS + AGES	LS + LNS	AGES + LNS
100	First	53	54	41	52	39
	Second	1	2	0	0	0
	Third	2	0	1	1	1
	Fourth	0	0	4	1	7
	Fifth	0	0	10	2	9
200	First	36	38	17	25	17
	Second	12	10	0	10	4
	Third	7	11	6	5	6
	Fourth	4	1	15	2	22
	Fifth	1	0	22	18	11
400	First	25	28	10	18	14
	Second	14	20	2	5	6
	Third	18	7	5	2	17
	Fourth	3	5	30	0	16
	Fifth	0	0	13	35	7
600	First	27	23	6	9	19
	Second	18	18	6	7	7
	Third	14	11	8	4	15
	Fourth	1	8	33	1	11
	Fifth	0	0	7	39	8
800	First	24	18	8	17	16
	Second	22	19	3	1	12
	Third	12	15	7	3	14
	Fourth	1	7	35	2	8
	Fifth	1	1	7	37	10

Table 3 continued

Size	Rank	LS + AGES + LNS	LS + GES + LNS	LS + AGES	LS + LNS	AGES + LNS
1000	First	25	15	4	12	21
	Second	17	21	4	3	11
	Third	14	11	10	2	14
	Fourth	2	8	32	2	7
	Fifth	0	3	8	39	5
All	First	190	176	86	133	126
	Second	84	90	15	26	40
	Third	67	55	37	17	67
	Fourth	11	29	149	8	71
	Fifth	2	4	67	170	50

Table 4 Algorithm mean rankings and P values

Test	P value	Mean ranks				
		Algo1	Algo2	Algo3	Algo4	Algo5
All	< 0.01	2.25	2.38	3.69	3.65	3.03
100	< 0.01	2.72	2.68	3.38	2.80	3.41
200	< 0.01	2.3	2.21	3.84	3.23	3.42
400	< 0.01	2.26	2.11	3.76	3.78	3.10
600	< 0.01	2.04	2.31	3.68	4.11	2.87
800	< 0.01	2.11	2.46	3.69	3.88	2.87
1000	< 0.01	2.09	2.57	3.76	4.07	2.51

Looking at Table 2 adaptive GES produces solutions with less routes than the GES (apart from the 100 and 200 instances where they are the same). When we compare the rankings pairwise, GES is better on the smaller instances but AGES is better on the larger instances and over all instances. However, the mean rankings are too similar to say the difference is statistically significant.

Investigating the benefit of including the (A)GES phase, it is clear that it is very effective. Algo4 (no GES) is always worse than Algo1 and Algo2 (the full algorithms with AGES or GES) and the pairwise comparisons are statistically significant. Similarly, it is clear than the LNS is an important component of the algorithm. When the LNS phase is removed (Algo3), the full algorithms (Algo1 and Algo2) are significantly better. It is clear than giving extra time and more iterations to LS and GES is not as effective as including the LNS albeit with less time for each phase and less iterations. Algo5 is also statistically better than Algo3 showing that using LNS instead of LS is more effective. Finally, we can conclude that over all instances, including the LS phase is more effective than not including it (Algo5) but on the largest instances 800 and 1000, the superiority is still visible in the mean rankings but is not large enough to be statistically significant. These results show that combining the three components in this configuration, local search with specialised moves, streamlined large neighbourhood search and adaptive guided ejection search, is more effective than using just two of the components. Using all three components means there is less time available for each method but it still more effective than just using two of the components even if there is more time available for each individual phase.

Next, we compare the results against the best-known results in peer-reviewed publications and the current best knowns that have been verified on the SINTEF website but have not been published in peer-reviewed outlets and for which no information is available about computation times and methods used. For comparing against published methods (Tables 2, 5), we use the results of the adaptive LNS method of (Ropke and Pisinger 2006) and the GES method of (Nagata and Kobayashi 2010a). These are the current best-known published results. Comparing against these results is not simple though. For the best results of (Ropke and Pisinger 2006) we do not know the computation times. For their best results the authors "*report the best solutions obtained in several experiments with our ALNS heuristic*

Table 5 Comparing against other methods

Instances	LNS			GES			LS + AGES + LNS		
	Veh.	Dist.	t (s)	Veh.	Dist.	t (s)	Veh.	Dist.	t (s)
100	402	56,060	–	–	–		402	58,163.22	300
200	606	180,419	–	601	–	3000	601	186,158.57	900
400	1157	420,396	–	1139	–	3000	1142	447,627.39	900
600	1664	860,898	–	1636	–	3000	1643	935,948.36	1800
800	2181	1,423,063	–	2135	–	3000	2146	1,551,495.35	3600
1000	2646	2,122,922	–	2613	–	3000	2634	2,310,830.27	3600

and with various parameter settings". We do not know how many experiments were run but we have an indication of computation times which are from 66 s per run on the smallest instances to 5370 s per run on the largest instances. We also know that the heuristic was run at least "*5 or 10 times on each instance*" (not including the different parameter setting testing) and that a 1.5 GHz Pentium IV processor was used. Again, comparing against Nagata and Kobayashi is difficult because their algorithm only minimises the number of vehicles used and we know from our results that using less vehicles often increases the total distance. They used an Opteron 2.6 GHz processor. Due to the unknown computation times, the differences in computing power and the difficulty in comparing summed values for a problem with hierarchical objectives we cannot have strong conclusions. The GES method produces solutions with less vehicles in total but the algorithm uses all its time minimising this objective where as our method only uses a large proportion of its time also minimising distance. The ALNS uses both objectives but produces solutions with more vehicles in total. Comparing total distance is not helpful because often solutions with less vehicles have longer distance. To assist future researchers and facilitate future comparisons, we have included in this paper in Table 12. Results for a Single Run of Algo1 Table 12 our results for a single run for a fixed run time for a single configuration (Algo1).

For the next comparison, we compare against best knowns from published and unpublished methods. Tables 5, 6, 7, 8, 9 list the solutions found after applying the LS + AGES + LNS algorithm on all instances. The algorithm was allowed 1 h computation but the best reported may be the best from several tests with different random seeds. Each table lists the solutions for each set of instances grouped by the number of locations. The tables also list the previous best-known solutions for each instance, and the date it was found. The information is taken from SINTEF's website which is regularly updated.[2] Solutions in italics are equal to previous best knowns and solutions in bold italics are new best knowns.

The results show that the algorithm was able to find a large number of new best-known solutions. On the 100 site instances the algorithm equalled the best knowns

[2] Retrieved from http://www.sintef.no/projectweb/top/pdptw/li-lim-benchmark/ on 25-Feb-2016.

on all instances. On the 200 site instances 35 best knowns were equalled and seven new best knowns were found (out of 60). Of the seven new best knowns 3 were improvements in terms of the number of vehicles. For example, on the instance *LR2_2_6,* the new best known has a solution of three vehicles, whereas the previous had four vehicles. This was an impressive result because the previous best known had stood for 15 years. On the 400 site instances there are 19 equal best knowns and 22 new best knowns (out of 60). On the 600 site instances there are six equal best knowns and 33 new best knowns (out of 60). On the 800 site instances there are five equal best knowns and 45 new best knowns (out of 60). On the 1000 site instances there are four equal best knowns and 35 new best knowns (out of 58). For many of the new best knowns the primary objective of reducing the number of vehicles is improved. This is particularly noticeable on the larger instances where the number of vehicles is reduced by more than one vehicle. For example, on instance *LRC1_10_5,* the previous best known required 76 vehicles, whereas the new best known has only 72 vehicles. This demonstrates the benefit of using the AGES within the algorithm specifically for reducing the number of vehicles.

5 Conclusion

This paper proposes an effective and fast hybrid metaheuristic algorithm to tackle the pickup and delivery problem with time windows (PDPTW). The approach performs a large neighbourhood search (LNS) that incorporates mechanisms for intensification and diversification. The approach also incorporates mechanisms to perturb the current solution. Such perturbation can be greedy by removing a full route from the solution through guided ejection search, or random when such removal is not successful. Then, alternating the LNS with guided ejection search and local search has resulted in a relatively simple but demonstrably effective framework. The guided ejection search is specifically designed for minimising the number of routes within solutions. An adaptive heuristic is developed for the guided ejection search phase which provides more time to the heuristic when its progress suggests it is close to removing a route. The local search and large neighbourhood search are more focused on minimising travel distances. The aim is to combine these strengths into an overall robust and successful method. A new search neighbourhood operator was added to the local search method and the LNS was streamlined and simplified without loss of efficacy.

The results show that when any one of the components is removed the results are significantly worse. In other words, two components given more time is not as effective as the three components but with less time for each component. The adaptive heuristic for the GES phase is particularly effective on the larger instances. Including the local search phase benefits the smaller instances and including the LNS phase is better for all instance sizes. When tested on a large and well used benchmark dataset, the algorithm is able to find 142 (out of 354 instances) new best-known solutions, confirming its efficacy. The many new best-known solutions obtained with the LS + AGES + LNS heuristic algorithm proposed here have been already verified and hence published in the SINTEF's website.

Although a large amount of research, development and testing was required to develop this algorithm, there are still possibilities for further research. The Li and Lim benchmark instances are a very useful resource that have stimulated and enabled innovative research within a competitive and verifiable environment. Although that research forms the basis for many commercial vehicle routing problem solvers (Hall and Partyka 2016) it could be argued that more realistic benchmark instances could lead to even more effective methods for real-world problems. Datasets that contain requirements, such as driver break rules, maximum driving hours, working time constraints, and soft time windows, could have significant practical benefit. We believe that the algorithm presented could be adapted to handle these requirements but there would undoubtedly be new research required for such new challenges within benchmark datasets.

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Appendix

See Tables 6, 7, 8, 9, 10, 11, 12.

Table 6 Best solutions for 100 site instances

Instance	Best known			LS + AGES + LNS		Instance	Best known			LS + AGES + LNS	
	Veh	Distance	Date	Veh	Distance		Veh	Distance	Date	Veh	Distance
lc101	10	828.94	23-Jun-05	*10*	*828.94*	lr201	4	1253.23	28-Feb-03	*4*	*1253.23*
lc102	10	828.94	23-Jun-05	*10*	*828.94*	lr202	3	1197.67	23-Jun-05	*3*	*1197.67*
lc103	9	1035.35	27-Jun-03	*9*	*1035.35*	lr203	3	949.40	23-Jun-05	*3*	*949.40*
lc104	9	860.01	11-Apr-03	*9*	*860.01*	lr204	2	849.05	23-Jun-05	*2*	*849.05*
lc105	10	828.94	23-Jun-05	*10*	*828.94*	lr205	3	1054.02	23-Jun-05	*3*	*1054.02*
lc106	10	828.94	2001	*10*	*828.94*	lr206	3	931.63	23-Jun-05	*3*	*931.63*
lc107	10	828.94	23-Jun-05	*10*	*828.94*	lr207	2	903.06	2001	*2*	*903.06*
lc108	10	826.44	2001	*10*	*826.44*	lr208	2	734.85	2001	*2*	*734.85*
lc109	9	1000.60	27-Jun-03	*9*	*1000.60*	lr209	3	930.59	09-Mar-03	*3*	*930.59*
lc201	3	591.56	23-Jun-05	*3*	*591.56*	lr210	3	964.22	23-Jun-05	*3*	*964.22*
lc202	3	591.56	2001	*3*	*591.56*	lr211	2	911.52	15-May-03	*2*	*911.52*
lc203	3	591.17	11-Mar-03	*3*	*591.17*	lrc101	14	1708.80	23-Jun-05	*14*	*1708.80*
lc204	3	590.60	08-Mar-03	*3*	*590.60*	lrc102	12	1558.07	19-Feb-03	*12*	*1558.07*
lc205	3	588.88	2001	*3*	*588.88*	lrc103	11	1258.74	23-Jun-05	*11*	*1258.74*
lc206	3	588.49	23-Jun-05	*3*	*588.49*	lrc104	10	1128.40	23-Jun-05	*10*	*1128.40*
lc207	3	588.29	23-Jun-05	*3*	*588.29*	lrc105	13	1637.62	23-Jun-05	*13*	*1637.62*
lc208	3	588.32	23-Jun-05	*3*	*588.32*	lrc106	11	1424.73	28-Feb-03	*11*	*1424.73*
lr101	19	1650.80	2001	*19*	*1650.80*	lrc107	11	1230.14	18-Feb-03	*11*	*1230.14*
lr102	17	1487.57	23-Jun-05	*17*	*1487.57*	lrc108	10	1147.43	28-Feb-03	*10*	*1147.43*
lr103	13	1292.68	23-Jun-05	*13*	*1292.68*	lrc201	4	1406.94	28-Feb-03	*4*	*1406.94*
lr104	9	1013.39	2001	*9*	*1013.39*	lrc202	3	1374.27	23-Jun-05	*3*	*1374.27*
lr105	14	1377.11	23-Jun-05	*14*	*1377.11*	lrc203	3	1089.07	23-Jun-05	*3*	*1089.07*
lr106	12	1252.62	23-Jun-05	*12*	*1252.62*	lrc204	3	818.66	23-Mar-03	*3*	*818.66*

Table 6 continued

Instance	Best known			LS + AGES + LNS		Instance	Best known			LS + AGES + LNS	
	Veh	Distance	Date	Veh	Distance		Veh	Distance	Date	Veh	Distance
lr107	10	1111.31	23-Jun-05	*10*	*1111.31*	lrc205	4	1302.20	23-Jun-05	*4*	*1302.20*
lr108	9	968.97	23-Jun-05	*9*	*968.97*	lrc206	3	1159.03	12-Mar-03	*3*	*1159.03*
lr109	11	1208.96	27-Feb-03	*11*	*1208.96*	lrc207	3	1062.05	04-Jan-04	*3*	*1062.05*
lr110	10	1159.35	23-Jun-05	*10*	*1159.35*	lrc208	3	852.76	23-Jun-05	*3*	*852.76*
lr111	10	1108.90	23-Jun-05	*10*	*1108.90*						
lr112	9	1003.77	23-Jun-05	*9*	*1003.77*						

Italics indicates equal to previous best known

Table 7 Best solutions for 200 site instances

Instance	Best known			LS + AGES + LNS		Instance	Best known			LS + AGES + LNS	
	Veh	Distance	Date	Veh	Distance		Veh	Distance	Date	Veh	Distance
LC1_2_1	20	2704.57	23-Jun-05	*20*	*2704.57*	LR2_2_1	5	4073.10	09-Dec-03	*5*	*4073.10*
LC1_2_2	19	2764.56	23-Jun-05	*19*	*2764.56*	LR2_2_2	4	3796.00	14-Feb-03	*4*	*3796.00*
LC1_2_3	17	3128.61	08-Jul-03	*17*	*3128.61*	LR2_2_3	4	3098.36	08-Jul-03	*4*	*3098.36*
LC1_2_4	17	2693.41	27-Jun-03	*17*	*2693.41*	LR2_2_4	3	2486.14	08-Jul-03	3	2491.73
LC1_2_5	20	2702.05	23-Jun-05	*20*	*2702.05*	LR2_2_5	4	3438.39	13-Dec-03	*4*	*3438.39*
LC1_2_6	20	2701.04	2001	*20*	*2701.04*	LR2_2_6	4	3201.54	2001	**3**	**4639.85**
LC1_2_7	20	2701.04	23-Jun-05	*20*	*2701.04*	LR2_2_7	3	3124.57	19-Nov-15	3	3201.68
LC1_2_8	19	3379.97	19-Nov-15	19	3397.65	LR2_2_8	2	2552.16	14-Apr-15	2	2586.42
LC1_2_9	18	2724.24	23-Jun-05	*18*	*2724.24*	LR2_2_9	3	3930.49	25-Feb-05	**3**	**3927.13**
LC1_2_10	17	2942.13	19-Nov-15	17	2947.00	LR2_2_10	3	3282.45	19-Nov-15	**3**	**3274.96**
LC2_2_1	6	1931.44	2001	*6*	*1931.44*	LRC1_2_1	19	3606.06	14-Dec-03	*19*	*3606.06*
LC2_2_2	6	1881.40	23-Jun-05	*6*	*1881.40*	LRC1_2_2	15	3671.02	19-Nov-15	15	3683.24
LC2_2_3	6	1844.33	15-Apr-03	*6*	*1844.33*	LRC1_2_3	13	3161.44	19-Nov-15	**13**	**3154.92**
LC2_2_4	6	1767.12	2001	*6*	*1767.12*	LRC1_2_4	10	2631.82	08-Jul-03	*10*	*2631.82*
LC2_2_5	6	1891.21	23-Jun-05	*6*	*1891.21*	LRC1_2_5	16	3715.81	27-Jun-03	*16*	*3715.81*
LC2_2_6	6	1857.78	29-Dec-03	*6*	*1857.78*	LRC1_2_6	16	3572.16	19-Nov-15	*16*	*3572.16*
LC2_2_7	6	1850.13	10-Dec-03	*6*	*1850.13*	LRC1_2_7	14	3666.34	13-Apr-15	14	3697.71
LC2_2_8	6	1824.34	2001	*6*	*1824.34*	LRC1_2_8	13	3167.23	19-Nov-15	13	3202.16
LC2_2_9	6	1854.21	01-Sep-03	*6*	*1854.21*	LRC1_2_9	13	3157.34	19-Nov-15	*13*	*3157.34*
LC2_2_10	6	1817.45	23-Jun-05	*6*	*1817.45*	LRC1_2_10	12	2928.90	19-Nov-15	12	2948.60
LR1_2_1	20	4819.12	2001	*20*	*4819.12*	LRC2_2_1	6	3595.18	14-Apr-15	*6*	*3595.18*
LR1_2_2	17	4621.21	08-Jul-03	*17*	*4621.21*	LRC2_2_2	5	3184.23	17-Nov-15	5	3342.00
LR1_2_3	14	4656.11	02-Nov-15	**14**	**4402.38**	LRC2_2_3	4	2907.35	17-Nov-15	4	2948.56

Table 7 continued

Instance	Best known			LS + AGES + LNS		Instance	Best known			LS + AGES + LNS	
	Veh	Distance	Date	Veh	Distance		Veh	Distance	Date	Veh	Distance
LR1_2_4	10	3031.20	15-Jul-03	10	3044.69	LRC2_2_4	3	2861.74	26-May-14	3	2908.50
LR1_2_5	16	4760.18	27-Jun-03	*16*	*4760.18*	LRC2_2_5	5	2776.93	27-Jun-03	*5*	*2776.93*
LR1_2_6	14	4175.16	27-Jun-03	**13**	**4800.94**	LRC2_2_6	5	2707.96	21-Dec-03	*5*	*2707.96*
LR1_2_7	12	3550.61	08-Jul-03	*12*	*3550.61*	LRC2_2_7	4	3018.05	17-Nov-15	4	3057.23
LR1_2_8	9	2766.42	19-Nov-15	9	2814.32	LRC2_2_8	4	2399.89	17-Nov-15	4	2400.19
LR1_2_9	14	4343.86	14-Apr-15	**13**	**5050.75**	LRC2_2_9	4	2208.49	08-Jul-03	*4*	*2208.49*
LR1_2_10	11	3692.20	19-Nov-15	11	3748.06	LRC2_2_10	3	2442.59	17-Nov-15	3	2664.99

Bold indicates a new best known and italics indicate equal to previous best known

Table 8 Best solutions for 400 site instances

Instance	Best known			LS + AGES + LNS		Instance	Best known			LS + AGES + LNS	
	Veh	Distance	Date	Veh	Distance		Veh	Distance	Date	Veh	Distance
LC1_4_1	40	7152.06	16-Jun-03	*40*	*7152.06*	LR2_4_1	8	9726.88	27-Jun-03	*8*	*9726.88*
LC1_4_2	38	8007.79	08-Oct-15	*38*	*8007.79*	LR2_4_2	7	9440.93	16-Nov-15	7	9473.11
LC1_4_3	33	8162.80	09-Oct-15	**32**	**9252.95**	LR2_4_3	6	8116.53	25-Feb-05	**5**	**10,658.64**
LC1_4_4	30	6451.68*	2001	30	6918.00	LR2_4_4	4	6649.78	08-Jul-03	4	6721.54
LC1_4_5	40	7150.00	19-Apr-03	*40*	*7150.00*	LR2_4_5	6	10,084.44	11-Jan-16	6	10,152.96
LC1_4_6	40	7154.02	2001	*40*	*7154.02*	LR2_4_6	5	9044.03	14-Dec-15	5	9145.93
LC1_4_7	40	7149.43	15-Mar-03	*40*	*7149.43*	LR2_4_7	5	6729.67	14-Dec-15	5	6964.65
LC1_4_8	39	7111.16	2001	*39*	*7111.16*	LR2_4_8	4	5356.37	02-Oct-15	4	5454.27
LC1_4_9	36	7451.20	09-Oct-15	*36*	*7451.20*	LR2_4_9	6	7930.55	04-Jan-16	6	7998.74
LC1_4_10	35	7387.13	08-Jul-03	**35**	**7325.01**	LR2_4_10	5	7846.99	16-Nov-15	5	8349.58
LC2_4_1	12	4116.33	2001	*12*	*4116.33*	LRC1_4_1	36	9127.15	25-Feb-05	**36**	**9124.52**
LC2_4_2	12	4144.29	15-May-03	*12*	*4144.29*	LRC1_4_2	31	8346.06	08-Jul-03	*31*	*8346.06*
LC2_4_3	12	4424.08	27-May-14	**12**	**4418.88**	LRC1_4_3	25	7307.09	27-Jun-03	**24**	**7856.72**
LC2_4_4	12	3743.95*	2001	12	4038.00	LRC1_4_4	19	5806.20	30-Jun-11	19	5841.95
LC2_4_5	12	4030.63	01-May-03	*12*	*4030.63*	LRC1_4_5	32	8867.38	30-Jun-11	32	8872.08
LC2_4_6	12	3900.29	20-Apr-03	*12*	*3900.29*	LRC1_4_6	30	8423.70	30-Jun-11	**30**	**8396.08**
LC2_4_7	12	3962.51	27-Jun-03	*12*	*3962.51*	LRC1_4_7	28	8037.87	30-Jun-11	28	8295.76
LC2_4_8	12	3844.45	2001	*12*	*3844.45*	LRC1_4_8	27	7563.09	30-Jun-11	**26**	**8173.63**
LC2_4_9	12	4188.93	08-Jul-03	*12*	*4188.93*	LRC1_4_9	26	7790.26	30-Jun-11	**25**	**8181.32**
LC2_4_10	12	3828.44	27-Jun-03	*12*	*3828.44*	LRC1_4_10	24	7065.73	08-Jul-03	**23**	**7222.97**
LR1_4_1	40	10,639.75	2003	*40*	*10,639.75*	LRC2_4_1	12	7454.14	20-Oct-15	*12*	*7454.14*
LR1_4_2	31	10,015.85	25-Feb-05	**31**	**9985.28**	LRC2_4_2	10	7424.72	28-Jan-16	10	7605.61
LR1_4_3	22	9458.99	25-Apr-15	**22**	**9291.25**	LRC2_4_3	9	5410.19	19-Jan-16	**8**	**6576.48**

Table 8 continued

Instance	Best known			LS + AGES + LNS		Instance	Best known			LS + AGES + LNS	
	Veh	Distance	Date	Veh	Distance		Veh	Distance	Date	Veh	Distance
LR1_4_4	16	6744.33	08-Jul-03	**16**	**6710.99**	LRC2_4_4	5	5322.43	08-Jul-03	5	5779.02
LR1_4_5	29	10,599.54	25-Feb-05	**28**	**11,374.06**	LRC2_4_5	11	6120.13	27-Jun-03	**10**	**7462.66**
LR1_4_6	24	10,326.45	21-Apr-15	**24**	**9891.02**	LRC2_4_6	9	6337.08	21-Jan-16	9	6342.74
LR1_4_7	19	8200.37	25-Feb-05	**18**	**8999.97**	LRC2_4_7	8	6326.50	20-Jan-16	8	6704.21
LR1_4_8	14	5946.44	08-Jul-03	**14**	**5944.67**	LRC2_4_8	7	5814.93	19-Jan-16	7	6070.90
LR1_4_9	24	9886.14	25-Feb-05	**24**	**9862.65**	LRC2_4_9	7	5259.20	20-Jan-16	**6**	**6877.02**
LR1_4_10	21	8016.62	08-Jul-03	**20**	**8364.66**	LRC2_4_10	6	5585.18	22-Jan-16	6	5840.81

Bold indicates a new best known and italics indicate equal to previous best known

* Indicates solution not verified

Table 9 Best solutions for 600 site instances

Instance	Best known			LS + AGES + LNS		Instance	Best known			LS + AGES + LNS	
	Veh	Distance	Date	Veh	Distance		Veh	Distance	Date	Veh	Distance
LC1_6_1	60	14,095.60	23-Jun-05	60	14,095.64	LR2_6_1	11	21,945.30	14-Jul-03	11	21,955.29
LC1_6_2	57	15,120.97	10-Apr-15	**57**	**15,048.16**	LR2_6_2	10	19,666.59	25-Feb-05	**9**	**23,903.03**
LC1_6_3	50	14,683.43	14-Jul-03	50	14,724.64	LR2_6_3	8	15,609.96	14-Jul-03	**7**	**19,183.41**
LC1_6_4	47	13,648.03	14-Jul-03	48	13,348.84	LR2_6_4	6	10,819.45	14-Jul-03	6	11,994.47
LC1_6_5	60	14,086.30	23-Jun-05	*60*	*14,086.30*	LR2_6_5	9	19,567.41	14-Jul-03	**9**	**19,411.73**
LC1_6_6	60	14,090.79	14-Jul-03	*60*	*14,090.79*	LR2_6_6	8	17,262.96	14-Jul-03	**7**	**22,570.45**
LC1_6_7	60	14,083.76	13-Jul-03	*60*	*14,083.76*	LR2_6_7	6	15,812.42	14-Jul-03	**6**	**15,526.81**
LC1_6_8	59	14,554.27	13-Jul-03	**58**	**14,880.70**	LR2_6_8	5	10,950.90	14-Jul-03	5	11,410.69
LC1_6_9	54	14,706.12	14-Jul-03	**54**	**14,661.73**	LR2_6_9	8	18,799.36	14-Jul-03	8	19,838.35
LC1_6_10	52	18,762.62	21-Apr-15	**52**	**15,204.30**	LR2_6_10	7	17,034.63	14-Jul-03	7	17,129.58
LC2_6_1	19	7977.98	14-Jul-03	*19*	*7977.98*	LRC1_6_1	52	27,262.21	18-Apr-15	**52**	**18,312.60**
LC2_6_2	18	9914.10	18-Jan-16	18	9940.34	LRC1_6_2	44	16,302.54	25-Feb-05	**43**	**17,063.21**
LC2_6_3	17	8718.22	04-Jan-16	18	7436.50	LRC1_6_3	36	14,060.31	14-Jul-03	36	14,115.00
LC2_6_4	17	7902.66	04-Jan-16	17	8022.96	LRC1_6_4	25	10,950.52	14-Jul-03	25	11,006.02
LC2_6_5	19	8047.37	27-Jun-03	*19*	*8047.37*	LRC1_6_5	47	16,742.55	14-Jul-03	**46**	**17,067.17**
LC2_6_6	19	8094.11	14-Jul-03	**18**	**8859.78**	LRC1_6_6	44	16,894.37	14-Jul-03	**42**	**17,405.48**
LC2_6_7	19	7997.96	15-Jan-16	*19*	*7997.96*	LRC1_6_7	39	15,394.87	14-Jul-03	**38**	**15,609.86**
LC2_6_8	18	7579.93	14-Jul-03	18	7580.88	LRC1_6_8	36	15,154.79	14-Jul-03	**33**	**15,919.78**
LC2_6_9	18	8864.29	12-Jan-16	18	9019.78	LRC1_6_9	35	15,134.24	14-Jul-03	**34**	**15,236.23**
LC2_6_10	17	7965.41	14-Jan-16	17	8064.71	LRC1_6_10	31	13,925.51	14-Jul-03	**29**	**14,607.38**
LR1_6_1	59	22,838.30	27-Jun-03	**59**	**22,824.32**	LRC2_6_1	16	14,817.72	14-Jul-03	16	14,892.18
LR1_6_2	45	20,246.18	14-Jul-03	45	20,355.13	LRC2_6_2	14	12,758.77	14-Jul-03	**13**	**15,649.64**
LR1_6_3	37	18,073.14	14-Jul-03	**37**	**17,987.49**	LRC2_6_3	10	12,812.67	25-Feb-05	10	14,845.21

Table 9 continued

Instance	Best known			LS + AGES + LNS		Instance	Best known			LS + AGES + LNS	
	Veh	Distance	Date	Veh	Distance		Veh	Distance	Date	Veh	Distance
LR1_6_4	28	13,269.71	14-Jul-03	**28**	**13,191.79**	LRC2_6_4	7	10,574.87	25-Feb-05	7	11,282.95
LR1_6_5	38	22,562.81	25-Feb-05	**38**	**22,489.30**	LRC2_6_5	14	13,009.52	14-Jul-03	**13**	**15,196.60**
LR1_6_6	32	20,641.02	14-Jul-03	**31**	**22,188.80**	LRC2_6_6	13	12,642.68	04-Jan-04	**12**	**17,149.19**
LR1_6_7	25	17,162.90	14-Jul-03	**24**	**18,531.68**	LRC2_6_7	11	11,975.28	24-Apr-15	**10**	**16,094.11**
LR1_6_8	19	11,957.59	14-Jul-03	**18**	**12,255.29**	LRC2_6_8	10	12,163.43	14-Jul-03	**9**	**15,024.47**
LR1_6_9	32	21,423.05	14-Jul-03	**32**	**21,117.75**	LRC2_6_9	9	13,768.01	14-Jul-03	9	14,560.69
LR1_6_10	27	18,723.13	14-Jul-03	**26**	**19,028.25**	LRC2_6_10	8	12,016.94	14-Jul-03	**7**	**15,098.15**

Bold indicates a new best known and italics indicate equal to previous best known

Table 10 Best solutions for 800 site instances

Instance	Best known			LS + AGES + LNS		Instance	Best known			LS + AGES + LNS	
	Veh	Distance	Date	Veh	Distance		Veh	Distance	Date	Veh	Distance
LC1_8_1	80	25,184.38	16-Jun-03	*80*	*25,184.38*	LR2_8_1	15	33,816.90	22-Aug-03	**14**	**46,452.00**
LC1_8_2	77	33,329.26	04-May-15	**77**	**26,864.13**	LR2_8_2	12	32,575.97	22-Aug-03	12	35,732.86
LC1_8_3	65	25,918.45	22-Aug-03	**63**	**27,459.81**	LR2_8_3	10	25,310.53	22-Aug-03	**9**	**30,485.19**
LC1_8_4	60	22,970.88	22-Aug-03	**60**	**22,943.54**	LR2_8_4	7	19,506.42	25-Feb-05	**6**	**24,285.15**
LC1_8_5	80	25,211.22	12-Jul-03	*80*	*25,211.22*	LR2_8_5	12	32,634.29	22-Aug-03	**11**	**37,332.53**
LC1_8_6	80	25,164.25	12-Jul-03	*80*	*25,164.25*	LR2_8_6	10	27,870.80	22-Aug-03	**9**	**31,372.05**
LC1_8_7	80	25,158.38	13-Jul-03	*80*	*25,158.38*	LR2_8_7	8	25,077.85	25-Feb-05	**7**	**30,605.35**
LC1_8_8	78	25,348.45	22-Aug-03	78	25,381.04	LR2_8_8	5	19,256.79	22-Aug-03	6	19,390.75
LC1_8_9	73	25,541.94	22-Aug-03	**72**	**26,360.69**	LR2_8_9	10	30,791.77	22-Aug-03	10	34,699.60
LC1_8_10	71	25,712.12	22-Aug-03	**70**	**26,811.45**	LR2_8_10	9	28,265.24	22-Aug-03	9	30,147.88
LC2_8_1	24	11,687.06	19-May-03	*24*	*11,687.06*	LRC1_8_1	66	35,901.74	04-May-15	**66**	**32,302.57**
LC2_8_2	24	14,358.92	22-Aug-03	**24**	**14,039.96**	LRC1_8_2	56	28,843.10	27-Jun-03	**56**	**28,042.91**
LC2_8_3	24	13,198.29	22-Aug-03	24	13,265.98	LRC1_8_3	48	29,276.29	04-May-15	**48**	**24,693.73**
LC2_8_4	23	13,376.82	22-Aug-03	24	12,373.83	LRC1_8_4	34	23,099.15	04-May-15	**34**	**18,806.89**
LC2_8_5	25	12,298.90	27-Jun-03	25	12,329.80	LRC1_8_5	59	34,909.37	04-May-15	**58**	**31,457.69**
LC2_8_6	24	12,702.87	22-Aug-03	24	12,835.00	LRC1_8_6	56	29,971.97	02-Jun-03	**54**	**29,836.27**
LC2_8_7	25	11,855.86	22-Aug-03	**25**	**11,854.44**	LRC1_8_7	52	32,430.86	10-Jul-15	**51**	**28,705.17**
LC2_8_8	24	11,482.88	22-Aug-03	**24**	**11,454.33**	LRC1_8_8	47	29,814.93	04-May-15	**45**	**27,374.52**
LC2_8_9	24	11,629.61	22-Aug-03	**24**	**11,629.41**	LRC1_8_9	46	28,955.40	10-Jul-15	**44**	**25,980.07**
LC2_8_10	24	11,578.58	22-Aug-03	24	11,583.30	LRC1_8_10	42	24,271.52	22-Aug-03	**40**	**24,582.28**
LR1_8_1	80	39,315.92	22-Aug-03	**80**	**39,292.13**	LRC2_8_1	20	23,289.40	22-Aug-03	**20**	**23,157.34**
LR1_8_2	59	34,370.37	22-Aug-03	**59**	**34,325.92**	LRC2_8_2	18	21,786.62	22-Aug-03	**17**	**22,686.62**
LR1_8_3	44	29,718.09	22-Aug-03	**44**	**29,676.42**	LRC2_8_3	15	36,127.35	04-May-15	**14**	**21,651.20**

Table 10 continued

Instance	Best known			LS + AGES + LNS		Instance	Best known			LS + AGES + LNS	
	Veh	Distance	Date	Veh	Distance		Veh	Distance	Date	Veh	Distance
LR1_8_4	25	21,197.65	22-Aug-03	**25**	**21,189.75**	LRC2_8_4	11	38,856.21	10-Jul-15	**11**	**16,232.51**
LR1_8_5	50	39,046.06	22-Aug-03	**49**	**39,624.94**	LRC2_8_5	17	41,062.80	04-May-15	**16**	**24,404.69**
LR1_8_6	41	40,488.25	04-May-15	**40**	**35,042.41**	LRC2_8_6	16	21,088.57	22-Aug-03	**15**	**23,854.87**
LR1_8_7	31	33,431.22	04-May-15	**30**	**28,252.49**	LRC2_8_7	15	19,695.96	22-Aug-03	**14**	**24,514.06**
LR1_8_8	21	19,570.21	22-Aug-03	**20**	**20,037.07**	LRC2_8_8	13	19,009.33	22-Aug-03	**12**	**21,508.49**
LR1_8_9	42	36,126.69	22-Aug-03	**40**	**40,077.86**	LRC2_8_9	12	19,003.68	22-Aug-03	**11**	**21,998.18**
LR1_8_10	32	30,200.86	22-Aug-03	**31**	**32,241.06**	LRC2_8_10	10	19,766.78	22-Aug-03	**10**	**19,712.20**

Bold indicates a new best known and italics indicate equal to previous best known

Table 11 Best solutions for 1000 site instances

Instance	Best known			LS + AGES + LNS		Instance	Best known			LS + AGES + LNS	
	Veh	Distance	Date	Veh	Distance		Veh	Distance	Date	Veh	Distance
LC1_10_1	100	42,488.66	23-Apr-03	*100*	*42,488.66*	LR2_10_1	19	45,422.58	25-Feb-05	**18**	**56,089.88**
LC1_10_2	95	43,858.42	10-Apr-15	**94**	**45,238.29**	LR2_10_2	15	47,824.44	25-Feb-05	**14**	**58,654.95**
LC1_10_3	82	42,631.11	25-Feb-05	**80**	**45,175.07**	LR2_10_3	11	39,894.32	25-Feb-05	11	45,477.21
LC1_10_4	74	39,217.18	23-May-15	**74**	**38,376.00**	LR2_10_4	8	28,314.95	25-Feb-05	8	29,113.98
LC1_10_5	100	42,477.40	10-Aug-03	100	42,477.41	LR2_10_5	14	53,209.98	25-Feb-05	14	54,053.67
LC1_10_6	101	42,838.39	11-Aug-03	*101*	*42,838.39*	LR2_10_6	12	43,792.11	25-Feb-05	**11**	**53,051.42**
LC1_10_7	100	42,854.99	25-Jun-05	*100*	*42,854.99*	LR2_10_7	9	36,728.20	25-Feb-05	9	41,524.99
LC1_10_8	98	42,951.56	25-Feb-05	98	42,965.59	LR2_10_8	7	26,278.09	25-Feb-05	7	30,358.26
LC1_10_9	92	42,391.98	25-Feb-05	**91**	**43,426.76**	LR2_10_9	13	48,447.49	25-Feb-05	13	50,126.78
LC1_10_10	90	42,435.16	25-Feb-05	**88**	**42,873.50**	LR2_10_10	11	44,155.66	25-Feb-05	**11**	**43,889.20**
LC2_10_1	30	16,879.24	2003	*30*	*16,879.24*	LRC1_10_1	84	49,315.30	27-Jun-03	**82**	**49,285.19**
LC2_10_2	31	18,980.98	25-Feb-05	31	19,342.00	LRC1_10_2	72	58,291.72	10-Jul-15	**72**	**45,289.03**
LC2_10_3	30	17,772.49	25-Feb-05	30	17,943.81	LRC1_10_3	54	43,435.85	10-Jul-15	**53**	**36,499.96**
LC2_10_4	29	18,089.98	25-Feb-05	29	18,231.53	LRC1_10_4	40	27,680.12	09-Apr-15	40	27,987.45
LC2_10_5	31	17,137.53	25-Feb-05	31	17,289.55	LRC1_10_5	76	49,816.18	25-Feb-05	**72**	**51,733.03**
LC2_10_6	31	17,198.01	25-Feb-05	31	17,204.18	LRC1_10_6	69	44,469.08	25-Feb-05	**68**	**44,444.34**
LC2_10_7	31	19,117.67	25-Feb-05	31	19,347.93	LRC1_10_7	63	49,951.54	10-Jul-15	**61**	**41,917.62**
LC2_10_8	30	17,018.63	25-Feb-05	**30**	**17,015.41**	LRC1_10_8	58	50,171.34	10-Jul-15	**56**	**42,640.89**
LC2_10_9	31	17,565.95	25-Feb-05	**30**	**20,057.42**	LRC1_10_9	56	46,710.65	10-Jul-15	**53**	**40,848.27**
LC2_10_10	29	17,425.55	25-Feb-05	29	17,898.25	LRC1_10_10	50	45,156.36	10-Jul-15	**48**	**36,092.22**
LR1_10_1	100	56,903.88	25-Feb-05	**100**	**56,875.21**	LRC2_10_1	22	35,059.40	30-Nov-15	**22**	**34,960.69**
LR1_10_2	80	49,652.10	25-Feb-05	**80**	**49,627.16**	LRC2_10_2	21	30,932.74	25-Feb-05	**20**	**33,576.15**
LR1_10_3	54	42,124.44	25-Feb-05	54	42,346.86	LRC2_10_3	16	28,403.51	25-Feb-05	16	29,495.48

Table 11 continued

Instance	Best known			LS + AGES + LNS		Instance	Best known			LS + AGES + LNS	
	Veh	Distance	Date	Veh	Distance		Veh	Distance	Date	Veh	Distance
LR1_10_4	28	32,133.36	25-Feb-05	**28**	**31,617.58**	LRC2_10_4	12	23,083.20	25-Feb-05	**11**	**26,239.60**
LR1_10_5	61	59,135.86	25-Feb-05	**59**	**60,616.90**	LRC2_10_5	18	33,451.16	30-Nov-15	**17**	**37,312.43**
LR1_10_6	49	57,674.00	10-Jul-15	**48**	**51,842.12**	LRC2_10_6	17	31,470.58	30-Nov-15	17	32,745.90
LR1_10_7	37	38,936.54	25-Feb-05	**36**	**40,127.52**	LRC2_10_7	17	29,499.79	30-Nov-15	**16**	**32,537.87**
LR1_10_8	26	29,452.32	25-Feb-05	**26**	**29,099.22**		–	–	–		
LR1_10_9	50	52,223.15	25-Feb-05	**49**	**53,353.22**		–	–	–		
LR1_10_10	40	46,218.35	25-Feb-05	**39**	**47,290.00**	LRC2_10_10	12	29,380.12	30-Nov-15	**11**	**31,334.49**

Bold indicates a new best known and italics indicate equal to previous best known

Table 12 Results for a single run of Algo1

Instance	Veh.	Dist.	Time (s)	Instance	Veh.	Dist.	Time (s)
lc101	10	828.94	300	lr201	4	1253.23	300
lc102	10	828.94	300	lr202	3	1197.67	300
lc103	9	1038.35	300	lr203	3	952	300
lc104	9	861.95	300	lr204	2	849.05	300
lc105	10	828.94	300	lr205	3	1054.02	300
lc106	10	828.94	300	lr206	3	931.63	300
lc107	10	828.94	300	lr207	2	903.06	300
lc108	10	826.44	300	lr208	2	734.85	300
lc109	9	1000.6	300	lr209	3	930.59	300
lc201	3	591.56	300	lr210	3	964.22	300
lc202	3	591.56	300	lr211	2	911.52	300
lc203	3	591.17	300	lrc101	14	1708.8	300
lc204	3	638.18	300	lrc102	12	1558.07	300
lc205	3	588.88	300	lrc103	11	1258.74	300
lc206	3	588.49	300	lrc104	10	1128.4	300
lc207	3	588.29	300	lrc105	13	1637.62	300
lc208	3	588.32	300	lrc106	11	1424.73	300
lr101	19	1650.8	300	lrc107	11	1230.14	300
lr102	17	1487.57	300	lrc108	10	1147.43	300
lr103	13	1292.68	300	lrc201	4	1455.54	300
lr104	9	1013.39	300	lrc202	3	1374.27	300
lr105	14	1377.11	300	lrc203	3	1089.07	300
lr106	12	1252.62	300	lrc204	3	818.66	300
lr107	10	1111.31	300	lrc205	4	1302.2	300
lr108	9	968.97	300	lrc206	3	1159.03	300

Table 12 continued

Instance	Veh.	Dist.	Time (s)	Instance	Veh.	Dist.	Time (s)
lr109	11	1208.96	300	lrc207	3	1062.05	300
lr110	10	1159.35	300	lrc208	3	852.76	300
lr111	10	1108.9	300				
lr112	9	1003.77	300				
LC1_2_1	20	2704.57	900	LR2_2_1	5	4073.1	900
LC1_2_2	19	2764.56	900	LR2_2_2	4	3796	900
LC1_2_3	17	3127.78	900	LR2_2_3	4	3098.36	900
LC1_2_4	17	2695.35	900	LR2_2_4	3	2511.77	900
LC1_2_5	20	2702.05	900	LR2_2_5	4	3438.39	900
LC1_2_6	20	2701.04	900	LR2_2_6	3	5079.87	900
LC1_2_7	20	2701.04	900	LR2_2_7	3	3226.37	900
LC1_2_8	19	3453.1	900	LR2_2_8	2	2907.04	900
LC1_2_9	18	2724.24	900	LR2_2_9	3	3998.73	900
LC1_2_10	17	3114.95	900	LR2_2_10	3	3372.39	900
LC2_2_1	6	1931.44	900	LRC1_2_1	19	3606.06	900
LC2_2_2	6	1881.4	900	LRC1_2_2	15	3678.89	900
LC2_2_3	6	1844.7	900	LRC1_2_3	13	3194.84	900
LC2_2_4	6	1767.12	900	LRC1_2_4	10	2633.25	900
LC2_2_5	6	1891.21	900	LRC1_2_5	16	3715.81	900
LC2_2_6	6	1857.78	900	LRC1_2_6	16	3713.7	900
LC2_2_7	6	1850.13	900	LRC1_2_7	14	3769.84	900
LC2_2_8	6	1824.34	900	LRC1_2_8	13	3209.72	900
LC2_2_9	6	1854.21	900	LRC1_2_9	13	3250.5	900
LC2_2_10	6	1817.45	900	LRC1_2_10	12	2963.71	900
LR1_2_1	20	4819.12	900	LRC2_2_1	6	3595.18	900

Table 12 continued

Instance	Veh.	Dist.	Time (s)	Instance	Veh.	Dist.	Time (s)
LR1_2_2	17	4621.21	900	LRC2_2_2	5	3253.69	900
LR1_2_3	14	4405.43	900	LRC2_2_3	4	3003.99	900
LR1_2_4	10	3034.17	900	LRC2_2_4	3	3092.38	900
LR1_2_5	16	4773	900	LRC2_2_5	5	2776.93	900
LR1_2_6	13	4800.94	900	LRC2_2_6	5	2707.96	900
LR1_2_7	12	3550.61	900	LRC2_2_7	4	3080.74	900
LR1_2_8	9	2810.26	900	LRC2_2_8	4	2399.99	900
LR1_2_9	14	4372.32	900	LRC2_2_9	4	2208.52	900
LR1_2_10	11	3714.16	900	LRC2_2_10	3	2691.21	900
LC1_4_1	40	7152.06	900	LR2_4_1	8	9842.79	900
LC1_4_2	38	8199.22	900	LR2_4_2	7	10,510.74	900
LC1_4_3	33	8512.53	900	LR2_4_3	6	9449.97	900
LC1_4_4	30	7166.68	900	LR2_4_4	4	7447.74	900
LC1_4_5	40	7150	900	LR2_4_5	6	10,658.54	900
LC1_4_6	40	7154.02	900	LR2_4_6	5	9663.35	900
LC1_4_7	40	7149.43	900	LR2_4_7	5	7438.04	900
LC1_4_8	39	7111.16	900	LR2_4_8	4	6178.83	900
LC1_4_9	36	8240.48	900	LR2_4_9	6	9741.13	900
LC1_4_10	35	7785.96	900	LR2_4_10	5	8588.43	900
LC2_4_1	12	4116.33	900	LRC1_4_1	36	9148.44	900
LC2_4_2	12	4144.29	900	LRC1_4_2	31	8366.84	900
LC2_4_3	12	4427.69	900	LRC1_4_3	24	7940.56	900
LC2_4_4	12	4155.54	900	LRC1_4_4	19	5851.35	900
LC2_4_5	12	4030.63	900	LRC1_4_5	32	8973.9	900
LC2_4_6	12	3900.29	900	LRC1_4_6	30	8561.18	900

Table 12 continued

Instance	Veh.	Dist.	Time (s)	Instance	Veh.	Dist.	Time (s)
LC2_4_7	12	4290	900	LRC1_4_7	28	8552.88	900
LC2_4_8	12	3844.45	900	LRC1_4_8	26	8341.27	900
LC2_4_9	12	4189.25	900	LRC1_4_9	25	8475.36	900
LC2_4_10	12	3829.34	900	LRC1_4_10	23	7552.75	900
LR1_4_1	40	10,639.75	900	LRC2_4_1	12	7587.81	900
LR1_4_2	31	10,009.1	900	LRC2_4_2	10	8488.36	900
LR1_4_3	22	9437.76	900	LRC2_4_3	8	6739.24	900
LR1_4_4	16	7101.42	900	LRC2_4_4	5	6086	900
LR1_4_5	29	10,621.62	900	LRC2_4_5	10	7835.13	900
LR1_4_6	24	9872.59	900	LRC2_4_6	9	6522.83	900
LR1_4_7	19	8501.8	900	LRC2_4_7	8	6780.16	900
LR1_4_8	14	5953.51	900	LRC2_4_8	7	5969.35	900
LR1_4_9	24	10,052.08	900	LRC2_4_9	7	5420.5	900
LR1_4_10	20	8584.19	900	LRC2_4_10	6	7590.79	900
LC1_6_1	60	14,095.64	1800	LR2_6_1	11	22,004.07	1800
LC1_6_2	57	15,062.27	1800	LR2_6_2	9	23,804.19	1800
LC1_6_3	50	14,684.81	1800	LR2_6_3	7	19,376.93	1800
LC1_6_4	48	13,346.44	1800	LR2_6_4	6	13,085.37	1800
LC1_6_5	60	14,086.3	1800	LR2_6_5	9	22,058.19	1800
LC1_6_6	60	14,090.79	1800	LR2_6_6	7	23,302.97	1800
LC1_6_7	60	14,083.76	1800	LR2_6_7	6	16,431.55	1800
LC1_6_8	58	15,205.1	1800	LR2_6_8	8	21,904.29	1800
LC1_6_9	54	14,859.61	1800	LR2_6_9	8	22,699.46	1800
LC1_6_10	52	15,811.84	1800	LR2_6_10	7	17,914.13	1800
LC2_6_1	19	7977.98	1800	LRC1_6_1	52	18,467.73	1800

Table 12 continued

Instance	Veh.	Dist.	Time (s)	Instance	Veh.	Dist.	Time (s)
LC2_6_2	18	10,685.95	1800	LRC1_6_2	43	17,092.29	1800
LC2_6_3	18	7440.4	1800	LRC1_6_3	36	14,110.35	1800
LC2_6_4	17	8068.58	1800	LRC1_6_4	25	11,099.15	1800
LC2_6_5	19	8047.37	1800	LRC1_6_5	46	17,167.51	1800
LC2_6_6	18	9622.26	1800	LRC1_6_6	42	17,576.85	1800
LC2_6_7	19	8010.02	1800	LRC1_6_7	38	15,557.42	1800
LC2_6_8	18	7579.93	1800	LRC1_6_8	33	16,049.69	1800
LC2_6_9	18	11,084.98	1800	LRC1_6_9	34	15,476.3	1800
LC2_6_10	18	7493.7	1800	LRC1_6_10	29	15,213.25	1800
LR1_6_1	59	22,821.65	1800	LRC2_6_1	16	15,118.51	1800
LR1_6_2	45	20,458.42	1800	LRC2_6_2	13	17,675.41	1800
LR1_6_3	37	18,157.15	1800	LRC2_6_3	10	13,508.6	1800
LR1_6_4	28	13,419.2	1800	LRC2_6_4	7	15,185.64	1800
LR1_6_5	38	22,777.17	1800	LRC2_6_5	13	15,438.84	1800
LR1_6_6	31	23,145.06	1800	LRC2_6_6	12	15,897.94	1800
LR1_6_7	25	17,481.34	1800	LRC2_6_7	10	16,674.47	1800
LR1_6_8	18	12,683.97	1800	LRC2_6_8	9	15,397.81	1800
LR1_6_9	32	21,878.25	1800	LRC2_6_9	9	14,019.72	1800
LR1_6_10	26	19,555.51	1800	LRC2_6_10	8	12,924.28	1800
LC1_8_1	80	25,184.38	3600	LR2_8_1	15	37,159.43	3600
LC1_8_2	77	27,053.48	3600	LR2_8_2	12	37,143.89	3600
LC1_8_3	64	27,080.22	3600	LR2_8_3	9	31,195.51	3600
LC1_8_4	60	23,046.35	3600	LR2_8_4	12	46,347.34	3600
LC1_8_5	80	25,211.22	3600	LR2_8_5	11	38,199.48	3600
LC1_8_6	80	25,164.25	3600	LR2_8_6	9	36,157.03	3600

Table 12 continued

Instance	Veh.	Dist.	Time (s)	Instance	Veh.	Dist.	Time (s)
LC1_8_7	80	25,158.38	3600	LR2_8_7	8	28,099.62	3600
LC1_8_8	78	25,525.38	3600	LR2_8_8	11	35,100.74	3600
LC1_8_9	72	26,309.29	3600	LR2_8_9	10	34,160.16	3600
LC1_8_10	70	26,896.55	3600	LR2_8_10	9	28,650.21	3600
LC2_8_1	24	11,687.06	3600	LRC1_8_1	66	32,343.63	3600
LC2_8_2	24	14,432.89	3600	LRC1_8_2	56	28,028.22	3600
LC2_8_3	24	13,489.52	3600	LRC1_8_3	48	24,868.53	3600
LC2_8_4	24	12,724.81	3600	LRC1_8_4	34	18,651.8	3600
LC2_8_5	25	12,329.8	3600	LRC1_8_5	58	31,743.18	3600
LC2_8_6	24	12,938.93	3600	LRC1_8_6	55	28,986.21	3600
LC2_8_7	25	13,313.06	3600	LRC1_8_7	51	28,684.47	3600
LC2_8_8	24	11,481.82	3600	LRC1_8_8	45	27,386.97	3600
LC2_8_9	24	11,630.33	3600	LRC1_8_9	44	25,541.71	3600
LC2_8_10	24	11,586.62	3600	LRC1_8_10	40	24,775.03	3600
LR1_8_1	80	39,314.59	3600	LRC2_8_1	20	23,280.89	3600
LR1_8_2	59	34,639.58	3600	LRC2_8_2	17	23,662.24	3600
LR1_8_3	44	29,672.24	3600	LRC2_8_3	15	19,558.58	3600
LR1_8_4	25	21,351.41	3600	LRC2_8_4	11	21,502.55	3600
LR1_8_5	49	39,911.37	3600	LRC2_8_5	16	26,300.04	3600
LR1_8_6	40	35,319.74	3600	LRC2_8_6	15	23,003.29	3600
LR1_8_7	30	28,180.3	3600	LRC2_8_7	14	23,874.42	3600
LR1_8_8	20	20,352.08	3600	LRC2_8_8	12	23,407.84	3600
LR1_8_9	41	36,802.82	3600	LRC2_8_9	11	21,294.5	3600
LR1_8_10	31	31,612.63	3600	LRC2_8_10	10	22,986.75	3600
LC1_10_1	100	42,488.66	3600	LR2_10_1	18	58,315.37	3600

Table 12 continued

Instance	Veh.	Dist.	Time (s)	Instance	Veh.	Dist.	Time (s)
LC1_10_2	95	43,813.15	3600	LR2_10_2	14	62,490.96	3600
LC1_10_3	82	43,103.5	3600	LR2_10_3	11	47,933.54	3600
LC1_10_4	74	38,849.97	3600	LR2_10_4	14	42,660.35	3600
LC1_10_5	100	42,477.41	3600	LR2_10_5	14	54,270.3	3600
LC1_10_6	101	42,838.39	3600	LR2_10_6	12	51,141.9	3600
LC1_10_7	100	42,854.99	3600	LR2_10_7	10	64,292.78	3600
LC1_10_8	98	43,012.3	3600	LR2_10_8	14	64,986.32	3600
LC1_10_9	91	43,146.18	3600	LR2_10_9	13	52,350.2	3600
LC1_10_10	88	44,889.54	3600	LR2_10_10	11	48,354.75	3600
LC2_10_1	30	16,879.24	3600	LRC1_10_1	82	49,267.82	3600
LC2_10_2	31	19,572.89	3600	LRC1_10_2	72	45,408	3600
LC2_10_3	30	18,045.44	3600	LRC1_10_3	53	37,085.36	3600
LC2_10_4	30	18,637.19	3600	LRC1_10_4	40	28,722.74	3600
LC2_10_5	31	17,289.55	3600	LRC1_10_5	72	52,609.93	3600
LC2_10_6	31	17,212.05	3600	LRC1_10_6	68	44,559.84	3600
LC2_10_7	31	18,893.71	3600	LRC1_10_7	61	42,347.71	3600
LC2_10_8	30	17,310.82	3600	LRC1_10_8	56	42,529.31	3600
LC2_10_9	30	19,420.8	3600	LRC1_10_9	53	40,648.11	3600
LC2_10_10	29	17,567.64	3600	LRC1_10_10	48	37,535.48	3600
LR1_10_1	100	57,060.03	3600	LRC2_10_1	22	35,357.39	3600
LR1_10_2	80	49,688.13	3600	LRC2_10_2	21	31,843.7	3600
LR1_10_3	54	42,999.25	3600	LRC2_10_3	16	34,504.22	3600
LR1_10_4	29	30,784.3	3600	LRC2_10_4	24	40,740.78	3600
LR1_10_5	60	58,370.14	3600	LRC2_10_5	17	38,958.91	3600
LR1_10_6	48	50,117.58	3600	LRC2_10_6	17	31,476.76	3600

Table 12 continued

Instance	Veh.	Dist.	Time (s)	Instance	Veh.	Dist.	Time (s)
LR1_10_7	36	39,091.89	3600	LRC2_10_7	16	32,788.83	3600
LR1_10_8	26	29,338.14	3600				
LR1_10_9	49	53,212.61	3600				
LR1_10_10	40	46,634.81	3600	LRC2_10_10	11	32,048.61	3600

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