Automata and Logic Engineering 1

Logic in Professional Practice

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# Introduction

This document describes the assignments for the course ALE1 (3 ECTS) and the innovation route Academic Preparations as part of the pre-master program for the Tue (5 ECTS).

The assignments can be done in any modern object oriented language (we advise C# and Java).

The assignments differ in difficulty. The next table gives a global indication (1 = relatively easy; 4 = relatively difficult), together with an advised week planning (YMMV).

|  |  |  |  |
| --- | --- | --- | --- |
| **assignment** | **difficulty** | **week (for 5 ECTS)** | **week (for 3 ECTS)** |
| 1 | 3 | 1 | 1+2 |
| 2 | 1 | 2 | 3 |
| 3 | 2 | 2 | 4+5 |
| 4 | 2 | 3 | 6 |
| 5 | 1 | 3 | 7 |
| 6 | 3 | 4+5 | - |
| 7 | 2 | 6 | - |
| 8 | 4 | 7 + 8 | - |

# Learning goals

Besides the specific contents of the assignments, the following general aspects of software engineering play a role:

* UML modelling
* refactoring (in particular when your initial UML modelling was not that optimal)
* testing (module tests and system tests)
* code analysis (coverage, complexity)
* user interface design

# Proposition logic

## Input format for proposition logic

Each capital letter is a predicate. Formulas for predicate logic are built with predicates and connectives like ¬, ⇒, ⇔, ⋀ and ⋁. For simplification in this course we use the following ASCII prefix notation of the formulas:

|  |  |
| --- | --- |
| Logica notatie | ASCII |
| ¬A | ~(A) |
| A ⇒ B | >(A,B) |
| A ⇔ B | =(A,B) |
| A ⋀ B | &(A,B) |
| A ⋁ B | |(A,B) |

So the formula

(A⇒B)⇔(¬A⋁B)

will be written as

=( >(A,B), |( ~(A) ,B) )

There are two special predicates: 0 which always has value false, and 1 which always has value true.

## Tips

Probably, you will make Assignment 1 with a text box and a button, and the results will be put in another text box. For all next assignments, it's convenient to start them by the same button (or by a new button) and to put the results in separate text boxes.

## Real life examples

Propositions can be realized with electronic logic circuits. In this way, a complete processor can be build. NAND circuits (see Assignment 5) are used in chip development because they are smaller than other circuits (and they use less energy).

See also:

* <https://en.wikipedia.org/wiki/Logic_gate>
* <https://en.wikipedia.org/wiki/Adder_%28electronics%29>
* <http://www.neuroproductions.be/logic-lab/> (and various other logic gate simulators)

## Test vectors

For a proper test of your system you need a set of formulas for which you know the results, preferably formulas which touch the difficult aspects of your formulas.

Testing can be done by module tests, but in this case every participant of this course has to write the tests again. It would be better if you all have a system, in which you can exchange your test formulas (and their results) with each other. This system should be easily extendible with new formulas and can be executed by each implementation.

Develop such a system within your group (for example: a common database or shared Dropbox files with a well described syntax)

## Assignment 1

In the Logic and Set Theory course you have learnt that formulas are in fact tree structures. Write a program that reads a formula in ASCII format (as described in Input format for proposition logic) and builds a tree of objects internally. Note: spaces are allowed but must be ignored.

Tip: use *recursion* to read the formula.

Use one base class (or interface) for each node and let method ToString() return the formula in infix notation.

Furthermore: make a method which returns a list of all variables of the formula (each variable should appear only once).

And show a picture of the tree (see Graphical representation of a logic formula).

## Assignment 2

Extend your program such that the truth table of a given proposition formula is calculated.

Put the names of the variables in alphabetic order when printing the table. Concatenate the digits in the last column and print them as a hexadecimal number (where the first line is the LSB). We call this the hash code and it is an identification of the formula. Example:

|  |  |  |  |
| --- | --- | --- | --- |
| A | B | C | (A ⋁ ¬B) ⋀ C |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

The last column yields: 10100010, or: A2. See also <https://en.wikipedia.org/wiki/Hexadecimal>.

## Assignment 3

If a truth table contains two rules with the same truth value where all-but-one variables have the same value then the value of this predicate with the different values is not important. Both lines can be rewritten with a \* as value for that variable (also called 'don't care'). The truth table of (A ⋁ B) ⋁ C can be simplified from

|  |  |  |  |
| --- | --- | --- | --- |
| A | B | C | (A ⋁ B) ⋁ C |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

into

|  |  |  |  |
| --- | --- | --- | --- |
| A | B | C | (A ⋁ B) ⋁ C |
| 0 | 0 | 0 | 0 |
| \* | \* | 1 | 1 |
| \* | 1 | \* | 1 |
| 1 | \* | \* | 1 |

Extend your program such that simplified truth tables are shown as well.

## Assignment 4

From a truth table you can construct unambiguously a disjunctive normal form of the formula. Take for every line with a truth value of 1 the conjunction of the predicates with value 1 with the conjunction of the negation of the predicates with value 0. Finally, take the disjunction of those formulas. For example:

|  |  |  |
| --- | --- | --- |
| A | B | A ⇒ B |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

When applying the rules, we first get the formulas (¬A ⋀ ¬B); (¬A ⋀ B)and (A ⋀ B). Grouping them as disjoints together gives us the disjunctive normal form:   
(¬A ⋀ ¬B) ⋁ (¬A ⋀ B) ⋁ (A ⋀ B).

When we strictly apply the rules ('\*' is not '0' and not '1') then we can use it for simplified truth tables as well. The truth table of (A ⋁ B) ⋁ C yields (A ⋁ B) ⋁ C. This formula is of course simpler than the disjunctive normal form of the full truth table.

Extend your program such that the disjunctive normal form is displayed (for both original and simplified truth tables). Those disjunctive normal forms can be read again by your program and must deliver the same truth table and hash code.

Tip: a contradiction and a tautology are nice test cases.

## Assignment 5

Each truth table can be created only with either the connectives ~ and &, or only with the connectives ~ and | (e.g. apply De Morgan on a disjunctive normal form). You can obtain a formula with only ~ and & by rewriting the connectives as follows:

|  |  |
| --- | --- |
| formula | becomes |
| A ⇒ B | ¬A ⋁ B |
| A ⇔ B | (¬A ⋀ ¬B) ⋁(A ⋀ B) |
| A ⋁ B | ¬(¬A ⋀ ¬B) |

If we define the connective NAND ("Not AND") as ¬(A⋀B), then we can create all truth tables only with this connective. We use the following ASCII notation: %(A,B).

Extend your program to convert a proposition in an equivalent proposition with only NANDs. The new NAND proposition can be read again by your program and must deliver the same truth table and hash code.

## Recapitulation

Your program has probably 2 data structures: a proposition tree and a truth table. In the assignments you might have written methods to convert one data structure into the other. Perhaps you have different names, but you might see the following relationships between data structures and methods:

nandify

normalize

evaluate

**proposition tree**

**truth   
table**

simplify

parse

Given the original proposition tree, calculate the hash code of 6 truth tables which are constructed as follows:

1. original + evaluate
2. original + evaluate + simplify
3. original + nandify + evaluate + simplify
4. original + evaluate + normalize + evaluate
5. original + evaluate + simplify + normalize + evaluate
6. original + evaluate + simplify + normalize + nandify + evaluate

Of course, install an automated test to validate that all hash codes are identical.

# Semantic Tableaux

## Assignment 6

Implement the proof procedure with semantic tableaux for propositions. The output indicates if the formula is a tautology, with a complete tree of the steps.

## Input format for predicate logic

Besides proposition logic we are going to work with predicate logic. Therefor we enhance our input syntax:

A lowercase letter is a variable. A predicate can be applied on 0 or more variables which are listed behind the predicate symbol (in braces, separated by commas). The number of variables for a predicate symbol is not pre-determined, but it stays the same in the formula (so: it's not allowed to have both P(a) en P(b,c) in one formula). Predicates without variables may be written without braces (so P instead of P()).

The following formulas are valid formulas for predicate logic:

1. predicates, applied on 0 or more variables are valid formulas
2. formulas build from (smaller) formulas and connectives ¬, ⇒, ⇔, ⋀ and ⋁ are valid formulas
3. if x is a variable and F is a valid formula (which might contain x), then ∀x.(F) (read: “for all x yields F”) and ∃x.(F) (read: “there exists an x such that F holds”) are valid formulas as well.

The ASCII notation for quantifier formulas are:

|  |  |
| --- | --- |
| Formula | ASCII |
| ∀x.(F) | @x.(F) |
| ∃x.(F) | !x.(F) |

## Assignment 7

Extend your program such that predicate formulas can be read and can be presented as a tree. Moreover, write methods to:

1. if the formula is a proposition formula (so: it doesn't contain variables nor quantifiers)
2. in a formula change all appearances of an unbound variable into another  
   so if we change x into y, then ∀z.(P(x,z)) becomes ∀z.(P(y,z))

## Assignment 8

Extend your program to proof predicate formulas with the help of sematic tableaux. The output indicates if the formula is a tautology, with a complete tree of the steps..

# Graphical representation of a logic formula

If you want to add a graphical picture of your automaton, please follow the following steps:

1. install GraphViz
2. adapt your $PATH environment variable
3. in your application:
   1. generate a text file (e.g. abc.dot), similar to this:

graph logic {

node [ fontname = "Arial" ]

node1 [ label = "=" ]

node1 -- node2

node2 [ label = ">" ]

node2 -- node4

node4 [ label = "A" ]

node2 -- node5

node5 [ label = "B" ]

node1 -- node3

node3 [ label = "|" ]

node3 -- node6

node6 [ label = "~" ]

node6 -- node13

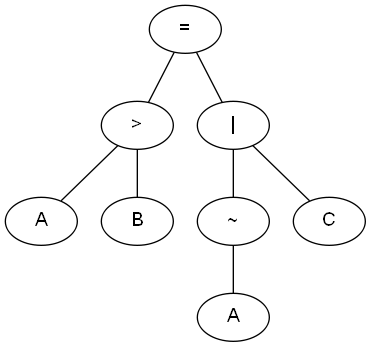
node13 [ label = "A" ]

node3 -- node7

node7 [ label = "C" ]

}

* 1. start the GraphViz executable  
     dot -Tpng -oabc.png abc.dot
  2. (wait until this executable is finished)
  3. show a picture (e.g. abc.png), for example in a PictureBox



In C#, the steps 3.2 - 3.4 *could* look like:

WriteDot("abc.dot"); // your method to write a proposition tree

// into a dot-format file

Process dot = new Process();

dot.StartInfo.FileName = @"C:\Program Files\<your path>\dot.exe";

dot.StartInfo.Arguments = "-Tpng -oabc.png abc.dot";

dot.Start();

dot.WaitForExit();

myPictureBox.ImageLocation = "abc.png";

In Java, the steps *could* look like:

String[] cmd = { " C:\\Program Files (x86)\\ <your path>\\dot.exe",

"-Tpng", "-oabc.png", "abc.dot" };

Process p = Runtime.getRuntime().exec(cmd);

p.waitFor();

File file = new File("abc.png");

Image image = new Image(file.toURL().toString());

myWidget.setImage(image);

(you can do some experiments for steps 3.1 - 3.4 in a text editor and on the command line)

Ideas for other nice features of GraphViz are welcome.