Number picking game analysis report

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***Abstract*—This document describes a multi-player number picking game and models it in normal form. The game is analyzed using the game theory. The data collected from a real play with over 40 participants are examined.**

***Index Terms*—game theory, normal form, Nash Equilibrium**

I. INTRODUCTION

Game theory became an independent field of study when John von Neumann published the paper *On the Theory of Games of Strategy* in 1928. While in 1950s, John Nash extended the scope of games with his new theorem known as Nash Equilibrium. Now days game theory is a vital field of study to make strategic desicions in humans and computers [1]. This document describes a simple game to explore the key concepts of modern game theory and the scientific approach to model real world problem with game theory.

The game in question is a multi-player game that each player has to figure out a number according to the rules are follows:

- Each player names an integer between 1 and 10.
- The player who names the integer closest to two thirds of the average wins a prize (2 marks to his/her final grade), the other players get nothing.
- If there are multiple winners, then the prize is shared by winners.

II. GAME REPRESENTATION

In game theory, we represent a finite, n-player game as $< \mathbb{N}, \mathbb{A}, u >$, where the set $\mathbb{N}$ stands for the set of players and the set $\mathbb{A}$ denotes the actions or strategies that players can take. Finally, u is a set of utility functions corresponding to each player. In our case, we reprensent the players as follows:

$$\mathbb{N} = \{1, 2, \cdots, n\} \tag{1}$$

The actions, with consideration of all participants, can be represented as:

$$\mathbb{A} = \mathbb{A}_1 \times \mathbb{A}_2 \times \cdots \times \mathbb{A}_n, \vec{a} = (a_1, a_2, \cdots, a_n) \in \mathbb{A} \tag{2}$$

TABLE I
TWO-PLAYER NUMBER PICKING GAME PAYOFF MATRIX

	1	2	3	4	5	6	7	8	9	10
1	1,1	2,0	2,0	2,0	2,0	2,0	2,0	2,0	2,0	2,0
2	0,2	1,1	2,0	2,0	2,0	2,0	2,0	2,0	2,0	2,0
3	0,2	0,2	1,1	2,0	2,0	2,0	2,0	2,0	2,0	2,0
4	0,2	0,2	0,2	1,1	2,0	2,0	2,0	2,0	2,0	2,0
5	0,2	0,2	0,2	0,2	1,1	2,0	2,0	2,0	2,0	2,0
6	0,2	0,2	0,2	0,2	0,2	1,1	2,0	2,0	2,0	2,0
7	0,2	0,2	0,2	0,2	0,2	0,2	1,1	2,0	2,0	2,0
8	0,2	0,2	0,2	0,2	0,2	0,2	0,2	1,1	2,0	2,0
9	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	1,1	2,0
10	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	1,1

where:

$$\forall i \in \mathbb{N}, a_i \in \{1, 2, 3, \cdots, 10\} \tag{3}$$

$\vec{a}$ denotes an action profile. In this game, it represents a particular combination of actions that all players take.

$$u_i : \mathbb{A} \rightarrow \mathbb{R}, \vec{u} = (u_1, u_2, \cdots, u_n) \tag{4}$$

The u_i is the payoff function to specify the award to the player at the end of game. It is usually speficied by using matrix.

A. Two-player Representation

For simplicity, we start with the two-player case. We use a 10×10 matrix, presented in [TABLE. I], to represent the payoff functions for the two-player scenario of our game. The row label stands for the number picked by the first player, while the column label is the number selected by the second player. Each cell lists a pair of numbers, which are the awards to the players. The first number is the award for the first player. Likewise, the second number is the award for the second player.

We can generalize the [TABLE I] to get equation 5.

$$u(a_1, a_2) = \begin{cases} 2, 0 & if \quad a_1 < a_2 \\ 1, 1 & if \quad a_1 = a_2 \\ 0, 2 & if \quad a_1 > a_2 \end{cases} \tag{5}$$

B. Three player or more Representation

When the number of players is greater than 2, we can still use the matrix to represent the payoff function. The basic idea is to use action profile vector instead of a single action as row or column label. To compact the representation, we use two action profile vectors, where $\vec{a_r} = (a_1, a_2, \cdots, a_{\frac{n}{2}})$ denotes action profile for the first half group of players. And $\vec{a_c} = (a_{\frac{n}{2}+1}, a_{\frac{n}{2}+2}, \cdots, a_n)$ is for the second half group of players. We put all possible permutations for $\vec{a_r}$ on the row, $\vec{a_r}$ on the column label. The cell is an n-element vector which specifies the payoff for each player. The size of matrix is determined by 10^n, where n is the number of player.

III. Game Analysis

Rationality means that every player is motivated to pursuit his max payoff. At the same time, he knows the fact that his adversaries are also aware of this fact, which could trigger a chain of actions. In this game, every player should pick a number close to $2/3$ of the average. According to the rationality premise, everyone should multiply this number with another $2/3$, and this process continues until no smaller number could be chosen, when the number 1 is the only choice. And this is what Nash Equilibrium of this game is reached. So picking number 1 is the best response for everyone in this game because nobody can achieve more gains by changing his choice unilaterally.

IV. Game Data Analysis

We explore the result of a trial play of this game by a group of 43 students. The data is published on Github at A jupyter notebook program is developed to process and visualize the trial play. The program is published on Github at [3]. The key findings of the trial play is presented in Fig. 1. The number picked by each player is on the y axis, while the x axis displays the response to the "$\frac{2}{3}$ of 10" question. We categorize the rationality into four levels with corresponding bubble size in Fig. 1 as follows:

- irrational (tiny size) – not motivated to win
- semi-rational(small size) – motivated to win but don't know how
- quasi-rational (medium size) – motivated to win, know basic rules, but don't take adversaries' move into account
- rational (large size) – motivated to win and know how to handle interplay of adversaries

As indicated by Fig. 1, most bubbles are scattered around and far from the green triangle which is the Nash Equilibrium of this game [1]. The three gold bubbles represents winners. The winners won the game by luck as two of them failed to answer the "$\frac{2}{3}$ of 10" question correctly. The reason given by the other player indicates the win was a random choice.

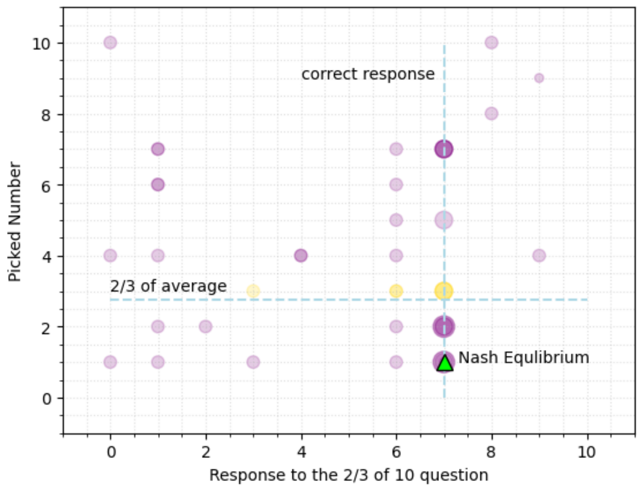

Fig. 1. Number picking Game Result

V. Conclusion

The people's rationality is of great importance to the game. If most participants are not rational, which is the case of this trial play, then the result is very difficult to predict. To increase the rationality for all participants, we could add one more question to complement the "$\frac{2}{3}$ of 10" question, asking the players what he will do if a few players has decreased the number to $\frac{2}{3}$ of 10. This helps to trigger a chain of actions to reduce the number.

Repeated plays are also good method to educate the participants and make them find the best strategy.

Regardless the rationality of the players, the change of upper limit 10 to 4 is likely to have more winners as the chances of mistakes become smaller. On the contray, changing the upper limit to 100 is likely to have fewer winners. Before all players become rational, change of the reward value might impact the game result. For instance, when the reward is doubled, some player might be more enanged in this game and take more effetive strategy and improve the chance to win. When all player are educated, changing reward has no impact on the result as everyone can win and their relative gains remain intact even if the overall reward changes.

[1]See analysis in section III

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