The GAPic Package

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The elements in X_0 are called *vertices*, the elements in X_1 are called *edges* and the elements in X_2 are called *faces*.

Definition

Let $(\prec, X_0, X_1, X_2)$ be a triangular complex.
Then we call $(\prec, X_0, X_1, X_2)$ *simplicial surface* if

(i) $\forall e \in X_1 : |\{f \in X_2 \mid e \prec f\}| \leq 2$

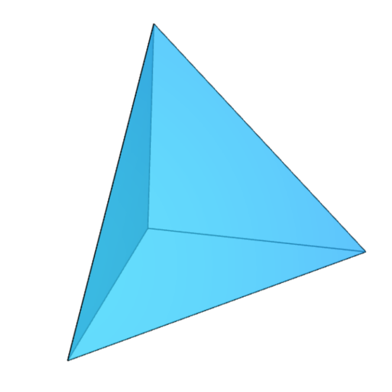
(ii) $\forall v \in X_0 : |\{f \in X_2 \mid v \prec f\}| < \infty$

(iii) $\forall v \in X_0$: there is an ordering of the $e_i, f_j \prec v$ such that

$$e_1 \prec f_1 \prec e_2 \prec f_2 \prec \cdots \prec f_{n-1} \prec e_n \prec f_n \prec e_1$$

the last condition is called the *umbrella condition*.

Example



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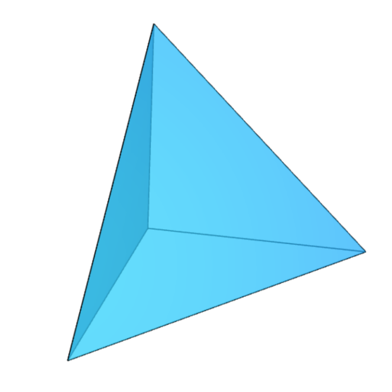

Simplicial Surface

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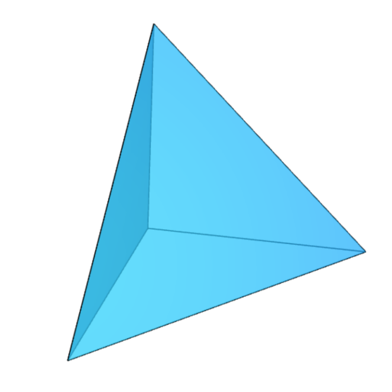

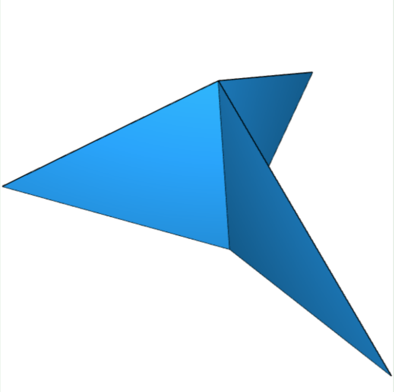

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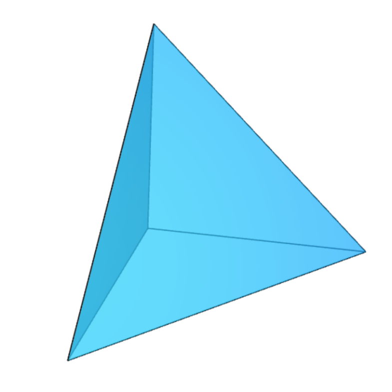

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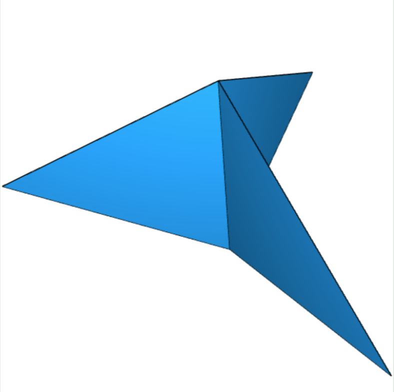

Triangular complex

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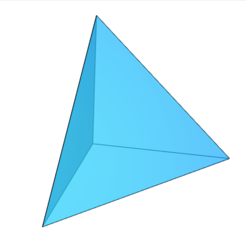

Is an embedded triangular complex.

Simplicial Surfaces Package

- Has functionality for displaying surfaces
  - Generates a .html file
  - Uses three.js

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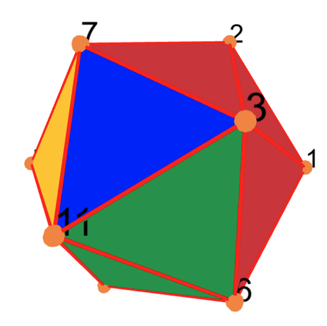
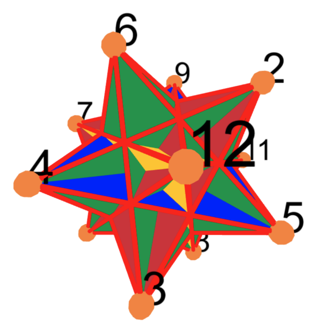
Example (Number 2.1 and 2.2 from [1])





Fachpraktikum

Was a project with the goal to improve the visualizations by adding shading/local lighting

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THREE.Geometry will be removed from core with r125

Discussion geometry

Mugen87 3 Jan '21

The upcoming release `r125` will contain a major, potentially breaking change. The class `THREE.Geometry` will be no longer part of the core but moved to `jsm/deprecated/Geometry.js`. It will only be available as an ES6 module and not as a global script.

Goal

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Implement shading in the visualizations of the Simplicial Surface package.

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First Approach: Implement directly
$\rightarrow$ Learn how the output is generated
Uses a class called `THREE.Geometry`

Workflow

But: In newer revisions of three.js shading is already implemented.
$\rightarrow$ After some promisiong tests: Decided to rewrite the entire function.

Demo

We need to switch to the browser for this.
For one example we use [1]

Results

ToDos

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Advantages

- New security requirements of javascript and modern browsers: need to load the code from server $\rightarrow$ way smaller file sizes (for small examples 9kB vs. 539kB)
- More efficient Animations, faster loading, fewer memory (Demo in Browser)
- Also works for triangular complexes
  $\rightarrow$Does not depend on incidence structure for visualization (Demo in Browser)

Future

- More functions in the GUI, e.g.
  - Turning the vertices on and off
  - Changing location of a vertex on the fly
- More options materials
  e.g. Color dependent on the normal of the polygon
  (Demo in browser)
- Intersection planes
  (Demo in browser)

Thank You for your attention

Are there Questions?

[1] Karl-Heinz Brakhage et al. *The icosahedra of edge length 1*. 2019. DOI: 10.48550/ARXIV.1903.08278. URL: https://arxiv.org/abs/1903.08278.