The GAPic Package

Lukas Schnelle

GAPDays Spring 2024

Definition

Let X_0 ("vertices"), X_1 ("edges") and X_2 ("faces") subsets of $\mathbb{N}_0$ and $\prec$ a symmetric relation between the sets called *incidence*.

Definition

Let X_0 ("vertices"), X_1 ("edges") and X_2 ("faces") subsets of $\mathbb{N}_0$ and $\prec$ a symmetric relation between the sets called *incidence*.
We call $(\prec, X_0, X_1, X_2)$ *triangular complex* if

Definition

Let X_0 ("vertices"), X_1 ("edges") and X_2 ("faces") subsets of $\mathbb{N}_0$ and $\prec$ a symmetric relation between the sets called *incidence*.
We call $(\prec, X_0, X_1, X_2)$ *triangular complex* if

(i) $\prec$ is transitive, i.e. $\forall v \in X_0, e \in X_1, f \in X_2$:

$$(v \prec e) \wedge (e \prec f) \Rightarrow v \prec f$$

Definition

Let X_0 ("vertices"), X_1 ("edges") and X_2 ("faces") subsets of $\mathbb{N}_0$ and $\prec$ a symmetric relation between the sets called *incidence*.
We call $(\prec, X_0, X_1, X_2)$ *triangular complex* if

(i) $\prec$ is transitive, i.e. $\forall v \in X_0, e \in X_1, f \in X_2$:

$$(v \prec e) \wedge (e \prec f) \Rightarrow v \prec f$$

(ii) $\forall e \in X_1 \exists f \in X_2$ such that $e \prec f$

Definition

Let X_0 ("vertices"), X_1 ("edges") and X_2 ("faces") subsets of $\mathbb{N}_0$ and $\prec$ a symmetric relation between the sets called *incidence*.
We call $(\prec, X_0, X_1, X_2)$ *triangular complex* if

(i) $\prec$ is transitive, i.e. $\forall v \in X_0, e \in X_1, f \in X_2$:

$$(v \prec e) \wedge (e \prec f) \Rightarrow v \prec f$$

(ii) $\forall e \in X_1 \exists f \in X_2$ such that $e \prec f$

(iii) $\forall e \in X_1 \exists! \, v_1 \neq v_2 \in X_0$ such that $v_1, v_2 \prec e$

Definition

Let X_0 ("vertices"), X_1 ("edges") and X_2 ("faces") subsets of $\mathbb{N}_0$ and $\prec$ a symmetric relation between the sets called *incidence*.
We call $(\prec, X_0, X_1, X_2)$ *triangular complex* if

(i) $\prec$ is transitive, i.e. $\forall v \in X_0, e \in X_1, f \in X_2$:

$$(v \prec e) \wedge (e \prec f) \Rightarrow v \prec f$$

(ii) $\forall e \in X_1 \exists f \in X_2$ such that $e \prec f$

(iii) $\forall e \in X_1 \exists !\, v_1 \neq v_2 \in X_0$ such that $v_1, v_2 \prec e$

(iv) $\forall f \in X_2 \exists !\, v_1 \neq v_2 \neq v_3 \in X_0$ such that $v_1, v_2, v_3 \prec f$

Definition

Let X_0 ("vertices"), X_1 ("edges") and X_2 ("faces") subsets of $\mathbb{N}_0$ and $\prec$ a symmetric relation between the sets called *incidence*.
We call $(\prec, X_0, X_1, X_2)$ *triangular complex* if

(i) $\prec$ is transitive, i.e. $\forall v \in X_0, e \in X_1, f \in X_2$:

$$(v \prec e) \land (e \prec f) \Rightarrow v \prec f$$

(ii) $\forall e \in X_1 \exists f \in X_2$ such that $e \prec f$

(iii) $\forall e \in X_1 \exists! \, v_1 \neq v_2 \in X_0$ such that $v_1, v_2 \prec e$

(iv) $\forall f \in X_2 \exists! \, v_1 \neq v_2 \neq v_3 \in X_0$ such that $v_1, v_2, v_3 \prec f$

(v) $\forall f \in X_2 \exists! \, e_1 \neq e_2 \neq e_3 \in X_1$ such that $e_1, e_2, e_3 \prec f$

Definition

Let X_0 ("vertices"), X_1 ("edges") and X_2 ("faces") subsets of $\mathbb{N}_0$ and $\prec$ a symmetric relation between the sets called *incidence*.
We call $(\prec, X_0, X_1, X_2)$ *triangular complex* if

(i) $\prec$ is transitive, i.e. $\forall v \in X_0, e \in X_1, f \in X_2$:

$$(v \prec e) \wedge (e \prec f) \Rightarrow v \prec f$$

(ii) $\forall e \in X_1 \exists f \in X_2$ such that $e \prec f$

(iii) $\forall e \in X_1 \exists! \, v_1 \neq v_2 \in X_0$ such that $v_1, v_2 \prec e$

(iv) $\forall f \in X_2 \exists! \, v_1 \neq v_2 \neq v_3 \in X_0$ such that $v_1, v_2, v_3 \prec f$

(v) $\forall f \in X_2 \exists! \, e_1 \neq e_2 \neq e_3 \in X_1$ such that $e_1, e_2, e_3 \prec f$

Definition

Let X_0 ("vertices"), X_1 ("edges") and X_2 ("faces") subsets of $\mathbb{N}_0$ and $\prec$ a symmetric relation between the sets called *incidence*.
We call $(\prec, X_0, X_1, X_2)$ *triangular complex* if

(i) $\prec$ is transitive, i.e. $\forall v \in X_0, e \in X_1, f \in X_2$:

$$(v \prec e) \wedge (e \prec f) \Rightarrow v \prec f$$

(ii) $\forall e \in X_1 \exists f \in X_2$ such that $e \prec f$

(iii) $\forall e \in X_1 \exists! \, v_1 \neq v_2 \in X_0$ such that $v_1, v_2 \prec e$

(iv) $\forall f \in X_2 \exists! \, v_1 \neq v_2 \neq v_3 \in X_0$ such that $v_1, v_2, v_3 \prec f$

(v) $\forall f \in X_2 \exists! \, e_1 \neq e_2 \neq e_3 \in X_1$ such that $e_1, e_2, e_3 \prec f$

$\rightarrow$ think of triangulated polyhedra

Definition

Let $(\prec, X_0, X_1, X_2)$ be a triangular complex.
Then we call $(\prec, X_0, X_1, X_2)$ *simplicial surface* if

Definition

Let $(\prec, X_0, X_1, X_2)$ be a triangular complex.
Then we call $(\prec, X_0, X_1, X_2)$ *simplicial surface* if

(i) $\forall e \in X_1 : |\{f \in X_2 \mid e \prec f\}| \leq 2$

(ii) $\forall v \in X_0 : def(v) := |\{f \in X_2 \mid v \prec f\}| < \infty$

(iii) $\forall v \in X_0$: there is an ordering of $e_1, f_1, \ldots, e_{deg(v)}, f_{deg(v)} \prec v$ such that

$$e_1 \prec f_1 \prec e_2 \prec f_2 \prec \cdots \prec f_{deg(v)} \prec e_{deg(v)}$$

with $e_1 = e_{deg}(v)$ if (i) is an equality.

Definition

Let $(\prec, X_0, X_1, X_2)$ be a triangular complex.
Then we call $(\prec, X_0, X_1, X_2)$ *simplicial surface* if

(i) $\forall e \in X_1 : |\{f \in X_2 \mid e \prec f\}| \leq 2$

(ii) $\forall v \in X_0 : def(v) := |\{f \in X_2 \mid v \prec f\}| < \infty$

(iii) $\forall v \in X_0$: there is an ordering of $e_1, f_1, \ldots, e_{deg(v)}, f_{deg(v)} \prec v$ such that

$$e_1 \prec f_1 \prec e_2 \prec f_2 \prec \cdots \prec f_{deg(v)} \prec e_{deg(v)}$$

with $e_1 = e_{deg}(v)$ if (i) is an equality.

Condition (iii) is called the *umbrella condition*.

Definition

Let $(\prec, X_0, X_1, X_2)$ be a triangular complex.
Then we call $(\prec, X_0, X_1, X_2)$ *simplicial surface* if

(i) $\forall e \in X_1 : |\{f \in X_2 \mid e \prec f\}| \leq 2$

(ii) $\forall v \in X_0 : def(v) := |\{f \in X_2 \mid v \prec f\}| < \infty$

(iii) $\forall v \in X_0$: there is an ordering of $e_1, f_1, \ldots, e_{deg(v)}, f_{deg(v)} \prec v$ such that

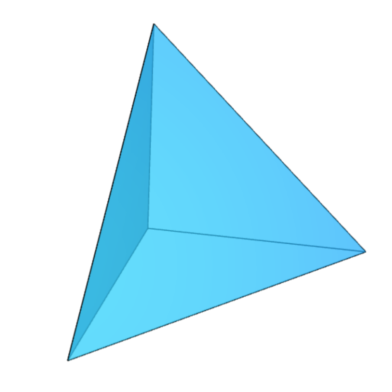
$$e_1 \prec f_1 \prec e_2 \prec f_2 \prec \cdots \prec f_{deg(v)} \prec e_{deg(v)}$$

with $e_1 = e_{deg}(v)$ if (i) is an equality.

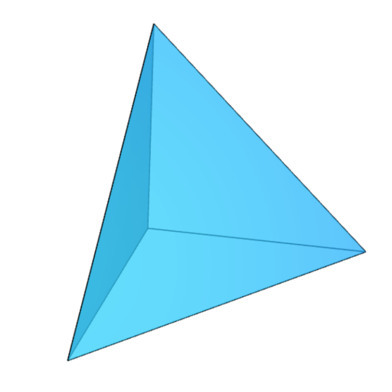
Condition (iii) is called the *umbrella condition*.

$\rightarrow$ almost all "nice" polyhedra fulfill these properties as well

Example

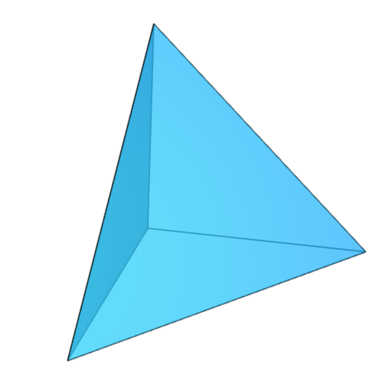
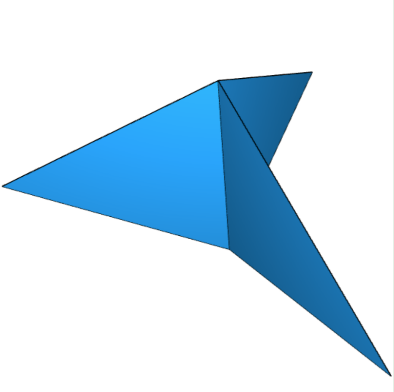


Example



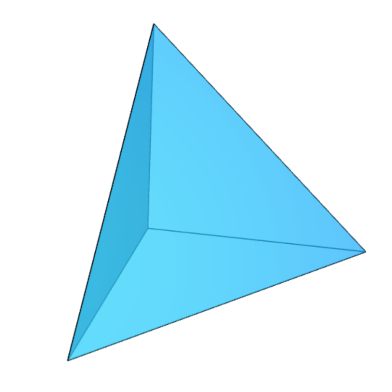
Simplicial Surface

Example

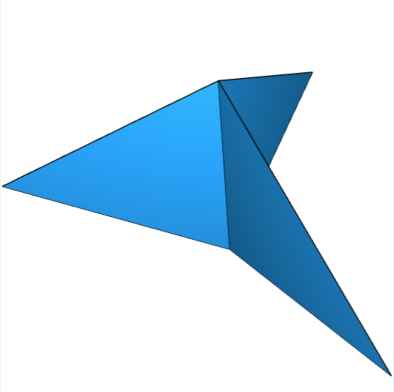
 

Simplicial Surface

Example



Simplicial Surface



Triangular complex

Definition

Let $(\prec, X_0, X_1, X_2)$ be a triangular complex.

Definition

Let $(\prec, X_0, X_1, X_2)$ be a triangular complex.
Then we define an *embedding* of $(\prec, X_0, X_1, X_2)$ as a map

$$c : X_0 \to \mathbb{R}^3$$

Definition

Let $(\prec, X_0, X_1, X_2)$ be a triangular complex.
Then we define an *embedding* of $(\prec, X_0, X_1, X_2)$ as a map

$$c : X_0 \to \mathbb{R}^3$$

The image of $v \in X_0$ is called *coordinate of v*.

Definition

Let $(\prec, X_0, X_1, X_2)$ be a triangular complex.
Then we define an *embedding* of $(\prec, X_0, X_1, X_2)$ as a map

$$c : X_0 \to \mathbb{R}^3$$

The image of $v \in X_0$ is called *coordinate of v*.

Example

Embedded triangular complex

Definition

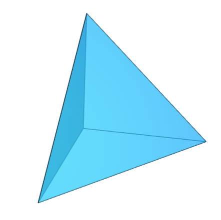
Let $(\prec, X_0, X_1, X_2)$ be a triangular complex.
Then we define an *embedding* of $(\prec, X_0, X_1, X_2)$ as a map

$$c : X_0 \to \mathbb{R}^3$$

The image of $v \in X_0$ is called *coordinate of v*.

Example

Embedded triangular complex



Embedded simplicial surface

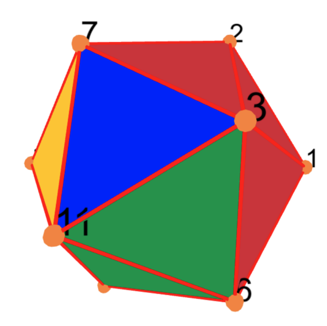
Simplicial Surfaces Package

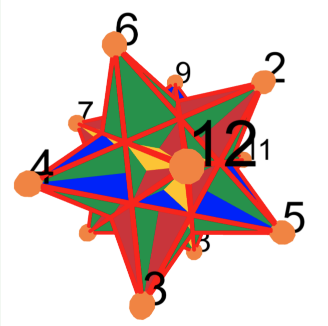
- Has functionality for displaying simplicial surfaces
    - Generates a .html file
    - Uses three.js

Simplicial Surfaces Package

- Has functionality for displaying simplicial surfaces
  - Generates a .html file
  - Uses three.js

Example (Number 2.1 and 2.2 from [1])





Fachpraktikum

Was a project with the goal to improve the visualizations by adding shading/local lighting.

Fachpraktikum

Was a project with the goal to improve the visualizations by adding shading/local lighting.
After some work it turns out: central class used in implementation is deprecated.

THREE.Geometry will be removed from core with r125

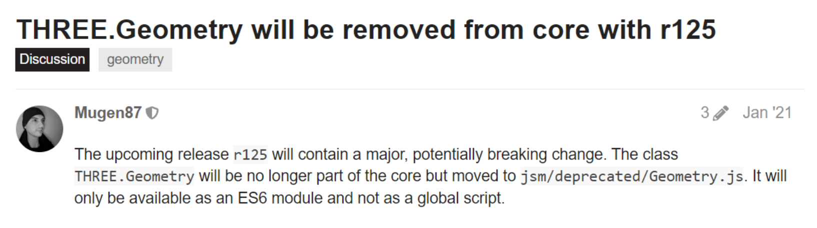
Discussion geometry

Mugen87 3 Jan '21

The upcoming release `r125` will contain a major, potentially breaking change. The class `THREE.Geometry` will be no longer part of the core but moved to `jsm/deprecated/Geometry.js`. It will only be available as an ES6 module and not as a global script.

Fachpraktikum

Was a project with the goal to improve the visualizations by adding shading/local lighting.
After some work it turns out: central class used in implementation is deprecated.



THREE.Geometry will be removed from core with r125

Discussion geometry

Mugen87 3 Jan '21

The upcoming release `r125` will contain a major, potentially breaking change. The class `THREE.Geometry` will be no longer part of the core but moved to `jsm/deprecated/Geometry.js`. It will only be available as an ES6 module and not as a global script.

→Decided to rewrite whole functionality

Advancements after rewrite

- New security requirements of JavaScript and modern browsers: need to load the code from some server $\rightarrow$ way smaller file sizes (for small examples 9kB vs. 539kB)

Advancements after rewrite

- New security requirements of JavaScript and modern browsers: need to load the code from some server $\rightarrow$ way smaller file sizes (for small examples 9kB vs. 539kB)
- More efficient Animations, faster loading, lower memory usage

Advancements after rewrite

- New security requirements of JavaScript and modern browsers: need to load the code from some server $\rightarrow$ way smaller file sizes (for small examples 9kB vs. 539kB)
- More efficient Animations, faster loading, lower memory usage
- Also works for triangular complexes
  $\rightarrow$Does not depend on umbrella condition for visualization

Afterwards decided to roll this feature into new package:

GAP **i**mage **c**reator

Afterwards decided to roll this feature into new package:

GAP **i**mage **c**reator

Goal is to divide up working with triangular complexes/simplicial surfaces in `SimplicialSurfaces` and to visualize them in `GAPic`.

New features since creating `GAPic`

- Most settings available via GUI in real time

New features since creating `GAPic`

- Most settings available via GUI in real time
- `normalsMaterial` for visualizing angles of faces
  → maps colors of a face smoothly depending on the normal

New features since creating `GAPic`

- Most settings available via GUI in real time
- `normalsMaterial` for visualizing angles of faces
  $\rightarrow$ maps colors of a face smoothly depending on the normal
- Intersection planes
  $\rightarrow$ allows seeing inside complexes

New features since creating `GAPic`

- Most settings available via GUI in real time
- `normalsMaterial` for visualizing angles of faces
  $\rightarrow$ maps colors of a face smoothly depending on the normal
- Intersection planes
  $\rightarrow$ allows seeing inside complexes
- Parameterized coordinates
  $\rightarrow$ allows coordinates to be defined as any equation JavaScript can evaluate

Future plans

- Currently in beta
  $\rightarrow$ need to finish manual

Future plans

- Currently in beta
  $\rightarrow$ need to finish manual
- Partial rewrite with new data structure
  $\rightarrow$ will reduce storage especially for big animations
- Add library of often used coordinates
  $\rightarrow$ e.g. icosahedra of length 1[1]

Future plans

- Currently in beta
  $\rightarrow$ need to finish manual
- Partial rewrite with new data structure
  $\rightarrow$ will reduce storage especially for big animations
- Add library of often used coordinates
  $\rightarrow$ e.g. icosahedra of length 1[1]
- Make compatible with objects from other packages
  $\rightarrow$ e.g. `simpcomp` package or

Future plans

- Currently in beta
  $\rightarrow$ need to finish manual
- Partial rewrite with new data structure
  $\rightarrow$ will reduce storage especially for big animations
- Add library of often used coordinates
  $\rightarrow$ e.g. icosahedra of length 1[1]
- Make compatible with objects from other packages
  $\rightarrow$ e.g. `simpcomp` package or **your package?**

Time for Demonstrations

Time for Demonstrations

- Triangular complex
- Improved performance
- Normals material
- Intersection planes
- Parameterized coordinates

Thank you for your attention

Want to get involved?
→ `github.com/GAP-ART-RWTH/GAPic`

References:

[1] Karl-Heinz Brakhage et al. *The icosahedra of edge length 1.* 2019. DOI: 10.48550/ARXIV.1903.08278. URL: https://arxiv.org/abs/1903.08278.