The GAPic Package

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GAPDays Spring 2024

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The elements in X_0 are called *vertices*, the elements in X_1 are called *edges* and the elements in X_2 are called *faces*.

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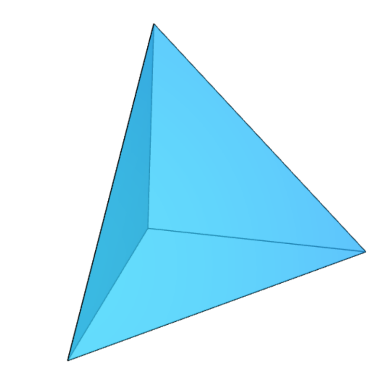
the last condition is called the *umbrella condition*.

Example



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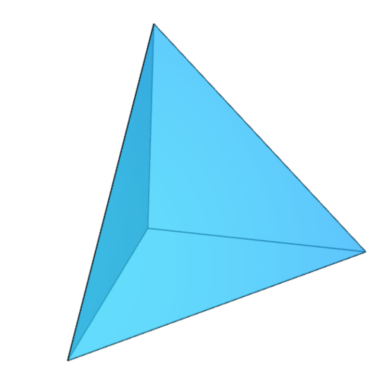

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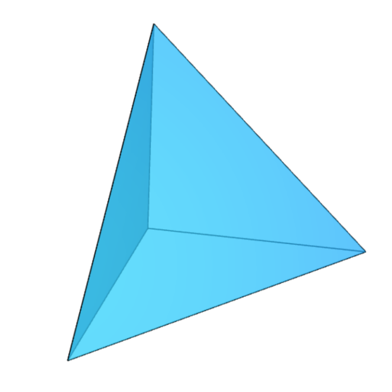

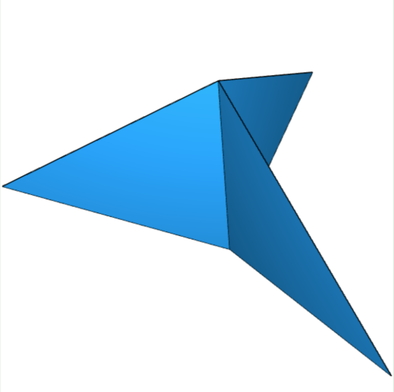

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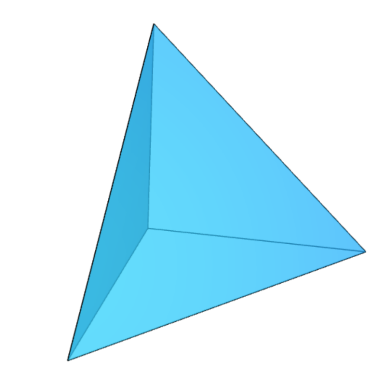

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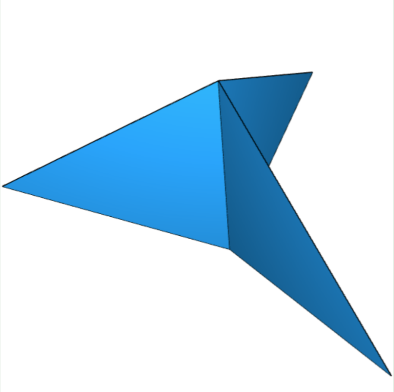

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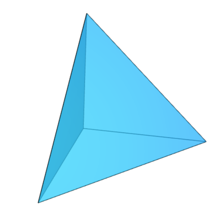

Embedded simplicial surface

Simplicial Surfaces Package

- Has functionality for displaying simplicial surfaces
  - Generates a .html file
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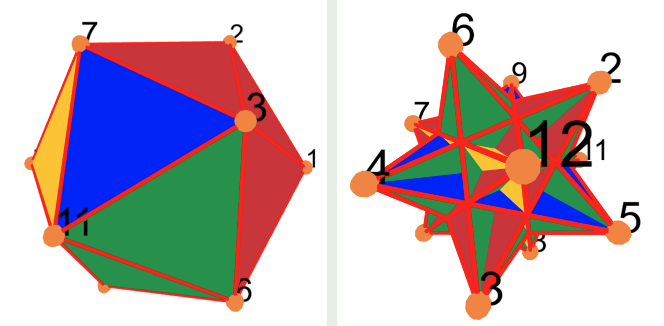
Example (Number 2.1 and 2.2 from [1])



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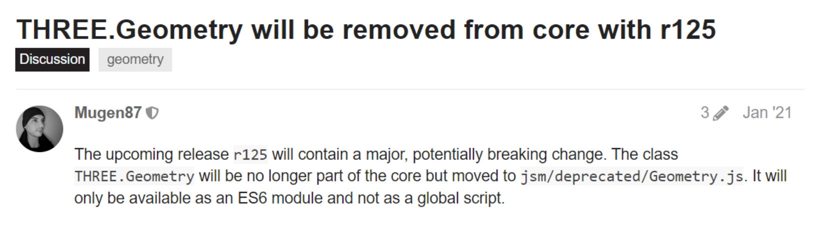

THREE.Geometry will be removed from core with r125

Discussion geometry

Mugen87 3 Jan '21

The upcoming release `r125` will contain a major, potentially breaking change. The class `THREE.Geometry` will be no longer part of the core but moved to `jsm/deprecated/Geometry.js`. It will only be available as an ES6 module and not as a global script.

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$\rightarrow$Decided to rewrite whole functionality

Advancements after rewrite

- New security requirements of JavaScript and modern browsers: need to load the code from some server $\rightarrow$ way smaller file sizes (for small examples 9kB vs. 539kB)

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- More efficient Animations, faster loading, less memory (Demo in Browser)
- Also works for triangular complexes
  $\rightarrow$Does not depend on incidence structure for visualization (Demo in Browser)

Afterwards decided to roll this feature into new package:

GAP **i**mage **c**reator

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Goal is to divide up working with triangular complexes/simplicial surfaces in `SimplicialSurfaces` and to visualize them in `GAPic`.

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- Intersection planes
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- Parameterized coordinates
  $\rightarrow$ allows coordinates to be defined as any equation JavaScript can evaluate

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Thank you for your attention

Want to get involved? → `github.com/GAP-ART-RWTH/GAPic`

References:

[1] Karl-Heinz Brakhage et al. *The icosahedra of edge length 1*. 2019. DOI: 10.48550/ARXIV.1903.08278. URL: https://arxiv.org/abs/1903.08278.