Mechanical Comparison of Arrangement Strategies for Topological Interlocking Assemblies

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Topological Interlocking

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Here:

- planar topological interlocking assemblies, i.e. between two parallel planes in 3D-space,
- use perimeter as the frame,
- only copies of the same block differently arranged.

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$$\begin{aligned} B_1 := \{&\{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_1, v_5\}, \{v_1, v_9\}, \{v_2, v_3\}, \{v_2, v_5\}, \\ &\{v_2, v_6\}, \{v_2, v_7\}, \{v_3, v_4\}, \{v_3, v_7\}, \{v_4, v_7\}, \{v_4, v_8\}, \{v_4, v_9\}, \\ &\{v_5, v_6\}, \{v_5, v_7\}, \{v_5, v_9\}, \{v_6, v_7\}, \{v_7, v_8\}, \{v_7, v_9\}, \{v_8, v_9\}\}, \end{aligned}$$

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and triangular faces

$$\begin{aligned} B_2 := \{ & \{v_1, v_2, v_3\}, \{v_1, v_2, v_5\}, \{v_1, v_3, v_4\}, \{v_1, v_4, v_9\}, \{v_1, v_5, v_9\}, \\ & \{v_2, v_3, v_7\}, \{v_2, v_5, v_6\}, \{v_2, v_6, v_7\}, \{v_3, v_4, v_7\}, \{v_4, v_7, v_8\}, \\ & \{v_4, v_8, v_9\}, \{v_5, v_6, v_7\}, \{v_5, v_7, v_9\}, \{v_7, v_8, v_9\}\}. \end{aligned}$$

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The Versatile Block is a polyhedron embedded in $\mathbb{R}^3$, given by B_0, B_1, B_2 together with coordinates

$$v_1 = (0, 0, 0), v_2 = (1, 1, 0), v_3 = (2, 0, 0),$$
$$v_4 = (1, -1, 0), v_5 = (0, 1, 1), v_6 = (1, 1, 1),$$
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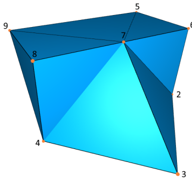

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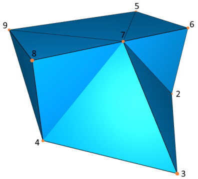

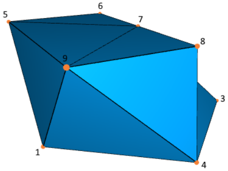

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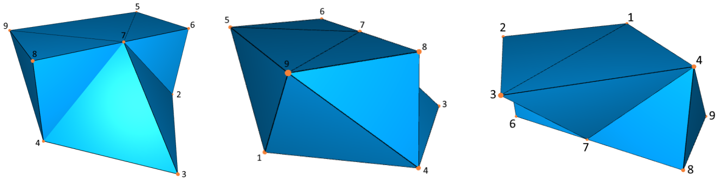

Isometries

Definition

Let $n \in \mathbb{N}$, V a euclidean K vectorspace (with metric $d : V \times V \to K$). Then $\varphi : V \to V$ is called an *isometry* if

$$\forall x, y \in V : d(x, y) = d(\varphi(x), \varphi(y)).$$

Note: An isometry is always a combination of an affine transformation and an orthogonal matrix. Such an isometry $\varphi = (A, a)$ operates as

$$(A, a)(x) := A \cdot x + a.$$

The isometries of a given euclidean vectorspace are a group with the operation

$$((A, a) \circ (B, b))(x) := (A \cdot B, A \cdot a + b)$$

denoted by $E(n)$.

Wallpaper Groups

Definition

Crystallograhpic/Wallpaper Groups Let $\Gamma \leq E(n)$ a subgroup of the group of isometries of dimension n.
Then Γ is called a crystallographic group if Γ is cocompact and discrete.

Planar Assemblies of the Versatile Block

Finite Element Method

Problem formulation

Simulation Setup

Stresses

Combinatorial Method

Combinatorial Results