Mechanical Comparison of Arrangement Strategies for Topological Interlocking Assemblies

Lukas Schnelle
This is joint work with Tom Goertzen, Domen Macek, Meike Weiß, Stefanie Reese, Hagen Holthusen and Alice C. Niemeyer

Based on [2]

Dec 2023

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- planar topological interlocking assemblies, i.e. between two parallel planes in 3D-space,
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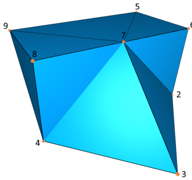

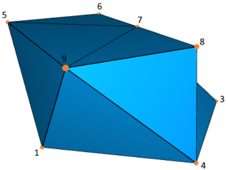

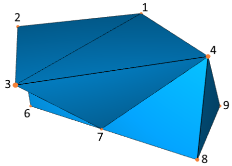

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The isometries of a given euclidean vectorspace are a group with the concatenation

$$((A, a) \circ (B, b))(x) := (A \cdot B, A \cdot a + b)$$

denoted by $E(n)$ ($\implies E(n) = O(n) \ltimes \mathbb{R}^n$).

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Let $\Gamma \leq E(n)$.
Then Γ is cocompact if the space $E(n)/\Gamma$ is compact.

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The translation lattice is always a fundamental domain.

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Theorem (3. Bieberbach) [3, Satz 1.6]

An isomorphism between two crystallographic groups in $E(n)$ is induced by conjugation with an affine map.

Examples of wallpaper groups

$p1 :=$

$$\left\langle \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right), \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right) \right\rangle$$

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$$p4 :=$$

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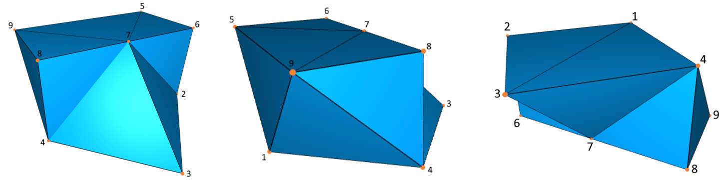
crystallographic groups on the Versatile Block



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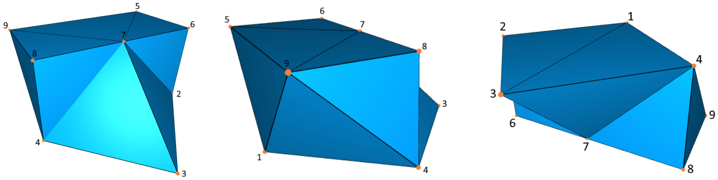

Notice: Versatile Block has only vertices with $z = 0$ and $z = 1$.
Now consider the an isometry φ from before lifted to $\mathbb{R}^3$ by

$$\hat{\varphi} : \mathbb{R}^3 \to \mathbb{R}^3, \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} \varphi \begin{pmatrix} x \\ y \end{pmatrix} \\ z \end{pmatrix}$$

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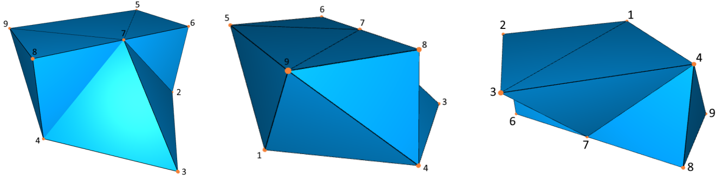

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Applied to all coordinates of the vertices of the Versatile Block we get a Versatile Block between the same planes.

Planar Assemblies of the Versatile Block

With the groups $p1$, pg and $p4$ from before we get three interlocking assemblies consisting only of copies of the Versatile Block:

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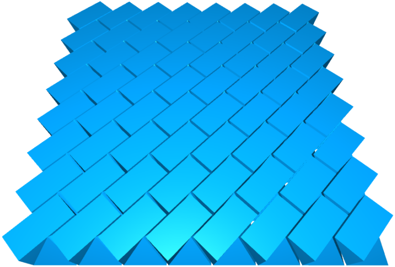

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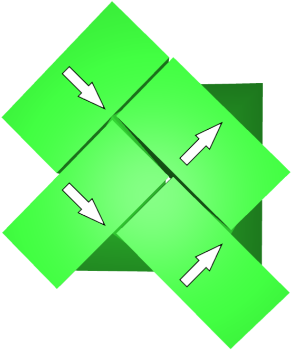

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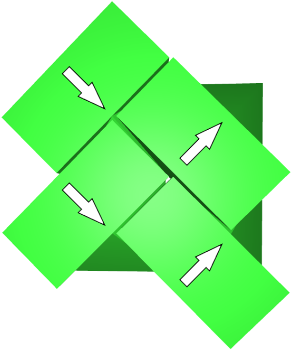

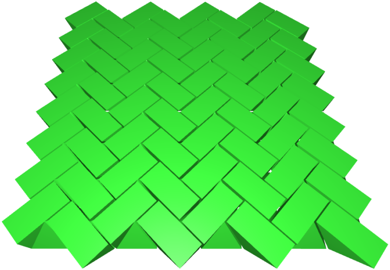

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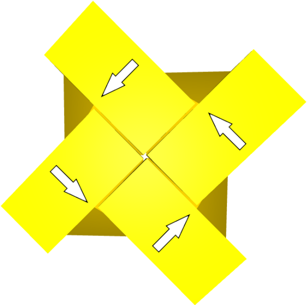

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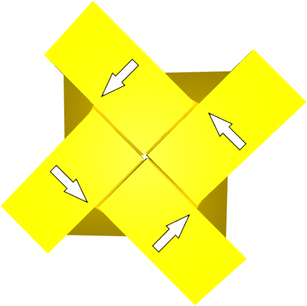

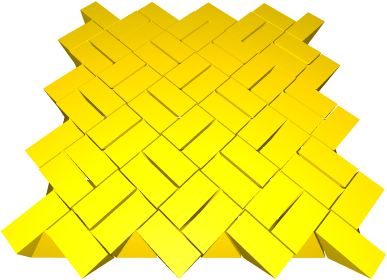

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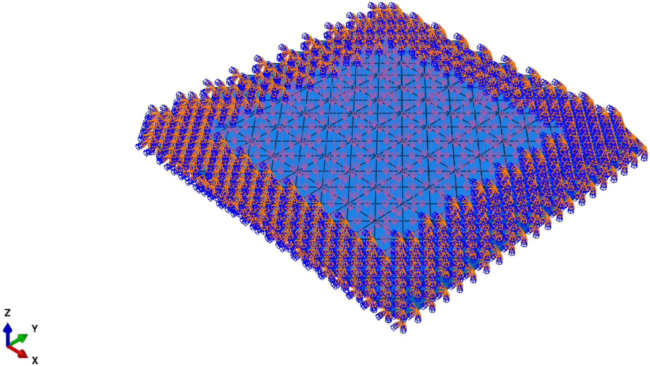

Finite Element Method

How to answer such a question?

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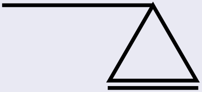

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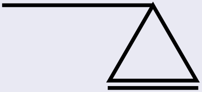

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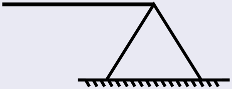

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   Solve remaining unknowns with principle of minimum potential. E.g. priciple of virtual work

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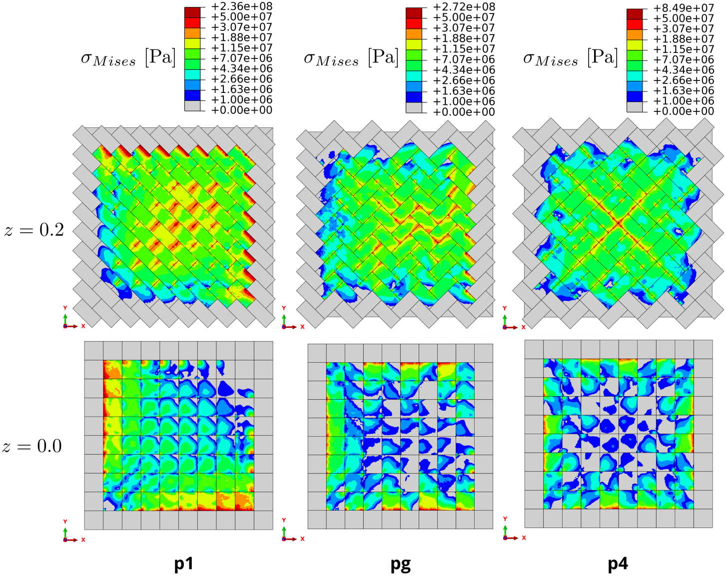
Run simulation

Stresses



σ_{Mises} [Pa]
+2.36e+08
+5.00e+07
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+4.34e+06
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+5.00e+07
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+1.88e+07
+1.15e+07
+7.07e+06
+4.34e+06
+2.66e+06
+1.63e+06
+1.00e+06
+0.00e+00
σ_{Mises} [Pa]
+8.49e+07
+5.00e+07
+3.07e+07
+1.88e+07
+1.15e+07
+7.07e+06
+4.34e+06
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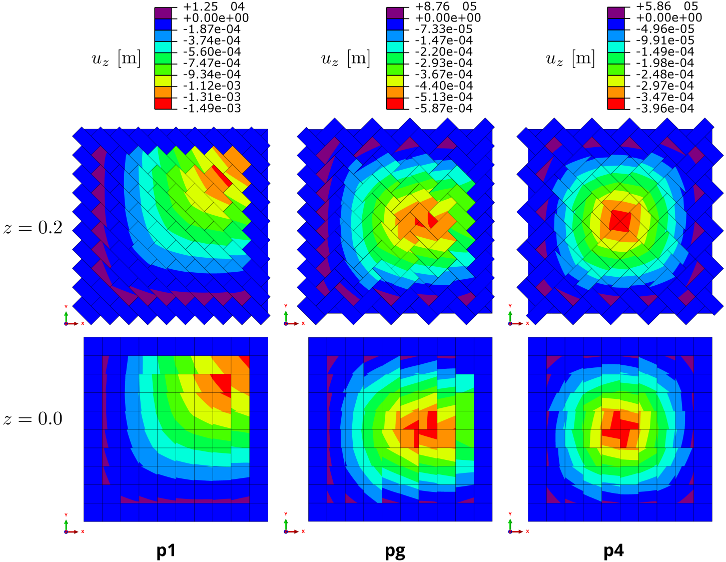
+1.63e+06
+1.00e+06
+0.00e+00
$z = 0.2$
$z = 0.0$
Y
X
p1
pg
p4

Deformations



Combinatorial Method

Definition

Let $A = \{X_i \mid i \in I\}$, a topological interlocking assembly consisting of blocks $X_i \subset \mathbb{R}^3$ indexed by a finite index set I with a frame indexed by $J \subset I$, the core indexed by $C := I \setminus J$ and $d \in \mathbb{R}^3$ a vector.

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- arcs $i \to j$ if X_i is restrained by X_j in direction d for $i, j \in I$.

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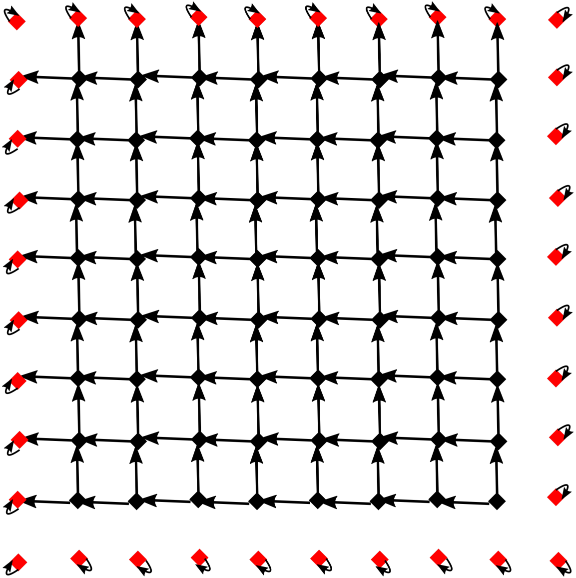
For k high enough $A^k \cdot x$ yields the amount of load transferred into the frame.

Combinatorial Results



Combinatorial Results

0	6.43	5.93	5.32	4.61	3.81	2.92	1.98	1.00	0
6.43	0	0	0	0	0	0	0	0	0
5.93	0	0	0	0	0	0	0	0	0
5.32	0	0	0	0	0	0	0	0	0
4.61	0	0	0	0	0	0	0	0	0
3.81	0	0	0	0	0	0	0	0	0
2.92	0	0	0	0	0	0	0	0	0
1.98	0	0	0	0	0	0	0	0	0
1.00	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

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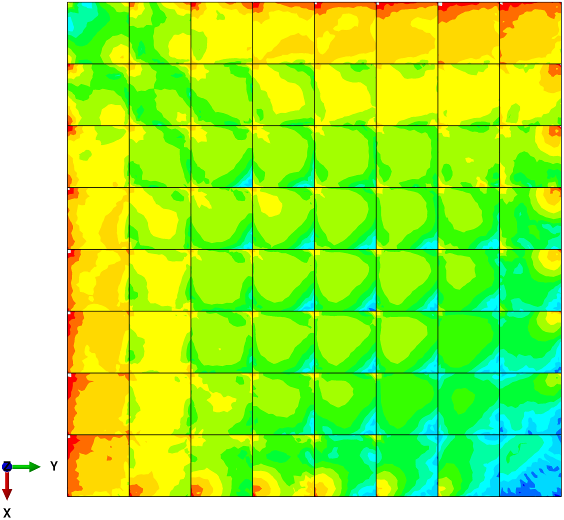

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Thank you for your attention

[1] Reymond Akpanya et al. "Topological Interlocking, Truchet Tiles and Self-Assemblies: A Construction-Kit for Civil Engineering Design". In: *Proceedings of Bridges 2023: Mathematics, Art, Music, Architecture, Culture*. Ed. by Judy Holdener et al. Phoenix, Arizona: Tessellations Publishing, 2023, pp. 61–68.

[2] Tom Goertzen et al. "Mechanical Comparison of Arrangement Strategies for Topological Interlocking Assemblies". In: *arXiv e-prints*, arXiv:2312.01958 (Dec. 2023), arXiv:2312.01958. DOI: 10.48550/arXiv.2312.01958. arXiv: 2312.01958 [cs.CE].

[3] Wilhelm Plesken. *Kristallographische Gruppen*. SS 2014.