Project 11 - Power Series
Jacob Schnoor

Problem 1

In[44]:=
```
Series[E^(I*θ), {θ, 0, 10}]
Series[Cos[θ], {θ, 0, 10}] + Series[I*Sin[θ], {θ, 0, 10}]
```

Out[44]= $1 + i\,\theta - \frac{\theta^2}{2} - \frac{i\,\theta^3}{6} + \frac{\theta^4}{24} + \frac{i\,\theta^5}{120} - \frac{\theta^6}{720} - \frac{i\,\theta^7}{5040} + \frac{\theta^8}{40\,320} + \frac{i\,\theta^9}{362\,880} - \frac{\theta^{10}}{3\,628\,800} + O[\theta]^{11}$

Out[45]= $1 + i\,\theta - \frac{\theta^2}{2} - \frac{i\,\theta^3}{6} + \frac{\theta^4}{24} + \frac{i\,\theta^5}{120} - \frac{\theta^6}{720} - \frac{i\,\theta^7}{5040} + \frac{\theta^8}{40\,320} + \frac{i\,\theta^9}{362\,880} - \frac{\theta^{10}}{3\,628\,800} + O[\theta]^{11}$

Problem 2

In[63]:=
```
Series[Log[x], {x, 1, 4}]
```

Out[63]= $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + O[x-1]^5$

Ln(0) has no solution meaning a power series cannot accurately estimate the result at x=0

Problem 3

In[60]:=
```
logXatTwo = Normal[Series[Log[x], {x, -1, 10}]] /. {x → -1}
Log[-1]
```

Out[60]= $i\,\pi$

Out[61]= $i\,\pi$

The answer is correct because if ln(y)=x, then e^x=y. According to the power series from problem 1, $e^{i\theta}$=Cos[θ]+iSin[θ]. Plugging in π for θ results in $e^{i\pi}$=Cos[π]+iSin[π]= -1. Therefore ln(-1)=$i\pi$

Problem 4

In[65]:=
```
Normal[Series[Log[x], {x, 1, 4}]]
Normal[Series[Log[x], {x, 1, 4}]] /. {x → -1}
```

Out[65]= $-1 - \frac{1}{2}(-1+x)^2 + \frac{1}{3}(-1+x)^3 - \frac{1}{4}(-1+x)^4 + x$

Out[66]= $-\frac{32}{3}$

The approximation for ln(-1) centered around x=1 is different than the previous approximation because power series have only a limited range of accuracy. As a point gets further from the center, the power series becomes less accurate. Putting the center at x=1 and evaluating ln(-1) gives -32/3 rather than $i\pi$ because the polynomial above can only output real answers.

Problem 5

In[190]:=
```
myMat = {{0.2 x + 1.7}, {-3 x + 9}, {3 x - 3}, {2 - Sqrt[1 - (x - 2)^2]}, {2 + Sqrt[1 - (x - 2)^2]}};
Plot[myMat, {x, 0, 6}, PlotRange → {{0, 4}, {0.5, 3.5}},
 PlotStyle → {{Thickness[0.03], Red}}]
```

Out[191]=