Structural Equation Models for Between, Within, and Mixed Factorial Designs

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Outline

- SEM factorial designs
- Contrast codes
- Measurement invariance simulation
- Real data examples

Factorial designs

- Used with two or more categorical independent variables
- Usually used with a fully crossed design
  - Each level of 1 independent variable is combined with each level of the other independent variable
- In this talk I will focus on factorial designs with 2 fully crossed independent variables
  - All the methods can be applied to designs with 3+ independent variables or partially crossed designs

Factorial designs

- Independent variables can be either between or within subject variables
- Between subjects: observations in different groups are independent
  - e.g., treatment/control, racial groups
- Within subjects: there is dependence in observations across groups
  - e.g., repeated measures, twin studies
- Designs can be any combination of between, and within subjects factors
  - i.e., between, within, or mixed factorial designs

Factorial designs

- Traditionally analyzed using ANOVA
  - Or regression/multilevel modeling
- One-way between or within subjects designs have been extended to SEM models

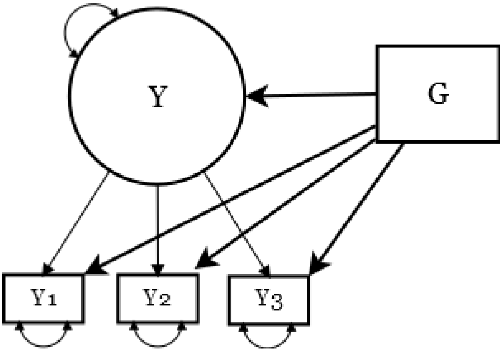
SEM for between subjects designs

- Two approaches to SEM for between subjects design
  - MIMIC models
  - Multiple groups

MIMIC model

- Use dummy coded variables to represent groups
  - Dummy coded variables predict both indicators and latent variables
  - Only can compare means/intercepts across groups

MIMIC model



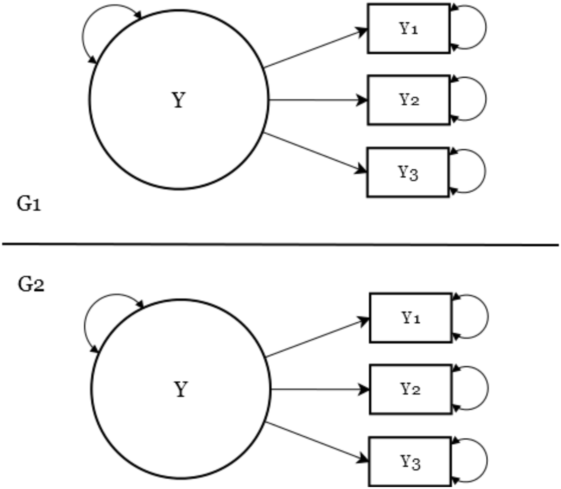
Multiple group model

- Modeling technique where the same models are simultaneously fit to different groups of participants
- Model parameters can be estimated separately in each group or constrained to equality across groups
  - Testing parameters is accomplished by comparing models with a parameter freely estimate or constrained to equality across groups

Multiple Group Modeling

- *Any* parameter in a model can be compared across groups
  - Factor loadings/item intercepts (factorial invariance)
  - Means (equivalent to t-test/ANOVA)
  - Variances (equivalent to F/Levene's test)
- Multiple group modeling provides several advantages over traditional methods
  - Reduced assumptions
  - Improved performance with non-normal data (Fan & Hancock 2012)
  - Easy missing data handling

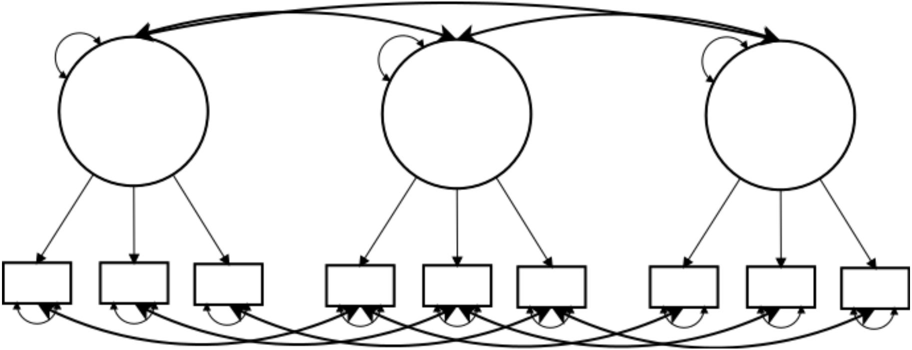
Multiple Group Modeling



SEM for within subjects designs

- Uses a "longitudinal" model
  - Wide format data
  - Each observation is a latent variable
  - Estimate covariances between latent variables

SEM for within subjects designs



SEM for within subjects designs

- How to compare parameters across conditions?
- Langenberg, Helm, & Mayer (2020): growth curve approach
  - Estimate latent variables representing contrasts between conditions
  - Factor loadings for latent variables are functions of specified contrasts
  - Estimates interindividual variability in contrasts
  - Can also be used for estimating main effects and interactions in factorial designs
  - Limited to comparisons of means

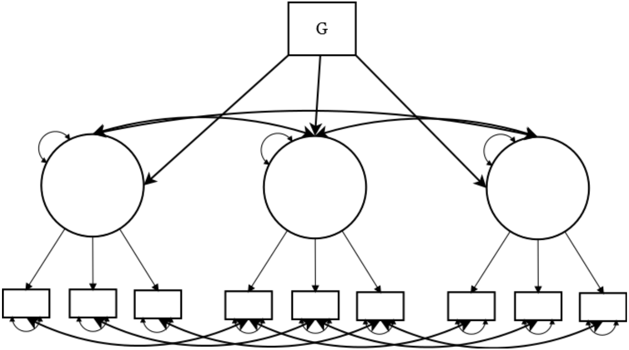
SEM for within subjects designs

- How to compare parameters across conditions?
  - Nested model tests
  - Can compare any parameter across conditions
    - e.g., factor loadings, latent means, latent variance, covariances
  - Similar process to longitudinal invariance testing

SEM for factorial designs

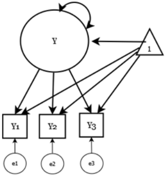
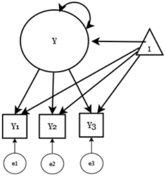
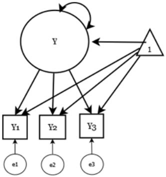
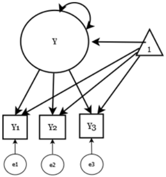
- Past research has focused on comparing means across factorial designs
  - Within subjects designs: Langenberg, Helm, & Mayer (2020) growth curve approach
  - Mixed or between subjects design: hybrid MIMIC model design

SEM for factorial designs



SEM for factorial designs

- Fit model with a group for each cell of the design (or each latent variables for within subjects):

	Z1	Z2
X1	Group 1: 	Group 2: 
X2	Group 3: 	Group 4: 

SEM for factorial designs

- Main effects: Constrain parameters to equality across levels of an IV (e.g. main effect of Z constrain latent means to equality across Groups 1 & 2 and 3 & 4)
- Interactions: Use orthogonal contrast codes
    - Similar to ANOVA, orthogonal interaction contrast codes can be used as model constraints on latent means (or any other parameter).
    - Each contrast adds one degree of freedom (df)

SEM for factorial designs

- Contrast Codes
- 2 x 2 Design

Factor X	X1		X2	
Factor Z	Z1	Z2	Z1	Z2
Int. Contrast 1	1	-1	-1	1

- 2 x 3 Design

Factor X	X1			X2		
Factor Z	Z1	Z2	Z3	Z1	Z2	Z3
Int. Contrast 1	1	0	-1	-1	0	1
Int. Contrast 2	1	-2	1	-1	2	1

SEM for factorial designs: Simulation

- Monte Carlo simulation investigating factorial invariance
- 2 x 2 between subjects design
- Single latent variable with six indicators
- n = 200 per group, overall n = 800
- 1000 replications per condition
- Three factor loadings differed across groups
  - Default is all factor loadings of 0.7
- All analyses used the *simsem* package (v.05-16)
  - Model fitting via *lavaan* (v. 0.6-13)

SEM for factorial designs: Simulation

- Factor loading conditions:
- No main effects or interaction
  - Loadings: 0.7, 0.7, 0.7, 0.7
- Main effects and interaction
  - Loadings: 0.5, 0.7, 0.7, 0.7
- No main effects but an interaction
  - Loadings: 0.6, 0.7, 0.7, 0.6

SEM for factorial designs: Simulation

- Fit three models:
  1. Configural invariance model
  2. Model with factor loadings constrained to equality across all groups (weak invariance)
  3. Model with interaction constraints fixed to 0 (interaction invariance)
- Compared models 1 & 2 and 1 & 3 for tests of invariance
  - Focus on Type I error rate/power

SEM for factorial designs: Simulation

- No main effects or interactions (Type I error rate)
  - Weak invariance model: 0.039
  - Interaction invariance model: 0.049
- Main effects and interaction
  - Weak invariance model: 0.76
  - Interaction invariance model: 0.42
- No main effects but interaction
  - Weak invariance model: 0.26
  - Interaction invariance model: 0.43

Example: 2 x 3 Mixed Factorial Design

- Example from Kilgus, Riley-Tillman, Stichter, Schoemann, & Bellesheim (2016)
- 60 students identified as possessing social competence deficits
  - Ages 11-14
  - 55 male students, 5 female students
  - 43.33% IEP for Autism, 25% IEP for Emotional Disturbance, 20% IEP for Other Health Impairment
  - IQ: $M = 95.75$ ($SD = 14.92$)
  - 71.66% identified as white, 15% declined to report race/ethnicity
- Students randomly assigned to receive a Social Competency Intervention for Adolescents (SCI-A) or business as usual control (BAU)

Example: 2 x 3 Mixed Factorial Design

- Classroom teachers completed daily behavior ratings of social competence (DBR-SC) for each student across three intervention phases.
  - Pre-intervention (January)
  - Mid-intervention (February & March)
  - Post-intervention (April & May)
  - On average teachers completed 28.93 ratings in each phase
- Assessed five target behaviors
  - Disruptive Behavior (DB)
  - Respectful Behavior (RB)
  - Academic Engagement (AE)
  - Appropriate social interaction with teacher (AT)
  - Appropriate social interaction with peers (AP)

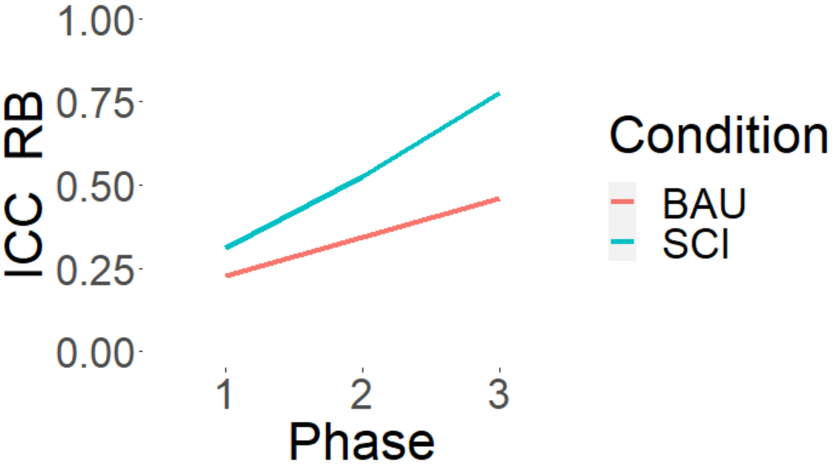
Example: 2 x 3 Mixed Factorial Design

- 2 (Intervention) X 3 (Phase) factorial design
  - Nested data: observations nested within students
  - Fit models using Multilevel SEM (MSEM) in MPlus
  - Intervention modeled as a multiple group variable
  - Phase modeled as a longitudinal variable
- Compare means and ICCs across condition and phase
  - ICC: ratio of between student variance to within student variance
  - Higher ICC: more variance in the outcome variable due to differences between students
  - Lower ICC: more variance in the outcome variable due to differences within students

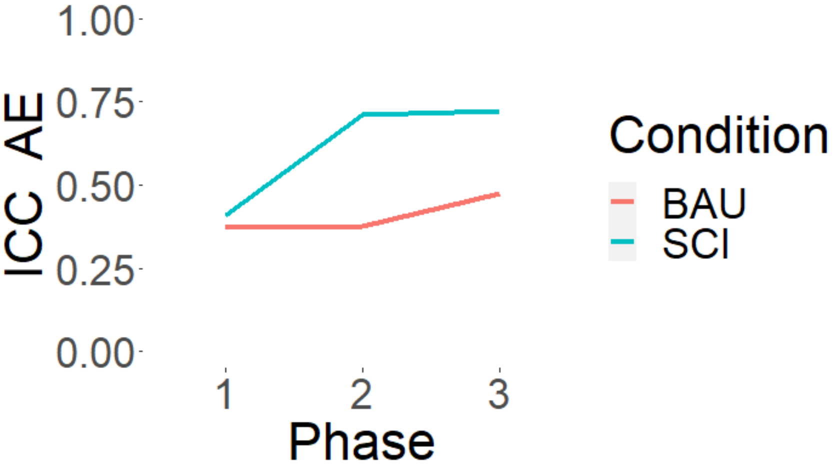
Example: 2 x 3 Mixed Factorial Design

- No differences in DBR means across conditions, time, or their interaction
- No differences in ICC across conditions
- ICC increased across phases
- Interaction between condition and phase for AE and RB measures

Example: 2 x 3 Mixed Factorial Design



Example: 2 x 3 Mixed Factorial Design



Example: 2 x 3 Mixed Factorial Design

- In the SCI condition ICCs increased across phases
- In the BAU condition ICCs did not increase across phases
- SCI results in students exhibiting more consistent RB and AE behaviors
    - Less day to day variability in RB and AE

Example: Correlations between affect and prejudice

- Create prejudice toward a novel group
  - Participants read about Gibraltar and Malta and rated each group. One was described positively and one negatively
- Prime participants with feelings of disgust or neutral primes
- Examine how correlations between specific affect and prejudice change over time
  - e.g., does the correlation between disgust and prejudice fade over time
- Thanks to Angela Bahns and Chris Crandall for sharing these data

Example: Correlations between affect and prejudice

- 2 x 2 x 2 Mixed Factorial design
  - Condition: Between subjects factor: disgust or neutral priming
  - Group: Within subjects factor, rating of group was described positively or negatively
  - Time: Within subjects factor: immediately following priming or one week later
- n = 290
  - Randomly assigned to condition, 145 in each group

Example: Correlations between affect and prejudice

- Estimate correlations between affect and prejudice at each time.
- Compute difference between T1 and T2 correlations
- Use contrast codes to test for main effects and interactions in the difference between correlations
  - No expected main effect for group
  - Expected main effect for condition
  - Expected interaction: Correlations decrease the most for the disgust prime/negative affect condition

Example: Correlations between affect and prejudice

- Correlations for disgust

	Disgust	Neutral
Negative	-0.32	-0.12
Positive	-0.12	-0.03

- Note: T1 correlations were $\sim$0.4 for all conditions but positive/neutral ($r = 0.12$)
- Interaction contrast is significant, $p = .032$

Summary

- Contrast codes provide a method of testing for main effects and interactions in factorial designs using SEM models
    - All the advantages of SEM models can be integrated into factorial designs
- Future directions
    - Simulation studies comparing methods of testing means in factorial designs

Thank you!

- Questions?
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