In [1]:
```
import matplotlib
from datascience import *
%matplotlib inline
import matplotlib.pyplot as plots
import numpy as np
plots.style.use('fivethirtyeight')
```

Lecture 26

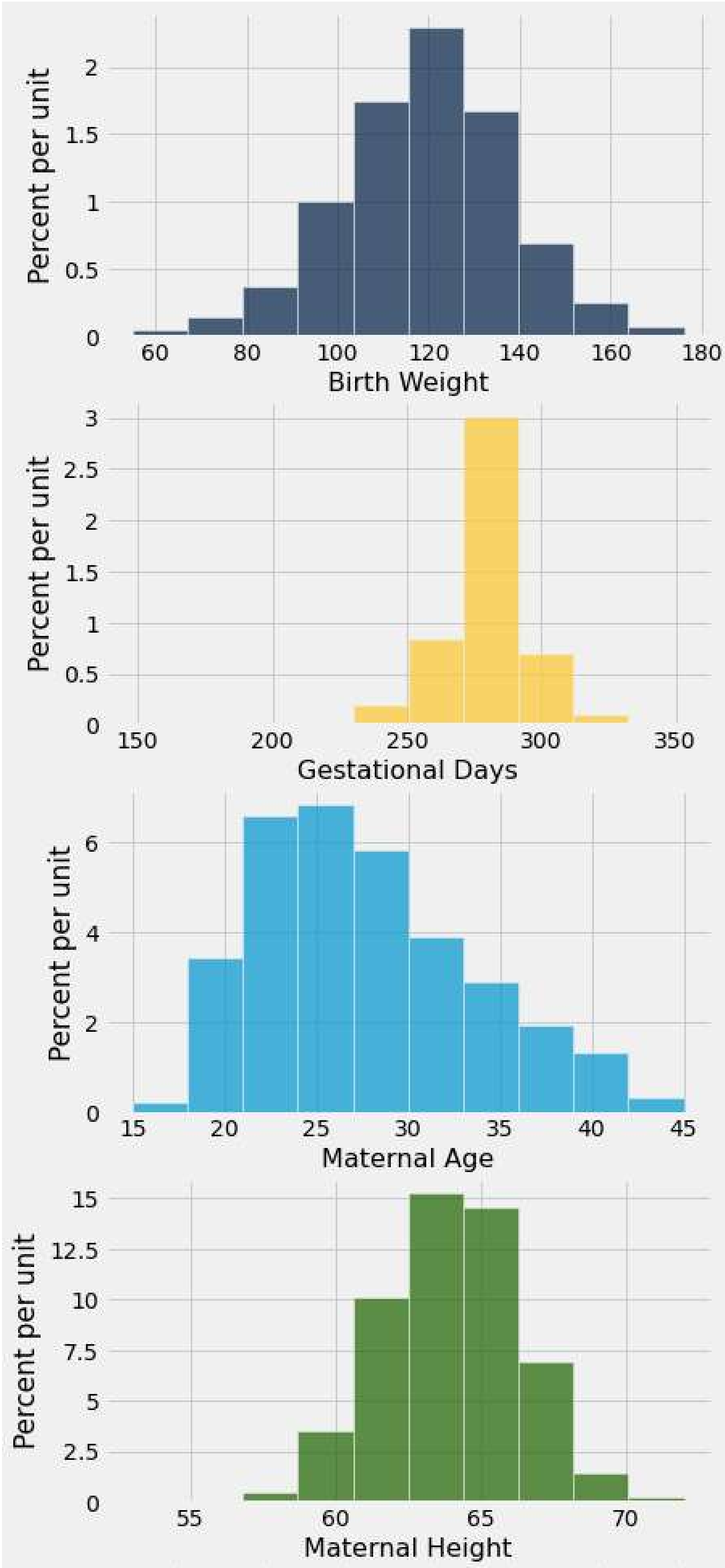
Chebyshev's Bounds

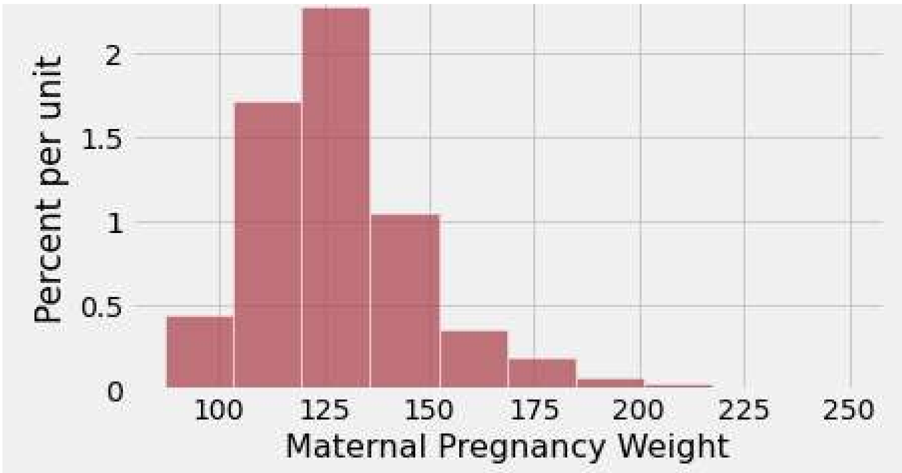
In [2]:
```
births = Table.read_table('baby.csv')
```

In [3]:
```
births.labels
```

Out[3]:
```
('Birth Weight',
 'Gestational Days',
 'Maternal Age',
 'Maternal Height',
 'Maternal Pregnancy Weight',
 'Maternal Smoker')
```

In [4]:
```
births.drop('Maternal Smoker').hist(overlay = False)
```


Percent per unit
Birth Weight
Percent per unit
Gestational Days
Percent per unit
Maternal Age
Percent per unit
Maternal Height



In [5]:
```
mpw = births.column('Maternal Pregnancy Weight')
mean = np.mean(mpw)
sd = np.std(mpw)
mean, sd
```

Out[5]:
```
(128.4787052810903, 20.725449704280411)
```

In [6]:
```
within_3_SDs = births.where('Maternal Pregnancy Weight', are.between(mean - 3*sd, mear
```

In [7]:
```
within_3_SDs.num_rows/births.num_rows
```

Out[7]:
```
0.9863713798977853
```

In [8]:
```
1 - 1/9
```

Out[8]:
```
0.8888888888888888
```

In [9]:
```
# See if Chebyshev's bounds work for different distributions

for k in births.labels:
    values = births.column(k)
    mean = np.mean(values)
    sd = np.std(values)
    print()
    print(k)
    for z in np.arange(2, 6):
        chosen = births.where(k, are.between(mean - z*sd, mean + z*sd))
        proportion = chosen.num_rows/births.num_rows
        percent = round(proportion * 100, 2)
        print('Mean plus or minus', z, 'SDs:', percent, '%')
```

```
Birth Weight
Mean plus or minus 2 SDs: 94.89 %
Mean plus or minus 3 SDs: 99.57 %
Mean plus or minus 4 SDs: 100.0 %
Mean plus or minus 5 SDs: 100.0 %

Gestational Days
Mean plus or minus 2 SDs: 93.78 %
Mean plus or minus 3 SDs: 98.64 %
Mean plus or minus 4 SDs: 99.57 %
Mean plus or minus 5 SDs: 99.83 %

Maternal Age
Mean plus or minus 2 SDs: 94.89 %
Mean plus or minus 3 SDs: 99.91 %
Mean plus or minus 4 SDs: 100.0 %
Mean plus or minus 5 SDs: 100.0 %

Maternal Height
Mean plus or minus 2 SDs: 97.19 %
Mean plus or minus 3 SDs: 99.66 %
Mean plus or minus 4 SDs: 99.91 %
Mean plus or minus 5 SDs: 100.0 %

Maternal Pregnancy Weight
Mean plus or minus 2 SDs: 95.06 %
Mean plus or minus 3 SDs: 98.64 %
Mean plus or minus 4 SDs: 99.49 %
Mean plus or minus 5 SDs: 99.91 %

Maternal Smoker
Mean plus or minus 2 SDs: 100.0 %
Mean plus or minus 3 SDs: 100.0 %
Mean plus or minus 4 SDs: 100.0 %
Mean plus or minus 5 SDs: 100.0 %
```

Standard Units

In [10]:
```
def standard_units(x):
    """Convert array x to standard units."""
    return (x - np.mean(x))/np.std(x)
```

In [11]:
```
ages = births.column('Maternal Age')
```

In [12]:
```
ages_standard_units = standard_units(ages)
```

In [13]:
```
np.mean(ages_standard_units), np.std(ages_standard_units)
```

Out[13]:
```
(-7.868020072300939e-17, 1.0)
```

In [14]:
```
both = Table().with_columns(
    'Age in Years', ages,
    'Age in Standard Units', ages_standard_units
)
both
```

Out[14]:

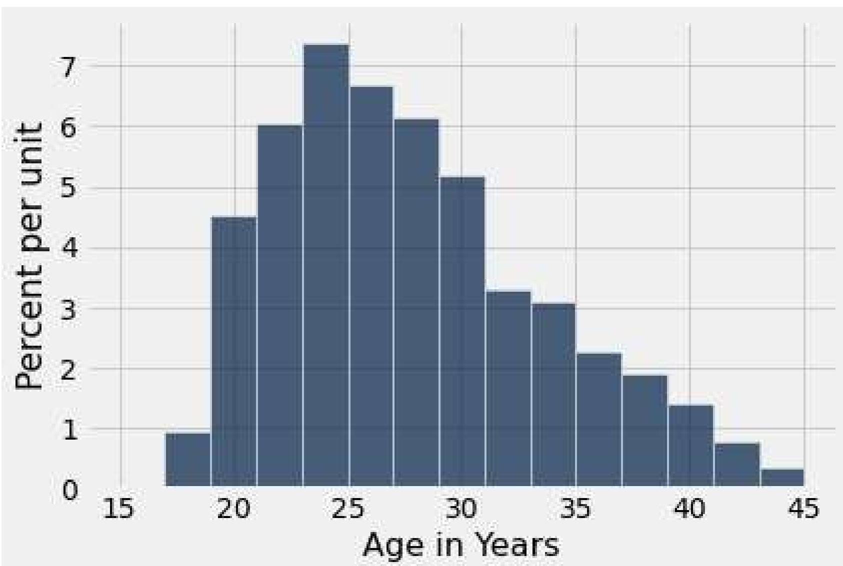
Age in Years	Age in Standard Units
27	-0.0392546
33	0.992496
28	0.132704
23	-0.727088
25	-0.383171
33	0.992496
23	-0.727088
25	-0.383171
30	0.476621
27	-0.0392546

... (1164 rows omitted)

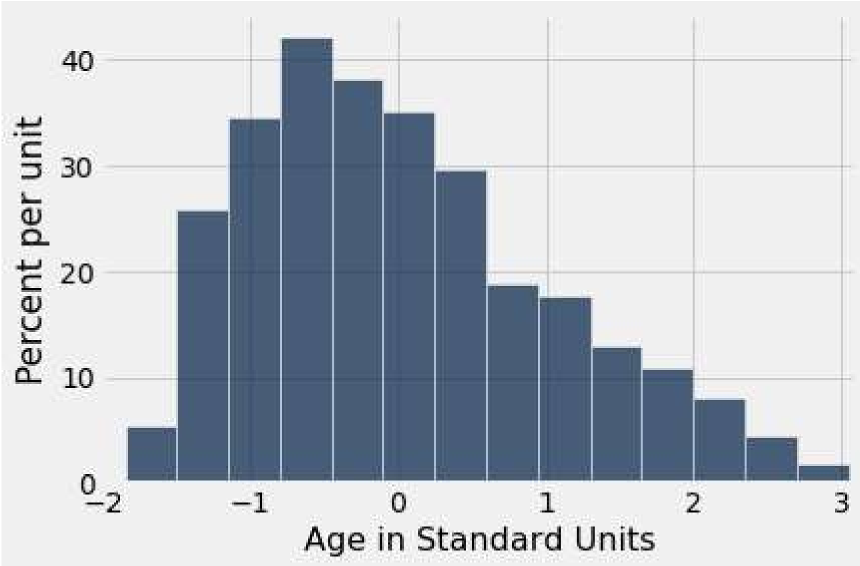
In [15]:
```
np.mean(ages), np.std(ages)
```

Out[15]:
```
(27.228279386712096, 5.8153604041908968)
```

In [16]:
```
both.hist('Age in Years', bins = np.arange(15, 46, 2))
```

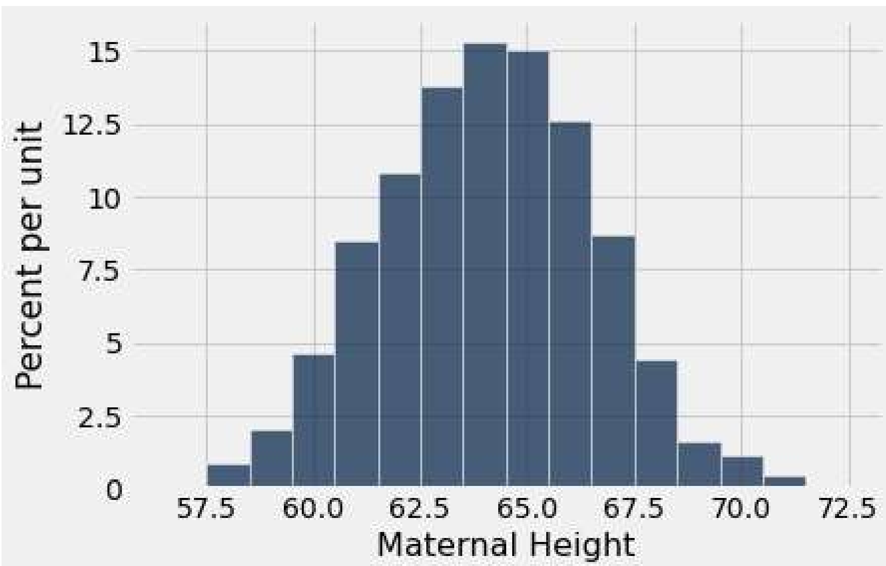


In [17]:
```
both.hist('Age in Standard Units', bins = np.arange(-2.2, 3.4, 0.35))
plots.xlim(-2, 3.1);
```



The SD and Bell Shaped Curves

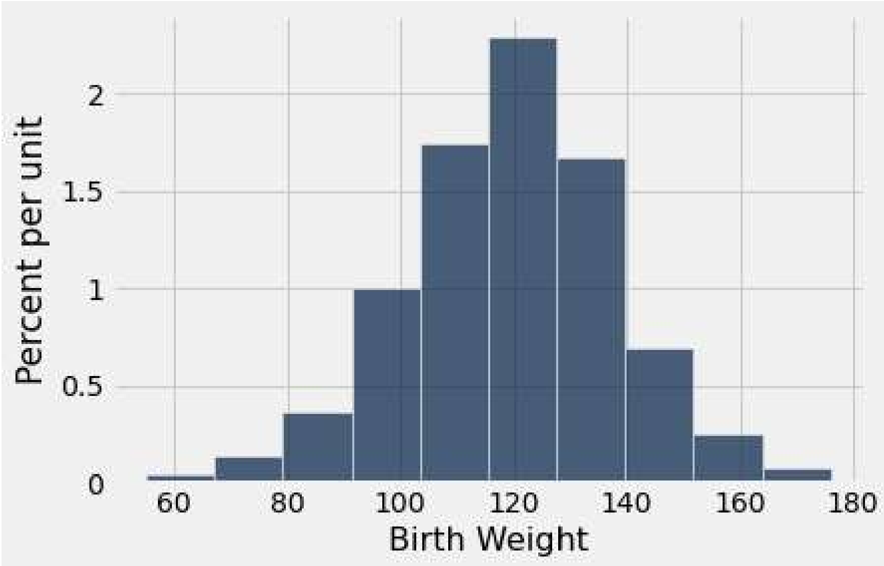
In [18]:
```
births.hist('Maternal Height', bins = np.arange(56.5, 72.6, 1))
```



In [19]:
```
heights = births.column('Maternal Height')
np.mean(heights), np.std(heights)
```

Out[19]:
```
(64.049403747870528, 2.5250254409674375)
```

In [20]:
```
births.hist('Birth Weight')
```



In [21]:
```
bw = births.column('Birth Weight')
mean_w = np.mean(bw)
sd_w = np.std(bw)
mean_w, sd_w
```

Out[21]:
```
(119.46252129471891, 18.320863702202779)
```

The Normal curve

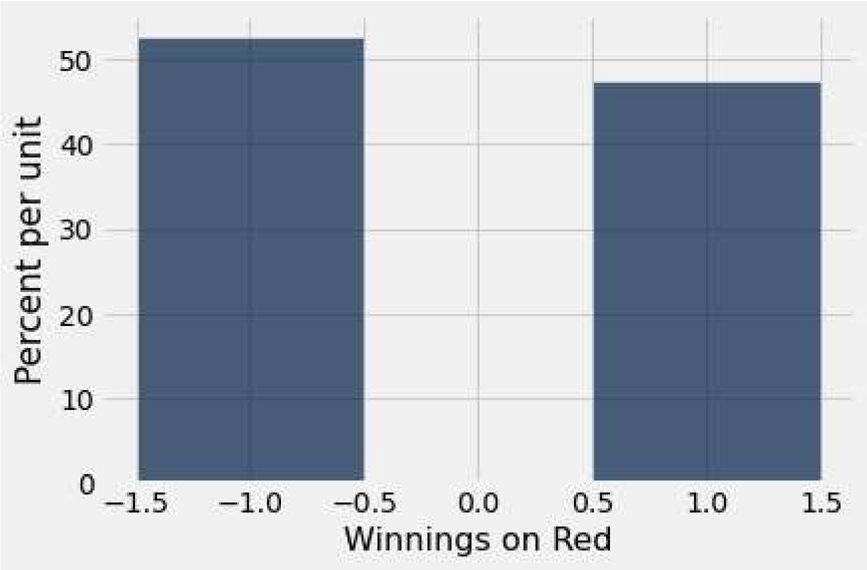
In [22]:
```
red_winnings = np.append(1*np.ones(18), -1*np.ones(20))
red = Table().with_columns('Winnings on Red', red_winnings)
```

In [23]:
```
red.show()
```

Winnings on Red
1
1
1
1
1
1
1
1
1
1
1
1
1
1
1
1
1
1
-1
-1
-1
-1
-1
-1
-1
-1
-1
-1
-1
-1
-1
-1
-1
-1

Winnings on Red
-1
-1
-1
-1

In [24]:
```
red.hist(bins = np.arange(-1.5, 1.6, 1))
```



In [25]:
```
18/38
```

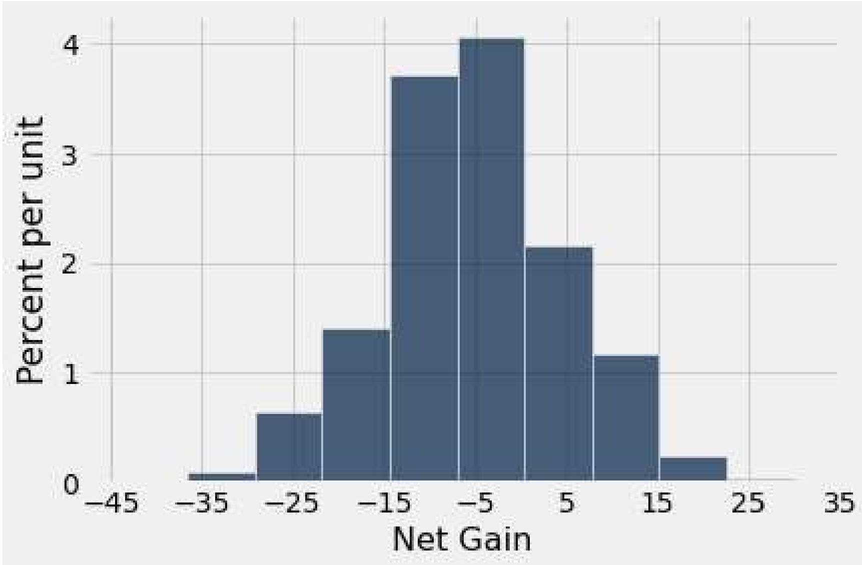
Out[25]:
```
0.47368421052631576
```

In [26]:
```
num_bets = 100

net_gains = make_array()

for i in np.arange(20000):
    spins = red.sample(num_bets)
    new_net_gain = sum(spins.column('Winnings on Red'))
    net_gains = np.append(net_gains, new_net_gain)
```

In [27]:
```
Table().with_columns('Net Gain', net_gains).hist()
plots.xticks(np.arange(-45, 36, 10));
```



In [28]:
```
np.average(net_gains)
```

Out[28]:
```
-5.2323000000000004
```

Central Limit Theorem and Simulating Sample Mean

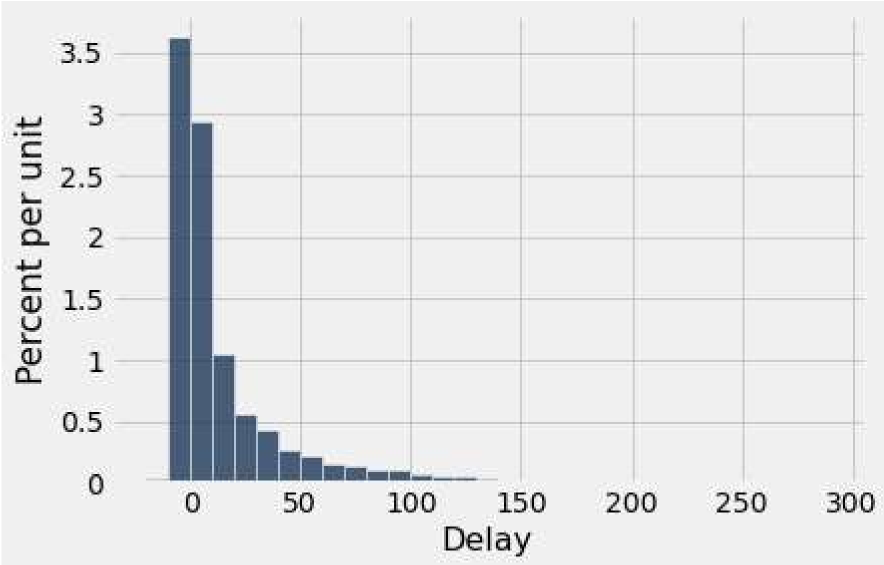
In [29]:
```
united = Table.read_table('united_summer2015.csv')
united
```

Out[29]:

Date	Flight Number	Destination	Delay
6/1/15	73	HNL	257
6/1/15	217	EWR	28
6/1/15	237	STL	-3
6/1/15	250	SAN	0
6/1/15	267	PHL	64
6/1/15	273	SEA	-6
6/1/15	278	SEA	-8
6/1/15	292	EWR	12
6/1/15	300	HNL	20
6/1/15	317	IND	-10

... (13815 rows omitted)

In [30]:
```
united.hist('Delay', bins = np.arange(-20, 300, 10))
```



In [31]:
```
delays = united.column('Delay')
mean_delay = np.mean(delays)
sd_delay = np.std(delays)

mean_delay, sd_delay
```

Out[31]:
```
(16.658155515370705, 39.480199851609314)
```

In [32]:
```
united = united.with_columns(
    'Delay in Standard Units', standard_units(delays)
)
united.sort('Delay', descending=True)
```

Out[32]:

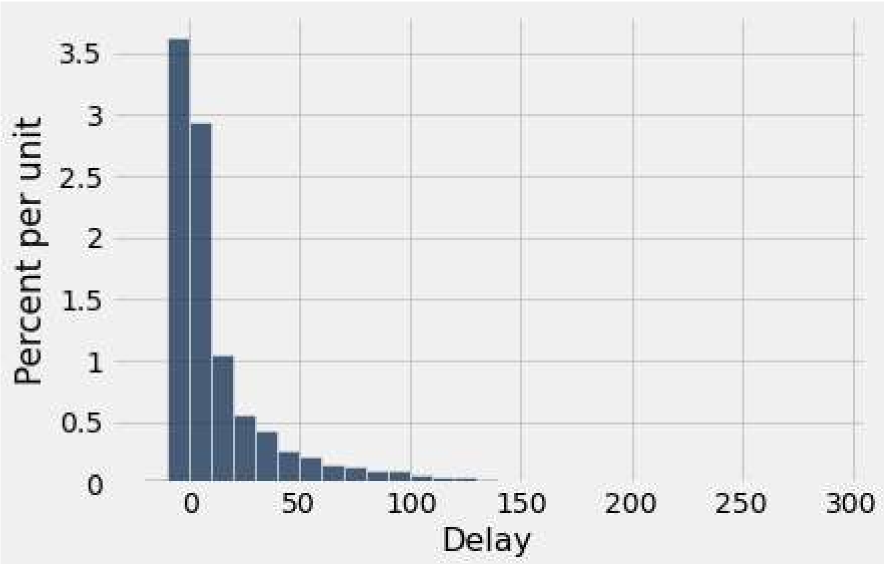
Date	Flight Number	Destination	Delay	Delay in Standard Units
6/21/15	1964	SEA	580	14.269
6/22/15	300	HNL	537	13.1798
6/21/15	1149	IAD	508	12.4453
6/20/15	353	ORD	505	12.3693
8/23/15	1589	ORD	458	11.1788
7/23/15	1960	LAX	438	10.6722
6/23/15	1606	ORD	430	10.4696
6/4/15	1743	LAX	408	9.91236
6/17/15	1122	HNL	405	9.83637
7/27/15	572	ORD	385	9.32979

... (13815 rows omitted)

In [33]:
```
chosen = united.where('Delay in Standard Units', are.between(-3, 3))
chosen.num_rows/united.num_rows
```

Out[33]:
```
0.9790235081374322
```

In [34]:
```
united.hist('Delay', bins = np.arange(-20, 300, 10))
```

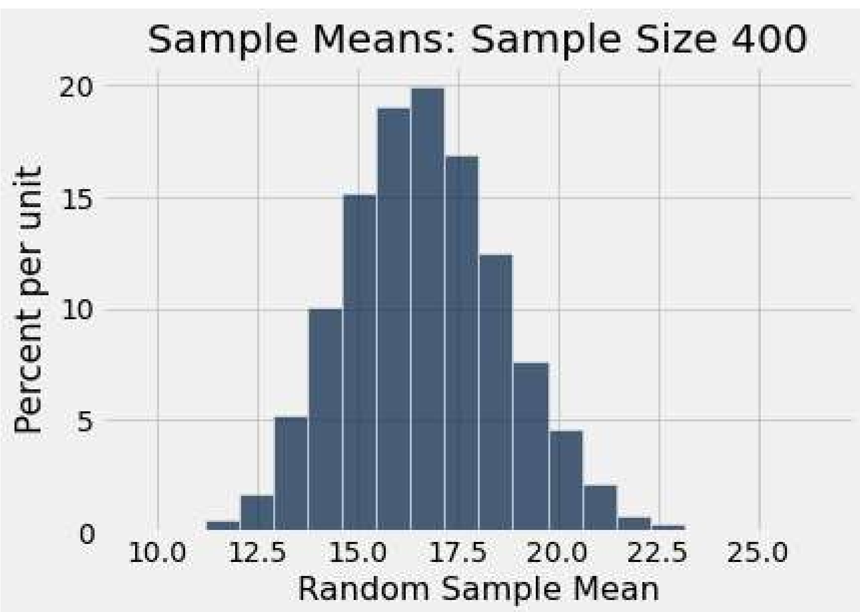


In [35]:
```
sample_size = 400

means = make_array()

for i in np.arange(10000):
    sampled_flights = united.sample(sample_size)
    sample_mean = np.mean(sampled_flights.column('Delay'))
    means = np.append(means, sample_mean)
```

In [36]:
```
Table().with_columns('Sample Mean', means).hist(bins = 20)
plots.title('Sample Means: Sample Size ' + str(sample_size))
plots.xlabel('Random Sample Mean');
```



In [37]:
```
#report the mean and the standard deviation of the
#sample means of 10000 samples of size 400

mean_of_sample_means = np.mean(means)
sd_of_sample_means = np.std(means)
mean_of_sample_means, sd_of_sample_means
```

Out[37]: (16.667014750000003, 2.0008327416746852)

In [38]:
```
#to empirical illustrate/validate the Central Limit Theorem
#compare the mean of the 13,000+ delays
#with the mean of the 10,000 means of random samples of size 400
mean_delay, mean_of_sample_means
```

Out[38]: (16.658155515370705, 16.667014750000003)

In [39]:
```
#and compare the
#standard deviation of the 13,000+ delays / sqrt(400)
#with the
#standard deviation of the 10,000 means of random samples of size 400

import math
sd_delay/math.sqrt(400), sd_of_sample_means
```

Out[39]: (1.9740099925804657, 2.0008327416746852)

In [ ]: