In [1]:
```
import matplotlib
from datascience import *
%matplotlib inline
import matplotlib.pyplot as plots
import numpy as np
plots.style.use('fivethirtyeight')
```

Lecture 27 and 28

Central Limit Theorem:

Mean and Variability of Sample Means

In [2]:
```
united = Table.read_table('united_summer2015.csv')
united
```

Out[2]:

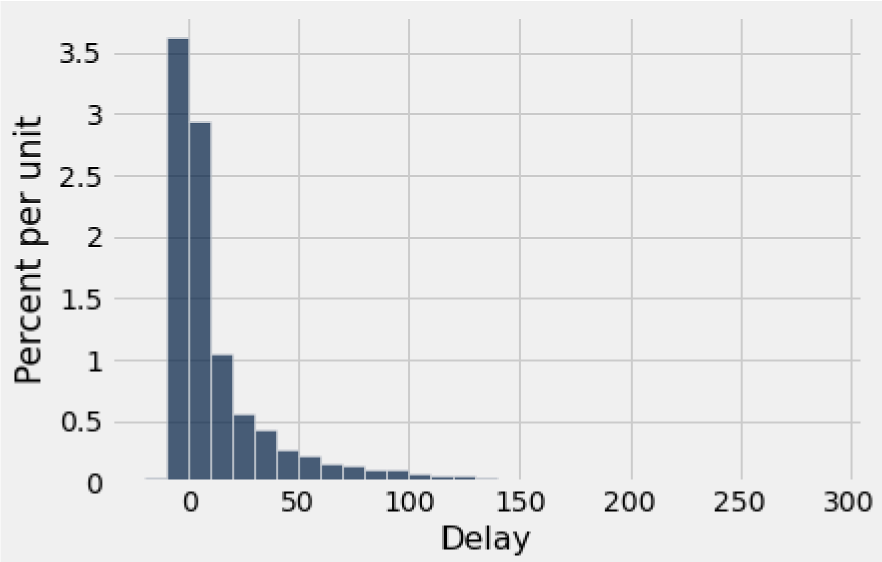
Date	Flight Number	Destination	Delay
6/1/15	73	HNL	257
6/1/15	217	EWR	28
6/1/15	237	STL	-3
6/1/15	250	SAN	0
6/1/15	267	PHL	64
6/1/15	273	SEA	-6
6/1/15	278	SEA	-8
6/1/15	292	EWR	12
6/1/15	300	HNL	20
6/1/15	317	IND	-10

... (13815 rows omitted)

In [3]:
```
united.num_rows
```

Out[3]:
```
13825
```

In [4]:
```
united.hist('Delay', bins = np.arange(-20, 300, 10))
```



In [5]:
```
delays = united.column('Delay')
mean_delay = np.mean(delays)
sd_delay = np.std(delays)

mean_delay, sd_delay
```

Out[5]:
```
(16.658155515370705, 39.480199851609314)
```

In [6]:
```
percentile(50, delays)
```

Out[6]:
```
2
```

In [7]:
```
sample_size = 400

means_400 = make_array()
```

```
for i in np.arange(10000):
    sampled_flights = united.sample(sample_size)
    sample_mean = np.mean(sampled_flights.column('Delay'))
    means_400 = np.append(means_400, sample_mean)
```

In [8]:

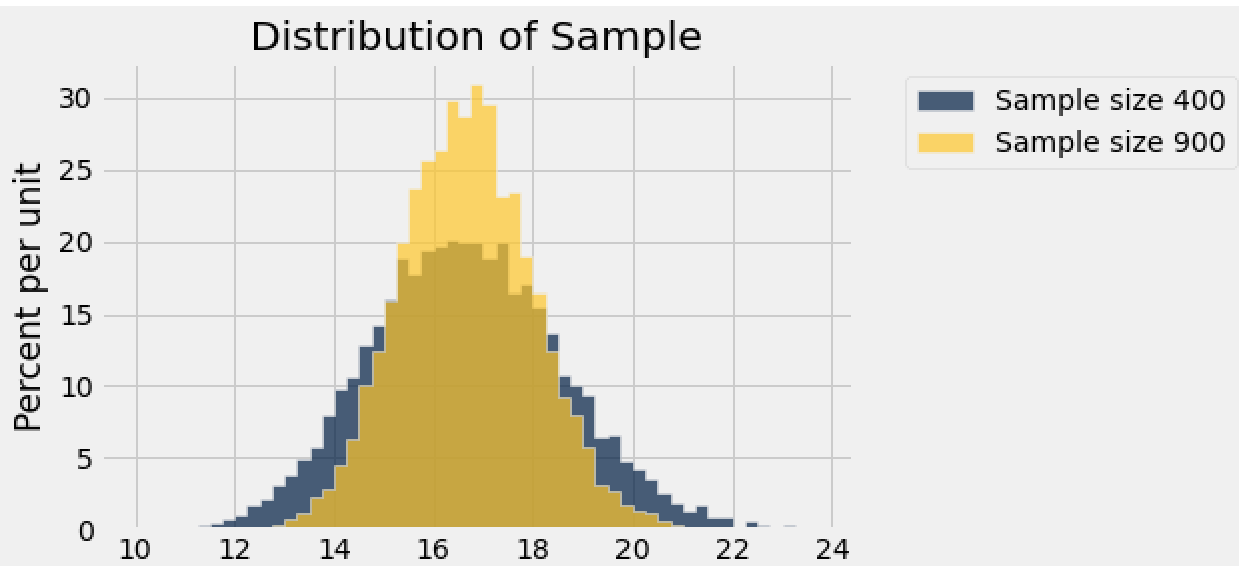
```
sample_size = 900

means_900 = make_array()

for i in np.arange(10000):
    sampled_flights = united.sample(sample_size)
    sample_mean = np.mean(sampled_flights.column('Delay'))
    means_900 = np.append(means_900, sample_mean)
```

In [9]:

```
means_tbl = Table().with_columns(
    'Sample size 400', means_400,
    'Sample size 900', means_900
)
```

In [10]:

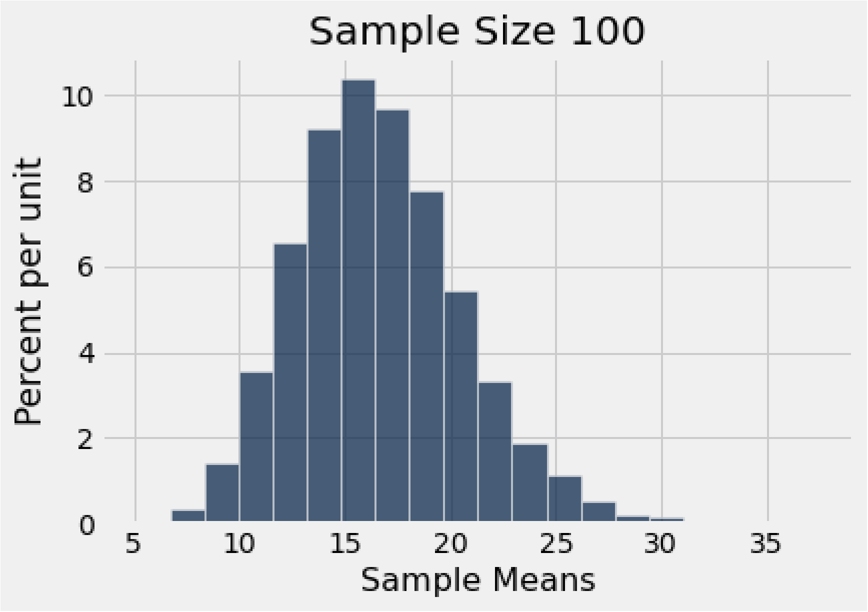
```
means_tbl.hist(bins = np.arange(10, 24, 0.25))
plots.title('Distribution of Sample');
```



In [11]:
```
"""Empirical distribution of random sample means"""

def sample_means(sample_size):

    repetitions = 10000
    means = make_array()

    for i in range(repetitions):
        sampled_flights = united.sample(sample_size)
        sample_mean = np.mean(sampled_flights.column('Delay'))
        means = np.append(means, sample_mean)

    sample_means = Table().with_column('Sample Means', means)

    # Display empirical histogram and print all relevant quantities
    sample_means.hist(bins=20)
    plots.xlabel('Sample Means')
    plots.title('Sample Size ' + str(sample_size))
    print("Sample size: ", sample_size)
    print("Population mean:", np.mean(united.column('Delay')))
    print("Average of sample means: ", np.mean(means))
    print("Population SD:", np.std(united.column('Delay')))
    print("SD of sample means:", np.std(means))
```

In [12]:
```
sample_means(100)
```

```
Sample size:  100
Population mean: 16.658155515370705
Average of sample means:  16.619591
Population SD: 39.480199851609314
SD of sample means: 3.9536497951537135
```



In [13]:
```
sample_sizes = np.arange(100, 401, 50)

mean_of_sample_means = make_array()
sd_of_sample_means = make_array()

for n in sample_sizes:
    means = make_array()
    for i in np.arange(10000):
        means = np.append(means, np.mean(united.sample(n).column('Delay')))
    sd_of_sample_means = np.append(sd_of_sample_means, np.std(means))
    mean_of_sample_means = np.append(mean_of_sample_means, np.mean(means))
```

In [14]:
```
mean_comparison = Table().with_columns(
    'Sample Size n', sample_sizes,
    'Pop_Mean', mean_delay,
    'Mean of 10,000 Sample Means', mean_of_sample_means

)
```

In [15]:
```
mean_comparison
```

Out[15]:

Sample Size n	Pop_Mean	Mean of 10,000 Sample Means
100	16.6582	16.695
150	16.6582	16.6455
200	16.6582	16.6286
250	16.6582	16.6639
300	16.6582	16.6457
350	16.6582	16.67
400	16.6582	16.6524

In [16]:

```
sd_comparison = Table().with_columns(
    'Sample Size n', sample_sizes,
    'Pop_SD', sd_delay,
    'SD of 10,000 Sample Means', sd_of_sample_means,
    'Pop_SD/sqrt(n)', sd_delay/np.sqrt(sample_sizes)
   )
```

In [17]:

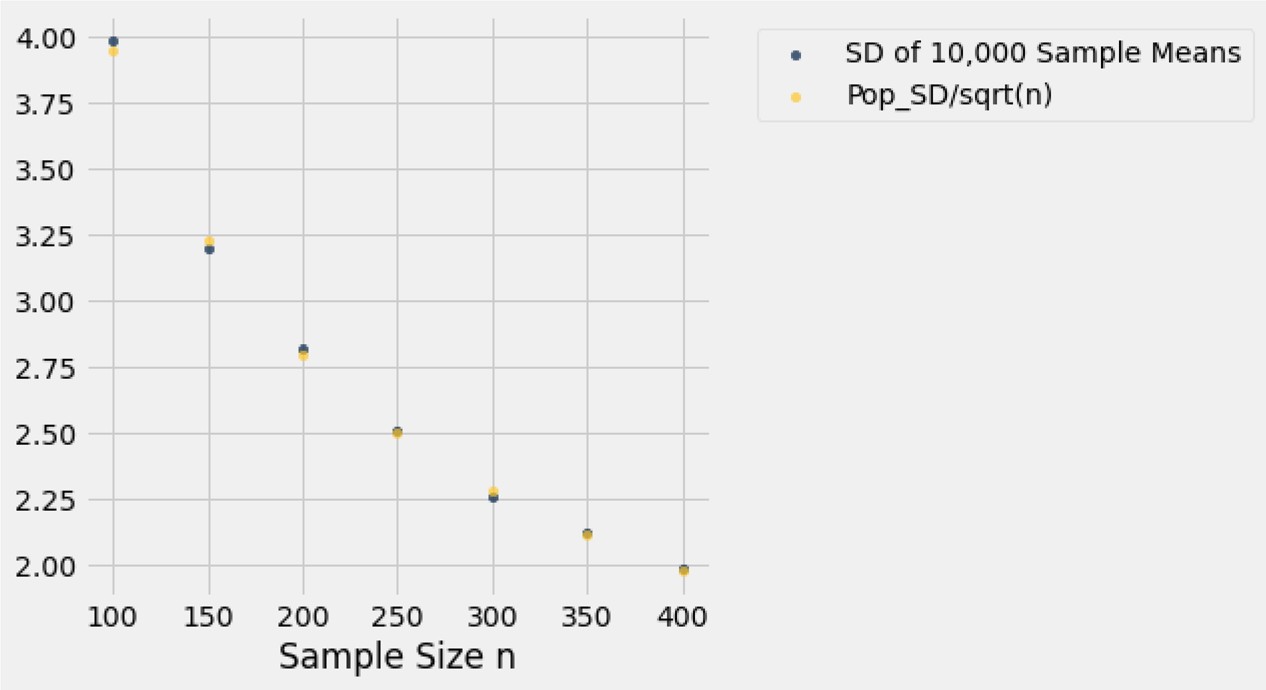
```
sd_comparison
```

Out[17]:

Sample Size n	Pop_SD	SD of 10,000 Sample Means	Pop_SD/sqrt(n)
100	39.4802	3.98263	3.94802
150	39.4802	3.19522	3.22354
200	39.4802	2.81596	2.79167
250	39.4802	2.50806	2.49695
300	39.4802	2.25817	2.27939
350	39.4802	2.11707	2.11031
400	39.4802	1.97835	1.97401

In [18]:

```
sd_comparison.drop('Pop_SD').scatter('Sample Size n')
```



Lecture 28

To illustrate that SD of 0/1 Population is 0.5 or less

In [19]:

```
# Population of size 10

ones = 5
zero_one_population = np.append(np.ones(ones), np.zeros(10 - ones))
zero_one_population
```

Out[19]:

```
array([1., 1., 1., 1., 1., 0., 0., 0., 0., 0.])
```

In [20]:

```
np.std(zero_one_population)
```

Out[20]:

```
0.5
```

In [21]:

```
pop_proportions = make_array()
```

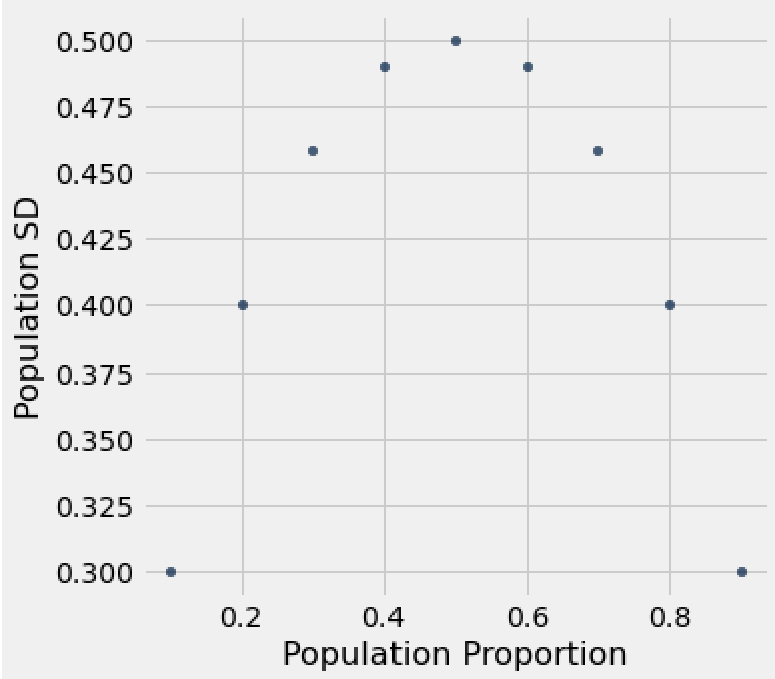
```
pop_SDs = make_array()

for k in np.arange(1, 10):
    population = np.append(np.ones(k), np.zeros(10 - k))
    population_SD = np.std(population)
    pop_SDs = np.append(pop_SDs, population_SD)
    pop_proportions = np.append(pop_proportions, k/10)

sd_table = Table().with_columns(
    'Population Proportion', pop_proportions,
    'Population SD', pop_SDs
)

sd_table
```

Out[21]:

Population Proportion	Population SD
0.1	0.3
0.2	0.4
0.3	0.458258
0.4	0.489898
0.5	0.5
0.6	0.489898
0.7	0.458258
0.8	0.4
0.9	0.3

In [22]:

```
sd_table.scatter(0)
```


Population SD
0.500
0.475
0.450
0.425
0.400
0.375
0.350
0.325
0.300
0.2
0.4
0.6
0.8
Population Proportion

In [ ]:

In [ ]: