Part I
Math Fundamentals (Pre-Algebra)

1 Numbers and negative numbers

There are identity numbers for addition and multiplication: $x + 0 = x$ and $x * 1 = x$ therefore 0 is the identity number for addition and 1 is the identity number for multiplication. Adding/multiplication by the respective identity number will always result in the origin value value.

The opposite of a number is the number multiplied by -1. An even number of negative signs is equal to a positive sign and an odd number of negative signs is equal to a negative sign: $1 + - - 1 = 2$ (or $1 - -1 = 2$) and $1 - - - 1 = 0$. The sign of a number shows in which direction you'd have to go on the number line. Multiplication of *two* numbers that have two different signs will result in a negative number, when both signs are the same the result will be positive. Dividing a negative number by a negative number will result in a positive number, dividing a positive by a negative will result in a negative number. When more than two numbers are divided an odd number of negative signed numbers will have a negative result and an even number of negative numbers will have a positive result: $\frac{6}{-3} = -\frac{6}{3}$ and $-\frac{-6}{3} = \frac{-1}{-1} * \frac{6}{3} = \frac{1}{1} * \frac{6}{3}$

An absolute value is referring to the distance from the origin (0): $|2| = 2$ and $|-2| = 2$ so it will always be positive since absolute is just the units of distance.

2 Factors and multiples

2.1 Divisibility

A number is evenly divisible by 2 if the last number of it is even $(0, 2, 4, 6, 8)$: $120 \div 2 = 60, 126 \div 2 = 63, 128 \div 2 = 64$

To find out if a number is evenly by 3 you have to add all of it individual numbers together and see if that is divisible by 3: $120 \div 3 = 40$ is evenly divisible because: $1 + 2 + 0 = 3$

To see if a number is evenly by 4 you have to check if the last two numbers are evenly divisble by 4: $120 \div 4 = 30$ is evenly divisible because: $20 \div 4 = 5$

To see if a number is evenly by 5 you have to check if the last number is either 0 or 5: $120 \div 5 = 24$ is evenly divisible by 5 because the last number is a 0, $126 \div 5$ is not.

The divisibility rule for 6 is achieved by testing if the number is divisble by 2 *and* 3: $120 \div 6 = 20$

To find out if a number is evenly divisble by 7 you have to multiply the last number of it by 5 and then add the result to the rest of the numbers and

check if that is evenly divisble by 7. $120 : 0 * 5 = 0 + 12 = 12 \div 7$↯(not even) $126 : 6 * 5 = 30 + 12 = 42 \div 7 = 6$✓

The divisibility for 8 is more attractive for larger numbers because you have to check the last 3 digits and see if they as a whole number are evenly divisible by 8: $120 \div 8 = 15$✓

For 9 you sum up all the individual digits and if the sum is divisble by 9 then the whole number is divisible by 9. For 120: $1 + 2 + 0 = 3 \div 9$↯, for 126: $1 + 2 + 6 = 9 \div 9 = 1$✓

A number is evenly divisible by 10 if it ends with 0.

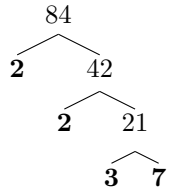
The multiples of a number are just the numbers that can be evenly divided by the number. To find for example the first 3 multiples of a number just multiply the number with 1, with 2, and with 3.

2.2 Prime and composite numbers

A prime number is a number that is evenly divisble by 1 and iteself only. Composite numbers are all other numbers so a number is either prime or composite. For example 1 and 5 both go into 5 but not any other number between these two. And numbers higher than 5 would return a decimal, therefore 5 is a prime number. For 35 there are other factors then 1 and the number itself (35), e.g. 5 or 7 so it is an composite number.

2.2.1 Prime Factorization

$a * b$ a and b are factors (a = Multiplikator, b = Multiplikand). Prime factorization is the representation of a number n as a product of prime numbers. For example: $102 : 2 = 51$ Now 2 cant be factorized further but 51 can be seperated into factors of 3 and 17. Now 3 is (like 2) a prime number and cant be factorized any furhter, the same goes for 17. Now to find the product of primes you have to take all the primes and multiply them together: $2 * 3 * 17 = 102$. No matter what the number is, the steps are break down the number into irreduceable prime numbers:

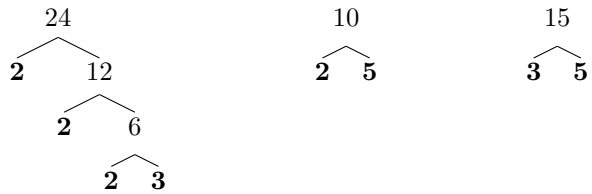


And then multiply these prime numbers: $2 * 2 * 3 * 7 = 2^2 * 3 * 7 = 84$

2.3 Least common multiple

The least common multiple of two numbers is the smallest number that is a multiple of both: $10 * 3 = 30$ and $15 * 2 = 30 \rightarrow 30$ would be the LCM for 10 and 15. To find the least common multiple of a set of numbers you have

to factorize the numbers into their primes, find the highest of each factor and mulitplies these together:



$24 = 2 * 2 * 2 * 3 = 2^3 * 3 \rightarrow 2^3$ is the largest of all factors of 2

$10 = 2 * 5$

$15 = 3 * 5$

The factors of 3 and 5 are all equal so I have to take one of each: $2^3 * 3 * 5 = 120 \rightarrow$ is the least common multiple of 24, 10 and 15.

3 Decimals

3.1 Place-value notation (Stellenwertsystem) and expanded notation

Describing a place value to the left of the decimal point, is talking about the 1s place, 10s place or the hundreds place and so on (Einerstelle, Zehnerstelle, etc). Talking about a number to the right of the decimal point is about the tenth place, the one hundredth place, the 1000th place and so on (Zehntel, Hundertstel, Tausendstel).

To write a number in expanded notation you take each number and multiply it by its corresponding place value e.g: $500 = 5 * 100 + 0 * 10 + 0 * 1$ and $1281 = 1 * 1000 + 2 * 100 + 8 * 10 + 1 * 1$ Of course the same thing is applicable to decimals (values to the right of the decimal point): $0.55 = 5 * \frac{1}{10} + 5 * \frac{1}{100} = \frac{5}{10} + \frac{5}{100}$. In contrast to whole numbers where each number is multiplied by its place ($50 = 5 * 10$) with decimals you divide each number by its place ($0.5 = \frac{5}{10}$)

3.2 Decimal arithmetic

To add decimals together the decimals have to line up with each other:

$$4.5 + 3.34 = \begin{array}{r} 4.50 \\ +\ 3.34 \\ \hline 7.84 \end{array}$$

If the numbers do not align a 0 can be added at the missing place behind the decimal point.

The same thing goes for subtraction:

$$16.7 - 2.26 = \begin{array}{r} 16.70 \\ -\ 2.26 \\ \hline 14.44 \end{array}$$

4 Fractions