Lemma 0.1. *Let C be a CW-complex in a Hausdorff space X as in the definition in the formalisation. Then C is a CW-complex as in the paper definition.*

Proof. Properties (i), (ii), (iii) and (v) of the definition are immediate. Thus let us look at property (iv). We assume that

$$A \subseteq C \text{ is closed in } X \iff \overline{e}_i^n \cap A \text{ is closed in } X \text{ for all } n \in \mathbb{N} \text{ and } i \in I_n$$

and need to show that

$$A \subseteq C \text{ is closed in } C \iff \overline{e}_i^n \cap A \text{ is closed in } C \text{ for all } n \in \mathbb{N} \text{ and } i \in I_n.$$

It is easy to see that the forward direction is true. For the backwards direction take $A \subseteq C$ such that $A \cap \overline{e}_i^n$ is closed in C for all $n \in \mathbb{N}$ and $i \in I_n$. That means that for every $n \in \mathbb{N}$ and $i \in I_n$ there is a closed set $B_i^n \subseteq X$ such that $B_i^n \cap C = A \cap \overline{e}_i^n$. But since C is closed that means that $A \cap \overline{e}_i^n$ was already closed for every $n \in \mathbb{N}$ and $i \in I_n$. Thus we are done by assumption. □

Lemma 0.2. *If $X \times Y$ is a k-space then it has weak topology, i.e. $A \subseteq X \times Y$ is closed iff $\overline{e}_i^n \times \overline{f}_j^m \cap A$ is closed for all $n, m \in \mathbb{N}$, $i \in I_n$ and $j \in J_m$.*

Proof. The forward direction is easy.

Moving on to the backward direction we know that A is closed if for every compact set $C \subseteq (X \times Y)_c$, $A \cap C$ is closed in C. Take such a compact set C. The projections $\mathrm{pr}_1(C)$ and $\mathrm{pr}_2(C)$ are compact as images of a compact set. There are finite sets $E \subseteq \{e_i^n \mid n \in \mathbb{N}, i \in I_n\}$ and $F \subseteq \{f_j^m \mid m \in \mathbb{N}, j \in J_m\}$ s.t $\mathrm{pr}_1(C) \subseteq \bigcup_{e \in E} e$ and $\mathrm{pr}_2(C) \subseteq \bigcup_{f \in F} f$. Thus

$$C \subseteq \mathrm{pr}_1(C) \times \mathrm{pr}_2(C) \subseteq \bigcup_{e \in E} e \times \bigcup_{f \in F} f = \bigcup_{e \in E} \bigcup_{f \in F} e \times f.$$

So C is included in a finite union of cells of $(X \times Y)_c$. Therefore

$$A \cap C = A \cap \left(\bigcup_{e \in E} \bigcup_{f \in F} e \times f \right) \cap C = \left(\bigcup_{e \in E} \bigcup_{f \in F} A \cap (e \times f) \right) \cap C$$

is closed since by assumption $A \cap (e \times f)$ is closed for every e and f and the union is finite. Thus $A \cap C$ is in particular closed in C. □