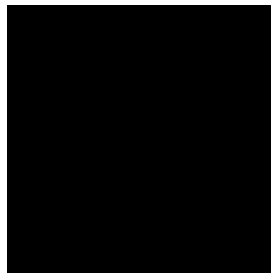


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# Midterm I - Section QA

ENGR 233

Fall 2023

This exam is worth 10% of your final grade.

All seven questions are equally valued at 10 point each. The maximum mark is 60.

Solve any six questions or try all seven questions. The best six marks will be recorded. Only faculty-approved calculators are allowed. No other electronic device is allowed. No documentation is allowed besides a formula sheet, that is provided.

You cannot give nor receive help from anyone besides the instructor.

Print your name and student ID legibly at the top of your exam booklet.

You must explain what you are doing, and show your work. You will be graded on your work, not just on your answer. Make it clear and legible so we can follow it.

1. Consider the planes P, given by the equation  $4x + 2y - z = 7$ , and Q, the plane of equation  $3x - 2y + 5z = -4$ .
  - (a) Find the angle formed by P and Q.
  - (b) Give the equation of the intersection of P and Q. *Hint: Consider the point  $(1, 1, -1)$ .*

Show all your work.

2. An artist needs for a project a long ribbon that will be arranged in the shape of the curve  $t^2, \frac{2t^3}{3}, \frac{t^4}{4}$ , with the z-axis pointing up. The total creation should be 4 meters high, and the ribbon will cover the total height of the creation, from the top of the creation to the floor. How long should the ribbon be?

Show all your work.

3. A line is given by a vector equation  $r(t) = -1, 2, 4 + t \, 3, 4, 2$ . Find the value of a parameter  $t$  such that the corresponding point on the line has its position vector orthogonal to  $1, 3, 2$ .

Show all your work.

4. Consider the function  $f(x, y) = xy^2 + e^x \sin x$ .

(a) Find  $f_x$  and  $f_y$ .

(b) Write an equation for the tangent plane to  $f(x, y)$  at the point  $\frac{1}{2}, 2, f(\frac{1}{2}, 2)$ .

Show all your work.

5. Let  $F(x, y, z) = (\frac{3x}{y^2} + 5)i - (\frac{x^2}{y^2})j + 3zk$ .

(a) Find the curl of  $F$ . Show your work.

(b) Does there exist a scalar function  $f(x, y, z)$  whose gradient is  $F$ ? Explain your answer.

6. Suppose  $r(t) = t^2i + (t^3 - 2t)j + (t^2 - 5t)k$  is the position vector of a moving particle.

(a) At what points does the particle pass through the xy-plane?

(b) What is its acceleration at these points?

Show all your work.

7. (Short answer.) For this problem, you don't need to show your work, and partial credit will be given only based on the answers provided.

$$f(x, y) = x^2 - 4y^2$$

- (a) Compute the gradient:  $\nabla f(x, y)$ .
- (b) Give the gradient at point  $(4, 2)$ .
- (c) Which of the following pictures represents the level curves of  $f$ ?

A)

B)

C)

D)

