2020 Winter Midterm - PRACTICE

Introduction to Theoretical Computer Science (Concordia University)

ID:

Part A There are 8 **multiple-choice** questions in this part.

You **must** use the supplied multiple-choice answer sheet to record your answers for this part (Part A). Only answers on that sheet will be considered (answers on the exam booklet itself will be **discarded**). You must highlight your ID on that multiple-choice sheet as well. A correct choice for one question will get you **4** points. An incorrect answer, marking several answers or selecting nothing for a question will get you **0**. If you believe that more than one answer is correct, select the best answer.

1. What is the best asymptotic ("big-O") characterization of the following function:

$$f(n) = (14logn)^2 + \log(3^n)$$

   a) $O(3^n)$
   b) $O(n^2)$
   c) $O(n)$
   d) $O(2^n)$
   e) $O(logn)$

2. Give the best asymptotic ("big-Oh") characterization of the worst case *and* the best case time complexities of the algorithm `DoAgain(A, n)`.

Algorithm *DoAgain(A,n)*
Input: Array A storing integers and of size $n > 1$.

```
sum ← 0
for c ← 0 to n^2 do
    if A[0] < 0 then
        for k ← 0 to n − 1 do
            sum ← sum + c · A[k]
```

(a) Best case $O(n)$ and worst case $O(n^2)$
(b) Best case $O(n)$ and worst case $O(n^3)$
(c) Best case $O(n^2)$ and worst case $O(n^3)$
(d) Best case $O(n)$ and worst case $O(n^4)$
(e) Best case $O(n^3)$ and worst case $O(n^4)$

3. You have implemented the queue with a linked list, keeping track of a front node and a rear node with two reference variables. Which of these reference variables will change during an insertion into a NONEMPTY queue?

   a) Neither changes
   b) Only front changes.
   c) Only rear changes.
   d) Both change.

4. Here is an incorrect kind of pseudo code a student provided for the algorithm which is supposed to determine whether a sequence of parentheses is balanced:

```
declare a character stack
while ( more input is available)
{
   read a character
   if ( the character is a '(' )
      push it on the stack
   else if ( the character is a ')' and the stack is not empty )
      pop a character off the stack
   else
      print "unbalanced" and exit
 }
 print "balanced"
```

Which of these unbalanced sequences does the above code think is balanced?

a) ((()))
b) ())(()
c) (()()))
d) ((())()

5. Consider the algorithm *Multiply(A,n)* below. *A* is an array of size *n* storing integer values. What is the best characterization of the best and worst case asymptotic time complexity of the following algorithm?

```
Algorithm Multiply(A,n)
    result = 0
    for i = 0 to n/2 do
        j = 0
        while A[j] < 0 and j < n/2 do
            result = result+A[i]*A[j]
            j = j + 1
    return result
```

a) Best case $O(1)$, worst case $O(n)$.
b) Best case $O(logn)$, worst case $O(n)$.
c) Best case $O(n)$, worst case $O(n)$.
d) Best case $O(1)$, worst case $O(n^2)$.
e) Best case $O(n)$, worst case $O(n^2)$.

6. Consider the following pairs of functions: $f(n), g(n)$. For which pair, the functions are such that $f(n)$ *is* $O(g(n))$ *and* $g(n)$ ***is not*** $O(f(n))$*?*

(a) $f(n) = n^2, g(n) = 2n^2 + 7$
(b) $f(n) = \log n, g(n) = \log(n^2)$
(c) $f(n) = 17, g(n) = n + 1$
(d) $f(n) = n, g(n) = 3000n + 1$
(e) $f(n) = \log n, g(n) = 1/n$

7. What is the best asymptotic ("big-O") characterization of the following function:

$$f(n) = 6n\ log\ log(n^2) + 2nlog^2n + nlogn$$

a) $O(nlog^2n)$
b) $O(nlogn)$
c) $O(n^2logn)$
d) $O(n^3logn)$
e) $O(nlog(n^2))$

8. Suppose we have a circular array implementation of the queue class, with ten items in the queue stored at data [2] through data [11]. The current capacity is 42. Where does the insert method place the new entry in the array?

a) data[1]
b) data[2]
c) data[11]
d) data[12]

Part B

Important Note

- Efficiency refers to time and memory.
- You will be graded as follows:
  - A mark (100%) if you provide a **correct and most efficient** algorithm.
  - A mark (%80) if you provide a **correct and second efficient** algorithm.
  - A mark (%50) if you provide a **correct** and missed to consider the efficiency of the algorithm.

B.1 [33 Points] A palindrome is a string that reads the same forward and backward, capitalization and space are ignored. For example *deed, go dog, level*…. are palindromes.

a) **[10 Points]** Write an iterative algorithm in pseudo code that tests whether a string is a palindrome.

b) **[10 Points]** Write a recursive algorithm in pseudocode that tests whether a string is a palindrome.

c) **[8 Points]** Describe how you could use a *Stack* or *Queue* to check whether a string is a palindrome, and write the pseudo code for that.

d) **[5 Points]** What are the worst-case runtimes of above algorithms (a), (b), (c) (use the big-Oh notation). Justify your answers.

ID:

Part C [24 Points]

For each of the **4** questions in this part, mark **T** if the given statement is **ALWAYS** true. Otherwise mark **F** and **justify** your answer. If you do not justify the FALSE case you will lose $^4/_6$ of the mark. There is **no penalty** for selecting a wrong answer. **Hint**: a correct counter example and/or correct specification will give you better marks. A correct answer will get you **6** points.

1. If $f(n) = 5n^2$ then $f(n) \in \Omega(2^n)$ □T □F

2. The worst-case asymptotic running time for the best algorithm for finding something in a sorted array? is $O(n)$ □T □F

3. The worst-case asymptotic running time of finding and removing all values greater than 12 from a stack implemented with a linked-list (leaving the rest of the stack in its original order) is O(1) □T □F

4. Performing a remove operation at the tail in a list ADT implemented as a singly LinkedList that keeps track of the head, is very efficient: performed in a constant time O(1). □T □F