Department of Computer Science & Software Engineering

COMP335 Introduction to Theoretical Computer Science

Winter 2017

Midterm Exam March 2

- Your Name:
- Your Student ID:
- Your Signature:

Instructions:

- There are 10 multiple-choice questions (Questions 1 – 10, each worth 3 points) and one proof question (Question 11, worth 9 points).
- Answer questions 1 – 10 on the scan sheet.

  NOTE: Only the scan sheet will be graded. Anything written in the booklet will be ignored.
- Answer Question 11 in the three boxes on page 6.

  NOTE: Only answers written in the three boxes will be graded. Anything else written in the booklet will be ignored.
- Use provided scrap paper for your rough work. DO NOT hand in the scrap paper!

1. Let $L_1 = \{a^i : i \geq 0\}$ and $L_2 = \{a^i b^j : j \geq i \geq 0\}$. Then the language $L_1 L_2$ is

(a) $\{a^{2i} b^j : i, j \geq 0\}$

(b) $\{a^i b^j : i \geq j\}$

(c) $\{a^i b^j : i, j \geq 0\}$

(d) $\{a^i b^j : j \geq i\}$

2. Which of the following strings is NOT in the Kleene closure of the language $\{011, 10, 110\}$

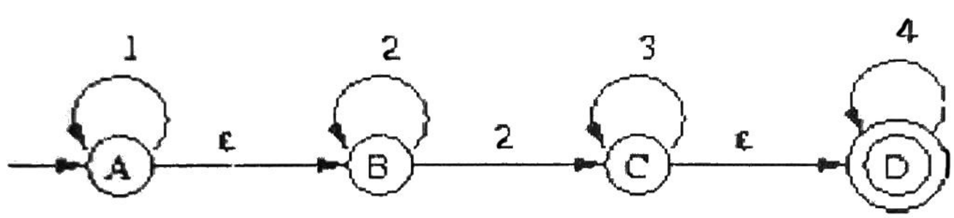
(a) 10111011

(b) 10110011

(c) 1001110

(d) 1010110

3. Consider the following ϵ-NFA



When you convert this ϵ-NFA to a DFA, which of the following would be a transition of the DFA?

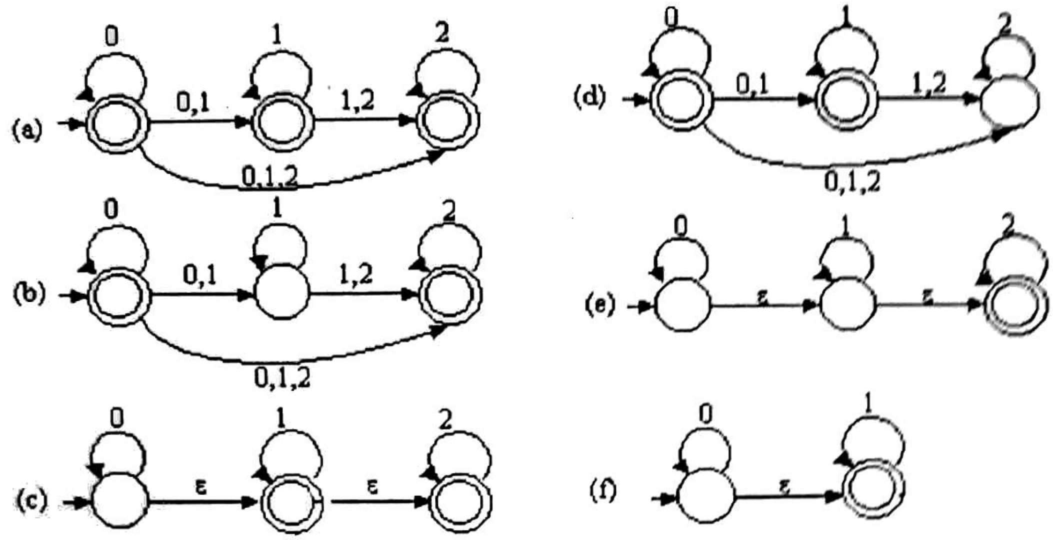
(a) $\delta(\{A, B\}, 1) = \{A, B\}$

(b) $\delta(\{A, B\}, 1) = \{A\}$

(c) $\delta(\{A, B\}, 3) = \{C, D\}$

(d) $\delta(\{A, B\}, 2) = \{A, B, C\}$

4. Consider the following six ϵ-NFA's



Which of these accept the same language?

(a) (a) and (f)
(b) (a) and (c)
(c) (c) and (d)
(d) (b) and (f)

5. Consider the two regular expressions

$$R = \mathbf{0}^* + \mathbf{1}^* \qquad S = \mathbf{0}\mathbf{1}^* + \mathbf{1}\mathbf{0}^* + \mathbf{1}^*\mathbf{0} + (\mathbf{0}^*\mathbf{1})^*$$

Then, consider the languages

$$L_1 = L(R) \setminus L(S),\ L_2 = L(S) \setminus L(R),\ L_3 = L(R) \cap L(S),\ L_4 = \overline{L(R) \cup L(S)}.$$

and the strings

$$w_1 = 011,\ w_2 = 111,\ w_3 = 000,\ w_4 = 1100$$

Which of the following is correct?

(a) $w_1 \in L_1,\ w_2 \in L_2,\ w_3 \in L_3,\ w_4 \in L_4$
(b) $w_3 \in L_1,\ w_1 \in L_2,\ w_2 \in L_3,\ w_4 \in L_4$
(c) $w_4 \in L_1\ \ w_2 \in L_2,\ w_1 \in L_3,\ w_3 \in L_4$
(d) $w_2 \in L_1,\ w_1 \in L_2,\ w_4 \in L_3,\ w_3 \in L_4$

6. How many strings of length less than 4 is contained in $L\Big((a+b)^*b(c+cd)^*\Big)$

(a) 10
(b) 11
(c) 12
(d) 13

7. Which of the following regular expressions defines the complement of the language $L\Big((0+10)^*\Big)$

(a) $(0+1)^*(11+1+\epsilon)^*$
(b) $(0+1)^*(1+11)(0+1)^*$
(c) $(0+10)^*1(\epsilon+11(0+1)^*)$
(d) $(0+1)^*11(0+1)^*+(0+10)^*1$

8. Let h be a homomorphism from $\{a,b,c\}$ to $\{0,1\}$, where $h(a)=01$, $h(b)=0$, and $h(c)=10$. Which of the following strings is in $h^{-1}(010010)$.

(a) $bcab$
(b) $abcb$
(c) $bcba$
(d) $babc$

9. Let $A=(Q,\Sigma,\delta,q_0,\{q_f\})$ be an ϵ-NFA that accepts language $L(A)$. Consider the following modifications of A.

- The automaton B constructed from A by adding ϵ-transitions from q_0 to every state, for which there is a path in A from q_0 to that state.
- The automaton C constructed from A by adding ϵ-transitions to q_f from every state, for which there is a path in A to q_f from that state.
- The automaton D constructed from A by doing both of the above modifications.

Here are three candidate languages:

- $L_1=\{x : xy\in L(A) \text{ for some } y\in\Sigma^*\}$
- $L_2=\{y : xyz\in L(A) \text{ for some } x,z\in\Sigma^*\}$
- $L_3=\{y : xy\in L(A) \text{ for some } x\in\Sigma^*\}$

Which of the following is correct?

(a) $L(B)=L_1,\ L(C)=L_2,\ L(D)=L_3$
(b) $L(B)=L_3,\ L(C)=L_1,\ L(D)=L_2$
(c) $L(B)=L_3,\ L(C)=L_2,\ L(D)=L_3$
(d) $L(B)=L_2,\ L(C)=L_3,\ L(D)=L_1$

10. Let A be the following DFA.

	0	1
$\rightarrow A$	E	B
$* B$	D	A
C	G	A
$* D$	G	E
E	A	D
F	B	E
$* G$	B	A

When you minimize A using the table-filling algorithm, the following are the sets of indistinguishable (equivalent) states.

(a) $\{A, C\}, \{B, D, G\}, \{E, F\}$
(b) $\{A, C, E\}, \{B, D, G\}, \{F\}$
(c) $\{A\}, \{B, D, G\}, \{C, E, F\}$
(d) $\{A, E\}, \{B, D, G\}, \{C, F\}$

11. Let L be the language of those strings over $\{0,1\}$ where the number of 0's differ from the number of 1's by at most 5. Complete the proof below, showing L is not regular.

Proof:

- Suppose to the contrary that L is regular
  $\Rightarrow \exists$ DFA A, s.t. $L(A) = L$.
- Let n be the number of states in A.
  $\Rightarrow \forall w \in L(A)$, if $|w| \geq n$, then $w = xyz$,
  where x, y, and z as in Pumping Lemma.

(a) Choose a suitable $w \in L$, where $|w| \geq n$.

My solution: $w =$

(b) Find an i, such that $xy^iz \in L(A)$ and $xy^iz \notin L$.

My solution: $i =$ $xy^iz =$

(c) Reason that $xy^iz \notin L$

My reason:

Since $xy^iz \in L(A) \Rightarrow L(A) \neq L$.

- (e) contradicts (a):
  $\Rightarrow$ (a) cannot be true
  $\Rightarrow$ L cannot be regular