2017 Winter midterm

Introduction to Theoretical Computer Science (Concordia University)

Department of Computer Science & Software Engineering

Comp335 Introduction to Theoretical Computer Science

Winter 2017

Midterm Exam March 2

- Your Name:
- Your Student ID:
- Your Signature:

Instructions:

- There are 10 multiple-choice questions (Questions 1 – 10, each worth 3 points) and one proof question (Question 11, worth 9 points).
- Answer questions 1 – 10 on the scan sheet.

  NOTE: Only the scan sheet will be graded. Anything written in the booklet will be ignored.
- Answer Question 11 in the three boxes on page 6.

  NOTE: Only answers written in the three boxes will be graded. Anything else written in the booklet will be ignored.
- Use provided scrap paper for your rough work. DO NOT hand in the scrap paper!

1. Let $L_1 = \{a^i : i \geq 0\}$ and $L_2 = \{a^i b^j : j \geq i \geq 0\}$. Then the language $L_1 L_2$ is

   (a) $\{a^{2i} b^j : i, j \geq 0\}$

   (b) $\{a^i b^j : i \geq j\}$

   (c) $\{a^i b^j : i, j \geq 0\}$

   (d) $\{a^i b^j : j \geq i\}$

2. Which of the following strings is NOT in the Kleene closure of the language $\{011, 10, 110\}$

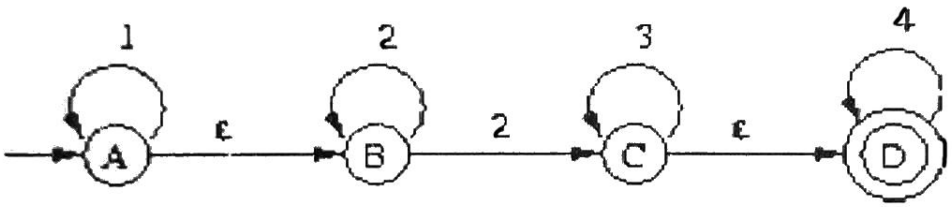
   (a) 10111011

   (b) 10110011

   (c) 1001110

   (d) 1010110

3. Consider the following ϵ-NFA



When you convert this ϵ-NFA to a DFA, which of the following would be a transition of the DFA?

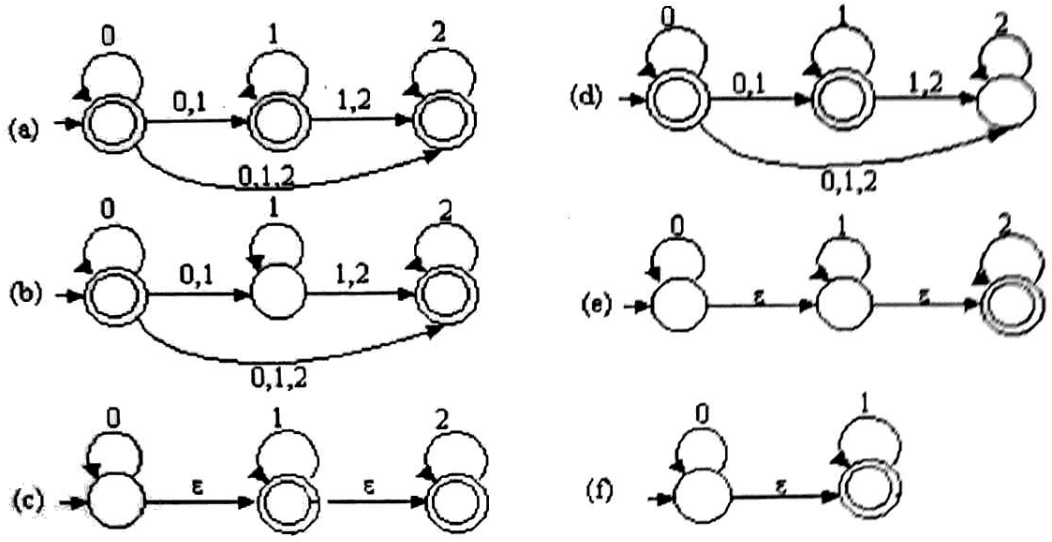
(a) $\delta(\{A, B\}, 1) = \{A, B\}$

(b) $\delta(\{A, B\}, 1) = \{A\}$

(c) $\delta(\{A, B\}, 3) = \{C, D\}$

(d) $\delta(\{A, B\}, 2) = \{A, B, C\}$

4. Consider the following six ϵ-NFA's



Which of these accept the same language?

(a) (a) and (f)

(b) (a) and (c)

(c) (c) and (d)

(d) (b) and (f)

5. Consider the two regular expressions

$$R = 0^* + 1^* \quad S = 01^* + 10^* + 1^*0 + (0^*1)^*$$

Then, consider the languages

$$L_1 = L(R) \setminus L(S),\ L_2 = L(S) \setminus L(R),\ L_3 = L(R) \cap L(S),\ L_4 = \overline{L(R) \cup L(S)}.$$

and the strings

$$w_1 = 011,\ w_2 = 111,\ w_3 = 000,\ w_4 = 1100$$

Which of the following is correct?

(a) $w_1 \in L_1,\ w_2 \in L_2,\ w_3 \in L_3,\ w_4 \in L_4$

(b) $w_3 \in L_1,\ w_1 \in L_2,\ w_2 \in L_3,\ w_4 \in L_4$

(c) $w_4 \in L_1\ \ w_2 \in L_2,\ w_1 \in L_3,\ w_3 \in L_4$

(d) $w_2 \in L_1,\ w_1 \in L_2,\ w_4 \in L_3,\ w_3 \in L_4$

6. How many strings of length less than 4 is contained in $L\Big((a+b)^*b(c+cd)^*\Big)$

   (a) 10
   (b) 11
   (c) 12
   (d) 13

7. Which of the following regular expressions defines the complement of the language $L\Big((0+10)^*\Big)$

   (a) $(0+1)^*(11+1+\epsilon)^*$
   (b) $(0+1)^*(1+11)(0+1)^*$
   (c) $(0+10)^*1(\epsilon+11(0+1)^*)$
   (d) $(0+1)^*11(0+1)^*+(0+10)^*1$

8. Let h be a homomorphism from $\{a,b,c\}$ to $\{0,1\}$, where $h(a)=01$, $h(b)=0$, and $h(c)=10$. Which of the following strings is in $h^{-1}(010010)$.

   (a) $bcab$
   (b) $abcb$
   (c) $bcba$
   (d) $babc$

9. Let $A=(Q,\Sigma,\delta,q_0,\{q_f\})$ be an ϵ-NFA that accepts language $L(A)$. Consider the following modifications of A.

   - The automaton B constructed from A by adding ϵ-transitions from q_0 to every state, for which there is a path in A from q_0 to that state.
   - The automaton C constructed from A by adding ϵ-transitions to q_f from every state, for which there is a path in A to q_f from that state.
   - The automaton D constructed from A by doing both of the above modifications.

   Here are three candidate languages:

   - $L_1=\{x : xy\in L(A) \text{ for some } y\in\Sigma^*\}$
   - $L_2=\{y : xyz\in L(A) \text{ for some } x,z\in\Sigma^*\}$
   - $L_3=\{y : xy\in L(A) \text{ for some } x\in\Sigma^*\}$

   Which of the following is correct?

   (a) $L(B)=L_1,\ L(C)=L_2,\ L(D)=L_3$
   (b) $L(B)=L_3,\ L(C)=L_1,\ L(D)=L_2$
   (c) $L(B)=L_3,\ L(C)=L_2,\ L(D)=L_3$
   (d) $L(B)=L_2,\ L(C)=L_3,\ L(D)=L_1$

10. Let A be the following DFA.

		0	1
→	A	E	B
*	B	D	A
	C	G	A
*	D	G	E
	E	A	D
	F	B	E
*	G	B	A

When you minimize A using the table-filling algorithm, the following are the sets of indistinguishable (equivalent) states.

(a) $\{A,C\}, \{B,D,G\}, \{E,F\}$

(b) $\{A,C,E\}, \{B,D,G\}, \{F\}$

(c) $\{A\}, \{B,D,G\}, \{C,E,F\}$

(d) $\{A,E\}, \{B,D,G\}, \{C,F\}$

11. Let L be the language of those strings over $\{0,1\}$ where the number of 0's differ from the number of 1's by at most 5. Complete the proof below, showing L is not regular.

Proof:

- Suppose to the contrary that L is regular
  $\Rightarrow$ $\exists$ DFA A, s.t. $L(A) = L$.
- Let n be the number of states in A.
  $\Rightarrow$ $\forall w \in L(A)$, if $|w| \geq n$, then $w = xyz$,
  where x, y, and z as in Pumping Lemma.

(a) Choose a suitable $w \in L$, where $|w| \geq n$.

My solution:
$w =$

(b) Find an i, such that $xy^iz \in L(A)$ and $xy^iz \notin L$.

My solution:
$i =$
$xy^iz =$

(c) Reason that $xy^iz \notin L$

My reason:

Since $xy^iz \in L(A) \Rightarrow L(A) \neq L$.

- (e) contradicts (a):
  $\Rightarrow$ (a) cannot be true
  $\Rightarrow$ L cannot be regular