## Stability analysis using Bode plot

**Aim:-** To analysis stability of a given control system using Bode plot

**Tools Required:-** A P.C loaded with MATLAB

**Theory:-**

Bode plots are fundamental tools in control systems engineering due to their ability to provide comprehensive insights into the frequency response characteristics of dynamic systems.

One significant aspect of Bode plots is their capacity to reveal how a system responds to different frequencies of input signals. This is crucial because many real-world systems operate within a range of frequencies, and understanding how they behave across this spectrum is essential for effective control, and stability.

By examining Bode plots, engineers can analyze the stability and performance of control systems. They can identify regions of instability, adjust controller parameters, and optimize system performance to meet desired specifications. Moreover, Bode plots facilitate the design of compensators and filters to improve system stability and response

| Term | Corner frequency | Slope | Change in slope |
|---|---|---|---|
| $(j\omega)^2$ | — | $+40$ db/dec | — |
| $\frac{1}{1+0.2j\omega}$ | $\frac{1}{0.2} = 5$ | $-20$ db/dec | $40-20 = 20$ db/dec |
| $\frac{1}{1+0.02j\omega}$ | $\frac{1}{0.02} = 50$ | $-20$ db/dec | $20-20 = 0$ db/dec |

Magnitude plot :

$\omega l = 0.5$ $\quad = 20 \log(\omega^2)$

$0.1, 0.2 - - - 1$

$\omega_1 = 5$ $\quad = 20 \log(\omega^2)$

$1, 2, 3 - - - 10$

$\omega_2 = 50$ $\quad = 20 \log\left(\frac{\omega_{c2}}{\omega_{c1}}\right) +$ previous gain

$10, 20, - - \cdot 100$

$\omega h = 500$ $\quad = 20 \log\left[\frac{\omega h}{\omega_{c2}}\right] +$ previous gain

$100, 200, - - 1k$

mag(dB), $\omega$.

| ~~~~ | ~~magnitude~~ |
|---|---|
| $-12$ | 0.5 |
| 28 | 5 |
| 48 | 50 |
| 48 | 500 |

phase :

$\frac{(j\omega)\cdot(j\omega)}{(1+0.2j\omega)(1+0.02j\omega)}$ : $\tan^{-1}\left(\frac{\omega}{0}\right) + \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{0.2\omega}{1}\right)$

$180^\circ - \tan^{-1}(0.2\omega) - \tan^{-1}(0.02\omega)$

| $\omega$ | Phase |
|---|---|
| 0.5 | $174^\circ$ |
| 1 | $168^\circ$ |
| 5 | $130^\circ$ |
| 50 | $50^\circ$ |
| 100 | $30^\circ$ |

$\omega gc = 1$ rad/sec & $\omega pc = 0$ rad/sec (from graph)

For $k_1 = 1$, $\omega gc = 1$ rad/sec

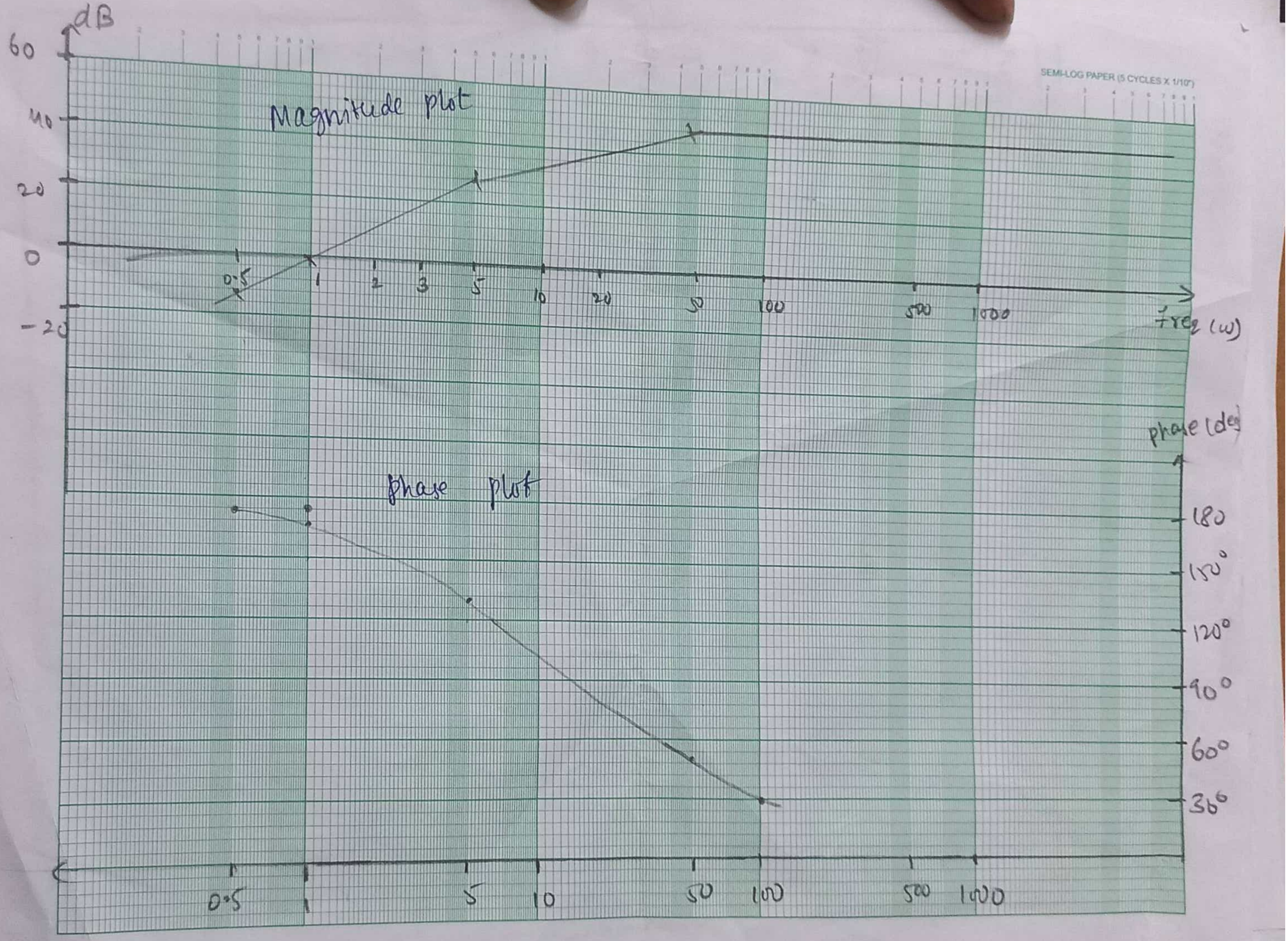
But given, $\omega gc = 0.5$ rad/sec

For $\omega gc$ to be 5 rad/sec, the magnitude at 5 rad/sec $= 0$ dB.

So, we need to shift graph by $-28$dB at $\omega gc = 5$rad/s

$\Rightarrow 20 \log k = -28$

$k = 0.0398$


SEMI-LOG PAPER (5 CYCLES X 1/10")
dB
60
40
20
0
-20
Magnitude plot
0.5
1
2
3
5
10
20
50
100
500
1000
freq (ω)
phase (deg)
Phase plot
180
150°
120°
90°
60°
30°
0.5
1
5
10
50
100
500
1000

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Program:-

```
% G(s) = k s^2 / (1+0.2s)(1+0.02s)
%Determine k if wgc = 5 rad/sec
clc;
clear;
close all;
num = [1 0 0];
den = conv([1 0.2], [1 0.02]);
sys = tf(num, den);
bode(sys);
margin(sys)
[Gm, pm, wc, wp] = margin(sys)
grid on;
figure;
num = [0.0398 0 0]
sys = tf(num, den);
bode(sys);
margin(sys)
[Gm, pm, wc, wp] = margin(sys)
grid on;
```

$$G(s)H(s) = \frac{1}{s(s+1)(s+2)}$$

Put $s = j\omega$

$$G(j\omega)H(j\omega) = \frac{100}{j\omega(j\omega+1)(j\omega+2)}$$

$$\text{Magnitude} = \frac{100}{\sqrt{\omega^2}\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$$

$$\phi = \tan^{-1}\left(\frac{\text{Img}}{\text{Real}}\right)$$

$$= -\tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

$$= -90^\circ - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

$\omega_{pc}$ :

$$-180^\circ = -90^\circ - \tan^{-1}\left(\frac{\omega_{pc}}{1}\right) - \tan^{-1}\left(\frac{\omega_{pc}}{2}\right)$$

$$90^\circ = \tan^{-1}\left(\frac{\omega_{pc}}{2}\right) + \tan^{-1}\left(\frac{\omega_{pc}}{1}\right)$$

$$= \tan^{-1}\left(\frac{\omega_{pc} + \frac{1}{2}\omega_{pc}}{1 - \omega_{pc}\frac{\omega_{pc}}{2}}\right)$$

$$\tan 90^\circ = \frac{\frac{3}{2}\omega_{pc}}{\frac{2-\omega_{pc}^2}{2}} = \frac{1}{0}$$

$$\frac{2-\omega_{pc}^2}{2} = 0 \Rightarrow \omega_{pc} = \sqrt{2}$$

$$M|_{\omega=\omega_{pc}=\sqrt{2}} = \frac{100}{\sqrt{2}\sqrt{3}\sqrt{6}} = \frac{100}{6}$$

$$\text{Gain margin} = \frac{1}{|M|_{\omega=\omega_{pc}}|} = \frac{6}{100}$$

$$\text{GM in dB} = -20\log(GM) = 15.56\,dB$$

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Question 2:

```
% G(s)H(s) = 100 / s(s+1)(s+2)
% Determine wgc, wpc, GM, PM
clc;
clear;
close all;
s=tf('s');
sys = 100/(s*(s+1)*(s+2))
bode(sys);
margin(sys)
[Gm, Pm, Wc, Wp] = margin(sys)
grid on;
```

Question 3:

$$\% \; G(s) = \frac{k}{s(s+2)(s+20)}$$

Determine:-

i) Limiting value of k for system to be stable

ii) k if GM = 10dB

iii) k if PM = 50°

(i) Limiting value : Marginally stable.

$$G(s) = \frac{k}{s \times 2\left[1+\frac{s}{2}\right] \times 20\left[1+\frac{s}{20}\right]}$$

$$\frac{k}{40} = 1$$

$w_l = 0.2$

$w_{c1} = 2$

$w_{c2} = 20$

$w_h = 200$

Magnitudes values : 14 db, −6db, −26db, −46db

Magnitude:

$$20\log\left(\frac{1}{w_{gc}}\right) - 20\log\left(\sqrt{1+\frac{w_{gc}^2}{4}}\right) - 20\log\left(\sqrt{1+\frac{w_{gc}^2}{4}}\right) = 0$$

By equating magnitude to 0, we get $w_{gc}$ = 1 rad/sec

Phase : $-90° - \tan^{-1}\left(\frac{w}{2}\right) - \tan^{-1}\left(\frac{w}{20}\right)$

$PM = 180° + \phi_{wgc}$

$\phi_{wgc} = -90° - \tan^{-1}(1/2) - \tan^{-1}(1/20) = -119.42 = -120°$

$P.M = 180° + \phi_{wgc} = 180° - 120° = 60°$

$w_{pc}$:

$w|_{\phi = -180°} = w_{pc}$

$-90° - \tan^{-1}\left(\frac{w_{pc}}{2}\right) - \tan^{-1}\left(\frac{w_{pc}}{2}\right) = -180° \Rightarrow w_{pc} = 6.4$ rad/s

$GM = 0\,db - \text{Gain}|_{w = w_{pc}}$

$$= 0 - \left[20\log\left(\frac{1}{w_{pc}}\right) - 20\log\left(\sqrt{1+\frac{w_{pc}^2}{4}}\right) - 20\log\left(\sqrt{1+\frac{w_{pc}^2}{400}}\right)\right]$$

$= 26\,db$.

For marginally stable, gain margin = 0db

So, we need to shift graph upwards by 26db

$20\log k_1 = 26$

$$k_1 = 10^{(26/20)} = \frac{k}{40} \Rightarrow k = 798$$

(ii) Gain margin = 10db

Shift graph upwards by 16db

($\because$ 26db − 16db = 10db)

$20 \log k_1 = 16$

$k_1 = 10^{16/20}$

$\Rightarrow k = 40 \times 10^{0.8}$

$= 252$

(iii) Phase margin = 50°

$\phi_{wgc} = -180° + 50° = -130°$

$\omega$ at $-130° = \omega_{gc}$ new = 2 rad/sec

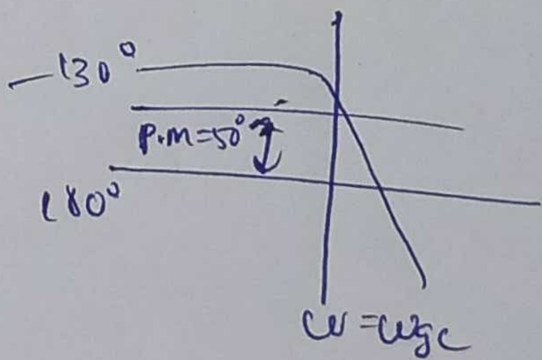
$\omega_{gc}$new = [magnitude = 0db]

(magnitude at 2 rad/sec = 5.5db)

$20 \log k_1 = 5.5$

$k = 40 \times 10^{5.5/20}$

$= 75$



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```
clc;
clear;
close all;
s=tf('s');
sys = 1/(s*(s+2)*(s+20))
bode(sys)
margin(sys)
[GM, PM, wc, wp]=margin(sys)
grid on;
figure;
sys1=252/(s*(s+2)*(s+20))
bode(sys1)
margin(sys1)
[GM, PM, wc, wp]= margin(sys1)
grid on;
figure;
sys2 = 75/(s*(s+2)*(s+20))
bode(sys2)
margin(sys2)
[GM, PM, wc, wp] = margin(sys2)
grid on;
```

Result:- Analyzed the stability of given system using bode plot