## Stability Analysis of a Control System (R-H Criteria)

**Aim:-** To analyze the stability of a control system using R-H criteria.

**Apparatus:-** A PC loaded with MATLAB

**Theory:-**

In control systems engineering, the "Routh-Hurwitz" (RH) criteria are essential for analyzing the stability of linear time-invariant systems. This criteria provide a systematic way to determine whether the system is stable based on the coefficients of its characteristic equation. By applying the RH criteria, engineers can predict stability without needing to solve the characteristic equation or compute eigen values. This simplies the analysis and design of control systems, ensuring their stability and robustness in real-world applications, such as in aerospace, automative, and industrial control systems. Overall, the RH criteria serve as a fundamental tool for ensuring the stability and reliability of control systems, which are crucial for their successful implementation and operation.

$$G(s) = \frac{K}{(s+2)(s+4)(s^2+6s+25)} \qquad H(s) = 1$$

characteristic equation: $1 + G(s)H(s) = 0$

$$1 + \frac{K}{(s+2)(s+4)(s^2+6s+25)} = 0$$

$$K = -(s+2)(s+4)(s^2+6s+25)$$

$$= -(s^2+6s+8)(s^2+6s+25)$$

$$= -(s^4+6s^3+25s^2+6s^3+36s^2+150s+8s^2+48s+200)$$

$$\Rightarrow s^4+12s^3+69s^2+198s+(200+K) = 0$$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 69 | 200+K |
| $s^3$ | 12 | 198 | 0 |
| $s^2$ | 52.5 | 200+K | 0 |
| $s^1$ | $\frac{10395-12(200+K)}{52.5}$ | 0 | |
| $s^0$ | 200+K | 0 | |

$200+K > 0$

$K > -200$

$10395 - 12(200+K) > 0$

$10395 > 12(200) + 12K$

$666.25 > K$

But for negative values of K, gain becomes negative; therefore the system becomes unstable.

$\therefore$ For system to be stable, the condition is $0 < K < 666.25$

To find frequency of oscillation,

$$\frac{10395-12(200+K)}{52.5} = 0$$

$$\Rightarrow K = 666.25$$

$$52.5s^2 + (200+K) = 0$$

$$52.5s^2 + 866.25 = 0$$

$$s = 4.06j$$

is the frequency of oscillation

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# VASAVI COLLEGE OF ENGINEERING
(AUTONOMOUS)
(Affiliated to Osmania University)
Hyderabad - 500 031.

DEPARTMENT OF : ECE

NAME OF THE LABORATORY : Control Systems Engineering

Name G.Sri Ganga Pranav Roll No. 1602-21-735-117 Page No.

Program:-

%Question 1:- Determine the range of k for which
i) the system is stable
ii) the system is oscillatory, also determine the frequency of oscillation

The open loop transfer function of the system is $G(s) = \frac{k}{(s+2)(s+4)(s^2+6s+25)}$

```
clc;
clear;
close all;
s=tf('s')
k1=20
g1= k1/((s+2)*(s+4)*(s^2+6*s+25))
h1= feedback(g1,1,-1)

k2=-1
g2= k2/((s+2)*(s+4)*(s^2+6*s+25))
h2= feedback(g2,1,-1)

k3=700
g3= k3/((s+2)*(s+4)*(s^2+6*s+25))
h3= feedback(g3,1,-1)

subplot(221)
impulse(h1)
title('Impulse response for k=20')
```

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```
subplot(222)
impulse(h2)
title('Impulse response for k=-1')

subplot(223)
impulse(h3)
title('Impulse response for k=700')

k4= 666.25
g4 = k4/((s+2)*(s+4)*(s^2+6*s+25))
h4 = feedback(g4,1,-1)
subplot(224)
stepplot(h4)
title('step response of oscillatory system')
```

% Question 2:-

The open loop transfer function of unity feedback system $G(s) = \frac{k(s+1)}{s^3+as^2+2s+1}$. Determine the value of k and a, so that the system is oscillatory at a frequency of 2 rad/sec

$$G(s) = \frac{K(s+1)}{s^3+as^2+2s+1} \qquad H(s)=1$$

Characteristic equation: $1+G(s)H(s)=0$

$$s^3+as^2+(K+2)s+(K+1)=0$$

| | | |
|---|---|---|
| $s^3$ | 1 | $K+2$ |
| $s^2$ | $a$ | $K+1$ |
| $s^1$ | $\frac{a(K+2)-(K+1)}{a}$ | 0 |
| $s^0$ | $(K+1)$ | 0 |

$$\frac{a(K+2)-(K+1)}{a}=0$$

$$a = \frac{K+1}{K+2} \quad \text{—— (1)}$$

$$as^2+(K+1)=0$$

$s=j\omega$ and $\omega = 2$ rad/sec

$$-a\omega^2+(K+1)=0$$

$$\Rightarrow K=4a-1 \quad \text{—— (2)}$$

From ①, and ②,

$$a = 3/4 \qquad K=2$$

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```
clc;
clear;
close all;
s=tf('s')
k1=0.75
a1=2
g1=k1*(s+1)/(s^3+a1*s^2+2*s+1)
h1=feedback(g1,1,-1)
subplot(211)
stepplot(h1)
title('step response for k=0.75 and a=2')
k2=2
a2=4
g2=k2*(s+1)/(s^3+a2*s^2+2*s+1)
h2=feedbak(g2,1,-1)
subplot(212)
stepplot(h2)
title('step response for k=2 and a=4')
```

Result:- Verified the stability conditions of a given control system using R-H criteria.