Imperfect Information in Health Care Markets

Exercise Session 7 - Rothschild-Stiglitz

Exercise 12

In this exercise we show that in the Rothschild-Stiglitz model only one contract per type can be sold in equilibrium. We do this by contradiction. Suppose this was not true, i.e. suppose there were two contracts (p_1, q_1) and (p_2, q_2) that are bought by consumers with high risk.

a) Draw in a coverage, premium diagram such two contracts and the indifference curve of the high risk consumers.

b) Draw the isoprofit lines of the insurers through these contracts.

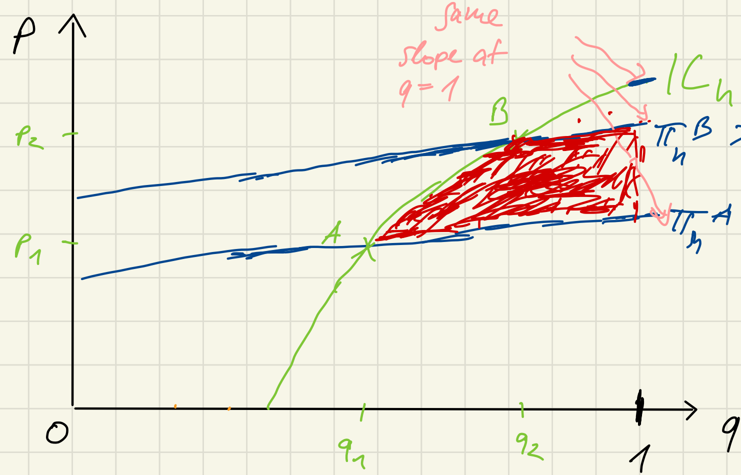
c) Find a deviation contract that yields strictly positive profit (and is bought by some players if offered).

d) Now suppose there were two contracts (p_1, q_1) and (p_2, q_2) that are bought by consumers with **low** risk. Do the same as above but be careful when arguing that the deviation contract is strictly profitable.

Exc. 12

Let's first show that the high risk types will not buy two different contracts.

By contradiction: Let us assume they do buy two different contracts in equilibrium.



a) indifference curve has to go through both contracts as instead they would only buy one of these contracts

b) as insurances want to make positive profits (they would not offer contract A if it made negative profits)

c) All contracts in red area yield positive profits for the insurance and are preferred by the high risk types (because they are below their IC)

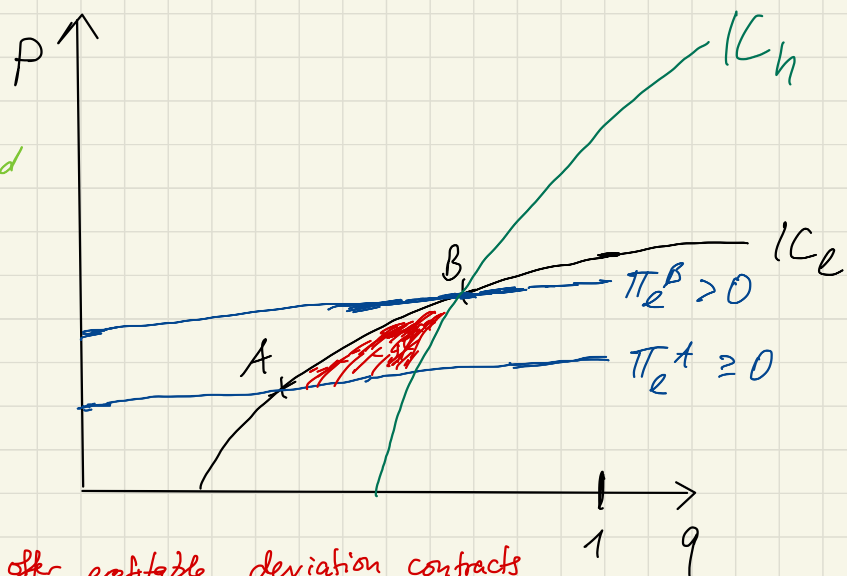
(We do not know whether the low risk type would buy these contracts or not, but we DO know that if she buys it, it will result in positive profits for the insurance - since $\alpha_e < \alpha_h$)

⟹ Contradiction, as a contract in the red region is at least weakly preferred by both the insurances and the consumers.

→ High types do not buy two contracts in equilibrium.

Exc. 12 d)

Now assume there are two contracts $A = (p_1, q_1)$ and $B = (p_2, q_2)$ that are bought by the low types in equilibrium. Want to show: This is impossible since there would be a profitable deviation for insurances.



(high types always prefer B over A since their ICs are steeper)

Note: Offering contracts A and B has to be profitable for insurances independent of what contract high types are buying.

=> Insurances could offer profitable deviation contracts in the red area*, so A and B being sold can not have been an equilibrium.

In conclusion (Exc. 12 a) - d)), no type will buy two (or more) different contracts in equilibrium.

* As the high risk type will not prefer these contracts over contract B.

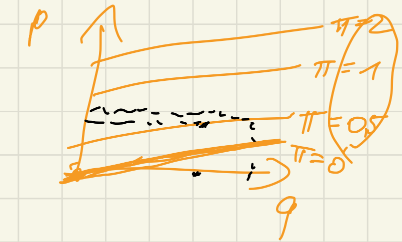
Exercise 13

In the Rothschild-Stiglitz model, assume that all consumers have the utility function $u(x) = -0.5x^2 + 10x$, that $W = 9$, $L = 5$, $\alpha_h = 1/2$ and $\alpha_l = 1/4$.

a) Derive the isoprofit curve of an insurance company insuring a consumer with risk α, i.e. if coverage is q what does the premium have to be to achieve expected profits of $\bar{\pi}$?

Exc. 13 a)

How do the profits of an insurance depend on p and q?



$$\overline{\pi} = p - \alpha \cdot L \cdot q$$

↳ some profit level

$$\Leftrightarrow \quad p = \overline{\pi} + \alpha \cdot L \cdot q$$

$$p = p(q)$$

⇒ isoprofit curves have slope $\alpha \cdot L$

Exercise 13 b)

Derive the consumer's indifference curve, i.e. if coverage is q what does the premium have to be to achieve an expected utility of $\bar{u}$?

Exc. 13 b)

$u(x) = -0{,}5 \cdot x^2 + 10x$

for the high type ($\alpha = \frac{1}{2}$):

$$\bar{u} = \underbrace{\frac{1}{2} \cdot u(9 - p - (1-q) \cdot 5)}_{\text{loss case}} + \underbrace{\frac{1}{2} \cdot u(9-p)}_{\text{no loss case}} = \frac{1}{2}\left(-\frac{1}{2} \cdot (9-p-(1-q) \cdot 5)^2 + 10 \cdot (9-p-(1-q) \cdot 5)\right) + \frac{1}{2} \cdot \left(-\frac{1}{2}(9-p)^2 + 10 \cdot (9-p)\right)$$

calculate and simplify

$$\Leftrightarrow \quad 4\bar{u} = 163 - 2p^2 + 60q - 25q^2 - 14p + 10pq$$

$$\Rightarrow \quad p = \frac{5q-7}{2} + \sqrt{375 - 25q^2 + 50q - 8\bar{u}} \cdot \frac{1}{2}$$

see next pages

for the low type ($\alpha = \frac{1}{4}$):

$$\bar{u} = \frac{1}{4} u(9 - p - (1-q) \cdot 5) + \frac{3}{4} u(9-p)$$

plug in u(x)

$$\Rightarrow \ldots \Rightarrow \quad p(q) = \frac{5q-9}{4} + \sqrt{1525 - 75q^2 + 150q - 32\bar{u}} \cdot \frac{1}{4}$$

equation from 13.b):

$$\bar{u} = \frac{1}{2}\left(-\frac{1}{2}\cdot(9-p-(1-q)\cdot 5)^2 + 10\cdot(9-p-(1-q)\cdot 5)\right) + \frac{1}{2}\left(-\frac{1}{2}(9-p)^2 + 10\cdot(9-p)\right)$$

$$\Leftrightarrow 4\bar{u} = 20(9-p-(1-q)\cdot 5) - (9-p-(1-q)\cdot 5)^2 - (9-p)^2 + 20(9-p)$$

$$= 40(9-p) - 100(1-q) - \left(81 - 9p - 45(1-q) - 9p + p^2 + p(1-q)5 - 45(1-q) + p(1-q)\cdot 5 + (1-q)^2\cdot 25\right) - (81-18p+p^2)$$

$$= 360 - 40p - 100 + 100q - \left(81 - 18p - 90(1-q) + p^2 + 10p(1-q) + (1-q)^2 25\right) - 81 + 18p - p^2$$

$$= 260 - 40p + 100q - 81 + 18p + 90(1-q) - p^2 - 10p(1-q) - (1-q)^2\cdot 25 - 81 + 18p - p^2$$

$$= 98 - 4p + 100q + 90(1-q) - 2p^2 - 10p(1-q) - 25(1-2q+q^2)$$

$$= 163 - 14p + 10pq + 60q - 2p^2 - 25q^2$$

$$= 163 - 2p^2 + 60q - 25q^2 - 14p + 10pq$$

equation from 13.b):

$$4\bar{u} = 163 - 2p^2 + 60q - 25q^2 - 14p + 10pq$$

$$\Leftrightarrow p^2 + \underbrace{(7-5q)}_{\text{"P"}}\cdot p \underbrace{- 81{,}5 - 30q + 12{,}5q^2 + 2\bar{u}}_{\text{"Q"}} = 0$$

$$\Rightarrow p = \frac{5q-7}{2} \pm \sqrt{\frac{(5q-7)^2}{4} - 12{,}5q^2 - 2\bar{u} + 81{,}5 + 30q}$$

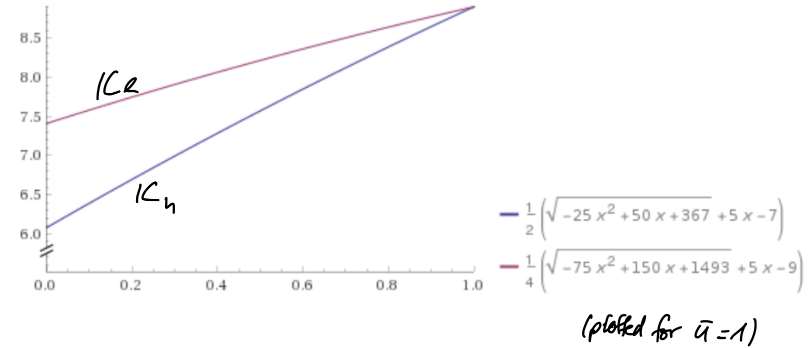
$$= \frac{5q-7}{2} \pm \sqrt{\frac{1}{4}\cdot(25q^2 - 70q + 49) - \frac{1}{4}\left(50q^2 + 8\bar{u} - 326 - 120q\right)}$$

$$= \frac{5q-7}{2} \pm \frac{1}{2}\cdot\sqrt{-25q^2 + 50q + 375 - 8\bar{u}}$$

pq - formula:

$$x^2 + px + q = 0$$

$$\Rightarrow x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$



8.5
8.0
7.5
7.0
6.5
6.0
0.0
0.2
0.4
0.6
0.8
1.0
IC_R
IC_H
$\frac{1}{2}\left(\sqrt{-25x^2+50x+367}+5x-7\right)$
$\frac{1}{4}\left(\sqrt{-75x^2+150x+1493}+5x-9\right)$
(plotted for $\bar{u}=1$)

Exercise 13 c)

Verify that the slope of the indifference curve of a consumer with higher risk is higher. Verify that the slope of the indifference curve is higher than the slope of the isoprofit curve for $q < 1$ and equal for $q = 1$.

Exc. 13 c)

Slope of isoprofit curve : $\alpha \cdot L$ → $\frac{5}{2}$ for h, → $\frac{5}{4}$ for e

for $\alpha = \frac{1}{2}$: $p'(q) = \frac{5}{2} + \underbrace{50(1-q) \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{375 - 25q^2 + 50q - 8\bar{u}}}}_{\geq 0}$

$\underset{q=1}{\Rightarrow} \frac{5}{2} = \alpha_h \cdot L$

for $\alpha = \frac{1}{4}$: $p'(q) = \frac{5}{4} + \underbrace{150(1-q) \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{1525 - 75q^2 + 150q - 32\bar{u}}}}_{\geq 0}$

$\underset{q=1}{=} \frac{5}{4} = \alpha_e \cdot L$

$\sqrt{x} = x^{\frac{1}{2}}$

$\Rightarrow \frac{\partial \sqrt{x}}{\partial x} = \frac{\partial x^{\frac{1}{2}}}{\partial x}$

$= \frac{1}{2} \cdot x^{-\frac{1}{2}}$

$= \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}}$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$

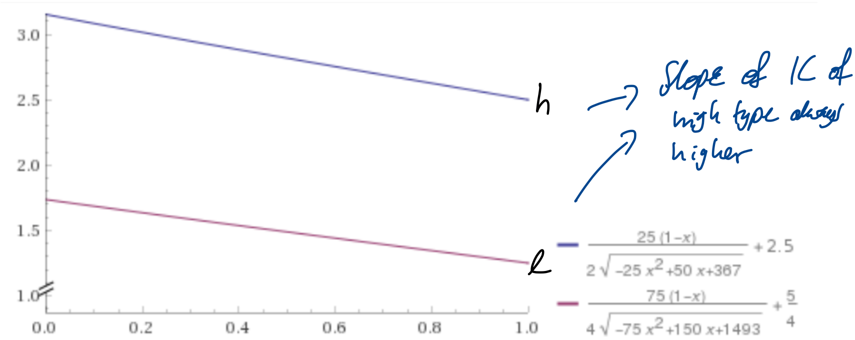
$x^1 \cdot x^1 = x^2$ (1+1 → 2)

$x^2 \cdot x = x^3$

$\sqrt{x} \cdot \sqrt{x} = x^1$

$\Rightarrow \sqrt{x} = x^{\frac{1}{2}}$

Slopes of the indifference curves at level $\bar{u} = 1$



Exercise 13 d)

If risk types were observable what would be the equilibrium contracts for the two risk types?

Exc. 13 d)

risk aversion, competition $\Rightarrow$ full coverage, zero profits

$\Rightarrow$ the contracts will have coverage $q = 1$ and premium $p = \alpha \cdot L$, with $\alpha = \alpha_L$ or $\alpha = \alpha_H$

Exercise 13 e)

What is the Rothschild-Stiglitz equilibrium (i.e. the equilibrium when risk types are not observed by the insurance companies)? For which shares of high risk types is there a full coverage pooling contract breaking this equilibrium?

Exc. 13e)

How to construct the RS-equilibrium (candidate, because sometimes equilibrium does not exist):

- high type gets full coverage contract at fair premium (here: $\alpha_H \cdot L = \frac{5}{2}$)
- contract (p_ℓ, q_ℓ) for the low type satisfies two things:
  1. yields zero profits for insurances from the ℓ-type $\Rightarrow p_\ell = q_\ell \cdot \alpha_L \cdot L = q_\ell \cdot \frac{5}{4}$
  2. high type is indifferent between his contract and (p_ℓ, q_ℓ)

$\Leftrightarrow$ utility of h-type of his contract: $u(9 - \frac{5}{2}) = u(6{,}5) = 43{,}875$

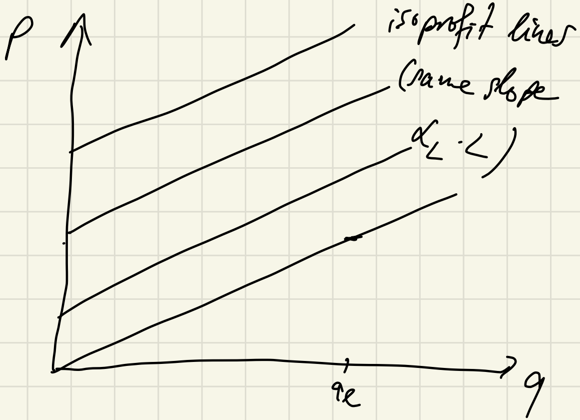
utility of (p_ℓ, q_ℓ)-contract:

$$\frac{1}{2} \cdot u(9 - p_\ell - 5(1-q_\ell)) + \frac{1}{2} u(9 - p_\ell) \overset{!}{=} 43{,}875$$

plug in $p_\ell = q_\ell \cdot \frac{5}{4}$

$\Leftrightarrow \quad q_\ell \approx 0{,}3355$

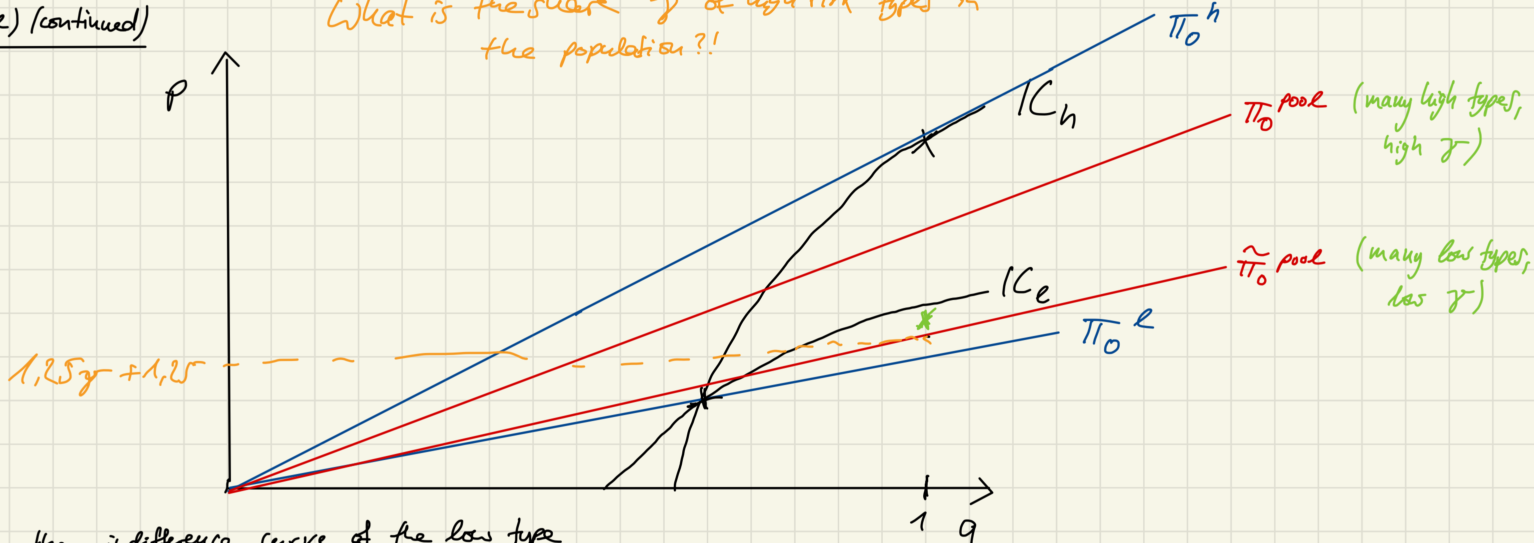
$\Rightarrow \quad p_\ell \approx 0{,}4193$



$\Rightarrow$ Contract of the low type is $(p_\ell, q_\ell) = (0{,}4193; 0{,}3355)$.

Exc. 13e) (continued)

What is the share γ of high risk types in the population?!



Whenever the indifference curve of the low type through her equilibrium candidate contract is above the zero-isoprofit pooling line (also partly above is sufficient), the RS-equilibrium candidate is broken by pooling.

Let us assume the share of high risk types is γ.

Then, the fair premium for a full coverage pooling contract would be $p = \gamma \cdot \alpha_h \cdot L + (1-\gamma)\cdot \alpha_\ell \cdot L$

$$= 1{,}25\gamma + 1{,}25$$

(expected) utility of low type from this full coverage contract:

$$u(9 - 1{,}25 - 1{,}25\gamma) = \ldots \approx 47{,}47 - 2{,}81\gamma - 0{,}78\gamma^2$$

exp. utility from his $(0{,}4193;\ 0{,}3355)$ contract: (p; q)

$$u((0{,}4193;\ 0{,}3355)) = \frac{1}{4}\cdot u(5{,}2582) + \frac{3}{4} u(8{,}5807) \approx 46{,}44$$

for which share γ is the low type indifferent between these two contracts?

$\rightarrow \quad 46{,}44 \overset{!}{=} 47{,}47 - 2{,}81\gamma - 0{,}78\gamma^2$

$\Leftrightarrow \quad \ldots \quad \Leftrightarrow \quad \gamma = 0{,}3365$ (since γ is a share between 0 and 1)

So, the equilibrium candidate is broken for $\gamma < 0{,}3365$ and no equilibrium exists in this case.

Exercise 14 (Briefly dealt with in the lecture)

Suppose the government mandates that coverage levels have to be at least $\bar{q}$. How does this affect the Rothschild-Stiglitz equilibrium? Who benefits/loses from this intervention?

Exc. 14

- If $\bar{q} \leq q_\ell^*$, there is no effect and the same contracts can be offered.
- If $\bar{q} > q_\ell^*$, the insurance offers $(\bar{q}, \tilde{p})$, where $\tilde{p}$ is on h's indifference curve (corresponding to $\bar{q}$).

  In this case, there are two possibilities:

  i) ℓ's indifference curve through $(\bar{q}, \tilde{p})$ is always below the zero-profit pooling line

  $\Rightarrow$ the two contracts form an equilibrium, the situation is more efficient and the low type is worse off and the insurance makes positive profits.

  ii) ℓ's IC through $(\bar{q}, \tilde{p})$ is partly above the zero-profit pooling line

  $\Rightarrow$ the equilibrium (candidate) is broken by pooling. No prediction possible.

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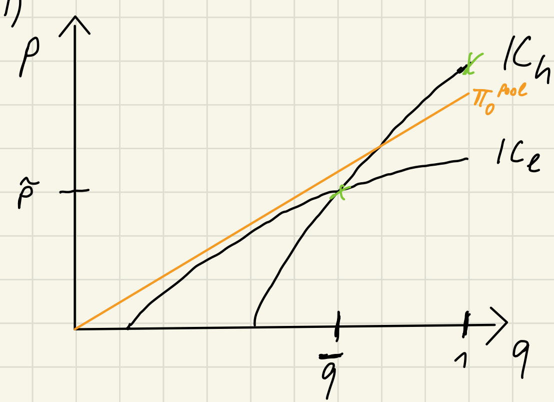
i)



ii)