Imperfect Information in Health Care Markets

Exercise Session 14 - SID, Mock Exam

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Exercise 34

A clinic offers 2 services. Demand for service i is $M_i + s_i$ where $M_i > 0$ is the primary demand and s_i is the induced demand for service $i \in \{1, 2\}$. Let the objective of the clinic be $u(y, s_1, s_2) = -e^{-\eta y} - 0.5s_1^2 - 0.5s_2^2$ where $y = (M_1 + s_1)p_1 + (M_2 + s_2)p_2$ is revenue of the clinic as p_i are the profit margins for the two services and $\eta > 0$ is a parameter.

a) What is the optimal proportion of inducement levels, i.e. s_1/s_2, that the clinic will choose?

b) Will an increase in p_1 in- or decrease the optimal inducement levels s_1 and s_2?

c) In Germany, the physician price for providing a given service to a patient insured in the private arm of the health insurance system is 2.3 times the price of providing the same service to a patient administered in the public arm. What does the model predict in terms of demand inducement?

Exc 34 a) $\quad (y = (\mu_1 + s_1)p_1 + (\mu_2 + s_2)p_2)$

Let's derive the first-order-conditions for s_1 and s_2:

$$\frac{\partial u(y, s_1, s_2)}{\partial s_1} = \frac{\partial(-e^{-\eta y} - 0{,}5\, s_1^2 - 0{,}5\, s_2^2)}{\partial s_1} = (-e^{-\eta y}) \cdot (-\eta) \cdot \underbrace{p_1}_{\frac{dy}{ds_1}} - s_1 \overset{!}{=} 0$$

$$\Leftrightarrow \quad s_1 = \eta \cdot p_1 \cdot e^{-\eta y}$$

$$\frac{\partial u(y, s_1, s_2)}{\partial s_2} = (-e^{-\eta y}) \cdot (-\eta) \cdot p_2 - s_2 \overset{!}{=} 0$$

$$\Leftrightarrow \quad s_2 = \eta \cdot p_2 \cdot e^{-\eta y}$$

From the two above equations, we see that the optimal solutions s_1 and s_2 will satisfy the following:

$$\frac{s_1}{s_2} = \frac{\eta \cdot p_1 \cdot e^{-\eta y}}{\eta \cdot p_2 \cdot e^{-\eta y}} = \frac{p_1}{p_2}$$

So, the optimal proportion is equal to the proportion of profit margins.

Exc. 34 b)

First, let us note that if p_1 increases, y has to increase as well. The reason is:

If y decreased, then by the FOCs from a), s_1 and s_2 would increase, which would yield a higher y.

So we know that y will increase if p_1 increases. But then, s_2 decreases from the FOC with s_2.

Hence: An increase in the profit margin of service 1 leads to less inducement of service 2.

The effect on s_1 is a priori unclear as a higher p_1 leads to a substitution effect increasing s_1 but also to an income effect reducing s_1 (as y increases). However, we could get a more precise result here using the implicit function theorem:

We know from the FOC that $\underbrace{\eta e^{-\eta y} \cdot p_1 - s_1}_{\text{define as } F(p_1, s_1)} = 0$

$\rightsquigarrow$ IFT gives: (implicit function theorem)

$$\frac{\partial s_1}{\partial p_1} = - \frac{\frac{\partial F}{\partial p_1}}{\frac{\partial F}{\partial s_1}} \overset{\text{see next page}}{=} - \frac{\eta e^{-\eta y}\left(1 - \eta p_1\left((M_1 + s_1) - \frac{s_1 p_2^2}{p_1^2}\right)\right)}{-\eta^2 e^{-\eta y}(p_1^2 + p_2^2) - 1} = \frac{\eta\left(1 - \eta p_1 \overbrace{\left((M_1+s_1) - \frac{s_1 p_2^2}{p_1^2}\right)}^{= \frac{\partial y}{\partial p_1} > 0 \text{ (see above)}}\right)}{\eta^2(p_1^2 + p_2^2) + e^{\eta y}}$$

The denominator of this term is always positive, while the numerator is positive for small η and negative for large η.

Hence, $\frac{\partial s_1}{\partial p_1}$ is positive for small η and $\frac{\partial s_1}{\partial p_1}$ is negative for large η.

Calculation using IFT: First, plug $s_2 = \frac{s_1 p_2}{p_1}$ into y to get $y = p_1(\mu_1 + s_1) + p_2\left(\mu_2 + \frac{s_1 p_2}{p_1}\right)$.

Hence, $F(p_1, s_1) = \eta\, p_1\, e^{-\eta\left(p_1(\mu_1+s_1) + p_2\left(\mu_2 + \frac{s_1 p_2}{p_1}\right)\right)} - s_1$. This yields:

$$\frac{\partial F(p_1, s_1)}{\partial s_1} = e^{-\eta y} \cdot \eta\, p_1 \cdot \left(-\eta\, p_1 - \frac{\eta\, p_2^2}{p_1}\right) - 1$$

$$= -\eta^2 e^{-\eta y}\left(p_1^2 + p_2^2\right) - 1$$

$$\frac{\partial F(p_1, s_1)}{\partial p_1} = \eta\, e^{-\eta y} + \eta\, p_1\, e^{-\eta y}\left(-\eta \underbrace{\left[(\mu_1 + s_1) - \frac{s_1 p_2^2}{p_1^2}\right]}_{\text{"inner derivative" } \frac{\partial y}{\partial p_1}}\right)$$

(Product rule for differentiation)

$$= \eta\, e^{-\eta y}\left(1 - \eta\, p_1\left((\mu_1 + s_1) - \frac{s_1 p_2^2}{p_1^2}\right)\right)$$

Exc 34 c)

Private patients will receive 2,3 times as much demand inducement as publicly insured patients (in the model).

If the 2,3 multiplier would be reduced, more demand inducement of publicly insured would result (as clinics want to keep their income).

Exercise 35

A clinic offer 2 services. Demand for service i is $M_i + s_i$ where $M_i > 0$ is the primary demand and s_i is the induced demand for service $i \in \{1, 2\}$. The two services are offered in separate units. Each unit has a leader who chooses the inducement level of this unit. The unit leader receives an income bonus that depends positively on the revenues of his own unit an negatively on the revenues of the other unit (e.g. there is some relative performance bonus). The head of unit i maximizes therefore the utility function $2\sqrt{p_i(1 + s_i) - \alpha p_j(1 + s_j)} - s_i$ where $\alpha \in (0, 1)$ is a parameter measuring the magnitude of relative performance pay.

a) Assume $p_1 = p_2 = 1$ and derive the optimal inducement levels the unit leaders will choose.

b) Assume $p_1 = 1.1$ and $p_2 = 1$ and let $\alpha = 1/2$. Derive the optimal inducement levels the unit leaders will choose.

c) Compare your results with the results in exercise 34.

Exc. 35 a)

$$u_i = 2\sqrt{p_i(1+s_i) - \alpha\, p_j(1+s_j)} - s_i \quad , \qquad p_1 = p_2 = 1$$

We consider the FOC of unit leader i:

inner derivative → outer derivative

$$\frac{\partial u_i}{\partial s_i} = \frac{2 \cdot p_i \cdot \frac{1}{2}}{\sqrt{p_i(1+s_i) - \alpha\, p_j(1+s_j)}} - 1 \overset{!}{=} 0$$

$$\overset{p_i = p_j = 1}{\Leftrightarrow} \quad 1 = \sqrt{1 + s_i - \alpha(1+s_j)}$$

$$\Rightarrow \quad 1 = 1 + s_i - \alpha(1+s_j)$$

As the problem is identical for both unit leaders, we can assume $s_i = s_j$:

$$\Rightarrow 1 = 1 + s_i - \alpha(1+s_i)$$

$$\Leftrightarrow \alpha = (1-\alpha)\, s_i$$

$$\Leftrightarrow s_i = \frac{\alpha}{1-\alpha}$$

Both leaders will induce the amount $\frac{\alpha}{1-\alpha}$.

Exc. 35 b)

We get the same FOC as before.

First, for unit 1:

$$\frac{du_1}{ds_1} = \frac{2 \cdot 1{,}1 \cdot \frac{1}{2}}{\sqrt{1{,}1(1+s_1) - \alpha(1+s_2)}} - 1 \overset{!}{=} 0$$

$$\Leftrightarrow 1{,}1 = \sqrt{1{,}1(1+s_1) - \alpha(1+s_2)}$$

$$\Rightarrow 1{,}21 = 1{,}1(1+s_1) - \alpha(1+s_2)$$

$$\Leftrightarrow 1{,}1(1+s_1) = 1{,}21 + \alpha(1+s_2)$$

Now, similar for unit 2:

$$1+s_2 - \alpha \cdot (1+s_1) \cdot 1{,}1 - 1 \overset{!}{=} 0$$

$$\Leftrightarrow s_2 - \alpha \underbrace{(1+s_1) \cdot 1{,}1}_{\text{plug in from above}} = 0$$

$$\Rightarrow s_2 - \alpha \cdot (1{,}21 + \alpha(1+s_2)) = 0$$

$$\Leftrightarrow 1{,}21\alpha + \alpha^2 = s_2(1-\alpha^2)$$

$$\Rightarrow s_2 = \frac{1{,}21\alpha + \alpha^2}{1-\alpha^2} \overset{\alpha=\frac{1}{2}}{=} 1{,}14$$

→ in Exc a), s_2 would have been 1 for $\alpha = \frac{1}{2}$.

What is s_1?

$$1{,}1(1+s_1) = 1{,}21 + \frac{1}{2}(1+1{,}14)$$

$$\Leftrightarrow 1{,}1 + 1{,}1 s_1 = 1{,}21 + 1{,}07 \quad \Leftrightarrow 1{,}1 s_1 = 1{,}17$$

$$\Leftrightarrow s_1 = \frac{1{,}17}{1{,}1} \approx 1{,}06$$

Exc. 35 c)

S_2 increases as a response to an increase in p_1 while it decreased in Exc. 34.

The reason is that a higher p_1 makes it easier for unit 1 to generate revenue and the unit leader of unit 2 tries to compensate by inducing more. This is a problem for study designs as in the second wave of SID studies: If decisions are made by competing units, price increases in one service can then increase inducement in other services.

Mock Exam, Exc. 2 b) - Moral Hazard

"Old Exam"

A consumer behaves as if maximizing the utility function $u(t) = \mathit{consumption}(t) - (s-t)^2$, where $s \in [0,5]$ is the health state of the consumer and $t \in [0,s]$ is the money amount spent on treatment. The consumer has an insurance that has a zero copayment rate up to treatment expenditures of 2 and a copayment rate of 0.5 for treatment expenditures above 2, i.e. the insurance pays t to the consumer if $t \leq 2$ and $2 + \frac{1}{2}(t-2)$ if $t > 2$. Assume that the initial wealth of the consumer is 10 and the insurance premium is 4. *consumption* is all the money the consumer has left over (after paying his insurance premium and his contribution to the treatment expenses), i.e. $\mathit{consumption} = 10 - 4$ if $t \leq 2$ and $\mathit{consumption} = 10 - 4 - \frac{1}{2}(t-2)$ if $t > 2$.

i) Determine the optimal treatment choice as a function of the health state. *(12 points)*

Mock Exam, Exc. 2 b) - Moral Hazard (cont.)

ii) Why can insurance contracts like the one above be used to empirically test whether moral hazard is relevant? (Also explain why this does not depend on the specific functional form of the utility function.) *(7 points)*

Solution to i) - Alternative 1: Heuristic Approach

Marginal utility of treatment is $2(s-t)$ (given $t \leq s$) or zero (for $t > s$). Marginal out of pocket expenses (=marginal costs of treatment) are either 0 (for $t < 2$) or $1/2$ (for $t > 2$). At the optimal treatment choice, marginal utility has to equal the marginal cost of treatment. Solution candidates are therefore $t = s$, $t = 2$ and $t = s - 1/4$. For $s \leq 2$, $t = s$ is clearly optimal ($u(t) = consumption(t) - (s-t)^2$ and $consumption(t)$ is weakly decreasing in t and the costs are zero for $s = t$). For $s \in (2, 2.25]$, 2 is closer to s than $s - \frac{1}{4}$, making the choice of 2 optimal in this case. This yields

$$t(s) = \begin{cases} s & \text{if } s \leq 2 \\ 2 & \text{if } s \in (2, 2.25] \\ s - 1/4 & \text{else.} \end{cases}$$

Solution to i) - Alternative 2: Mathematical Approach

We can also just take the standard approach and maximize the utility function with respect to t. However, we have to note that the utility function looks different for $t \leq 2$ and $t > 2$ and there might be corner solutions. Hence, we have to solve the following two problems:

1. for $t \leq 2$: $\max_t 10 - 4 - (s-t)^2$

$$FOC: -2(s-t) * (-1) = 2(s-t) \overset{!}{=} 0$$
$$\Leftrightarrow t = s \quad \text{(only consistent for } s \leq 2\text{)}$$

2. for $t > 2$: $\max_t 10 - 4 - \frac{1}{2}(t-2) - (s-t)^2$

$$FOC: -\frac{1}{2} + 2(s-t) \overset{!}{=} 0$$
$$\Leftrightarrow s = \frac{1}{4} + t \Leftrightarrow t = s - \frac{1}{4} \quad \text{(only consistent for } s > 2.25\text{)}$$

Solution to i) - Alternative 2: Mathematical Approach (cont.)

But what happens for $s \in (2, 2.25]$?! To answer this, we have to compare the utilities from the two solution candidates from above **and the corner solution t=2** in these cases.

$$u(t_1^*) = u(t = s) = 6 - \frac{1}{2}(s-2) - 0 = 6 - \frac{s-2}{2}$$

t_1^*

t_2^*

t_3^*

$$u(t = s - \frac{1}{4}) = 6 - (\frac{1}{4})^2$$

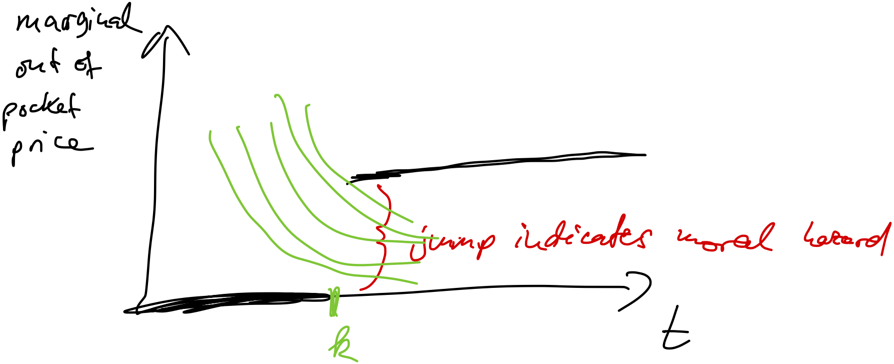
$$u(t = 2) = 6 - (s-2)^2$$

optimal for $s \in (2; 2{,}25]$

Solution to ii)

The model underlying moral hazard, i.e. a consumer chooses treatment to maximize expected utility taking financial consequences explicitly into account, predicts bunching at the kink. More precisely, suppose health states are distributed with some density. Then the distribution of treatment choices will spike at $t = 2$ (and follow a continuous distribution for other values) because all patients with $s \in [2, 2.25]$ choose this treatment level. If there was no moral hazard, i.e. if treatment choice was only driven by medical necessity and did not depend on payments, then there should not be spikes at kink points of the insurance tariff. The functional form of the utility is irrelevant because at a kink point k the marginal out-of-pocket-price jumps up (say from 0 to c) and therefore all consumers whose marginal utility of treatment at k is between 0 and c will consume treatment equal to k. This is a mass while for all other treatments it is not.

Solution to ii) - Graphical intuition



Mock Exam - Exc. 1 c)

Discuss the welfare implications of genetic tests for insurance markets.

Points you could mention here:

Arguments on welfare reduction

- The risk of having bad genetics can not be insured anymore

Arguments on an increase in welfare

- People might change their behavior when they learn their high risk (stop smoking, ...)
- Tests could allow treatment before the disease breaks out

Some conclusion weighing the pros and cons