Imperfect Information in Health Care Markets

Exercise Session 3 - Insurance Demand

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Questions?

Insurance Demand

In all exercises, let the person be an expected utility maximizer, i.e. the person's choices satisfy the assumptions of the von Neumann-Morgenstern expected utility theorem.

Exercise 4

Consider a person with utility of income $u(x) = \sqrt{x}$. Is this person risk averse? For the following lotteries, compute the expected income, the certainty equivalent, and the risk premium.

a) Probability $1/3$ for each 1600, 2500, and 3600 Euros.

b) Income is uniformly distributed between 1600 and 2500 Euros.

$\boxed{4a)}$ $u(x) = \sqrt{x} = x^{\frac{1}{2}}$

Is this person risk averse?

Risk aversion: $u''(x) < 0$ for all $x > 0$

↳ concave utility function

$$u'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$u''(x) = -\frac{1}{4} x^{-\frac{3}{2}} = -\frac{1}{4x^{\frac{3}{2}}} < 0 \text{ for } x > 0$$

⇒ Since $u''(x) < 0$, the person is risk averse

We have shown risk aversion by concavity of the utility function.

Expected income:

$u(X) = \sqrt{x}$

probability $1/3$ for each 1600, 2500 and 3600

$$\mathbb{E}(X) = \frac{1}{3} \cdot 1600 + \frac{1}{3} \cdot 2500 + \frac{1}{3} \cdot 3600$$

$$\approx 2566,66$$

Expected utility:

$$\mathbb{E}(u) = \frac{1}{3} \cdot u(1600) + \frac{1}{3} \cdot u(2500) + \frac{1}{3} \cdot u(3600)$$

$$= \frac{1}{3} \sqrt{1600} + \frac{1}{3} \sqrt{2500} + \frac{1}{3} \cdot \sqrt{3600}$$

$$= \frac{1}{3} (40 + 50 + 60) = 50$$

Certainty equivalent (CE):

→ measures the safe amount of income that makes me indifferent to playing the lottery or receiving the safe amount

$$\underbrace{u(CE)}_{\text{utility at the "safe amount"}} \overset{!}{=} \underbrace{\mathbb{E}(u)}_{\text{expected utility of lottery}}$$

$$\Leftrightarrow \quad \sqrt{CE} = 50 \quad | \; (\;)^2$$

$$\Leftrightarrow \quad CE = 50^2 = 2500$$

$$\mathbb{E}(x) = 2566{,}66 > 2500 = CE$$

Risk premium (RP):

→ difference between the expected payment from a lottery and the certainty equivalent of this lottery

$$RP = \mathbb{E}(x) - CE$$

$$\Leftrightarrow \quad RP = 2566{,}66 - 2500 = 66{,}66 > 0$$

⇒ indication that the person is risk-averse since $RP > 0$

$\boxed{4b)}$

Income is uniformly distributed between 1600 and 2500

$$\underbrace{X}_{\text{Income}} \sim U(1600, 2500)$$

General uniform distribution:

$$X \sim U(a, b)$$

cdf: $F(x) = \dfrac{x-a}{b-a}$ for $x \in [a, b]$

pdf: $f(x) = \dfrac{1}{b-a}$ for $x \in [a, b]$

Here: $X \sim U(1600, 2500)$

cdf: $F(x) = \dfrac{x-a}{b-a} = \dfrac{x-1600}{2500-1600} = \dfrac{x-1600}{900}$ for $x \in [1600, 2500]$

pdf: $f(x) = \dfrac{1}{b-a} = \dfrac{1}{2500-1600} = \dfrac{1}{900}$ for $x \in [1600, 2500]$

Expected income:

or $\dfrac{1}{900} \cdot \left[\dfrac{1}{2}x^2\right]_{1600}^{2500}$

$$\mathbb{E}(x) = \int_a^b x \cdot f(x)\,dx = \int_{1600}^{2500} x \cdot \frac{1}{900}\,dx = \underbrace{\left[\frac{1}{900} \cdot \frac{1}{2}x^2\right]_{1600}^{2500}}$$

$$= \frac{1}{900} \cdot \frac{1}{2} \cdot 2500^2 - \left(\frac{1}{900} \cdot \frac{1}{2} \cdot 1600^2\right)$$

$$= 2050$$

Expected utility:

// $x^{\frac{1}{2}}$

$$\mathbb{E}(u) = \int_a^b u(x)\, f(x)\, dx = \int_{1600}^{2500} \sqrt{x} \cdot \frac{1}{900}\, dx$$

$$= \frac{1}{900} \cdot \left[\frac{1}{\frac{1}{2}+1}\, x^{\frac{1}{2}+1} \right]_{1600}^{2500} = \frac{1}{900} \left[\frac{2}{3}\, x^{\frac{3}{2}} \right]_{1600}^{2500}$$

$$= \frac{1}{900} \left[\frac{2}{3} \cdot 2500^{\frac{3}{2}} - \frac{2}{3} \cdot 1600^{\frac{3}{2}} \right]$$

$$\approx 45{,}18\overline{5}$$

Certainty equivalent:

$u(x) = \sqrt{x}$

$$u(CE) \overset{!}{=} \mathbb{E}(u)$$

$$\sqrt{CE} = 45{,}185 \quad | (\;)^2$$

$$CE = 45{,}185^2 = 2041{,}7 < 2050 = \mathbb{E}(X)$$

↳ indication that person is risk-averse

Risk premium:

$$RP = \mathbb{E}(X) - CE = 2050 - 2041{,}7 = 8{,}3 > 0$$

Exercise 5

Consider a person with utility of income $u(x) = \sqrt{x}$. The person has an income of 2500 Euros but loses L Euros with probability α. Determine the certainty equivalent and the risk premium as a function of α and L. Is the risk premium increasing or decreasing in L? Is the risk premium increasing or decreasing in α?

$\boxed{5}$ $\quad u(x) = \sqrt{x}$

$W = 2500$ $\quad L = \text{loss}$ $\quad \alpha \in (0, 1)$

↳ probability of loss

2 cases:

1. no loss: income of 2500

↳ probability $(1-\alpha)$

2. loss occurs: income of $2500 - L$

↳ probability α

$$(1-\alpha) \cdot 2500 + \alpha \cdot 2500$$
$$= 2500 - \alpha \cdot 2500 + \alpha \cdot 2500$$
$$= 2500$$

Expected income:

$$\mathbb{E}(X) = \underbrace{(1-\alpha) \cdot 2500}_{\text{no loss}} + \underbrace{\alpha \cdot (2500 - L)}_{\text{loss occurs}} = 2500 - \alpha \cdot 2500 + \alpha \cdot 2500 - \alpha \cdot L$$

$$= 2500 - \alpha \cdot L$$

Expected utility:

$$E(u) = (1-\alpha)\cdot u(2500) + \alpha \cdot u(2500 - L)$$

$$= (1-\alpha)\cdot \sqrt{2500} + \alpha \cdot \sqrt{2500 - L}$$

$$= (1-\alpha)\cdot 50 + \alpha \cdot \sqrt{2500 - L}$$

Certainty equivalent:

binomial formula
$(a+b)^2 = a^2 + 2ab + b^2$

$$u(CE) \overset{!}{=} \mathbb{E}(u)$$

$$\sqrt{CE} = \alpha \cdot \sqrt{2500-L} + 50 \cdot (1-\alpha) \quad | \; (\;)^2$$

$$CE = (\underbrace{\alpha \cdot \sqrt{2500-L}}_{a} + \underbrace{50 \cdot (1-\alpha)}_{b})^2$$

$$= (\alpha \cdot \sqrt{2500-L})^2 + 2 \cdot \alpha \cdot \sqrt{2500-L} \cdot 50 \cdot (1-\alpha)$$

$$+ (50 \cdot (1-\alpha))^2$$

$$= \alpha^2 \cdot (2500-L) + 100 \cdot \sqrt{2500-L} \cdot \alpha \cdot (1-\alpha)$$

$$+ 2500 (1-\alpha)^2$$

Risk premium:

$$RP = \mathbb{E}(X) - CE$$

$$RP = 2500 - \alpha \cdot L - \overbrace{\alpha^2 (2500 - L)}^{-2500 \cdot \alpha^2 + \alpha^2 \cdot L} - 100 \cdot \sqrt{2500 - L} \cdot \alpha (1 - \alpha$$

$$\underbrace{-2500 (1 - \alpha)^2}$$

$$-2500 \cdot (1^2 - 2 \cdot 1 \cdot \alpha + \alpha^2) = -2500 \cdot (1 - 2\alpha + \alpha^2)$$

$$= -2500 + 5000\alpha - 2500\alpha^2$$

$$RP = -\alpha \cdot L + 5000\alpha \underbrace{- 2\alpha^2 \cdot 2500}_{-5000\alpha^2} + \alpha^2 \cdot L - 100 \cdot (1-\alpha) \cdot \alpha \sqrt{2500 - L}$$

Is the risk premium increasing or decreasing in L?

$$RP = -\alpha L + 5000\alpha - 2\alpha^2 \cdot 2500 + \alpha^2 \cdot L - 100(1-\alpha)\alpha\sqrt{2500-L}$$

$$\frac{\partial RP(\alpha,L)}{\partial L} = -\alpha + \alpha^2 - 100\overbrace{(1-\alpha)\alpha}^{=\alpha-\alpha^2}\frac{1}{2}\cdot(2500-L)^{-\frac{1}{2}}\cdot(-1)$$

$$= \alpha^2 - \alpha + 50\underbrace{(\alpha-\alpha^2)}_{-(\alpha^2-\alpha)}\frac{1}{\sqrt{2500-L}}$$

$$= \underbrace{(\alpha^2-\alpha)}_{<0}\underbrace{\left(1 - 50\cdot\frac{1}{\sqrt{2500-L}}\right)}_{<0} > 0$$

since $\alpha \in (0,1)$

since $\frac{50}{\sqrt{2500-L}} > 1$

as $\sqrt{2500-L} < 50$ for $L>0$

$\sqrt{2500} = 50$

$\Rightarrow$ The risk premium increases with a higher loss (L)