Imperfect Information in Health Care Markets

Exercise Session 4 - Insurance Demand

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Questions?

Exercise 5

Consider a person with utility of income $u(x) = \sqrt{x}$. The person has an income of 2500 Euros but loses L Euros with probability α. Determine the certainty equivalent and the risk premium as a function of α and L. Is the risk premium increasing or decreasing in L? Is the risk premium increasing or decreasing in α?

$\boxed{5}$ $\quad u(x) = \sqrt{x}$

$W = 2500$ $\quad L = \text{loss}$ $\quad \alpha \in (0, 1)$

↳ probability of loss

2 cases:

1. no loss: income of 2500

↳ probability $(1-\alpha)$

2. loss occurs: income of $2500 - L$

↳ probability α

$(1-\alpha) \cdot 2500 + \alpha \cdot 2500$
$= 2500 - \alpha \cdot 2500 + \alpha \cdot 2500$
$= 2500$

Expected income:

$$\mathbb{E}(X) = \underbrace{(1-\alpha) \cdot 2500}_{\text{no loss}} + \underbrace{\alpha \cdot (2500 - L)}_{\text{loss occurs}} = 2500 - \alpha \cdot 2500 + \alpha \cdot 2500 - \alpha \cdot L$$

$$= 2500 - \alpha \cdot L$$

Expected utility:

$$E(u) = (1-\alpha)\cdot u(2500) + \alpha \cdot u(2500-L)$$

$$= (1-\alpha)\cdot\sqrt{2500} + \alpha\cdot\sqrt{2500-L}$$

$$= (1-\alpha)\cdot 50 + \alpha\cdot\sqrt{2500-L}$$

Certainty equivalent:

binomial formula
$(a+b)^2 = a^2 + 2ab + b^2$

$$u(CE) \stackrel{!}{=} \mathbb{E}(u)$$

$$\sqrt{CE} = \alpha \cdot \sqrt{2500-L} + 50 \cdot (1-\alpha) \quad | \, (\;)^2$$

$$CE = (\underbrace{\alpha \cdot \sqrt{2500-L}}_{a} + \underbrace{50 \cdot (1-\alpha)}_{b})^2$$

$$= (\alpha \cdot \sqrt{2500-L})^2 + 2 \cdot \alpha \cdot \sqrt{2500-L} \cdot 50 \cdot (1-\alpha)$$

$$+ (50 \cdot (1-\alpha))^2$$

$$= \alpha^2 \cdot (2500-L) + 100 \cdot \sqrt{2500-L} \cdot \alpha \cdot (1-\alpha)$$

$$+ 2500 (1-\alpha)^2$$

Risk premium:

$$RP = \mathbb{E}(X) - CE$$

$$RP = 2500 - \alpha \cdot L \overbrace{- \alpha^2 (2500 - L)}^{-2500 \cdot \alpha^2 + \alpha^2 \cdot L} - 100 \cdot \sqrt{2500 - L} \cdot \alpha (1 - \alpha)$$

$$\underbrace{-2500 (1 - \alpha)^2}$$

$$-2500 \cdot (1^2 - 2 \cdot 1 \cdot \alpha + \alpha^2) = -2500 \cdot (1 - 2\alpha + \alpha^2)$$

$$= -2500 + 5000\alpha - \underline{2500\alpha^2}$$

$$RP = -\alpha \cdot L + 5000\alpha \underbrace{- 2\alpha^2 \cdot 2500}_{-5000\alpha^2} + \alpha^2 \cdot L - 100 \cdot (1-\alpha) \cdot \alpha \sqrt{2500 - L}$$

Is the risk premium increasing or decreasing in L?

$$RP = -\alpha L + 5000\alpha - 2\alpha^2 \cdot 2500 + \alpha^2 \cdot L - 100(1-\alpha)\alpha\sqrt{2500-L}$$

$$\frac{\partial RP(\alpha, L)}{\partial L} = -\alpha + \alpha^2 - \overbrace{100(1-\alpha)\alpha}^{=\alpha-\alpha^2}\,\frac{1}{2}\cdot(2500-L)^{-\frac{1}{2}}\cdot(-1)$$

$$= \alpha^2 - \alpha + 50\underbrace{(\alpha-\alpha^2)}_{-(\alpha^2-\alpha)}\frac{1}{\sqrt{2500-L}}$$

$$= \underbrace{(\alpha^2-\alpha)}_{<0}\underbrace{\left(1 - 50\cdot\frac{1}{\sqrt{2500-L}}\right)}_{<0} > 0$$

since $\alpha \in (0,1)$

since $\frac{50}{\sqrt{2500-L}} > 1$

$\Rightarrow$ The risk premium increases with a higher loss (L)

as $\sqrt{2500-L} < 50$ for $L>0$

$\sqrt{2500} = 50$

Is the risk premium increasing or decreasing in α?

$$RP = -\alpha L + 5000\alpha - \underbrace{2\alpha^2 \cdot 2500}_{-5000\alpha^2} + \alpha^2 \cdot L - 100 \overbrace{(1-\alpha)\alpha}^{(\alpha-\alpha^2)} \sqrt{2500-L}$$

$$- (\alpha - \alpha^2) \cdot 100 \cdot \sqrt{2500-L}$$

$$\frac{\partial RP(\alpha, L)}{\partial \alpha} = -L + 5000 - 10000\alpha + 2\alpha \cdot L - 100 \cdot \sqrt{2500-L}\,(1-2\alpha)$$

This is neither always ≥ 0 nor always ≤ 0.

For example, for $L = 900$, we get

$$\frac{\partial RP(\alpha, 900)}{\partial \alpha} = 100 - 200 \cdot \alpha \begin{cases} \to \geq 0 & \text{for } \alpha \leq \frac{1}{2} \\ \to < 0 & \text{for } \alpha > \frac{1}{2} \end{cases}$$

Exercise 6

Consider the utility function $u(x) = -e^{-\eta x}$. The person has an income of 1 and experiences a loss of 1 with probability α. The coefficient of absolute risk aversion is defined as $-u''(x)/u'(x)$. Compute this coefficient. Let now $\alpha = 0.5$ and check whether the certainty equivalent in- or decreases in η.

$\boxed{6}$ $u(x) = -e^{-rx}$

Two cases:

1. no loss: income of 1

$\hookrightarrow$ probability $(1-\alpha)$

2. loss occurs: income of $1-1 = 0$

$\hookrightarrow$ probability α

Coefficient of absolute risk aversion:

$$u(x) = -e^{-\eta x}$$

$$f(x) = e^{x}$$

$$f'(x) = e^{x}$$

$$-\frac{u''(x)}{u'(x)}$$

$$u'(x) = (-\eta) \cdot -e^{-\eta x} = \eta e^{-\eta x}$$

$$u''(x) = \eta \cdot (-\eta)\, e^{-\eta x} = -\eta^2 \cdot e^{-\eta x}$$

$$-\frac{u''(x)}{u'(x)} = -\frac{-\eta^2 \cdot e^{-\eta x}}{\eta e^{-\eta x}} = -\frac{-\eta^2}{\eta} = \eta$$

measures risk aversion

Certainty equivalent: for $\alpha = \frac{1}{2}$ $\quad u(x) = -e^{-rx}$

Expected utility:

$$\mathbb{E}(u) = (1-\alpha) \cdot u(1) + \alpha \cdot u(0) = \frac{1}{2} \cdot u(1) + \frac{1}{2} \cdot u(0)$$

$$= \frac{1}{2} \cdot \left(-e^{-r \cdot 1}\right) + \frac{1}{2} \cdot \left(-e^{\overbrace{-r \cdot 0}^{=0}}\right)$$

$$= -\frac{1}{2} e^{-r} + \frac{1}{2} \cdot (-1) = -\frac{1}{2} e^{-r} - \frac{1}{2}$$

$$u(x) = -e^{-rx}$$

$$u(CE) \stackrel{!}{=} \mathbb{E}(u)$$

$$\Leftrightarrow \quad -e^{-r \cdot CE} = -\frac{1}{2} e^{-r} - \frac{1}{2} \quad | \cdot (-1)$$

$$\Leftrightarrow \quad e^{-r \cdot CE} = \frac{1}{2} e^{-r} + \frac{1}{2} \quad | \ln(\,)$$

$$\ln(e^{x}) = x$$

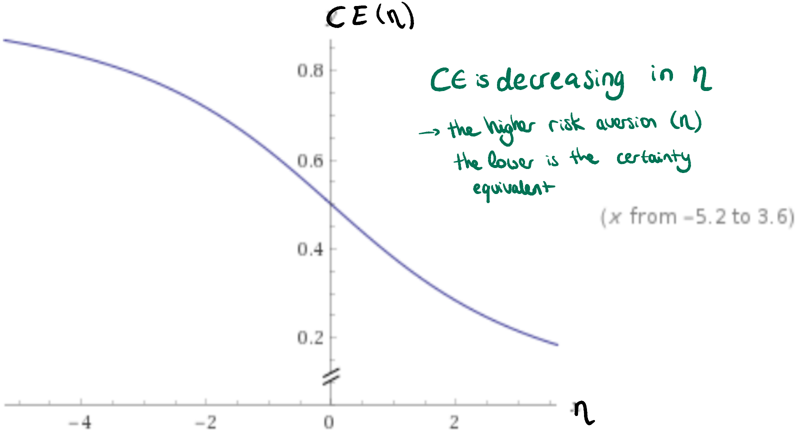
$$\Leftrightarrow \quad \ln(e^{-r \cdot CE}) = \ln\left(\frac{1}{2} e^{-r} + \frac{1}{2}\right)$$

$$\Leftrightarrow \quad -r \cdot CE = \ln\left(\frac{1}{2} e^{-r} + \frac{1}{2}\right) \quad | : (-r)$$

$$\Leftrightarrow \quad CE = -\frac{\ln\left(\frac{1}{2} e^{-r} + \frac{1}{2}\right)}{r}$$

CE depending on η for $\alpha = 0.5$

$$CE(\eta) = \frac{-\ln(0.5 + 0.5e^{-\eta})}{\eta}$$



CE is decreasing in η

→ the higher risk aversion (η) the lower is the certainty equivalent

Exercise 7

The Wall Street Journal reported in 2006 of "mini-medical insurance plans". These plans cover routine services, but little hospital coverage and usually have a cap on payouts (say of \$10.000). The premium, however, is only \$50 per month. Why might people buy a mini-medical plan? Why are such insurance plans not more popular (in a country where a substantial part of the population did/does not have health insurance)?

7 Mini-medical insurance plan

Why might people like such a plan?

- Affordability
  → they might have a low willingness to pay and 50$ is quite cheap
- access to care that's covered
- think they have lower risk for severe illnesses / expensive treatments
- act as "consumption smoothing"
  ↳ don't pay one big amount at once but several small amounts more often
- access to preventive care
  → maintain overall health which may help prevent more severe health issues down the line.

Why are such plans not more popular?

- If you think that you have a high risk of severe health issues you know these will most likely not be covered

- advantages of full coverage insurances
  - → higher premium but they provide higher protection
    - → not as vulnerable to high hospitalization or specialized care costs.

- Cap on payouts

  e.g. 10 000 could often be too low to cover significant medical expenses

  → risk/fear of significant out-of-pocket costs

$$50 > \underbrace{\alpha}_{\text{risk}} \cdot \underbrace{L}_{\text{loss}} + \text{administrative costs (+ profit margin)}$$

→ These plans could be too expensive for insurance companies

Exercise 8

Consider a person with utility of income $u(x) = \sqrt{x}$. The person has an income of 2500 Euros but loses 1500 Euros with probability $1/4$. Assume there is an insurance company that offers to insure an arbitrary coverage $C \in [0, 1500]$ at premium pC. Determine the amount of coverage $C(p)$ that the person will buy. (If you find this too hard, let p be 0.3.)

$\boxed{8}$ $u(x) = \sqrt{x}$

Two cases: (without insurance)

1. The loss occurs: Income of $2500 - 1500 = 1000$

   ↳ probability $\frac{1}{4}$

2. No loss: Income of 2500

   ↳ probability $1 - \frac{1}{4} = \frac{3}{4}$

There is an insurance company that offers to insure an arbitrary coverage $C \in [0, 1500]$ at premium pC.

What is the amount of coverage $C(p)$ that the person will buy?

$\rightarrow$ Maximize expected utility with respect to C

$$\mathbb{E}^{\text{noins.}}(u) = \frac{1}{4}\sqrt{2500-1500} + \frac{3}{4}\sqrt{2500} \approx 45{,}41$$

$\hookrightarrow$ Expected utility without insurance

Expected utility with insurance:

$$\mathbb{E}^{\text{ins.}}(u) = \frac{1}{4}\sqrt{1000+C-pC} + \frac{3}{4}\sqrt{2500-pC}$$

$$\max_{c} \mathbb{E}^{ins.}(u) = \frac{1}{4} \cdot \sqrt{1000 + c - pc} + \frac{3}{4} \sqrt{2500 - pc}$$

first-order-condition

FOC: $$\frac{\partial \mathbb{E}^{ins.}(u)}{\partial c} \overset{!}{=} 0$$

$$\frac{1}{4} \cdot \frac{1}{2} (1000 + c - pc)^{-\frac{1}{2}} \cdot (1 - p) + \frac{3}{4} \cdot \frac{1}{2} (2500 - pc)^{-\frac{1}{2}} \cdot (-p) \overset{!}{=} 0$$

$$\Leftrightarrow \frac{1}{8} \frac{1}{\sqrt{1000 + c - pc}} (1-p) - \frac{3}{8} \cdot p \frac{1}{\sqrt{2500 - pc}} = 0 \qquad | \cdot 8$$

$$\Leftrightarrow \frac{1-p}{\sqrt{1000 + c - pc}} - \frac{3p}{\sqrt{2500 - pc}} = 0$$

$$\Leftrightarrow \frac{1-p}{\sqrt{1000 + c - pc}} = \frac{3p}{\sqrt{2500 - pc}} \qquad | ()^2$$

$$\Leftrightarrow \left(\frac{1-p}{\sqrt{1000+C-pC}}\right)^2 = \left(\frac{3p}{\sqrt{2500-pC}}\right)^2$$

$$\Leftrightarrow \frac{(1-p)^2}{1000+C-pC} = \frac{(3p)^2}{2500-pC}$$

$\rightarrow 3^2p^2 = 9p^2$

For $p = 0{,}3$ this gives $C \approx 581{,}2$

$$\Leftrightarrow (1-p)^2 \cdot (2500-pC) = 9p^2 \cdot (1000 + C - pC)$$

$$\Leftrightarrow (1-p)^2 \cdot 2500 - (1-p)^2 \cdot pC = 9000p^2 + 9p^2C - 9p^3C$$

$$\Leftrightarrow -(1-p)^2 \cdot pC - 9p^2C + 9p^3C = 9000p^2 - (1-p)^2 \cdot 2500$$

$$\Leftrightarrow C \cdot \left(-(1-p)^2 \cdot p - 9p^2 + 9p^3\right) = 9000p^2 - (1-p)^2 \cdot 2500$$

$$\Leftrightarrow C = \frac{9000p^2 - (1-p)^2 \cdot 2500}{-(1-p)^2 \cdot p - 9p^2 + 9p^3}$$

$\rightarrow$ This is the coverge the person would choose