Imperfect Information in Health Care Markets

Exercise Session 7 - Rothschild-Stiglitz

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Questions?

Exercise 12

In this exercise we show that in the Rothschild-Stiglitz model only one contract per type can be sold in equilibrium. We do this by contradiction. Suppose this was not true, i.e. suppose there were two contracts (p_1, q_1) and (p_2, q_2) that are bought by consumers with high risk.

a) Draw in a coverage, premium diagram such two contracts and the indifference curve of the high risk consumers.

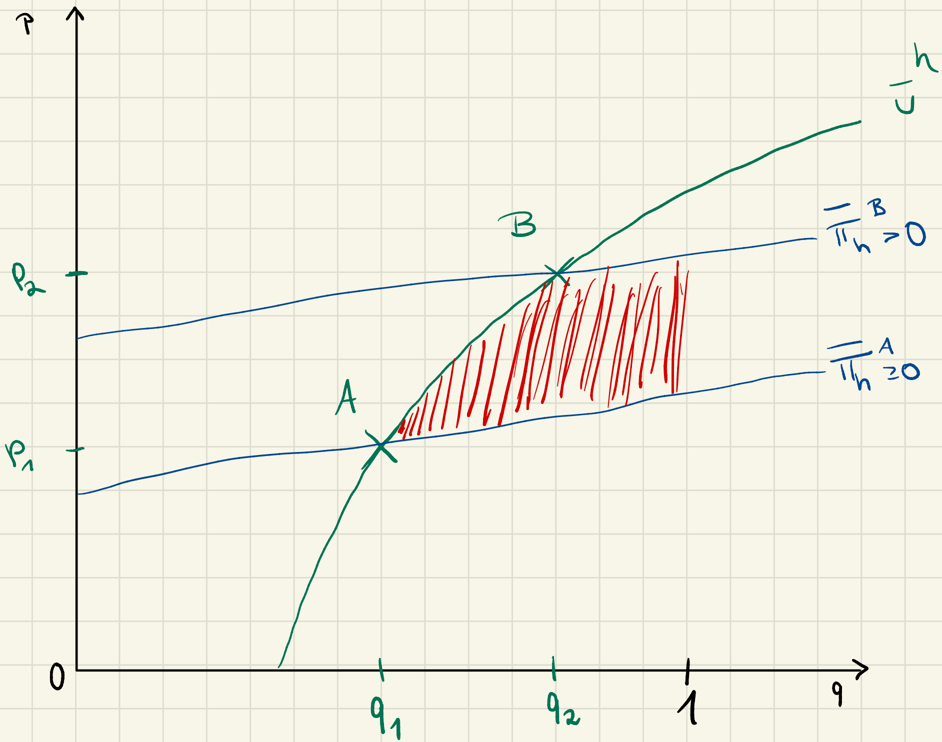
b) Draw the isoprofit lines of the insurers through these contracts.

c) Find a deviation contract that yields strictly positive profit (and is bought by some players if offered).

d) Now suppose there were two contracts (p_1, q_1) and (p_2, q_2) that are bought by consumers with **low** risk. Do the same as above but be careful when arguing that the deviation contract is strictly profitable.

12 Show that the high types will not buy two different contracts.

By contradiction: Start by assuming that they <u>do</u> buy different contracts in equilibrium.



a) The indifference curve has to go through both contract as otherwise they would only buy one of these contracts.

b) As insurances want to make positive profits, they would not offer contract A if it made negative profits. (iso-profit lines have to flatter than IC for $q < 1$)

c) All contracts in the red-area yield positive profits for the insurance and are preferred by the high risk types (because they are <u>below</u> their IC)

(We do not know whether the low risk type would buy these contracts or not, but we DO know that if he buys it, it will result in positive profits for the insurance since $\alpha_e < \alpha_h$)

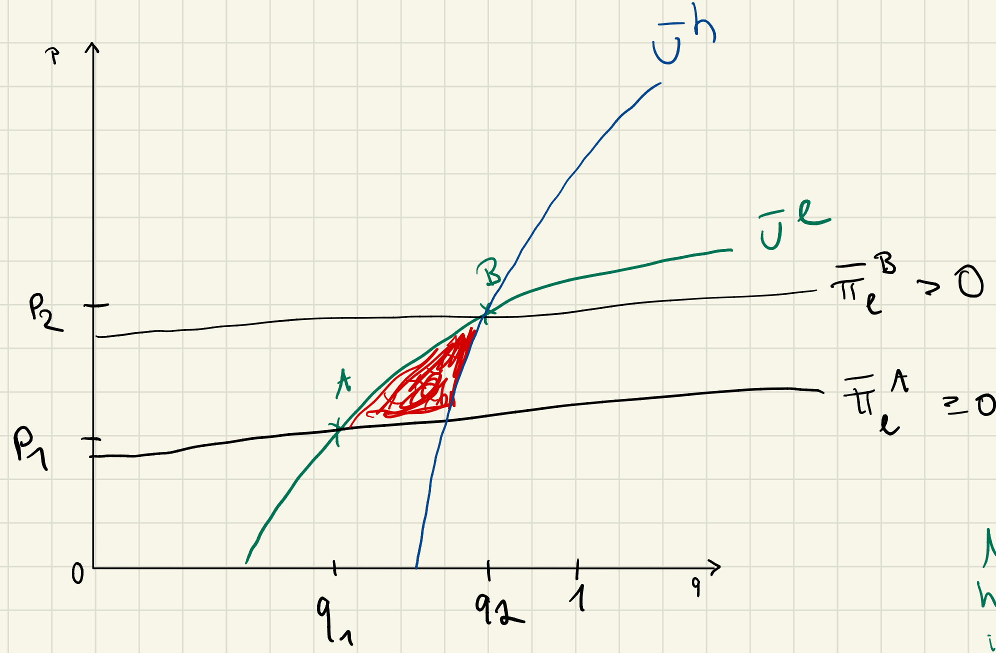
$\Rightarrow$ Contradiction, as a contract in the red region is at least weakly preferred by both the insurances and the consumers

$\rightarrow$ High types do not buy two contracts in equilibrium

12 d)

Now assume there are two contracts $A = (p_1, q_1)$ and $B = (p_2, q_2)$ that are bought by the low type in equilibrium.

Want to show: This is not possible since there would be profitable deviations



high types always prefer B over A since A is to the left of IC^h

Note: offering contracts A and B has to be profitable for insurances independent of what contract is bought

⇒ Insurances could offer profitable deviation contracts in the red area,

So A and B being sold cannot be an equilibrium.

The high type will <u>not</u> prefer these contracts over contract B

$\boxed{\text{Ex. 12 a)-d)}}$

⇒ In conclusion, no type will buy two (or more) different contracts

in equilibrium.

Exercise 13

In the Rothschild-Stiglitz model, assume that all consumers have the utility function $u(x) = -0.5x^2 + 10x$, that $W = 9$, $L = 5$, $\alpha_h = 1/2$ and $\alpha_l = 1/4$.

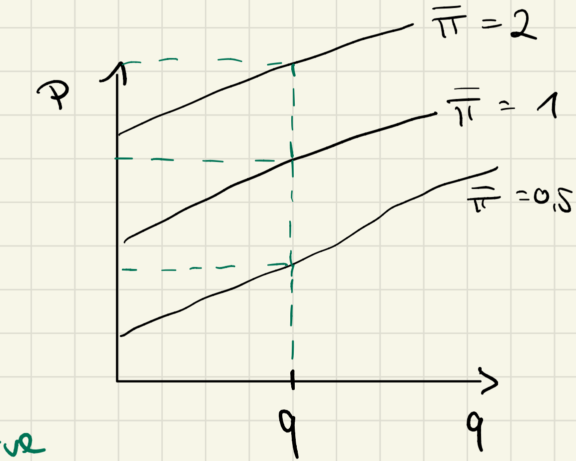
a) Derive the isoprofit curve of an insurance company insuring a consumer with risk α, i.e. if coverage is q what does the premium have to be to achieve expected profits of $\bar{\pi}$?

13 a)

$u(x) = -0{,}5x^2 + 10x, \quad W = 9, \quad L = 5, \quad \alpha_h = \frac{1}{2}, \quad \alpha_\ell = \frac{1}{4}$

Isoprofit curve:

What does the premium p have to be if coverage is q and the insurance wants to achieve expected profits of $\bar{\pi}$.



Profits:

$$\underbrace{\pi}_{\text{premium}} = p - \underbrace{\alpha \cdot L}_{\text{exp. loss}} \cdot \underbrace{q}_{\text{↳ coverage}} \overset{!}{=} \underbrace{\bar{\pi}}_{\text{↳ some fixed profit level}}$$

$$\Leftrightarrow \underbrace{p}_{p(q \mid \pi = \bar{\pi})} = \bar{\pi} + \underbrace{\alpha \cdot L}_{\text{slope of the isoprofit curve}} \cdot q \quad \rightarrow \text{Isoprofit curve}$$

Exercise 13 b)

Derive the consumer's indifference curve, i.e. if coverage is q what does the premium have to be to achieve an expected utility of $\bar{u}$?

$\boxed{13b)}$ $u(x) = -0{,}5x^2 + 10x$, $W = 9$, $L = 5$, $\alpha_h = \frac{1}{2}$, $\alpha_\ell = \frac{1}{4}$

Indifference curve:

What does the premium (p) have to be if coverage is q to achieve an expected utility of $\bar{U}$?

For the high type $(\alpha_h = \frac{1}{2})$:

$$\bar{U} = \underbrace{\frac{1}{2} U(\underbrace{9}_{W} - p - (1-q)\cdot \underbrace{5}_{L})}_{\text{loss case}} + \underbrace{\frac{1}{2} U(9 - p)}_{\text{no loss case}}$$

$$\Leftrightarrow \bar{U} = \frac{1}{2} \cdot \left[-\frac{1}{2} \cdot (9 - p - (1-q)\cdot 5)^2 + 10 \cdot (9 - p - (1-q)\cdot 5 \right]$$

$$+ \frac{1}{2} \left[-\frac{1}{2} (9-p)^2 + 10 (9-p) \right]$$

$$\Leftrightarrow \bar{v} = \frac{1}{4}\left[-(9-p-(1-q)\cdot 5)^2 + 20\cdot(9-p-(1-q)\cdot 5)\right]$$

↳ has to be multiplied by 2 to take out the $\frac{1}{2}$

$$+ \frac{1}{4}\left[-(9-p)^2 + 20\cdot(9-p)\right] \qquad | \cdot 4$$

$9-p-5+5q = 4-p+5q$

$$\Leftrightarrow 4\bar{v} = -(9-p-(1-q)\cdot 5)^2 + 20\cdot(9-p-(1-q)\cdot 5)$$
$$-(9-p)^2 + 20\cdot(9-p)$$

$$\Leftrightarrow 4\bar{v} = 40(9-p) - 100(1-q) - (4-p+5q)^2 - (81-18p+p^2)$$

$$\Leftrightarrow 4\bar{v} = 360 - 40p - 100 + 100q - 16 + 8p - 40q$$
$$-p^2 - 25q^2 + 10pq - 81 + 18p - p^2$$

$(4-p+5q)(4-p+5q)$
$= 16 - 4p + 20q - 4p$
$+p^2 - 5pq + 20q$
$-5pq + 25q^2$

$= 16 - 8p + 40p + p^2$
$+ 25q^2 - 10pq$

$$\Leftrightarrow 4\bar{v} = 163 - 14p + 60q - 2p^2 - 25q^2$$
$$+ 10pq$$

$$\Leftrightarrow \quad 2p^2 + 14p - 10pq - 163 - 60q + 25q^2 + 4\bar{u} = 0 \qquad |:2$$

$$\Leftrightarrow \quad p^2 + 7p - 5pq - 81{,}5 - 30q + 12{,}5q^2 + 2\bar{u} = 0$$

$$x^2 + px + q = 0$$

$$\Leftrightarrow \quad p^2 + \underbrace{(7-5q)}_{P}\, p \; \underbrace{- 81{,}5 - 30q + 12{,}5q^2 + 2\bar{u}}_{q} = 0$$

$$p_{1,2} = \frac{-(7-5q)}{2} \pm \sqrt{\left(\frac{7-5q}{2}\right)^2 + 81{,}5 + 30q - 12{,}5q^2 - 2\bar{u}}$$

$$p_{1,2} = \frac{5q-7}{2} \pm \sqrt{\frac{1}{4}\left(49 - 70q + 25q^2\right) - \frac{1}{4}\left(50q^2 + 8\bar{u} - 326 - 120q\right)}$$

$$p_{1,2} = \frac{5q-7}{2} \pm \frac{1}{2}\sqrt{-25q^2 + 50q + 375 - 8\bar{u}}$$

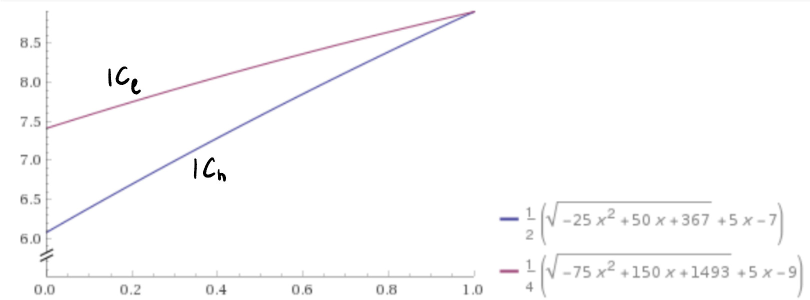
$$\rightarrow p(q) = \frac{5q-7}{2} + \frac{1}{2}\sqrt{-25q^2 + 50q + 375 - 8\bar{u}}$$

For the low type ($\alpha_e = \frac{1}{4}$):

$$\bar{U} = \frac{1}{4}\left(U(9-p-(1-q)\cdot 5\right) + \frac{3}{4} U(9-p)$$

=> plug in $U(x)$ ...

$$\Rightarrow p(q) = \frac{5q-9}{4} + \sqrt{1525 - 75q^2 + 150q - 32\bar{U}} \cdot \frac{1}{4}$$



plotted for $\bar{v} = 1$

Exercise 13 c)

Verify that the slope of the indifference curve of a consumer with higher risk is higher. Verify that the slope of the indifference curve is higher than the slope of the isoprofit curve for $q < 1$ and equal for $q = 1$.

13c)

Verify that the slope of the indifference curve of a consumer with higher risk is higher.

<u>For high type ($\alpha_h = \frac{1}{2}$):</u>

$$p(q) = \frac{5q-7}{2} + \frac{1}{2} \cdot \sqrt{375 - 25q^2 + 50q - 8\bar{u}}$$

$$p'(q) = \frac{5}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{375 - 25q^2 + 50q - 8\bar{u}}} \cdot (-50q + 50)$$

$$= \frac{5}{2} + \underbrace{\frac{1}{4} \cdot 50(1-q) \frac{1}{\sqrt{375 - 25q^2 + 50q - 8\bar{u}}}}_{\geq 0}$$

for $q = 1$

$$p'(1) = \frac{5}{2}$$

For the low type ($\alpha_e = \frac{1}{4}$):

$$p(q) = \frac{5q-9}{4} + \frac{1}{4} \cdot \sqrt{1525 - 75q^2 + 150q - 32\bar{v}}$$

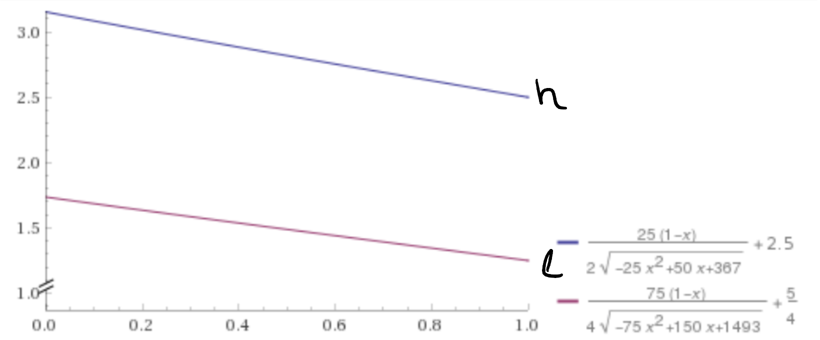
$$p'(q) = \frac{5}{4} + \frac{1}{4} \cdot \frac{1}{2} \frac{1}{\sqrt{1525 - 75q^2 + 150q - 32\bar{v}}} \cdot (-150q + 150)$$

$$= \frac{5}{4} + \underbrace{\frac{1}{8} \cdot 150(1-q) \frac{1}{\sqrt{1525 - 75q^2 + 150q - 32\bar{v}}}}_{\geq 0}$$

For $q = 1$

$$p'(1) = \frac{5}{4}$$

Slopes of the indifference curves at $\bar{u} = 1$



Slope of IC of high type is always higher

Verify that the slope of the IC is higher than the slope of the isoprofit curve for $q < 1$ and equal for $q = 1$.

Isoprofit curve:

$$p(q) = \overline{\pi} + \underbrace{\alpha \cdot L}_{\text{slope of isoprofit curve}} \cdot q$$

Slope of isoprofit curve for the high type:

$$\alpha_h \cdot L = \frac{1}{2} \cdot 5 = \frac{5}{2} < \text{Slope of } IC_h \text{ for } q < 1$$

and for $q = 1$ the slope of IC_h is equal to $\frac{5}{2}$.

Slope of isoprofit curve of the low type:

$$\alpha_e \cdot L = \frac{1}{4} \cdot 5 = \frac{5}{4} < \text{slope of } IC_e \quad \text{for } q < 1$$

For $q = 1$, the slope of IC_e is equal to $\frac{5}{4}$

Exercise 13 d)

If risk types were observable what would be the equilibrium contracts for the two risk types?

13d)

Observable risk types

Equilibrium contracts for both types:

↳ seperate markets for the two risk types

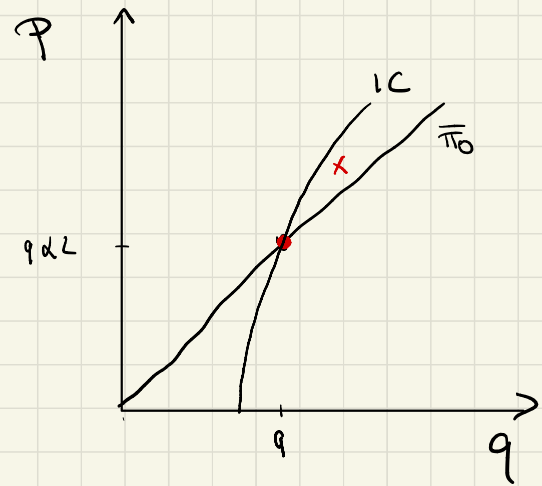
Competition → zero profits for insurances

The contracts will have a coverage of q=1

$$p = \underset{0}{\overline{\pi}} + \underset{1}{q} \cdot \alpha \cdot L$$

$$p_h = \alpha_h \cdot L = \frac{1}{2} \cdot 5 = \frac{5}{2}$$

$$p_e = \alpha_e \cdot L = \frac{1}{4} \cdot 5 = \frac{5}{4}$$



• is not an equilibrium as insurers could offer x instead, yielding higher profits and higher utility

→ q < 1 is not an equilibrium

Exercise 13 e)

What is the Rothschild-Stiglitz equilibrium (i.e. the equilibrium when risk types are not observed by the insurance companies)? For which shares of high risk types is there a full coverage pooling contract breaking this equilibrium?

$\boxed{13e)}$

$$u(x) = -\frac{1}{2}x^2 + 10x$$

Rothschild - Stiglitz equilibrium

↳ risk types are not observed by the insurance companies

Construct equilibrium candidate (sometimes the equilibrium does not exist):

- high type gets full coverage contract at premium where his zeroprofit curve intersects $q = 1$.

  → here: $p_h = \alpha_h \cdot L = \frac{5}{2}$

- Contract (p_l, q_l) for the low type satisfies two things:

  1. yields zero profits for insurances from the low type

$$p_l = q_l \cdot \alpha_l \cdot L = q_l \cdot \frac{5}{4}$$

2. high type is indifferent between his contract and (p_e, q_e)

→ Utility of high type of his contract:

$$U = \frac{1}{2} \cdot U(9 - p - (1-q) \cdot 5) + \frac{1}{2} U(9-p)$$

$$\rightarrow U(9-p) \overset{q=1}{=} U\left(9 - \frac{5}{2}\right) = U(6{,}5)$$

$$= -\frac{1}{2} \cdot 6{,}5^2 + 10 \cdot 6{,}5 = 43{,}875$$

Utility of (p_e, q_e) contract:

$$\frac{1}{2} U(9 - p_e - (1-q_e) \cdot 5) + \frac{1}{2} U(9 - p_e) \overset{!}{=} 43{,}875$$

$$\Leftrightarrow \frac{1}{2} \cdot U(9 - \frac{5}{4} q_e - 5(1-q_e)) + \frac{1}{2} U(9 - \frac{5}{4} q_e) = 43{,}875 \qquad 1 \cdot 2$$

$$\Leftrightarrow U(4 + \frac{15}{4} q_e) + U(9 - \frac{5}{4} q_e) = 87{,}75$$

$$\Leftrightarrow -\frac{1}{2}(4 + \frac{15}{4} q_e)^2 + 10 \cdot (4 + \frac{15}{4} q_e) - \frac{1}{2}(9 - \frac{5}{4} q_e)^2 + 10 \cdot (9 - \frac{5}{4} q_e) = 87{,}75$$

$$\Leftrightarrow (4 + \frac{15}{4} q_e)^2 - 20(4 + \frac{15}{4} q_e) + (9 - \frac{5}{4} q_e)^2 - 20(9 - \frac{5}{4} q_e) = -175{,}5$$

$$\Leftrightarrow 16 + 30 q_e + \frac{225}{16} q_e^2 - 80 - 75 q_e + 81 - \frac{45}{2} q_e + \frac{25}{16} q_e^2 - 180 + 25 q_e = -175{,}5$$

$$\Leftrightarrow \frac{125}{8} q_e^2 - \frac{85}{2} q_e + 12{,}5 = 0$$

$$\Leftrightarrow q_e^2 - 2{,}72 + 0{,}8 = 0$$

$$\Leftrightarrow \quad q_{e_{1,2}} = -\frac{-2{,}72}{2} \pm \sqrt{\left(\frac{-2{,}72}{2}\right)^2 - 0{,}8}$$

$$\Leftrightarrow \quad q_{e_{1,2}} = 1{,}36 \pm \sqrt{1{,}0496}$$

$$q_e \approx 0{,}3355$$

$$p_e = \frac{5}{4} \cdot q_e \approx 0{,}419$$

$\Rightarrow$ Contract of the low type is $(p_e, q_e) = (0{,}4193;\ 0{,}3355)$