Exercises

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1 Introduction

1. Assume that the utility function u_i represents i's preferences over a set of alternatives $X = \{x_1, x_2, \dots, x_n\}$. Show that
   (a) i's preferences are transitive;
   (b) the function v_i defined by $v_i(x) = f(u_i(x))$ also represents i's preferences if f is a strictly increasing function.
   (c) Assume now that there are only 2 alternatives, i.e. $X = \{x_1, x_2\}$. Assume that there are 2 people in the society and person 1 prefers x_1 over x_2 while person 2 prefers x_2 over x_1. Choose some utility functions u_1 and u_2 to represent their preferences. Assume that society chooses the alternative x maximizing $u_1(x) + u_2(x)$.
      - Which alternative does society choose with the utility functions you chose?
      - Show that a transformation as in the previous subquestion can change society's choice. What is the problem and how does it come about?

2. Assume that there are m people in society and society has to choose an option from $X = \{x_1, x_2, \dots, x_n\}$. The preferences of each member of society can be represented by a utility function u_i. Society chooses the alternative $x \in X$ maximizing $\sum_{i=1}^{m} u_i(x)$. Show that the chosen alternative is Pareto efficient.

3. Assume i's preferences over lotteries on the set of outcomes $\{healthy, ill, dead\}$ satisfy the assumptions of the von Neumann-Morgenstern expected utility theorem and can therefore be represented by three numbers $u^{healthy}$, u^{ill} and u^{dead}. Assume that $u^{healthy} = 1$, $u^{ill} = 0.75$ and $u^{dead} = 0$.
   (a) Treatment 1 leads to the probability distribution over $(0.3, 0.5, 0.2)$ (over $\{healthy, ill, dead\}$) while treatment 2 leads to the probability distribution $(0.4, 0.3, 0.3)$. Which treatment does i prefer?
   (b) Show that i's preferences over lotteries can also be represented by the three numbers $v^{healthy} = a * u^{healthy} + b$, $v^{ill} = a * u^{ill} + b$ and $v^{dead} = a * u^{dead} + b$ where $a > 0$ and $b \in \Re$ are some real numbers.
   (c) Show by means of an example that i's preferences are not necessarily represented by $v^{healthy} = f(u^{healthy})$, $v^{ill} = f(u^{ill})$ and $v^{dead} = f(u^{dead})$ for some strictly increasing function f. Why does this not contradict our result from exercise 1 above?

2 Insurance demand

In all exercises let the person be an expected utility maximizer, i.e. the person's choices satisfy the assumptions of the von Neumann-Morgenstern expected utility theorem.

1. Consider a person with utility of income $u(x) = \sqrt{x}$. Is this person risk averse? For the following lotteries, compute the expected income, the certainty equivalent and the risk premium.
   (a) Probability 1/3 for each 1600, 2500, and 3600 Euros.
   (b) Income is uniformly distributed between 1600 and 2500 Euros.

2. Consider a person with utility of income $u(x) = \sqrt{x}$. The person has an income of 2500 Euros but loses L Euros with probability α. Determine the certainty equivalent and the risk premium as a function of α and L. Is the risk premium increasing or decreasing in L? Is the risk premium increasing or decreasing in α?

3. Consider the utility function $u(x) = -e^{-\eta x}$. The person has an income of 1 and experiences a loss of 1 with probability α. The coefficient of absolute risk aversion is defined as $-u''(x)/u'(x)$. Compute this coefficient. Let now $\alpha = 0.5$ and check whether the certainty equivalent in- or decreases in η.

4. The Wall Street Journal reported in 2006 of "mini-medical insurance plans". These plans cover routine services, but little hospital coverage and usually have a cap on payouts (say of \$ 10.000). The premium, however, is only \$ 50 per month. Why might people buy a mini-medical plan? Why are such insurance plans not more popular (in a country where a substantial part of the population did/does not have health insurance)?

5. Consider a person with utility of income $u(x) = \sqrt{x}$. The person has an income of 2500 Euros but loses 1500 Euros with probability 1/4. Assume there is an insurance company that offers to insure an arbitrary coverage $C \in [0, 1500]$ at premium pC. Determine the amount of coverage $C(p)$ that the person will buy. (If you find this too hard, let p be 0.3.)

6. Consider the same person as in the previous exercise. Let $p = 0.3$ and suppose the government guarantees a minimum income of 1500. Will the person still buy insurance? Discuss what features of the health care sector are similar to a minimum income guarantee in the model.

3 Adverse selection

1. We now have a continuum of people of length 1/2. More precisely, we have a person i for each $i \in [0, 1/2]$. All people have the same utility function $u(x) = \sqrt{x}$ and the same income of 2500. However, they differ in terms of risk: Person i loses 1600 with probability i. We consider an insurance policy with full coverage, i.e. a policy that pays out 1600 in case of a loss. Every person knows his own risk but insurance companies cannot distinguish people (and will therefore have to offer the same premium to everybody).

   (a) Determine the willingness to pay for the insurance of person i.

   (b) For every possible insurance premium, how many people will buy insurance? Use your results to draw the demand for insurance.

   (c) Determine the marginal cost of person i.

   (d) Determine the average cost of insuring all people $i \geq j$, i.e. everyone in $[j, 1/2]$.

   (e) If many risk neutral insurance companies with no administrative costs are active on this market, what is the market equilibrium?

   (f) Is the market equilibrium efficient? If not, determine the size of the inefficiency. What would be welfare in "first best", i.e. in a situation in which everyone with a willingness to pay above marginal cost gets insurance? Determine the relative inefficiency due to adverse selection.

   (g) Consider an insurance subsidy to insurers, i.e. each insurer receives for each sold insurance a subsidy payment s. How high does s have to be to ensure efficiency?

   (h) Consider an insurance mandate (without subsidies), i.e. everyone is forced to buy an insurance contract. What is the equilibrium insurance premium? Who will benefit from the mandate? Who will lose out with the mandate?

   (i) Suppose insurers can now distinguish two groups: The people $i \geq 0.3$ and the people $i < 0.3$. Assume that insurers are allowed to offer different contracts to these two groups. Consequently, there are now two separate markets. What is the equilibrium on the "high risk market"? What is the equilibrium on the "low risk" market? Is the new situation more or less efficient than the one considered in the previous subquestions? Who benefits from groups discrimination and who does not?

   (j) With the previous subquestion in mind, what happens if insurers can identify people better? (For example, distinguish more and more subgroups as in the previous subquestion.) What are the consequences for welfare? Who benefits and who loses?

2. You work for a profit maximizing health insurer which recently understood the problem of adverse selection. Your boss asks you what to do to increase/maintain profits in light of the adverse selection problem. What do you answer?

3. In this exercise we show that in the Rothschild-Stiglitz model only one contract per type can be sold in equilibrium. We do this by contradiction. Suppose this was not true, i.e. suppose there were two contracts (p_1, q_1) and (p_2, q_2) that are bought by consumers with high risk.

   (a) Draw in a coverage, premium diagram such two contracts and the indifference curve of the high risk consumers.

   (b) Draw the isoprofit lines of the insurers through these contracts.

   (c) Find a deviation contract that yields strictly positive profit (and is bought by some players if offered).

   (d) Now suppose there were two contracts (p_1, q_1) and (p_2, q_2) that are bought by consumers with *low* risk. Do the same as above but be careful when arguing that the deviation contract is strictly profitable.

4. In the Rothschild-Stiglitz model, assume that all consumers have the utility function $u(x) = -0.5x^2 + 10x$, that $W = 9$, $L = 5$, $\alpha_h = 1/2$ and $\alpha_l = 1/4$.

   (a) Derive the isoprofit curve of an insurance company insuring a consumer with risk α, i.e. if coverage is q what does the premium have to be to achieve expected profits of $\bar{\pi}$?

   (b) Derive the consumer's indifference curve, i.e. if coverage is q what does the premium have to be to achieve an expected utility of $\bar{U}$?

   (c) Verify that the slope of the indifference curve of a consumer with higher risk is higher. Verify that the slope of the indifference curve is higher than the slope of the isoprofit curve for $q < 1$ and equal for $q = 1$.

   (d) If risk types were observable what would be the equilibrium contracts for the two risk types?

   (e) What is the Rothschild-Stiglitz equilibrium (i.e. the equilibrium when risk types are not observed by the insurance companies)? For which shares of high risk types is there a full coverage pooling contract breaking this equilibrium?

5. Suppose the government mandates that coverage levels have to be at least $\bar{q}$. How does this affect the Rothschild-Stiglitz equilibrium? Who benefits/loses from this intervention?

6. Suppose that a low risk type is indifferent between his contract in the Rothschild-Stiglitz equilibrium candidate and a full coverage contract at premium $(\gamma\alpha_h + (1-\gamma)\alpha_l) * L$. What interpretation does the premium $(\gamma\alpha_h + (1-\gamma)\alpha_l) * L$ have? Demonstrate that in this case the Rothschild-Stiglitz equilibrium does not exist.

7. In the Netherlands, health insurance contracts can only be changed at the end of the calendar year. Discuss why such a regulation may or may not be a good idea. Do you know of other similar provisions or regulations?

8. Consider an insurance market where everyone has the *same risk* of falling ill but people differ in their degree of risk aversion. More concretely, assume that everyone has the utility function $u(x) = -e^{-\eta x}$ and that a share γ of the population has a high degree of risk aversion, $\eta = \eta_h$, while a share $1 - \gamma$ of the population has a low degree of risk aversion, $\eta = \eta_l < \eta_h$. Everything else is as in the Rothschild-Stiglitz model. In particular, we assume a perfectly competitive market, i.e. many insurance companies competing in menus of coverage, premium pairs. What will be the equilibrium on this market?

3.1 Genetic tests

1. Assume that all people in our economy are similar and have the same Bernoulli utility function $u(x) = \sqrt{x}$. A person has wealth $W = 9$ and falls ill with probability 1/2. When falling ill the person needs treatment costing $L = 5$.

   (a) Determine the risk premium of a consumer for a full coverage contract. What contract will a profit maximizing insurance monopolist offer in this market?

   (b) Suppose a genetic test becomes available: The test results can be either "high risk" (h) or "low risk" (l). Those that test have a 50% chance of getting either result. High risk people have probability 3/4 and low risk people have the probability 1/4 of falling ill.

- Calculate the risk premium of an h type and the risk premium of an l type (again using a full coverage contract).
- Assume everyone gets tested and the insurance monopolist can make his contracts dependent of the test result. What contracts will he offer? How do profits and expected utility change compared to (a)?
- Assume that the monopolist is prohibited from making his contract contingent upon the test results. How do expected utility and insurance profits change compared to (a)? (Note: you do not have to calculate the profit maximizing contracts to answer this question qualitatively.)

(c) How does your answer to the previous subquestions change if we assume that the insurance market is perfectly competitive?

3.2 Premium risk

1. In Germany (private) health insurers are required to charge a constant premium over the life cycle. We use the premium risk model from the lecture: 2 periods, income W in each period, everyone has low risk α_l of a loss L in period 1, probability $1-\lambda$ of an increse of risk to α_h in period 2, perfect competition.

   (a) Calculate the constant premium that yields zero expected profits to insurers under the assumption that no one switches insurers in period 2.

   (b) Given the premium from the previous subquestion, what would happen if consumers could switch insurers in period 2?

   (c) Compare the premium of the first subquestion with the premiums under "guaranteed renewal". What are the implications?

   (d) Suppose now that in period 2 everyone's health deteriorates. More precisely, assume that the risk is $\alpha_m > \alpha_l$ with probability λ and $\alpha_h > \alpha_m$ with probability $1-\lambda$.

   - Calculate the constant premium that yields zero profits to insurers (without switching).
   - Compare it to the premiums with "guaranteed renewal".

2. Discuss the advantages and disadvantages of using "last year health care expenditures of insured" as an explanatory variable in a risk adjustment scheme.

3. Suppose the population consists of two types l and h with the expenditure distribution for each type as in the table below. In this exercise we measure the incentive of an insurance to engage in risk selection by the difference in expected expenditures.

   (a) Calculate the expected expenditures per risk type and the incentives to engage in risk selection.

   (b) Consider a risk adjustment scheme that covers all expenditures above 20 (i.e. all expenditures above 20 are covered by some common fund to the extent that they exceed 20). Calculate the expected expenditures per risk type that an insurer has to cover himself and the incentives to engage in risk selection. What is the idea behind such a risk adjustment scheme?

   (c) Consider a risk adjustment scheme that covers all expenditures up to 8 (i.e. all expenditures up to 8 are covered by some common fund). Calculate the expected expenditures per risk type that an insurer has to cover himself and the incentives to engage in risk selection.

   (d) Consider expenditure distributions that satisfy the following conditions: $p_h^{30} > p_l^{30}$ and $p_h^{10} + p_h^{30} \geq p_l^{10} + p_l^{30}$ where p_h^{30} is the probability that a high risk type has expenditures 30 and so on.

   - Show that the incentive to engage in risk selection are decreased by a risk adjustment scheme as in (b) for all such distributions.
   - Show that the incentive to engage in risk selection are decreased by a risk adjustment scheme as in (c) for all such distributions.

risk/expenditure	0	10	30
l	40%	10%	50%
h	10%	50%	40%

3.3 Advantageous selection

1. Compare adverse and advantageous selection.

2. Let consumers have the utility function $u(x) = -e^{-\eta x}$. Each consumer faces a loss L of his initial wealth W with probability α. While W and L are the same for all consumers, consumers differ in η and α. Let $W = 10$ and $L = 5$.

   (a) Compare the willingness to pay for a full coverage insurance contract of two consumers: Consumer A has risk $\alpha_A = 0.3$ and risk aversion $\eta_A = 1$. Consumer B has risk $\alpha_B = 0.2$ and risk aversion $\eta_B = 1.5$.

   (b) Using otherwise the same parameters as in (a), who would have the higher willingness to pay if η_B was 1 as well?

   (c) Using otherwise the same parameters as in (a), who would have the higher willingness to pay if α_B was 0.3 as well?

   (d) (PC exercise in spread sheet application or Julia) Let there be a continuum of consumers whose risk α is uniformly distributed on $[0.5, 0.75]$. Assume that $\eta(\alpha) = 3 - \alpha$ and consider a full coverage insurance contract. Is this a case of adverse or advantageous selection? Repeat with $\eta(\alpha) = 3 - 3.75\alpha$.

3. A consumer derives utility from consumption c and good health h according to the utility function $u(c, h) = 2c - c^2/2 + h$. If the consumer is ill his health state is 0 and can be increased to 1 by a certain treatment. This treatment costs 1.

   (a) Suppose the disposable income of the consumer is 2 and the consumer falls ill. Will he use the treatment if he has no insurance? (income not spent on the treatment is used for consumption)

   (b) Suppose the disposable income of the consumer is 1 and the consumer falls ill. Will he use the treatment if he has no insurance?

   (c) Consider a full coverage insurance at premium 1/2 and let the consumer have an income of 2 and a probability of 1/2 of falling ill. Will he buy insurance?

   (d) Consider a full coverage insurance at premium 0.6 and let the consumer have an income of 1 and a probability of 0.6 of falling ill. Will he buy insurance?

   (e) If there are people like in (c) and (d) (in equal proportion) what will be the correlation between risk and insurance purchases in a perfectly competitive market in which risk and income are private information? What drives this result?[1]

4 Moral Hazard

1. Ambulatory mental health care was the most price sensitive element of health care in the RAND health insurance experiment. How do you think the market for mental health care has changed since the 1970s? How does this affect the price sensitivity? What evidence would you look for to support your claims?

2. Dental care was quite price sensitive in the RAND health insurance experiment. This effect was particulalry large in the first year. What is the explanation for this? What are the implications?

3. Health insurance plans can often be described by a deductible D, a copayment rate c and a maximal out of pocket amount M: Up to D all expenditures are paid by the insured, for every \$ spent between D and M the insured pays c and the insurance bears all expenses above M.[2] Assume that consumers act as to maximize the utility function $cons - 0.5(2 - s - t)^2$ where $cons$ is consumption, i.e. all money left to the consumer after paying for treatment $t \in [0, 2 - s]$, and $s \leq 1$ is a health state. Assume that the consumer has an initial wealth of 4 (net of the insurance premium) and therefore consumption is $4 - t$ if he has no insurance.

[1] On first sight this seems to contradict the result in Boone and Schottmüller (2017) mentioned in the lecture, i.e. that under perfect competition the high risk type always gets more coverage. However, this result was obtained under the assumption that full coverage insurance is welfare maximizing for both types and this is not true in this exercise for the high risk type. To change this, one can assume that the risk of the high risk type is 0.55 instead of 0.6. In this case full coverage is welfare maximizing for this type and a monopolist insurer will sell full coverage to a low risk type but exclude the high risk type while the high risk type does receive full coverage under perfect competition.

[2] Hence, the total copayment if expenditures are x is x if $x \leq D$; is $D + c(x - D)$ if $D < x < M$ and is $D + cM$ for $x \geq M$.

(a) Suppose the consumer has no insurance (or equivalently $D > 4$). How much treatment will he buy in health state $s \in [0, 1]$?

(b) Suppose the consumer has a coinsurance rate of $c \in [0, 1)$ while $D = 0$ and $M = \infty$. How much treatment will he buy in health state $s \in [0, 1]$?

(c) Now let $D = 0.5$, $c = 1/2$ and $M = \infty$. How much treatment will the consumer buy in health state $s \in [0, 1]$?

(d) Think now about expected expenditure at the time of insurance purchase (i.e. we do not know the health state yet). Under which conditions on the distribution of health states will an increase in the deductible reduce expected expenditures? What does this imply for the effectiveness of small deductibles in reducing expected expenditures?

4. Suppose a study like the RAND health insurance experiment could be redone for \$ 200 million. On what should the new study focus, i.e. how should it be different from the old one? Do you think it would be worth the money?

5. A consumer has wealth $W = 64$ and face a potential loss of $L = 15$. The consumer has to decide whether to "be careful" or not. If he is careful, the loss realizes with probability $1/4$. If he is not careful, the loss realizes with probability $1/2$. Being careful costs (the money equivalent of) 1 unit of income. (The consumer is a risk averse expected utility maximizer and you can assume $u(x) = \sqrt{x}$.)

   (a) Consider the situation where the consumer is not insured. Will he be careful?

   (b) Consider the situation where the consumer is fully insured at premium $p > 0$. Will he be careful?

6. A consumer with Bernoulli utility $u(x) = -x^2 + 10x$ has wealth $W = 4$ and faces a potential (money equivalent) loss $L = 2$ which realizes with probability $\alpha = 1/2$. If the loss realizes the consumer can (partially) make up for the loss by treatment $M \in [0, 2]$. The insurance will cover cM of these treatment expenditures for some $c \in [0, 1]$. Treatment M will mitigate the loss to $L - 2M + M^2/2$.

   (a) If the consumer is ill, what treatment intensity $M^*(c)$ will he choose?

   (b) (numerical) Assume that the insurance premium is fair, i.e. $p = \alpha c M^*(c)$. Write down the consumers expected utility. Which c maximizes expected consumer utility? How and why does this result differ from models without moral hazard?

4.1 Empirical case study: Minimum deductible in the Netherlands

In the lecture we discussed some evidence for moral hazard. The main question here is whether we can find some empirical evidence for moral hazard ourselves.[3]

In the Netherlands health insurance is provided by a handful semi-private health insurers. The base coverage is fixed by law and the law also mandates, since 2008 , a minimum deductible for all insured *above age 18*, i.e. there is no deductible or other copayment for kids. Copayments that do not take the form of a deductible are not used and most insured have a deductible equal to the legal minimum. The minimum deductible has been increased over time from the initial 150€ per year. (2009: € 155; 2010: € 165; 2011: € 170; 2012: € 220; 2013: € 350; 2014: € 360; 2015: € 375; 2016, 2017 en 2018: € 385). Cost data for the Netherlands is available on `http://www.vektis.nl/index.php/vektis-open-data` where for each (age,gender,postcode) triple you can find the total health care costs split up into different categories. On the course website I provide a simplified version of the data sets from 2011 and 2014 in which I changed the variable names to English and aggregated all the costs that fall under the deductible into one variable.

You can do the exercises below in a spreadsheet app (like Microsoft Excel or *OpenOffice Calc*) but even better suited would be a statistics software (like *R* or Stata or SPSS) or a data analysis package in a general pupose programming language (like *Pandas for Python* or *DataFrames for Julia*). (in italics those options that are free, open source and available for all usual operating systems; as you might have guessed, I use Julia)

1. Download the data for 2011 and open it in your software of choice. Do you understand what the number in the different cells mean?

[3] This case study is based on material prepared by Jan Boone, see section "Regulation in health care markets" here.

2. Can you find out how many people had health insurance in the Netherlands in 2011?
3. Can you make a plot with age on the x-axis and average costs under the deductible on the y-axis? (If you are not familiar with the software this might be tricky and you might want to proceed without it.)
4. Can you find the average costs under the deductible of 17 year olds?
5. Can you find the average costs under the deductible of 19 year olds?
6. How do you interpret the difference in average costs between 17 and 19 year old?
7. To get a better idea of the difference plot the distribution of costs (this is called a "histogram") for 17 and 19 year olds. (again this can be a bit tricky)
8. Why would it make sense to repeat some of the analysis with the 2014 data?
9. Can you give a demand elasticity for the deductible, i.e. if we increase the deductible by 100% by how much do expenditures decrease?
10. Can the estimate of the previous exercise be compared to the famous -0.2 demand elasticity from the RAND health insurance experiment?

4.2 Utilization management

1. Assume for simplicity that a consumer needs to go to hospital exactly once per year. When he goes to hospital, a *long stay* is appropriate with probability 1/2 and a *short stay* is appropriate with probability 1/2. The costs of a long (short) stay are c_l (c_s) with $c_l > c_s$. The hospital has idle capacity and prefers if the consumer stays long. The consumer cannot judge whether a short or a long stay is more appropriate but the hospital knows this perfectly. Assume that there is perfect competition on the insurance market, i.e. insurance premia equal expected cost, that only full coverage contracts are allowed and that insurers have no administrative costs.
   (a) Assume that the hospital determines the length of the stay. What is the equilibrium on this market, i.e. how long will the consumer stay and what is the insurance premium?
   (b) Now assume that the insurer engages in utilization management, in particular assume that the insurer decides whether the stay is short or long. Assume that the insurer does not know which length of stay is appropriate but he has some information on this: More precisely, assume that the insurer's perception of which lenght of stay is appropriate is correct with probability $\alpha > 1/2$. What is the equilibrium insurance premium if the insurer uses his perception?
   (c) Assume that the consumer has utility 1 if the length of his stay is at least as long as appropriate but 0 if he has a short stay and a long one would have been appropriate. The consumer maximizes expected utility from health minus the insurance premium. Is the consumer better off with or without utilization management? Reconsider what the equilibrium is when utilization management is possible.