Imperfect Information in Health Care Markets

Exercise Session 2 - Introduction

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Questions about the lecture

Exercise 1

$f(x) = x^2 \quad u(x) = 2x$

$\Rightarrow f(u(3)) = f(2 \cdot 3) = f(6) = 36$

Assume that the utility function u_i represents i's preferences over a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$. Show that

$(f \circ u_i)(x) = f(u_i(x))$

1. i's preferences are transitive;
2. the function v_i defined by $v_i(x) = f(u_i(x))$ also represents i's preferences if f is a strictly increasing function.
3. Assume now that there are only 2 alternatives, i.e. $X = \{x_1, x_2\}$. Assume that there are 2 people in the society and person 1 prefers x_1 over x_2 while person 2 prefers x_2 over x_1. Choose some utility functions u_1 and u_2 to represent their preferences. Assume that society chooses the alternative x maximizing $u_1(x) + u_2(x)$. - Which alternative does society choose with the utility functions you chose? - Show that a transformation as in the previous subquestion can change society's choice. What is the problem and how does it come about?

Exc. 1

Definition: utility function u_i represents individual i's preferences means the following:

$$x_1 \succsim_i x_2 \iff u_i(x_1) \geq u_i(x_2) \quad \text{for all } x_1, x_2 \text{ in } X$$

"weakly preferred over"

is a number in $\mathbb{R}$

Definition 2: i's preferences are transitive, if

$$(x \succsim_i y \text{ and } y \succsim_i z) \Rightarrow x \succsim_i z \quad \text{for all } x, y, z \text{ in } X.$$

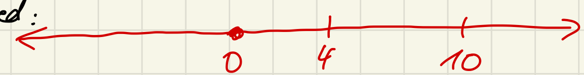
1.1 Want to show: If i's preferences can be represented by a utility function, then the preferences are transitive.

Proof: Let us assume that $x \succsim_i y$ and $y \succsim_i z$ for some arbitrary x, y, z in X.

Sufficient to show: $x \succsim_i z$.

As the preferences are represented by u_i, we know that $u_i(x) \geq u_i(y)$ and $u_i(y) \geq u_i(z)$.

Hence, $u_i(x) \geq u_i(y) \geq u_i(z)$ and in particular $u_i(x) \geq u_i(z)$ as the $\geq$-relation on the real numbers is (naturally) transitive. (Real numbers are ordered:



)

But this implies that $x \succsim_i z$. $\square$

You know from the lecture: Preferences over a finite set of objects can be represented by a utility function if the preferences are complete and transitive.

Exc. 1.2

Know: u_i represents i's preferences

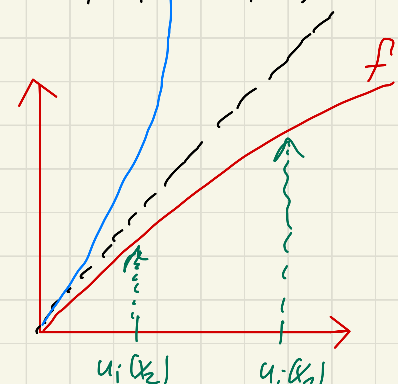
Want to show: $f(u_i)$ also represents i's preferences, for f being a strictly increasing function ($f'(x) > 0$ for all x).

$$\overset{!}{=} \left(x_1 \succsim_i x_2 \iff f(u_i(x_1)) \geq f(u_i(x_2)) \text{ for all } x_1, x_2 \text{ in } X \right)$$

Proof: We have to show both implications:

"$\Rightarrow$" We know that $x_1 \succsim_i x_2$. As u_i represents i's preferences, this means that $u_i(x_1) \geq u_i(x_2)$.

But this implies $f(u_i(x_1)) \geq f(u_i(x_2))$ as f is strictly increasing. ✓



"$\Leftarrow$" We know that $f(u_i(x_1)) \geq f(u_i(x_2))$. To this inequality, let us apply the inverse function f^{-1} of f.

$$\Rightarrow \quad f^{-1}(f(u_i(x_1))) \geq f^{-1}(f(u_i(x_2)))$$

(as f^{-1} is also strictly increasing, due to the derivative of the inverse function rule $(f^{-1})'(f(x)) = \frac{1}{f'(x)}$)

$$\iff \quad u_i(x_1) \geq u_i(x_2) \iff x_1 \succsim_i x_2$$

as u_i represents $\succsim_i$

$\square$

$$f(x) = a \cdot x + b$$

$$u(x) = 529.354 \cdot x - 278.113$$

transform via $f(x) = \frac{1}{529.354} \cdot x + \frac{278.113}{529.354}$

$$\tilde{u}(x) = x$$

Exc. 1.3

Assume $u_1(x_1) = 1$, $u_1(x_2) = 0$ (Person 1 prefers x_1)
$u_2(x_1) = 0$, $u_2(x_2) = 0{,}1$ (Person 2 prefers x_2)

⇝ Society would chose x_1, since $u_1(x_1) + u_2(x_1) = 1 + 0 = 1 > 0{,}1 = u_1(x_2) + u_2(x_2)$

But now, assume that person 2 reports the utility function $\hat{u}_2$ with $\hat{u}_2(x_1) = 0$, $\hat{u}_2(x_2) = 10$
(Person 2 still prefers x_2)

What happens? Society will choose x_2, since $u_1(x_1) + \hat{u}_2(x_1) = 1 < 10 = u_1(x_2) + \hat{u}_2(x_2)$.

$\Rightarrow$ Utility is an ordinal concept!