Demand for insurance

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Outline

Section 1

Gambles and utility functions

Certainty equivalent

Example

Flip a fair coin: If heads you have 700€ as monthly income, if tails 1.500€. What is your expected income?
If you could get X€ for sure as income, how high has X to be to make you indifferent to the lottery?

Definition (Certainty equivalent)

Take a lottery L. The *certainty equivalent* of L is the amount X such that the individual is indifferent between the lottery and receiving X for sure.

$$u(CE) = \text{expected utility of lottery}$$

Risk premium

Example (continued)

Flip a fair coin: If heads you have 700€ as monthly income, if tails 1.500€. What is your expected income?
How much expected income are you willing to sacrifice in order to avoid risk?

Definition (Risk premium)

The *risk premium* is the difference between the expected payout in a lottery L and the certainty equivalent of this lottery.

$$\text{risk premium} = \text{expected wealth in lottery} - \text{certainty equivalent}$$

note:
expected utility of lottery = u(expected wealth of lottery - risk premium)

Plotting your utility function

Section 2

Drivers of insurance demand

The simple standard model of insurance demand

- person has wealth W
- monetary loss L with probability $\alpha \in (0, 1)$
- person maximizes expected utility

$$(1 - \alpha)u(W) + \alpha u(W - L)$$

where u is a strictly increasing function (with inverse function u^{-1})

- what is the expected wealth of the person?
- what is the formula for the certainty equivalent and the risk premium?
- how could we interpret the loss L in the context of health insurance?

Risk aversion

Definition (Risk aversion)

A person is *risk averse* if the risk premium is non-negative for all lotteries.

- a person is risk averse if and only if his utility function is concave

Aside: Implicit function theorem

Implicit function theorem

Let the function $C(L)$ be implicitly defined by the equation

$$F(C, L) = 0$$

where F is a continuously differentiable function. Then,

$$C'(L) = -\frac{\partial F / \partial L}{\partial F / \partial C}$$

at all points where $\partial F / \partial C \neq 0$.

Example (IFT)

$3C - 4L = 0$ implicitly defines the function

$$C(L) =$$

Check $C'(L)$ according to IFT and by directly differentiating $C(L)$.

Size of the loss

- consider a wealth of W and a potential loss of $L \in (0, W)$ that occurs with probability α

Theorem (Size of the loss)

If a person is risk averse, the risk premium is increasing in L.

Proof:
definition of CE:

$$u(CE) = (1-\alpha)u(W) + \alpha u(W-L)$$
$$\Leftrightarrow u(CE) - (1-\alpha)u(W) - \alpha u(W-L) = 0$$

by implicit function theorem:

$$CE'(L) = -\frac{\alpha u'(W-L)}{u'(CE)}$$

definition of RP:

$$RP(L) = W - \alpha L - CE(L)$$
$$\Rightarrow RP'(L) = -\alpha - CE'(L) = \alpha\left(-1 + \frac{u'(W-L)}{u'(CE)}\right)$$

as $W - L < CE$, $u'(W-L) > u'(CE)$ by concavity of $u \Rightarrow RP'(L) > 0$ □

Probability of loss

Theorem (Probability of the loss)

If a person is risk averse, the risk premium is first increasing in α (when α is small) and then decreasing in α (when α is large).

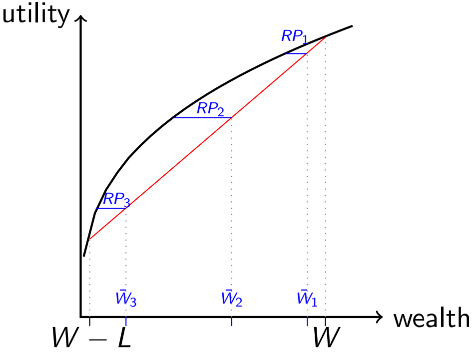

Figure: risk premia for different values of α where $\alpha_1 < \alpha_2 < \alpha_3$ and $\bar{W}_i = W - \alpha_i L$

Wealth effect

Theorem (Wealth effect)

If a person is risk averse, the effect of W on the risk premium is ambiguous. If the person becomes less risk averse as income increases – in the sense that $u''' \geq 0$ – then a higher W leads to a lower risk premium.

Access motive

- think of catastrophes:
  - probability of an illness α is small
  - costs of treatment L are higher than W
  - death without treatment

Summary: Drivers of insurance demand

insurance demand (and therefore the importance of insurance) is particularly high if

- people are risk averse
- the potential loss is high
- there is uncertainty whether the risk realizes or not is high
- (people are poor and $u''' \geq 0$)
- insurance allows access to otherwise unaffordable treatments.

Section 3

Choosing coverage

Choosing coverage

- loss of L with probability α from wealth W
- insurance covers C at premium pC and insuree chooses C
- $u' > 0$, $u'' < 0$

$$E[u] = (1-\alpha)u(W - pC) + \alpha u(W - pC - L + C)$$

- let $W_1 = W - pC$ and $W_2 = W - pC - L + C$

Theorem (Insurance demand)

The optimal decision C^* leads to

$$-\frac{(1-\alpha)u'(W_1^*)}{\alpha u'(W_2^*)} = -\frac{1-p}{p}$$

$$\Leftrightarrow \frac{u'(W_1^*)}{u'(W_2^*)} = \frac{\alpha(1-p)}{(1-\alpha)p}$$

Results:

- fair insurance ($p = \alpha$): demand full coverage
- "unfair" insurance ($p > \alpha$): demand partial coverage

Effects of minimum income/treatment

- suppose government guarantees income $\underline{W} > W - L$
- new option: $W_1 = W$ and $W_2 = \underline{W}$ (no insurance)
- no insurance is chosen if $(1-\alpha)u(W_1^*) + \alpha u(W_2^*) < (1-\alpha)u(W) + \alpha u(\underline{W})$, i.e. if $\underline{W}$ is sufficiently high

Results:

- government guarantees crowd out insurance
- insurance mandate necessary (?)