Imperfect Information in Health Care Markets

Exercise Session 3 - Insurance Demand

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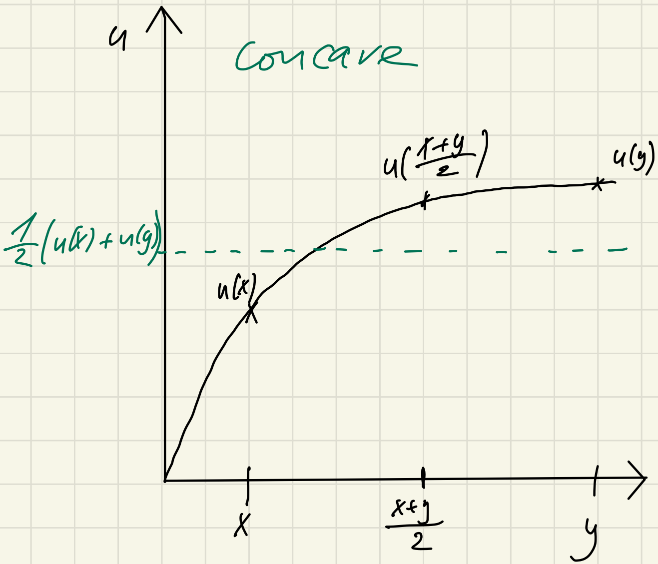
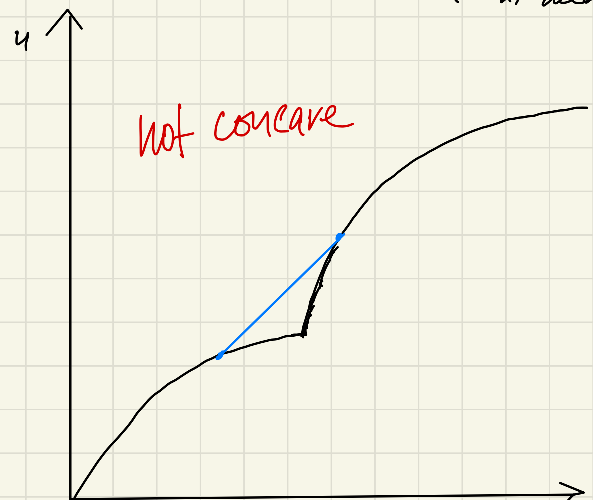
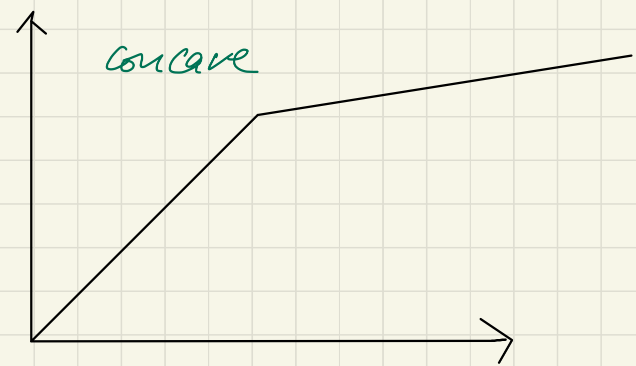
Questions about the lecture

Q: Risk averse people have concave utility functions. How to prove concavity if the function is not twice differentiable?
($u''(x)$ does not exist)

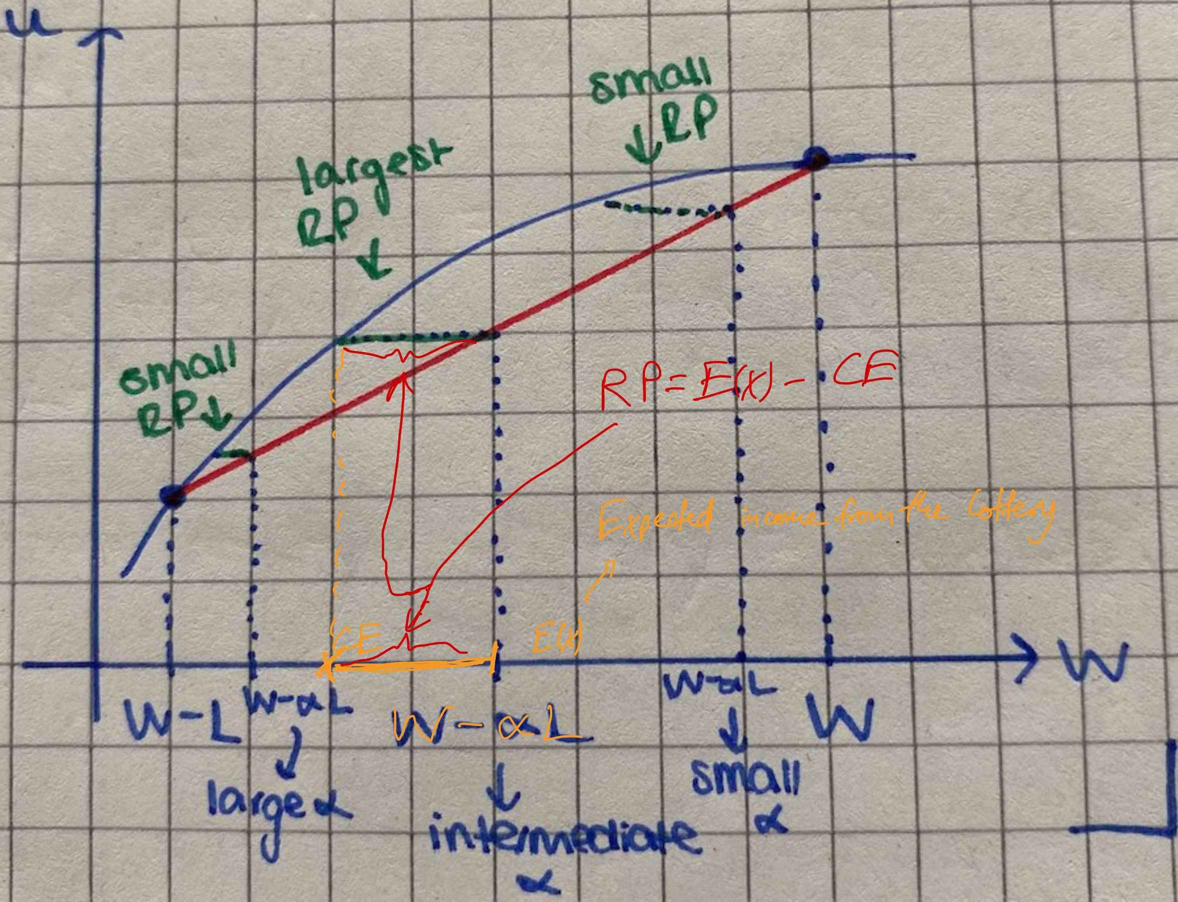




If the connecting line of any two points on the graph is never above the graph, then the function is concave.



Q: For a risk averse person, why is the risk premium first increasing and then decreasing in α (prob. of the loss)?


u
W
largest
RP
small
RP
small
RP
RP = E(x) - CE
Expected income from the lottery
CE
E(x)
W-L
W-αL
W-αL
W-αL
W-αL
W
large α
intermediate α
small α

Exercise 2

Assume that there are m people in society and society has to choose an option from $X = \{x_1, x_2, \ldots, x_n\}$. The preferences of each member of society can be represented by a utility function u_i. Society chooses the alternative $x \in X$ maximizing $\sum_{i=1}^{m} u_i(x)$. Show that the chosen alternative is Pareto efficient.

Exc. 2

Proof: Assume society chose a state $y \in X$ maximizing $\sum_{i=1}^{n} u_i(x)$.

Want to show: This state y is Pareto efficient.

Proof by contradiction: We assume that y is not Pareto efficient, this means that there exists some alternative $\tilde{x} \in X$ that makes at least one person strictly better off than y and that makes all the other persons not worse off. This means that one person has a higher utility from $\tilde{x}$ and all the others have at least the same utility.

$$\Longrightarrow \sum_{i=1}^{n} u_i(\tilde{x}) > \sum_{i=1}^{n} u_i(y) \quad ↯$$

This is a contradiction, since y was supposed to maximize $\sum_{i=1}^{n} u_i(x)$.

$\Longrightarrow$ y has to be Pareto efficient.

$\square$

Exercise 3

Assume i's preferences over lotteries on the set of outcomes $\{healthy, ill, dead\}$ satisfy the assumptions of the von Neumann-Morgenstern expected utility theorem and can therefore be represented by three numbers $u^{healthy}$, u^{ill} and u^{dead}. Assume that $u^{healthy} = 1$, $u^{ill} = 0.75$ and $u^{dead} = 0$.

a) Treatment 1 leads to the probability distribution $(0.3, 0.5, 0.2)$ (over $\{healthy, ill, dead\}$) while treatment 2 leads to the probability distribution $(0.4, 0.3, 0.3)$. Which treatment does i prefer?

b) Show that i's preferences over lotteries can also be represented by the three numbers $v^{healthy} = a * u^{healthy} + b$, $v^{ill} = a * u^{ill} + b$ and $v^{dead} = a * u^{dead} + b$ where $a > 0$ and $b \in \mathbb{R}$ are some real numbers.

Exc. 3 a)

To see which treatment i will choose, we compare its expected utility from the two alternatives / treatments:

Treatment 1: $0,3 \cdot u^{healthy} + 0,5 \cdot u^{ill} + 0,2 \cdot u^{dead}$

$$= 0,3 \cdot 1 + 0,5 \cdot 0,75 + 0,2 \cdot 0 = 0,675$$

Treatment 2: $0,4 \cdot u^{healthy} + 0,3\, u^{ill} + 0,3\, u^{dead}$

$$= 0,4 \cdot 1 + 0,3 \cdot 0,75 + 0,3 \cdot 0 = 0,4 + 0,225 = 0,625 < 0,675$$

$\Rightarrow$ Person i would choose treatment 1.

Exc. 3 b)

$$v^i = a \cdot u^i + b$$

To show that the individual's preferences can also be represented by the numbers v^i, let us compute its utility from some random lottery $(p, q, 1-p-q)$, with $p, q, 1-p-q$ in $[0,1]$.

prob. of "healthy" prob. of "ill" prob. of "dead"

expected utility person i gets from the v^i numbers

$$E(v(p,q,1-p-q)) = p \cdot v^{healthy} + q \cdot v^{ill} + (1-p-q) \cdot v^{dead}$$

plug in $v^i = a \cdot u^i + b$

$$= p \cdot (a \cdot u^{healthy} + b) + q \cdot (a \cdot u^{ill} + b) + (1-p-q) \cdot (a \cdot u^{dead} + b)$$

$$= p \cdot b + q \cdot b + (1-p-q) \cdot b + a \cdot \underbrace{(p \cdot u^{healthy} + q \cdot u^{ill} + (1-p-q) u^{dead})}_{\text{old utility with numbers } u^i}$$

$$= b + a \cdot E(u(p,q,1-p-q))$$

↳ old utility given by the u^i's

rearranging terms

"distributive property"!!

So we just applied the transformation / function $f(x) = a \cdot x + b$ to the old utility.

Since $f'(x) = a > 0$ (by assumption), this is a positive monotone transformation and results in the same preferences by Exercise 1.2.

Exercise 3 (cont.)

c) Show by means of an example that i's preferences are not necessarily represented by $v^{healthy} = f(u^{healthy})$, $v^{ill} = f(u^{ill})$ and $v^{dead} = f(u^{dead})$ for some strictly increasing function f. Why does this not contradict our result from exercise 1 above?

Exc. 3c)

Example: Let us take the function $f(x) = \sqrt{x}$, which is strictly increasing on $(0,\infty)$.

$\Rightarrow v^{healthy} = f(u^{healthy}) = \sqrt{1} = 1$ $\qquad f'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} > 0$ for $x > 0$

$v^{ill} = \sqrt{0{,}75} \approx 0{,}866$ $\qquad$ derivative of $\sqrt{x}$ - function

$v^{dead} = \sqrt{0} = 0$

Now let us compare two lotteries:

Lottery 1: $(0, 1, 0)$ $\qquad$ Lottery 2: $(0{,}4 ; 0{,}5 ; 0{,}1)$

Before the transformation:

$E(u(0,1,0)) = 1 \cdot 0{,}75$, $\quad E(u(0{,}4;0{,}5;0{,}1)) = 0{,}4 \cdot u^{healthy} + 0{,}5 \cdot u^{ill} + 0{,}1 \cdot u^{dead}$

$= 0{,}4 + 0{,}375 + 0 = 0{,}775 > 0{,}75$

$\rightarrow$ Here, the person chooses lottery 2.

After the transformation:

$E(v(0,1,0)) = 1 \cdot v^{ill} = 1 \cdot \sqrt{0{,}75} \approx 0{,}866$ $\neq \sqrt{0{,}4 \cdot 1 + 0{,}5 \cdot 0{,}75 + 0{,}1 \cdot 0}$

$= f(E(u(0{,}4;0{,}5;0{,}1)))$

$E(v(0{,}4;0{,}5;0{,}1)) = 0{,}4 \cdot \sqrt{1} + 0{,}5 \cdot \sqrt{0{,}75} + 0{,}1 \cdot \sqrt{0} \approx 0{,}4 + 0{,}433 = 0{,}833 < 0{,}866$

$\Rightarrow$ In this case, the person decides for lottery 1 !!!

The point is: The transformation was <u>not</u> applied to the whole utility function, but just to each of the u's separately.

Chapter 2 - Insurance Demand

In all exercises let the person be an expected utility maximizer, i.e. the person's choices satisfy the assumptions of the von Neumann-Morgenstern expected utility theorem.

Exercise 4

Consider a person with utility of income $u(x) = \sqrt{x}$. Is this person risk averse? For the following lotteries, compute the expected income, the certainty equivalent and the risk premium.

a) Probability $1/3$ for each 1600, 2500, and 3600 Euros.

b) Income is uniformly distributed between 1600 and 2500 Euros.

Exc. 4 a)

$u(x) = \sqrt{x}$

Expected income: $E(x) = \frac{1}{3} \cdot 1600 + \frac{1}{3} \cdot 2500 + \frac{1}{3} \cdot 3600 \approx 2566{,}66$

Expected utility: $E(u) = \frac{1}{3} \cdot \sqrt{1600} + \frac{1}{3} \cdot \sqrt{2500} + \frac{1}{3} \cdot \sqrt{3600}$

$$= \frac{1}{3}(40 + 50 + 60) = 50$$

Certainty Equivalent (CE) measures the safe income that makes me indifferent to playing the lottery or not. In mathematical terms: $u(CE) = E(u)$ defining equation of the CE

$$\Rightarrow u(CE) \overset{!}{=} E(u) = 50$$

$$\Leftrightarrow \sqrt{CE} = 50$$

$$\Rightarrow CE = 2500$$

$\Rightarrow$ the individual is <u>risk-averse</u> since this number is smaller than $E(x)$

The risk premium (RP) is just the difference in expected payment from playing the lottery or taking the certainty equivalent.

$$RP = E(x) - CE$$

[if $RP < 0$: individual is <u>risk-loving</u>]

Here: $RP = 2566{,}66 - 2500 = 66{,}66 > 0$ $\Rightarrow$ individual is <u>risk-averse</u> since the RP is greater than 0