Imperfect Information in Health Care Markets

Exercise Session 6 - Rothschild-Stiglitz

Marius Gramb

Exercise 11

You work for a profit maximizing health insurer which recently understood the problem of adverse selection. Your boss asks you what to do to increase/maintain profits in light of the adverse selection problem. What do you answer?

Exc. 11

Problem of adverse selection: attraction of high risk consumers

→ tailor insurance plan toward healthy people:

- bonus programs for e.g. fitness courses, ...
- pay back part of the premium in case no care was used
- maybe offer partial coverage contracts

→ make it unattractive for chronically ill and unfit people

- signing insurance contract online / 3rd floor without elevator
- build office in neighborhood with high socioeconomic status / or advertise especially there
- do not cover certain brands of medication for chronic diseases

Questions about the lecture

1. Adverse Selection Slide 17: Why is it a must that in the Rothschild-Stiglitz Model the slope of the indifference curve which shows the preferences of the consumers must be shown through the Implicit Function Theorem?

$$\bar{u} = \alpha u(W - p - (1-q)L) + (1-\alpha) u(W - p) \Leftrightarrow \underbrace{\alpha u(W - p - (1-q)L) + (1-\alpha) u(W-p) - \bar{u}}_{= F(p, q)} = 0$$

$\rightsquigarrow p'(q)$ from Implicit Function Theorem

2. Adverse Selection Slide 15: Why is the premium in the Equilibrium for full coverage for the low type p=alpha*L?

This corresponds to the insurance's expected costs.
→ They make zero profit due to perfect competition.

3. Slide 15: Is it a must that in the Equilibrium there has to be a full coverage q before we can say it is an Equilibrium?

No. As you will see, low types will get partial coverage in equilibrium.

The Rothschild-Stiglitz Model

Starting point: Insurers are aware of the adverse selection problem (People that buy insurances typically have higher risks)

Main idea: Insurers offer a menu of contracts (coverage-premium pairs) that are designed in such a way that different risk types self-select into the contract designed for them

→ The RS-Model analyses how these contracts are designed in the simple case of two possible risk types (high + low)

The Players: Insurances, High risk types, Low risk types

Insurances: Want to make profits (but will make zero profit due to perfect competition assumption)

→ Their zero-isoprofit-lines indicate which contracts are profitabel for them (depending on the type buying the contract)

High + Low risk types: Want to get the best possible contract (higher coverage + lower price)

→ Their indifference curves indicate which contracts they prefer over other contracts

Exercise 12

In this exercise we show that in the Rothschild-Stiglitz model only one contract per type can be sold in equilibrium. We do this by contradiction. Suppose this was not true, i.e. suppose there were two contracts (p_1, q_1) and (p_2, q_2) that are bought by consumers with high risk.

a) Draw in a coverage, premium diagram such two contracts and the indifference curve of the high risk consumers.

b) Draw the isoprofit lines of the insurers through these contracts.

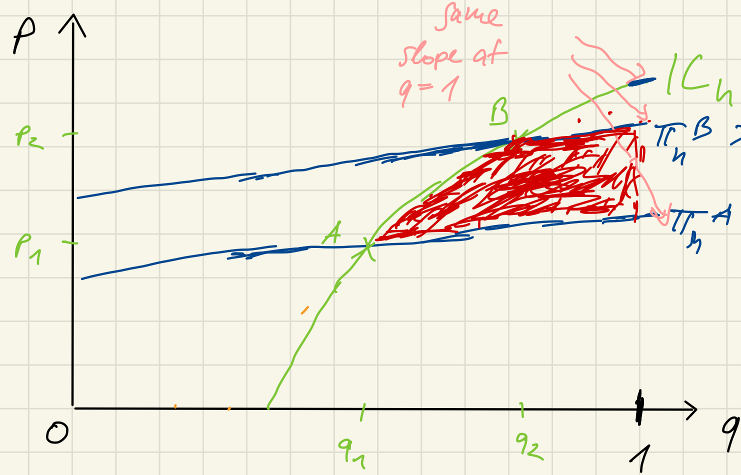
c) Find a deviation contract that yields strictly positive profit (and is bought by some players if offered).

d) Now suppose there were two contracts (p_1, q_1) and (p_2, q_2) that are bought by consumers with **low** risk. Do the same as above but be careful when arguing that the deviation contract is strictly profitable.

Exc. 12

Let's first show that the high risk types will not buy two different contracts.

By contradiction: Let us assume they do buy two different contracts in equilibrium.



a) indifference curve has to go through both contracts as instead they would only buy one of these contracts

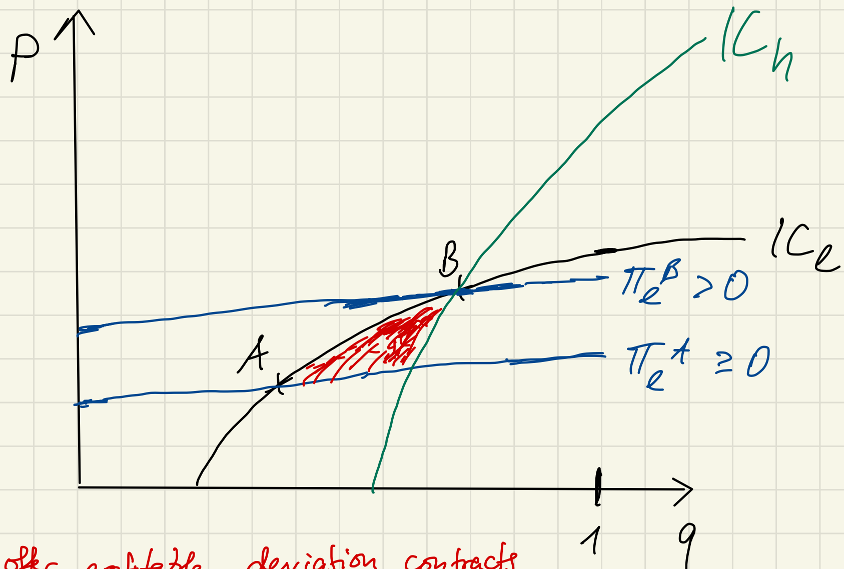
b) as insurances want to make positive profits (they would not offer contract A if it made negative profits)

c) All contracts in ~~red~~-area yield positive profits for the insurance and are preferred by the high risk types (because they are below their IC)

(We do not know whether the low risk type would buy these contracts or not, but we DO know that if she buys it, it will result in positive profits for the insurance - since $\alpha_e < \alpha_h$)

Exc. 12 d)

Now assume there are two contracts $A = (p_1, q_1)$ and $B = (p_2, q_2)$ that are bought by the low types in equilibrium. Want to show: This is impossible since there would be a profitable deviation for insurances.



(high types always prefer B over A since their ICs are steeper)

$\Rightarrow$ Insurances could offer profitable deviation contracts in the red area*, so A and B being sold can not have been an equilibrium.

In conclusion (Exc. 12 a) - d)), no type will buy two (or more) different contracts in equilibrium.

* As the high risk type will ~~not~~ prefer these contracts over contract B.

Exercise 13

In the Rothschild-Stiglitz model, assume that all consumers have the utility function $u(x) = -0.5x^2 + 10x$, that $W = 9$, $L = 5$, $\alpha_h = 1/2$ and $\alpha_l = 1/4$.

a) Derive the isoprofit curve of an insurance company insuring a consumer with risk α, i.e. if coverage is q what does the premium have to be to achieve expected profits of $\bar{\pi}$?

Exc. 13 a)

How do the profits of an insurance depend on p and q ?

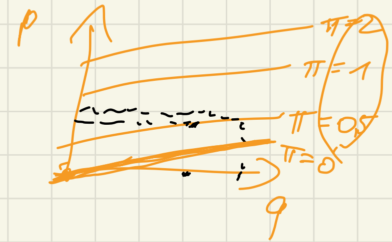
$$\overline{\pi} = p - \alpha \cdot L \cdot q$$

some profit level

$$\iff p = \overline{\pi} + \alpha \cdot L \cdot q$$

$$p = p(q)$$

$\Rightarrow$ isoprofit curves have slope $\alpha \cdot L$



Exercise 13 b)

Derive the consumer's indifference curve, i.e. if coverage is q what does the premium have to be to achieve an expected utility of $\bar{u}$?

Exc. 13 b)

$u(x) = -0{,}5 \cdot x^2 + 10x$

for the high type ($\alpha = \frac{1}{2}$):

$$\bar{u} = \frac{1}{2} \cdot \underbrace{u(9 - p - (1-q)\cdot 5)}_{\text{loss case}} + \underbrace{\frac{1}{2} \cdot u(9-p)}_{\text{no loss case}} = \frac{1}{2}\left(-\frac{1}{2} \cdot (9-p-(1-q)\cdot 5)^2 + 10 \cdot (9-p-(1-q)\cdot 5)\right)$$
$$+ \frac{1}{2} \cdot \left(-\frac{1}{2}(9-p)^2 + 10 \cdot (9-p)\right)$$

plug in $u(x) = \ldots$

$$\Leftrightarrow \quad 4\bar{u} = 163 - 2p^2 + 60q - 25q^2 - 14p + 10pq$$

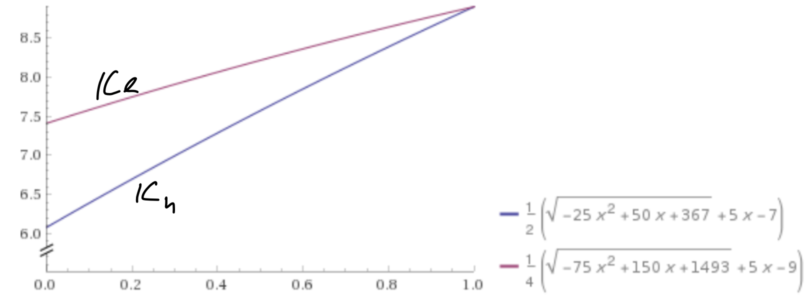
$$\Rightarrow \quad p = \frac{5q-7}{2} + \sqrt{375 - 25q^2 + 50q - 8\bar{u}} \cdot \frac{1}{2}$$

for the low type ($\alpha = \frac{1}{4}$):

$$\bar{u} = \frac{1}{4}\, u(9 - p - (1-q)\cdot 5) + \frac{3}{4}\, u(9-p)$$

plug in $u(x)$

$$\Rightarrow \ldots \Rightarrow \quad p(q) = \frac{5q-9}{4} + \sqrt{1525 - 75q^2 + 150q - 32\bar{u}} \cdot \frac{1}{4}$$



(plotted for $\bar{u}=1$)

Exercise 13 c)

Verify that the slope of the indifference curve of a consumer with higher risk is higher. Verify that the slope of the indifference curve is higher than the slope of the isoprofit curve for $q < 1$ and equal for $q = 1$.

Exc. 13 c)

Slope of isoprofit curve : $\alpha \cdot L$ $\nearrow \frac{5}{2}$ for h $\searrow \frac{5}{4}$ for ℓ

for $\alpha = \frac{1}{2}$: $p'(q) = \frac{5}{2} + \underbrace{50(1-q) \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{375 - 25q^2 + 50q - 8\bar{u}}}}_{\geq 0}$

$\overset{q=1}{=}$ $\frac{5}{2} = \alpha_h \cdot L$

for $\alpha = \frac{1}{4}$: $p'(q) = \frac{5}{4} + \underbrace{150(1-q) \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{1525 - 75q^2 + 150q - 32\bar{u}}}}_{\geq 0}$

$\overset{q=1}{=}$ $\frac{5}{4} = \alpha_\ell \cdot L$

$\sqrt{x} = x^{\frac{1}{2}}$

$\Rightarrow \frac{\partial \sqrt{x}}{\partial x} = \frac{\partial x^{\frac{1}{2}}}{\partial x}$

$= \frac{1}{2} \cdot x^{-\frac{1}{2}}$

$= \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}}$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$

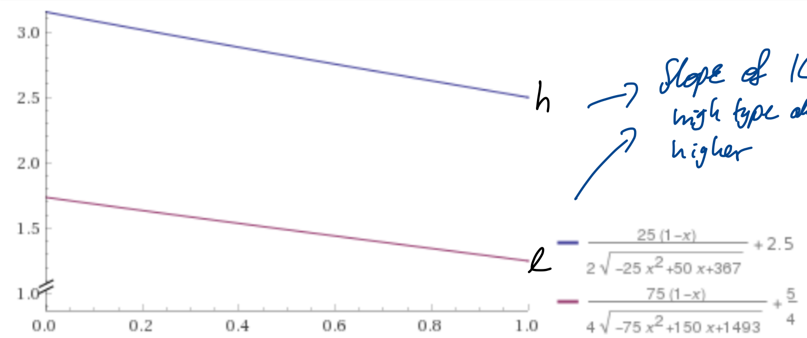
$x^1 \cdot x^1 = x^2$ ($1+1$)

$x^2 \cdot x = x^3$

$\sqrt{x} \cdot \sqrt{x} = x^1$

$\Rightarrow \sqrt{x} = x^{\frac{1}{2}}$

Slopes of the indifference curves at level $\bar{u} = 1$



Slope of IC of high type always higher