Imperfect Information in Health Care Markets

Exercise Session

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Why do people struggle with this course?

- Prerequisites for this course:
  - High school level math knowledge
- Problem: Students often lack important knowledge about basic math concepts (or forgot it over the years)
- This deflects them from the (economic) concepts taught in class

Suggested work flow for the process of understanding the lectures

- After each lecture, you will typically not understand the whole content directly (which is perfectly normal)
- This is why you need to study it carefully afterwards
- As a first step, try to identify all the mathematical steps/tools that are used within this lecture
- Recall these mathematical concepts and make sure you fully understand them (again)
- Once this is done, have a second look at the lecture and try to understand what was taught to you (not being distracted by mathematics that might have scared you off before)
- If you still have difficulties with the understanding, discuss the topic with your peers (or ask a concrete question in the exercise session in case you can formulate one)
- Repeat this procedure every week before the next lecture

Suggested work flow for the process of understanding the lectures (cont.)

- The exercises are also helpful in understanding the lecture material
- To get the most out of the exercise sessions, try to solve the exercises that are going to be discussed on your own or in groups
- This way, you will understand the correct solutions quicker, even if you have not reached these solutions yourself
- The final exam will be based on understanding of the lecture and you WILL be asked to transfer your built up knowledge to new exercises
- Therefore, it does not pay off to learn some exercises or lecture material by heart without understanding them!

Exercise 1

1. Recall the notion of a random variable and how to compute the expected value of discrete and continuous random variables.
2. Let X and Y be two random variables with $Y \sim \mathrm{U}([0,1])$ and X taking a value of 1 with the probability 0.4 and a value of 0 with the probability 0.6. Calculate $\mathbb{E}(X)$ and $\mathbb{E}(2Y)$.
3. Consider the random experiment "rolling a regular die once" and define a random variable Z that gives the number that is rolled. How is Z distributed? Assume that you receive a payment of z^2 when the die shows the number z. What is the expected payment you get?

Exc. 1.1

A random variable X is a map $X: \Omega \to \mathbb{R}$

greek "Omega"

the "universe" of a random experiment
= set of events that can occur in this experiment

How to compute the expected value of a random variable?

i) discrete random variables: number of possible outcomes/events is countable, that means finite or countably infinite.

more precisely: $\mathbb{P}(\omega : X(\omega) = x_i)$, probability that any ω leading to a value of x_i is realized

$$\Rightarrow \mathbb{E}(X) = \sum_{i=1}^{n} x_i \cdot \mathbb{P}(X = x_i)$$, where $x_1, x_2, \ldots, x_n$ are the values that X can take

expected value

ii) continuous random variable: there are uncountably infinitely many possible outcomes
→ this usually happens when possible values are in some interval of real numbers

expected value of a continuous random variable X defined on the interval $[a,b]$:

$$\Rightarrow \mathbb{E}(X) = \int_a^b x \cdot \rho(x)\, dx$$, where $\rho(x)$ is the density function of X

greek rho

Exc. 1.2

$$\mathbb{E}(X) = \sum_{i=0}^{1} i \cdot \mathbb{P}(X=i) = 0 \cdot \overset{\mathbb{P}(X=0)}{0{,}6} + 1 \cdot \overset{\mathbb{P}(X=0{,}4)}{0{,}4} = 0{,}4$$

$$Y \sim U([0,1]) \implies f_Y \overset{f_Y(y)}{=} 1 \text{ for } y \text{ in } [0,1] \text{ and } 0 \text{ instead}$$

↳ "is distributed as"

Uniform distribution

$$\implies \mathbb{E}(Y) = \int_0^1 y \cdot f_Y(y)\, dy = \int_0^1 y \cdot 1\, dy = \left[\frac{1}{2} y^2\right]_0^1 = \frac{1}{2} \cdot 1^2 - \frac{1}{2} \cdot 0^2 = \frac{1}{2}$$

$$\mathbb{E}(2Y) = \int_0^1 2y \cdot f_Y(y)\, dy = \int_0^1 2y \cdot 1\, dy = \left[y^2\right]_0^1 = 1^2 - 0^2 = 1$$

Exc. 1.3

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

$$Z : \Omega \to \mathbb{R}$$

$$\omega \mapsto \omega$$

small omega

How is Z distributed?

The die is regular $\Rightarrow$ $Z \sim \text{Unif}(\{1,2,3,4,5,6\})$

↳ uniform distribution in the discrete case

Z^2 is the random variable that defines our payment. What is our expected payment?

$$\mathbb{E}(Z^2) = \sum_{i=1}^{6} i^2 \cdot \mathbb{P}(Z=i) = 1^2 \cdot \frac{1}{6} + 2^2 \cdot \frac{1}{6} + 3^2 \cdot \frac{1}{6} + 4^2 \cdot \frac{1}{6}$$

$$+ 5^2 \cdot \frac{1}{6} + 6^2 \cdot \frac{1}{6}$$

$$= \frac{1}{6}(1+4+9+16+25+36) = \frac{1}{6} \cdot 91 = 15\frac{1}{6}$$

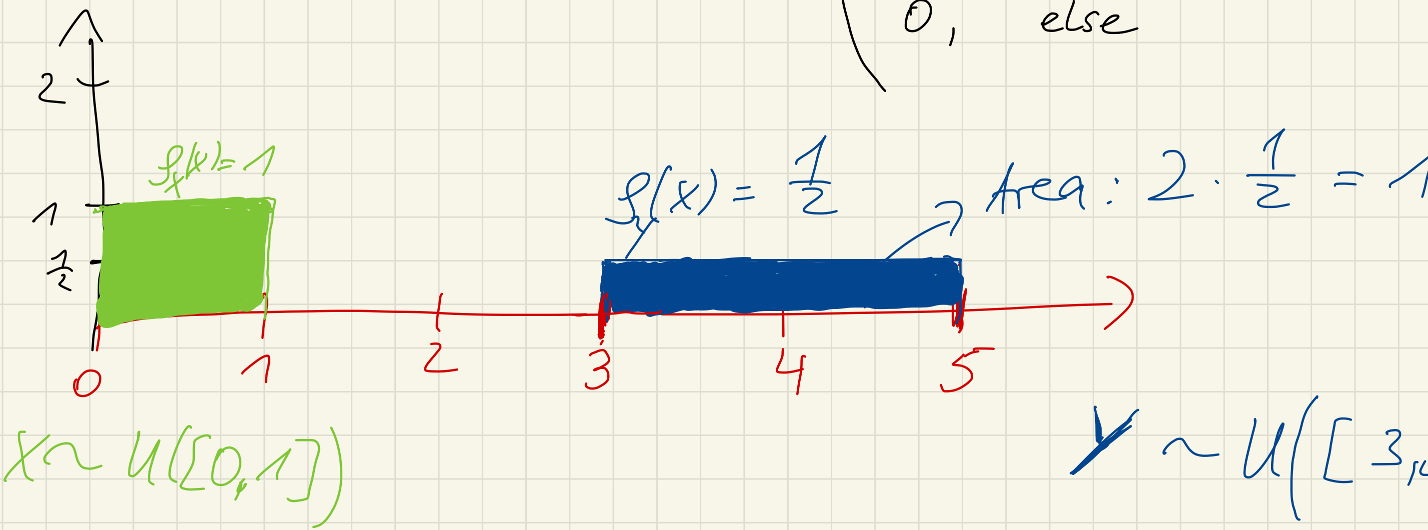
the continuous uniform distribution in general:

$$U([a,b]), \text{ with } a<b$$

$\Rightarrow$ the density of this distribution is

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{for } x \text{ in } [a,b] \\ 0, & \text{else} \end{cases}$$

length of the interval $[a,b]$



Exercise 2

1. Let $f(x, y) = y^2 \ln(x) - y$. Compute $\frac{\partial f(x,y)}{\partial x}$ and $\frac{\partial f(x,y)}{\partial y}$. What is the geometric interpretation of $\frac{\partial f(x,y)}{\partial x}$?
2. Recall the definition of concave functions in one real variable.
3. Compute $\max_{x \in \mathbb{R}} g(x)$ with $g(x) = -2x^2 + 32x + 7$.

Exc. 2.1

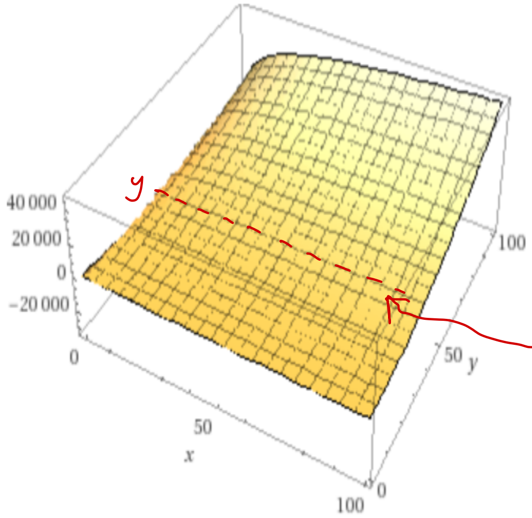
$$f(x,y) = y^2 \ln(x) - y$$

$$\frac{\partial f(x,y)}{\partial x} = y^2 \cdot \frac{1}{x} - 0 = \frac{y^2}{x}$$

$$\frac{\partial y}{\partial x}$$

$$\frac{\partial f(x,y)}{\partial y} = \ln(x) \cdot 2y - 1$$

3D plot:



for a fixed y,

$\frac{\partial f(x,y)}{\partial x}$ is

just the slope of

this line

Exc. 2.2

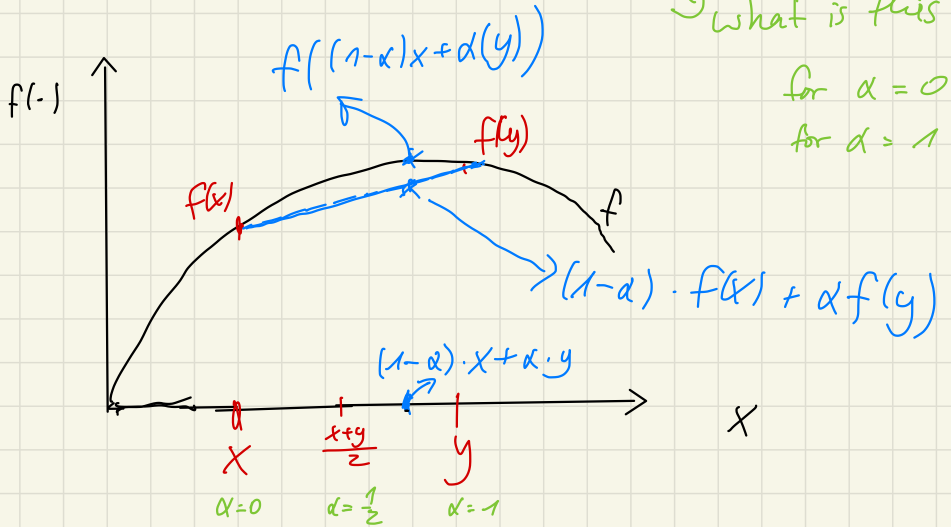
Def.: A function f is called <u>concave</u>, if $f(\underbrace{(1-\alpha)x + \alpha y}) \geq (1-\alpha) f(x) + \alpha f(y)$ for all $\alpha \in [0,1]$

what is this number?!

for $\alpha = 0$: x

for $\alpha = 1$: y

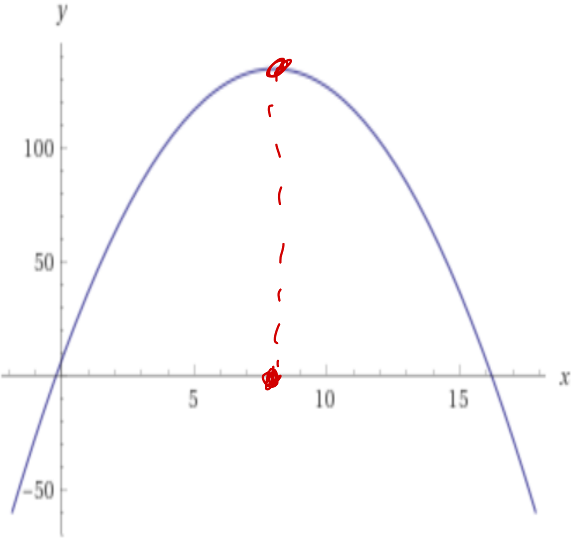
Graphically:



intuitive meaning of concavity: if you connect two points on the graph, the line will always be <u>below</u> the graph

for differentiable functions: f concave $\Leftrightarrow$ $f'' \leq 0$

Exc. 2.3



Exc. 2.3

$$\max_{x \in \mathbb{R}} \; g(x) = -2x^2 + 32x + 7$$

FOC: $g'(x) \overset{!}{=} 0$

↳ first-order condition

SOC: $g''(x) < 0$

↳ second-order condition

$g''(x) = -4 < 0$ ✓

$$g'(x) = -4x + 32 \overset{!}{=} 0$$

$$\Leftrightarrow \quad 32 = 4x$$

$$\Leftrightarrow \quad 8 = x$$

$\Rightarrow$ maximum is attained at $x = 8$

and this maximum value is $g(8) = -2 \cdot 8^2 + 32 \cdot 8 + 7$

$$= -128 + 256 + 7 = 135$$

Here are some of the mathematical concepts you should master:

- rearranging terms
- computing the average of two (or more) numbers
- binomial formulas
- basic algebraic notations, such as $x^{\frac{1}{2}} = \sqrt{x}$ and $x^{-a} = \frac{1}{x^a}$, as well as rules of calculation with fractions, exponents and (square-)roots
- extreme value problems (maximizing a function under some constraints)
- integration, e.g. $\int_0^1 \sqrt{x} dx = ?$
- differentiation, in particular product rule, quotient rule and chain rule
- understanding and interpreting graphs and functions, computing points of intersection of two curves
- In the course schedule, you will find mathematical prerequisites for the chapters