Imperfect Information in Health Care Markets

Exercise Session 2 - Introduction

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Questions about the lecture

Exercise 1

$f(x) = x^2$ $u(x) = 2x$

$\Rightarrow f(u(3)) = f(2 \cdot 3) = f(6) = 36$

Assume that the utility function u_i represents i's preferences over a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$. Show that

$(f \circ u_i)(x) = f(u_i(x))$

1. i's preferences are transitive;
2. the function v_i defined by $v_i(x) = f(u_i(x))$ also represents i's preferences if f is a strictly increasing function.
3. Assume now that there are only 2 alternatives, i.e. $X = \{x_1, x_2\}$. Assume that there are 2 people in the society and person 1 prefers x_1 over x_2 while person 2 prefers x_2 over x_1. Choose some utility functions u_1 and u_2 to represent their preferences. Assume that society chooses the alternative x maximizing $u_1(x) + u_2(x)$. - Which alternative does society choose with the utility functions you chose? - Show that a transformation as in the previous subquestion can change society's choice. What is the problem and how does it come about?

Exc. 1

Definition: utility function u_i <u>represents</u> individual i's preferences means the following:

$$x_1 \succsim_i x_2 \iff u_i(x_1) \geq u_i(x_2) \quad \text{for all } x_1, x_2 \text{ in } X$$

"weakly preferred over"   is a number in $\mathbb{R}$

Definition 2: i's preferences are <u>transitive</u>, if

$$(x \succsim_i y \text{ and } y \succsim_i z) \Rightarrow x \succsim_i z \quad \text{for all } x, y, z \text{ in } X.$$

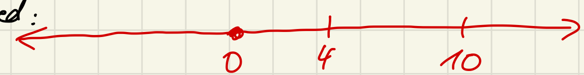
1.1 Want to show: If i's preferences can be represented by a utility function, then the preferences are transitive.

Proof: Let us assume that $x \succsim_i y$ and $y \succsim_i z$ for some arbitrary x, y, z in X.

Sufficient to show: $x \succsim_i z$.

As the preferences are represented by u_i, we know that $u_i(x) \geq u_i(y)$ and $u_i(y) \geq u_i(z)$.

Hence, $u_i(x) \geq u_i(y) \geq u_i(z)$ and in particular $u_i(x) \geq u_i(z)$ as the $\geq$-relation on the real numbers is (naturally) transitive. (Real numbers are ordered:



)

But this implies that $x \succsim_i z$. $\square$

You know from the lecture: Preferences over a finite set of objects can be represented by a utility function if the preferences are complete and transitive.

$$\underline{10} \longrightarrow \ln(10) \longrightarrow e^{\ln(10)} = 10$$

Exc. 1.2

Know: u_i represents i's preferences

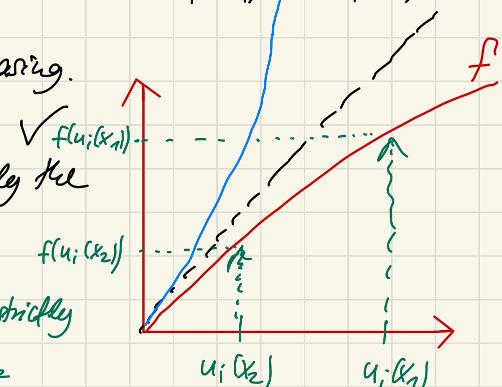
Want to show: $f(u_i)$ also represents i's preferences, for f being a strictly increasing function ($f'(x) > 0$ for all x).

$$\overset{!}{=} \left(x_1 \succsim_i x_2 \iff f(u_i(x_1)) \geq f(u_i(x_2)) \text{ for all } x_1, x_2 \text{ in } X \right)$$

Proof: We have to show both implications:

"$\Rightarrow$" We know that $x_1 \succsim_i x_2$. As u_i represents i's preferences, this means that $u_i(x_1) \geq u_i(x_2)$.

But this implies $f(u_i(x_1)) \geq f(u_i(x_2))$ as f is strictly increasing.



"$\Leftarrow$" We know that $f(u_i(x_1)) \geq f(u_i(x_2))$. To this inequality, let us apply the inverse function f^{-1} of f.

$$\Rightarrow \quad f^{-1}(f(u_i(x_1))) \geq f^{-1}(f(u_i(x_2)))$$

(as f^{-1} is also strictly increasing, due to the derivative of the inverse function rule $(f^{-1})'(f(x)) = \frac{1}{f'(x)}$)

$$\iff \quad \underbrace{u_i(x_1) \geq u_i(x_2) \iff x_1 \succsim_i x_2}_{\text{as } u_i \text{ represents } \succsim_i}$$

$\square$

Exc. 1.3

Assume $u_1(x_1) = 1$, $u_1(x_2) = 0$ (Person 1 prefers x_1)

$u_2(x_1) = 0$, $u_2(x_2) = 0{,}1$ (Person 2 prefers x_2)

$\rightsquigarrow$ Society would chose x_1, since $u_1(x_1) + u_2(x_1) = 1 + 0 = 1 > 0{,}1 = u_1(x_2) + u_2(x_2)$

But now, assume that person 2 reports the utility function $\hat{u}_2$ with $\hat{u}_2(x_1) = 0$, $\hat{u}_2(x_2) = 10$
($\rightarrow$ transformation by $f(x) = 100 \cdot x$) (Person 2 still prefers x_2)

What happens? Society will choose x_2, since $u_1(x_1) + \hat{u}_2(x_1) = 1 < 10 = u_1(x_2) + \hat{u}_2(x_2)$.

$\Rightarrow$ Utility is an <u>ordinal</u> concept!

Exercise 2

Assume that there are m people in society and society has to choose an option from $X = \{x_1, x_2, \ldots, x_n\}$. The preferences of each member of society can be represented by a utility function u_i. Society chooses the alternative $y \in X$ maximizing $\sum_{i=1}^{m} u_i(x)$. Show that the chosen alternative is Pareto efficient.

Exc. 2

Proof: Assume society chose a state $y \in X$ maximizing $\sum_{i=1}^{n} u_i(x)$.

Want to show: This state y is Pareto efficient.

Proof by contradiction: We assume that y is not Pareto efficient, this means that there exists some alternative $\tilde{x} \in X$ that makes at least one person strictly better off than y and that makes all the other persons not worse off. This means that one person has a higher utility from $\tilde{x}$ and all the others have at least the same utility.

$$\Longrightarrow \sum_{i=1}^{n} u_i(\tilde{x}) > \sum_{i=1}^{n} u_i(y) \quad ↯$$

This is a contradiction, since y was supposed to maximize $\sum_{i=1}^{n} u_i(x)$.

$\Longrightarrow$ y has to be Pareto efficient.

$\square$