Imperfect Information in Health Care Markets

Exercise Session 4 - Insurance Demand

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Questions about the lecture

Exercise 4

Consider a person with utility of income $u(x) = \sqrt{x}$. Is this person risk averse? For the following lotteries, compute the expected income, the certainty equivalent and the risk premium.

a) Probability $1/3$ for each 1600, 2500, and 3600 Euros. ✓

b) Income is uniformly distributed between 1600 and 2500 Euros.

Exc. 4 b)

in general: $U(a,b)$ uniform distribution $\Rightarrow$ density: $f(x) = \frac{1}{b-a}$ for $x \in [a,b]$

Now: Income is distributed as $U(\underline{1600; 2500})$ (uniform distribution)

$\Leftrightarrow$ density $f(x) = \frac{1}{900}$ for $x \in [1600, 2500]$

$$E(x) = \int_{1600}^{2500} x \, \frac{1}{900} \, dx = \frac{1}{900} \left[\frac{1}{2} x^2 \right]_{1600}^{2500} = 2050$$ expected income

$\hookrightarrow$ How to compute expected values of continuous random variables (and why?) ???

$$E(u) = \int_{1600}^{2500} \underbrace{\sqrt{x}}_{=u(x)} \cdot \frac{1}{900} \, dx = \frac{1}{900} \left[\frac{2}{3} x^{\frac{3}{2}} \right]_{1600}^{2500} = \frac{1}{900} \cdot \frac{2}{3} \left(125000 - 64.000 \right) = \frac{61000 \cdot 2}{2700}$$

expected utility

$$\approx 45,185$$

$\Rightarrow \quad u(CE) \stackrel{!}{=} E(u) = 45,185$

$\Leftrightarrow \sqrt{CE} = 45,185$

$\Rightarrow CE \approx 2041,7 < \overset{E(x)}{\overset{\shortparallel}{2050}}$ $\Rightarrow$ indication that person is <u>risk-averse</u>

Certainty Equivalent

$RP = E(x) - CE = 2050 - 2041,7 = 8,3 > 0.$ Risk premium

Exercise 5

Consider a person with utility of income $u(x) = \sqrt{x}$. The person has an income of 2500 Euros but loses L Euros with probability α. Determine the certainty equivalent and the risk premium as a function of α and L. Is the risk premium increasing or decreasing in L? Is the risk premium increasing or decreasing in α?

Exc. 5

First, let us compute the expected income:

$$E(x) = \alpha \cdot (2500 - L) + (1-\alpha) \cdot 2500 = 2500 - \alpha \cdot L$$

(loss occurs) (no loss)

$$E(u) = \alpha \cdot \underbrace{\sqrt{2500-L}}_{u(2500-L)} + (1-\alpha)\underbrace{\sqrt{2500}}_{u(2500)} = \alpha \cdot \sqrt{2500-L} + 50(1-\alpha)$$

$$u(CE) \overset{!}{=} E(u)$$

$$\Leftrightarrow \sqrt{CE} = E(u)$$

$$\Rightarrow CE = \left(\alpha \cdot \sqrt{2500-L} + 50(1-\alpha)\right)^2 = \alpha^2 \cdot (2500-L) + 100(1-\alpha)\alpha\sqrt{2500-L} + 2500(1-\alpha)^2$$

binomial formula !!!

$\alpha - \alpha^2 = -(\alpha^2 - \alpha)$

$g(L) = 2500 - L$

$f(x) = \sqrt{x}$

$\hookrightarrow f(g(L))$

$$RP = E(x) - CE = 2500 - \alpha L - \alpha^2(2500-L) - 100(1-\alpha)\alpha\sqrt{2500-L} - 2500(1-\alpha)^2$$

$$= -\alpha L + 5000\alpha - 2\alpha^2 \cdot 2500 + \alpha^2 \cdot L - 100(1-\alpha)\alpha \cdot \sqrt{2500-L}$$

Is this decreasing/increasing in α and L? $\Rightarrow$ Look at derivatives:

$$\frac{\partial RP(\alpha, L)}{\partial L} = -\alpha + \alpha^2 - 100\overbrace{(1-\alpha)\cdot\alpha}^{=\alpha-\alpha^2} \cdot \frac{1}{2} \cdot (-1) \cdot \frac{1}{\sqrt{2500-L}}$$

Chain rule for derivation

greek "del" (indicates partial derivatives)

$$= \underbrace{(\alpha^2 - \alpha)}_{<0 \text{ (as } \alpha \in (0,1))} \underbrace{\left(1 - \frac{50}{\sqrt{2500-L}}\right)}_{<0, \text{ since } \sqrt{2500-L} < 50 \text{ for } L>0} > 0$$

$\Rightarrow$ When L increases, RP will increase as well

$$\frac{\partial RP(\alpha, L)}{\partial \alpha} = -L + 5000 - 10000\alpha + 2\alpha L - (1-2\alpha) \cdot 100\sqrt{2500 - L}$$

This is neither always ≥ 0 nor always ≤ 0.

For example, for $L = 900$, we get

$$\frac{\partial RP(\alpha, 900)}{\partial \alpha} = 100 - 200\alpha \begin{cases} \geq 0 & \text{for } \alpha \leq \frac{1}{2} \\ < 0 & \text{for } \alpha > \frac{1}{2} \end{cases}$$

Exercise 6

greek "eta"

Consider the utility function $u(x) = -e^{-\eta x}$. The person has an income of 1 and experiences a loss of 1 with probability α. The coefficient of absolute risk aversion is defined as $-u''(x)/u'(x)$. Compute this coefficient. Let now $\alpha = 0.5$ and check whether the certainty equivalent in- or decreases in η.

Exc. 6

$u(x) = -e^{-\eta x}$

(derivative of exponential functions)

$u'(x) = -(-\eta)\, e^{-\eta x} = \eta \cdot e^{-\eta x}$

$u''(x) = \; = \eta \cdot (-\eta) \cdot e^{-\eta x} = -\eta^2 e^{-\eta x}$

$\Rightarrow \quad -\frac{u''(x)}{u'(x)} = -\frac{-\eta^2 e^{-\eta x}}{\eta\, e^{-\eta x}} = \eta$

$\Rightarrow \eta$ measures risk aversion!

$E(u) = \alpha \cdot u(0) + (1-\alpha)\, u(1)$

$= \alpha \cdot (-e^0) + (1-\alpha) \cdot (-e^{-\eta}) = -\alpha + (1-\alpha) \cdot (-e^{-\eta})$

$u(CE) \overset{!}{=} E(u)$

$\Rightarrow \quad -e^{-\eta \cdot CE} \overset{!}{=} -\alpha - (1-\alpha)\, e^{-\eta}$

Natural logarithm (ln)

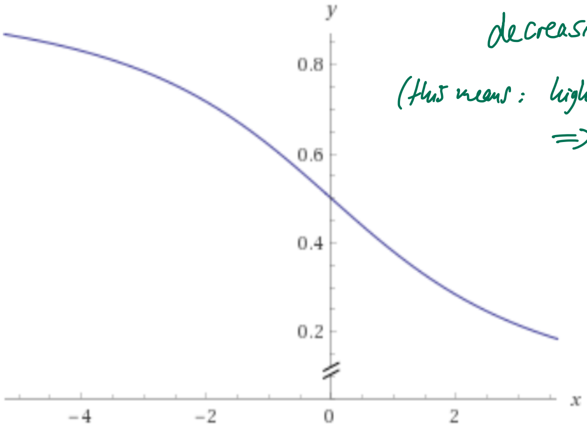
$\Leftrightarrow \quad e^{-\eta \cdot CE} = \alpha + (1-\alpha)\, e^{-\eta}$

$\Rightarrow \quad -\eta \cdot CE = \ln(\alpha + (1-\alpha) e^{-\eta})$

$\Leftrightarrow \quad CE = \frac{-\ln(\alpha + (1-\alpha) e^{-\eta})}{\eta}$,

for $\alpha = 0{,}5$: $\frac{-\ln(0{,}5 + 0{,}5 e^{-\eta})}{\eta}$

CE depending on η for $\alpha = 0.5$



decreasing in η

(this means: higher risk aversion
$\Rightarrow$ lower CE)

(x from -5.2 to 3.6)

Exercise 7

The Wall Street Journal reported in 2006 of "mini-medical insurance plans". These plans cover routine services, but little hospital coverage and usually have a cap on payouts (say of \$10.000). The premium, however, is only \$50 per month. Why might people buy a mini-medical plan? Why are such insurance plans not more popular (in a country where a substantial part of the population did/does not have health insurance)?

Exc. 7

Why people might like such a plan:

- they might have a low WTP and \$50 a month is quite cheap (also, they are risk averse)

- acts as a kind of "consumption smoothing" → don't pay one big amount once, but several small amounts more often

Why are these plans not much more popular?

- large risks are not covered, so it is not really an insurance in the proper sense (people might anticipate / fear higher payments)

- in practice, these plans would probably be too expensive for insurances, since they compare

$$50 \overset{?}{>} \underset{\text{risk}}{\alpha} \cdot \underset{\text{loss}}{L} + \text{administrative costs} \quad (+\text{ profit margin})$$