Imperfect Information in Health Care Markets

Exercise Session 5

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Exercise 8

Consider a person with utility of income $u(x) = \sqrt{x}$. The person has an income of 2500 Euros but loses 1500 Euros with probability $1/4$. Assume there is an insurance company that offers to insure an arbitrary coverage $C \in [0, 1500]$ at premium pC. Determine the amount of coverage $C(p)$ that the person will buy. (If you find this too hard, let p be 0.3.)

Exc. 8

Idea: Look at expected utility, maximize with respect to C.

$$E^{\text{no ins.}}(u) = \frac{1}{4}\underbrace{\sqrt{2500-1500}}_{\nearrow\text{ loss}} + \frac{3}{4}\underbrace{\sqrt{2500}}_{\nearrow\text{ no loss}} \quad \text{(with no insurance)}$$

With insurance:

$$E(u) = \frac{1}{4}\sqrt{2500-1500+C-pC} + \frac{3}{4}\sqrt{2500-pC}$$

Find $\max\limits_{C} E(u)$. FOC: $\frac{\partial E(u)}{\partial C} \overset{!}{=} 0$

$$\frac{\partial E(u)}{\partial C} = \frac{1}{4}(1-p)\cdot\frac{1}{2}\frac{1}{\sqrt{2500-1500+C-pC}} + \frac{3}{4}\cdot(-p)\cdot\frac{1}{2}\frac{1}{\sqrt{2500-pC}}$$

$$= \frac{(1-p)}{8}\frac{1}{\sqrt{2500-1500+C-pC}} - \frac{3}{8}\cdot p\frac{1}{\sqrt{2500-pC}} \overset{!}{=} 0$$

$$\Leftrightarrow \frac{1-p}{\sqrt{2500-1500+C-pC}} = \frac{3p}{\sqrt{2500-pC}}$$

$$\Rightarrow \frac{(1-p)^2}{1000+C-pC} = \frac{9p^2}{2500-pC}$$

$$\Leftrightarrow (2500-pC)(1-p)^2 = 9p^2(1000+C-pC)$$

$$\Leftrightarrow -9p^2C + 9p^3C - (1-p)^2pC = 9000p^2 - 2500(1-p)^2$$

$\Leftrightarrow \quad C \cdot \left(-9p^2 + 9p^3 - (1-p)^2 \cdot p\right) = 9000p^2 - 2500(1-p)^2$

$$\Leftrightarrow \quad C = \frac{9000p^2 - 2500(1-p)^2}{-9p^2 + 9p^3 - (1-p)^2 \cdot p}$$

$\Rightarrow$ This is the coverage the individual would choose

(For $p = 0{,}3$, this gives $C \approx 581{,}2$)

Exercise 9

Consider the same person as in the previous exercise. Let $p = 0.3$ and suppose the government guarantees a minimum income of 1500. Will the person still buy insurance? Discuss what features of the health care sector are similar to a minimum income guarantee in the model.

Exc. 9

exp. utility of buying an insurance

$$E^{ins}(u) = \frac{1}{4} \cdot \sqrt{2500 - 1500 + \underset{\downarrow \atop c}{581{,}2} - 0{,}3 \cdot 581{,}2} + \frac{3}{4}\sqrt{2500 - 0{,}3 \cdot 581{,}2} \approx 45{,}55$$

exp. utility of not buying an insurance

$$E^{no\ ins}(u) = \frac{1}{4}\sqrt{1500} + \frac{3}{4}\sqrt{2500} \approx 47{,}18 > 45{,}55$$

income guarantee, the government pays the person 500,
as $2500 - 1500 = 1000 < 1500$.

$\Rightarrow$ With an income guarantee, the person will not buy an insurance, as the government will pay part of the loss in case it occurs.

Similar feature of health care sector to this income guarantee:

Hospitals are obliged to treat you, even if you cannot (fully) pay the treatment.

Exercise 10

a) - e): What is the equilibrium? f) Why is it inefficient?
g) - j): What can be done to fix this?

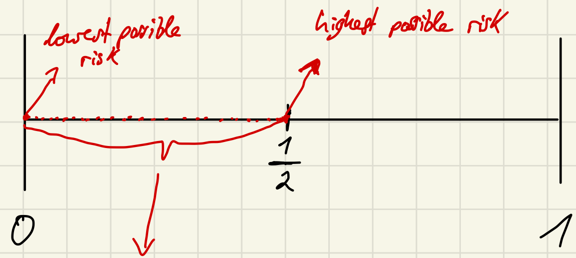
We now have a continuum of people of length $1/2$. More precisely, we have a person i for each $i \in [0, 1/2]$. All people have the same utility function $u(x) = \sqrt{x}$ and the same income of 2500. However, they differ in terms of risk: Person i loses 1600 with probability i. We consider an insurance policy with full coverage, i.e. a policy that pays out 1600 in case of a loss. Every person knows his own risk but insurance companies cannot distinguish people (and will therefore have to offer the same premium to everybody).

a) Determine the willingness to pay for the insurance of person i.

b) For every possible insurance premium, how many people will buy insurance? Use your results to draw the demand for insurance.

c) Determine the marginal cost of person i.

d) Determine the average cost of insuring all people $i \geq j$, i.e. everyone in $[j, 1/2]$.



these are all the different people, the position on the line corresponds to their risk

Exercise 10 (cont.)

e) If many risk neutral insurance companies with no administrative costs are active on this market, what is the market equilibrium?

f) Is the market equilibrium efficient? If not, determine the size of the inefficiency. What would be welfare in "first best", i.e. in a situation in which everyone with a willingness to pay above marginal cost gets insurance? Determine the relative inefficiency due to adverse selection.

g) Consider an insurance subsidy to insurers, i.e. each insurer receives for each sold insurance a subsidy payment s. How high does s have to be to ensure efficiency?

h) Consider an insurance mandate (without subsidies), i.e. everyone is forced to buy an insurance contract. What is the equilibrium insurance premium? Who will benefit from the mandate? Who will lose out with the mandate?

Exercise 10 (cont.)

i) Suppose insurers can now distinguish two groups: The people $i \geq 0.3$ and the people $i < 0.3$. Assume that insurers are allowed to offer different contracts to these two groups. Consequently, there are now two separate markets. What is the equilibrium on the "high risk market"? What is the equilibrium on the "low risk" market? Is the new situation more or less efficient than the one considered in the previous subquestions? Who benefits from group discrimination and who does not?

j) With the previous subquestion in mind, what happens if insurers can identify people better? (For example, distinguish more and more subgroups as in the previous subquestion.) What are the consequences for welfare? Who benefits and who loses?

Exc. 10 a)

WTP = number that, if paid, gives you the same utility as the lottery = (maximal) willingness to pay to avoid the lottery

Certainty Equivalent (⇒ CE = W − WTP)

prob. of a loss

$$\Rightarrow \quad u(\underbrace{2500 - WTP}_{\text{Certainty Equivalent}}) = i \cdot \sqrt{2500 - 1600} + (1-i) \cdot \sqrt{2500}$$

$$u(2500 - WTP) = \sqrt{2500 - WTP}$$

$$\Leftrightarrow \quad \sqrt{2500 - WTP} = 30i + 50(1-i) = 50 - 20i$$

$$\Rightarrow \quad 2500 - WTP = 2500 - 2000i + 400i^2$$

$$\Leftrightarrow \quad 2000i - 400i^2 = WTP \quad (= WTP(i))$$

Exercise 10 b)

For every possible insurance premium, how many people will buy insurance? Use your results to draw the demand for insurance.

Exc. 10 b)

as the WTP goes up when i goes up $\left(\frac{d\, 2000i - 400i^2}{di} = 2000 - 800i > 0 \text{ for } i \in [0; \tfrac{1}{2}] \right)$

First, note that $i = \frac{1}{2}$ has the highest WTP of $WTP(\frac{1}{2}) = 2000 \cdot \frac{1}{2} - 400\,(\frac{1}{2})^2$

$$= 1000 - 400 \cdot \tfrac{1}{4} = 900$$

So, nobody will buy an insurance if $p > 900$.

For every premium p, we know that someone with $WTP(i) = p$ will buy this insurance, as will everybody with $WTP > p$.

$\Rightarrow$ for every p, we look for the person i with $WTP(i) = p$.

$$p = WTP(i) = 2000i - 400i^2$$

$$\Leftrightarrow \quad 400i^2 - 2000i + p = 0$$

$$\Leftrightarrow \quad i^2 - 5i + \frac{p}{400} = 0$$

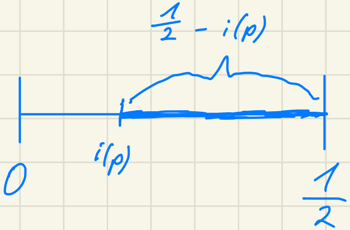
$$\Rightarrow \quad \underset{i(p)}{\underset{\parallel}{i}} = 2{,}5 \;\cancel{+}\!- \sqrt{6{,}25 - \frac{p}{400}}$$

we want $i \in [0, \frac{1}{2}]$

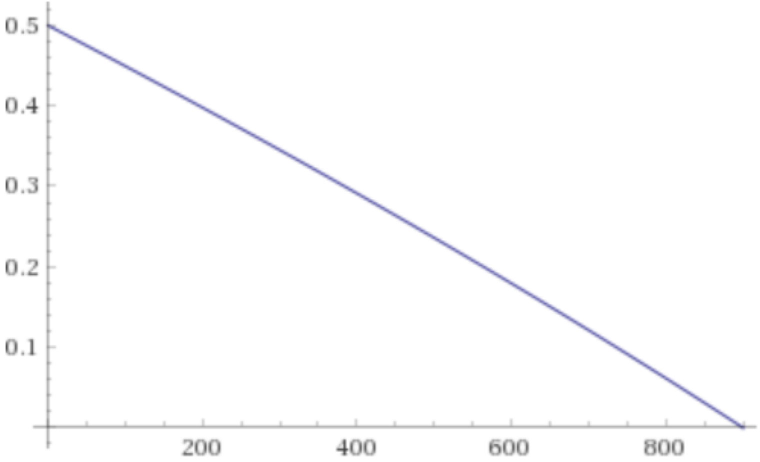
$\rightarrow$ for every p, this gives us the lowest person i that still buys insurance at premium p.

$$\Rightarrow \underbrace{D(p)}_{\text{demand of insurance at premium } p} = \frac{1}{2} - i(p) = \frac{1}{2} - 2{,}5 + \sqrt{6{,}25 - \frac{p}{400}}$$

$$= \sqrt{6{,}25 - \frac{p}{400}} - 2$$



Plot of $D(p) = \sqrt{6.25 - \frac{p}{400}} - 2$



Exercise 10 c)

Determine the marginal cost of person i.

Exc. 10c)

The $\underbrace{\text{marginal cost of person } i}_{=\ \text{cost of the ``marginal'' person } i \text{ for the insurance}}$ for the insurance is just $i \cdot 1600$.

Exercise 10 d)

Determine the average cost of insuring all people $i \geq j$, i.e. everyone in $[j, 1/2]$.

Exc. 10 d)

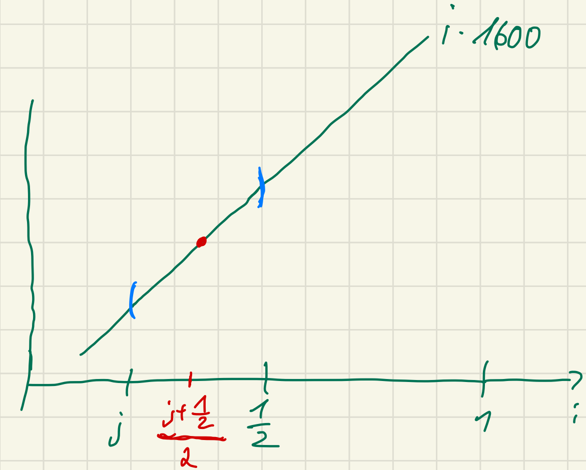
Let us denote the average costs of insuring all people in $[j, \frac{1}{2}]$ by $AC(j)$.

We know that for every i in $[j, \frac{1}{2}]$, the marginal costs are $i \cdot 1600$.

$$\Rightarrow \quad AC(j) = \frac{1}{2}\left(j \cdot 1600 + \frac{1}{2} \cdot 1600 \right) = 800\left(\frac{1}{2} + j\right)$$

Why?

This just takes the average between both borders of the interval $[j, \frac{1}{2}]$ (respectively both costs for these persons).



alternatively, you can also compute the integral

$$\frac{1}{\frac{1}{2} - j} \int_j^{\frac{1}{2}} i \cdot 1600 \, di \overset{\text{compute}}{=} 800\left(\frac{1}{2} + j\right)$$

Exercise 10 e)

If many risk neutral insurance companies with no administrative costs are active on this market, what is the market equilibrium?

Exc. 10e)

When there is perfect competition, the premium p will equal the average costs the insurance has from insuring everyone who wants to buy insurance at this premium.

$$AC(i^*) = p = WTP(i^*)$$

holds due to perfect competition (insurances will make zero profit in equilibrium)

i^* is the lowest person who buys insurance at premium p.

(we want to find this person)

this holds because the lowest person that buys the insurance is exactly indifferent between buying and not buying (because his WTP is exactly p)

$$\Rightarrow AC(i^*) \overset{!}{=} WTP(i^*)$$

$$\Leftrightarrow 800\left(\frac{1}{2} + i^*\right) = 2000\, i^* - 400\, i^{*2}$$

$$\Leftrightarrow 400\, i^{*2} - 1200\, i^* + 400 = 0$$

$$\Leftrightarrow i^{*2} - 3\, i^* + 1 = 0$$

$$\Rightarrow i^* = 1{,}5 - \sqrt{2{,}25 - 1} \approx 0{,}38$$

as we look for a solution in $[0, \frac{1}{2}]$

$$p = AC(0{,}38) = 800 \cdot \left(\frac{1}{2} + 0{,}38\right) = 704$$

$\Rightarrow$ In equilibrium, the premium will be 704 and the full insurance contract will be bought by everyone in $[0{,}38;\, 0{,}5]$ and everybody else remains uninsured.

Exercise 10 f)

Is the market equilibrium efficient? If not, determine the size of the inefficiency. What would be welfare in "first best", i.e. in a situation in which everyone with a willingness to pay above marginal cost gets insurance? Determine the relative inefficiency due to adverse selection.

Exc. 10 f)

Let us compute total welfare in the equilibrium from e):

$$\int_{0{,}38}^{0{,}5} \underbrace{WTP(i) - \cancel{p}}_{\text{welfare of person } i} + \underbrace{\cancel{p} - MC(i)}_{\text{welfare insurance gets from person } i} \, di = \int_{0{,}38}^{0{,}5} WTP(i) - MC(i) \, di$$

$$= \int_{0{,}38}^{0{,}5} 2000i - 400i^2 - 1600i \, di = 400 \cdot \int_{0{,}38}^{0{,}5} i - i^2 \, di = 400 \left[\tfrac{1}{2} i^2 - \tfrac{1}{3} i^3\right]_{0{,}38}^{0{,}5} \approx 11{,}584$$

Efficiency: Everyone with $WTP > MC$ is insured.

$$WTP(i) - MC(i) = 2000i - 400i^2 - 1600i = 400i - 400i^2 \geq 0 \quad \text{for all } i \text{ in } [0, \tfrac{1}{2}]$$

$\Rightarrow$ efficient to insure <u>everyone</u>.

Total welfare from insuring everyone:

$$\int_{0}^{0{,}5} WTP(i) - MC(i) \, di = 400 \left[\tfrac{1}{2} i^2 - \tfrac{1}{3} i^3\right]_0^{0{,}5} \approx 33{,}333$$

everyone insured $\rightarrow$ 0

Relative inefficiency: $\dfrac{33{,}333 - 11{,}584}{33{,}333} \approx 0{,}653$ $\Rightarrow$ There is quite an efficiency loss due to adverse selection