Mechanism Design

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Outline

Equilibrium

- Last time:

A mechanism $\Gamma = (S_1, \ldots, S_I, g)$ implements f if the game induced by the mechanism has an equilibrium $(s_1^*, \ldots, s_I^*)$ such that $g(s_1^*(\theta_1), \ldots, s_I^*(\theta_I)) = f(\theta)$.

- What kind of equilibrium?
- Today and next week: dominant strategy equilibrium

Recap: Dominant strategy

- dominant strategy s_i: s_i is utility maximizing for agent *i no matter what the other players do*

Definition (dominant strategy)

$s_i : \Theta_i \to S_i$ is a dominant strategy if for all $\theta_i \in \Theta_i$ $u_i(g(s_i(\theta_i), s_{-i}), \theta_i) \geq u_i(g(\hat{s}_i, s_{-i}), \theta_i)$ for all $\hat{s}_i \in S_i$ and all $s_{-i} \in S_{-i}$.

Dominant strategy equilibrium

- in a dominant strategy equilibrium every player plays a dominant strategy

Definition (dominant strategy equilibrium)

A strategy profile $(s_1^*, \ldots, s_I^*)$ is a dominant strategy equilibrium of mechanism $\Gamma = (S_1, \ldots, S_I, g)$ if for all i and all $\theta_i \in \Theta_i$ $u_i(g(s_i^*(\theta_i), s_{-i}), \theta_i) \geq u_i(g(\hat{s}_i, s_{-i}), \theta_i)$ for all $\hat{s}_i \in S_i$ and all $s_{-i} \in S_{-i}$.

Dominant strategy equilibrium

Example (Prisoner's dilemma)

	C	NC
C	-5,-5	-1,-10
NC	-10,-1	-2,-2

Dominant strategy equilibrium

Example (Second price sealed bid auction)

- one object is auctioned off
- each bidder has valuation θ_i which is his private information
- highest bidder gets the object but has to pay only the second highest bid as price
- Check: bidding one's true valuation θ_i is a dominant strategy equilibrium

Implementation in dominant strategies

- A mechanism implements f in dominant strategies if
  - the game induced by the mechanism has a dominant strategy equilibrium
  - the outcome in this equilibrium coincides with f

Definition (implementation in dominant strategies)

A mechanism $\Gamma = (S_1, \ldots, S_I, g)$ implements the social choice function f in dominant strategies if there exists a dominant strategy equilibrium $(s_1^*, \ldots, s_I^*)$ of Γ such that $g(s_1^*(\theta_1), \ldots, s_I^*(\theta_I)) = f(\theta)$ for all $\theta \in \Theta$.

Why implementation in dominant strategies?

- very robust equilibrium concept
  - no need to predict what the other players will play
  - no need to know the type distribution ϕ
  - works even if
- players don't know ϕ or even if players believe in different ϕ_i
- players doubt that other players are not rational

Towards revelation principle: Example

Example (Trade/Auction)

- one indivisible good (say a house)
- individual 0 owns the house and values it $\theta_0 = 1$
- each (potential) buyer i knows his valuation θ_i but not the valuation of the others
- ϕ: the θ_i's are independently and uniformly distributed on $[1, 2]$
- f: the seller wants to sell the good to the bidder j with the highest valuation and get a price equal to θ_j

- Is f truthfully implementable in dom. strat.?
- Is there another mechanism implementing f in dom. strat.?

Revelation principle (informal)

- trying out thousands of possible mechanisms is tedious
- revelation principle:
  - if f can be implemented in dominant strategies by some mechanism Γ, then f is truthfully implementable
  - if truthful implementation does not work, no mechanism will work

Revelation principle (formal)

Theorem (revelation principle for dominant strategies)

Suppose there exists a mechanism $\Gamma = (S_1, \ldots, S_I, g)$ *that implements the social choice function* f *in dominant strategies.*
Then f *is truthfully implementable in dominant strategies.*

Revelation principle: Intuition

Revelation principle: Proof

Proof (Revelation principle).

Let Γ implement f in dominant strategies, i.e. there is a strategy profile $(s_1^*, \ldots, s_I^*)$ such that $g(s_1^*(\theta_1), \ldots, s_I^*(\theta_I) = f(\theta)$ for all θ, and for all i and $\theta_i \in \Theta_i$ $u_i(g(s_i^*(\theta_i), s_{-i}), \theta_i) \geq u_i(g(\hat{s}_i, s_{-i}), \theta_i)$ for all $\hat{s}_i \in S_i$ and all $s_{-i} \in S_{-i}$. …

□

Revelation principle: Why is it awesome?

- easy tool to check whether a given social choice function can be implemented by any mechanism (in dominant strategies):

just check truthful implementation

- if we can find out which social choice functions are truthfully implementable, we know which outcomes (SCFs) we can achieve

Example

Example (trade/auction continued)

- can f be achieved with any mechanism?
- assume $I = 1$
  - a direct mechanism (i) asks for the type, (ii) determines a payment and (iii) determines whether the agent gets the good or not
  - which mechanisms are truthfully implementable?
  - what is the optimal selling mechanism?

Example

Example (Second price sealed bid auction continued)

- bidding one's true valuation, i.e. $bid_i(\theta_i) = \theta_i$, is dominant strategy equilibrium
- revelation principle: same outcome can be implemented with a direct mechanism (Check!)

Example

Example (The equal society)

- society of I people; half of them have high productivity
- everyone has either a high productivity θ^h or a low productivity $\theta^l < \theta^h$
- a θ^h (θ^l) type produces 4 (2) units per full day
- working full day leads to a utility loss of 1 unit
- working half day leads to a utiliy loss of 0.5 units
- the government can only observe the income=production but not the type (nor number of hours worked)
- the government can use taxation to redistribute income

- Can the government achieve an equal society in which everyone works full time and has an income of 3?

Weak preference reversal property I

- if a SCF f is incentive compatible in dominant strategies and agent i has type θ_i', then agent i must prefer announcing his true type θ_i' to announcing a false type θ_i'' (no matter what the other agents' types are!)
  for every θ_{-i} and all $\theta_i'' \in \Theta_i$
  $$u_i(f(\theta_i', \theta_{-i}), \theta_i') \geq u_i(f(\theta_i'', \theta_{-i}), \theta_i')$$

- and if agent i has type θ_i'', then for every θ_{-i} and all $\theta_i'' \in \Theta_i$
  $$u_i(f(\theta_i'', \theta_{-i}), \theta_i'') \geq u_i(f(\theta_i', \theta_{-i}), \theta_i'')$$

Weak preference reversal property II

- If f is incentive compatible in dominant strategies, then agent i's preference ranking over $f(\theta_i', \theta_{-i})$ and $f(\theta_i'', \theta_{-i})$ must weakly reverse when his type changes from θ_i' to θ_i''.
- In exercise, you are asked to show the opposite:

If the weak preference reversal property holds for all θ_{-i} and all θ_i', $\theta_i'' \in \Theta_i$, then truth telling is a dominant strategy for agent i.

- In short, weak preference reversal property and incentive compatibility in dominant strategies are equivalent.

Towards Gibbard Satterthwaite

BIG QUESTION:

Which social choice functions are incentive compatible in dominant strategies?

Towards Gibbard Satterthwaite

- One class of ic social choice functions that are however *not so nice* are dictatorial choice functions:

Definition (dictatorial social choice function)

The social choice function is dictatorial if there is an agent i (the dictator) such that for all $\theta \in \Theta$
$f(\theta) \in \{x \in X : \; u_i(x, \theta_i) \geq u_i(y, \theta_i) \; \text{ for all } \; y \in X\}$.

- roughly: if the social choice function always picks the alternative that i loves most, then i is a dictator
- Check: a dictatorial social choice function is incentive compatible and Pareto efficient

Gibbard Satterthwaite Theorem (informal) I

Assumptions

- X is a finite set with at least 3 elements, say $X = \{x_1, x_2, \dots, x_n\}$
- preferences are strict, i.e. no agent is indifferent between two alternatives x_m and x_k
  - all preferences over X are possible; e.g. for $n = 3$ this means that for each player i there is
  - a type θ_i^1 such that $u_i(x_1, \theta_i^1) > u_i(x_2, \theta_i^1) > u_i(x_3, \theta_i^1)$
  - a type θ_i^2 such that $u_i(x_1, \theta_i^2) > u_i(x_3, \theta_i^2) > u_i(x_2, \theta_i^2)$
  - a type θ_i^3 such that $u_i(x_2, \theta_i^3) > u_i(x_1, \theta_i^3) > u_i(x_3, \theta_i^3)$
  - a type θ_i^4 such that $u_i(x_2, \theta_i^4) > u_i(x_3, \theta_i^4) > u_i(x_1, \theta_i^4)$
  - a type θ_i^5 such that $u_i(x_3, \theta_i^5) > u_i(x_2, \theta_i^5) > u_i(x_3, \theta_i^5)$
  - a type θ_i^6 such that $u_i(x_3, \theta_i^6) > u_i(x_1, \theta_i^6) > u_i(x_2, \theta_i^6)$

Gibbard Satterthwaite Theorem (informal) II

Result

Only dictatorial social choice functions are truthfully implementable in dominant strategies.

Gibbard Satterthwaite Theorem (formal)

Theorem (Gibbard Satterthwaite Theorem)

Suppose X is finite and contains at least three elements. Suppose further that all preferences on X are possible for all agents i.
A social choice function f is then truthfully implementable in dominant stategies if and only if it is dictatorial.

Proof.

(skipped; see, for example, Lars-Gunnar Svensson, Alexander Reffgen, The proof of the Gibbard–Satterthwaite theorem revisited, Journal of Mathematical Economics, Volume 55, December 2014, Pages 11-14, `http://dx.doi.org/10.1016/j.jmateco.2014.09.007`.) □

Gibbard Satterhwaite Theorem: Interpretation and economics

- in connection with revelation principle:
  only dictatorial social welfare functions can be implemented by any mechanism
- original application of mechanism design: writing a constitution
  What does the Gibbard Satterthwaite theorem imply in this context?
- quite demoralizing!
- comment: similar result holds for infinite X

Gibbard Satterhwaite Theorem: What now?

- two ways to get out of this negative result:
  - don't allow all possible preferences (next week)
  - don't use dominant strategy implementation; i.e. use Bayesian Nash equilibrium instead of dominant strategy equilibrium (in two weeks)
- both ways out have their drawbacks!!!

Teaser

You rent a furnished flat with a friend of yours. You sit down with the landlord and discuss whether to buy a dish washer. You and your friend have a private valuation for the dishwasher of θ_{you} and θ_{friend}. You all agree that the dish washer should be bought if and only if $\theta_{you} + \theta_{friend} \geq c$ where c is the cost of the dish washer. Can you find a dominant strategy mechanism that achieves this goal? (there is the possibility of you paying money to the landlord for buying the dish washer and all of you have linear utility functions)

Review questions

- Reading: MWG p. 869-876
- What is a dominant strategy equilibrium? How is it defined?
- What is the main advantage of dominant strategy equilibrium?
- What does the revelation principle say?
- What is the idea behind the revelation principle?
- What are the implications of the Gibbard Satterthwaite theorem for mechanism design? Why is it a "negative result"?

Exercises

1. Consider a two person auction. Is there a payment rule ($t : \Theta \to \Re^2$) such that this payment rule together with the decision rule "agent 1 gets the good if and only if his valuation is at least twice as high as agent 2's valuation" is implementable in dominant strategies?
2. MWG 23.C.2