Mechanism Design

Christoph Schottmüller

Outline

1. Groves Clarke Mechanism
2. More general formulation
3. Review questions and exercises

Where we left last time

Gibbard Satterthwaite:

If we allow for all possible preferences and there are at least three alternatives, then only dictatorial social choice functions are implementable in dominant strategies.

- Today:
  - don't allow all possible preference
  - "quasilinear preferences"
  - can an efficient and non-dictatorial social choice function be implemented?

Public project example

Example (Provision of a public good)

- decision whether to provide a public good
- costs of public good are c
- I agents (citizens)
- agent i values the good θ_i which is his private information and possibly negative
- agent i has utility $t_i + \theta_i$ if the good is provided and t_i if it is not
- t_i is transfer to i (additional tax); most likely $t_i < 0$
- When is it efficient to provide the public good?
- Is there an efficient mechanism?

Setup

- alternative: $x = (k, t_1, t_2, \ldots, t_I)$ where $k \in \{0, 1\}$ is whether the good is provided or not
- set of alternatives
  $X = \{(k, t_1, t_2, \ldots, t_I) : \; k \in \{0, 1\}, \; \sum_i t_i \leq -k * c\}$
- utility function: $u_i(x, \theta_i) = k\theta_i + t_i + \bar{m}_i$ where $\bar{m}_i$ is the initial wealth of i
- a social choice function f assigns one alternative to every type profile θ of the players

$$f(\theta) = (k(\theta), t_1(\theta), \ldots, t_I(\theta))$$

Efficient mechanism

- a social choice rule f^* can be efficient if the good is provided if and only if the sum of valuations is higher than the costs, i.e. $k^*(\theta) = 1$ if and only if

$$\sum_{i=1}^{I} \theta_i \geq c$$

- is there an implementable social choice function that is efficient?
- revelation principle: we only need to consider direct revelation mechanisms
  - every agent announces his type
  - announcing his true type must be a dominant strategy for each agent

case $c = 0$

- assume for now $c = 0$
- example: whether to allow a multinational to build a new plant in a community
- efficiency in case $c = 0$: allow if $\sum_{i=1}^{I} \theta_i \geq 0$

The trick (case $c = 0$)

- we design the transfer payment such that every agent becomes a welfare maximizer when he decides about his type announcement!

Theorem (Groves mechanism)

The following transfers truthfully implement the efficient decision rule k^* in dominant strategies $t_i(\theta) = k^*(\theta)\left(\sum_{j \neq i} \theta_j\right) + h_i(\theta_{-i})$ where h_i is an arbitrary function of θ_{-i}.

Groves mechanism (case $c = 0$)

- agent i maximizes his utility over his type announcement $\hat{\theta}_i$

$$U(\hat{\theta}_i, \theta_i) = t_i(\hat{\theta}_i, \hat{\theta}_{-i}) + k^*(\hat{\theta}_i, \hat{\theta}_{-i})\theta_i + \bar{m}_i$$

with the Groves transfers we get

$$U(\hat{\theta}_i, \theta_i) = k^*(\hat{\theta}_i, \hat{\theta}_{-i}) \left(\theta_i + \sum_{j \neq i} \hat{\theta}_j\right) + h_i(\hat{\theta}_{-i}) + \bar{m}_i$$

- this last expression is maximized by announcing the true type because k^* is efficient decision also in case $\theta = (\theta_i, \hat{\theta}_{-i})$

What about this h? (case $c = 0$)

- h can be any function of θ_{-i}
- an interesting suggestion was made by Clarke
  - let $k^*_{-i}(\theta_{-i})$ be the efficient decision if i did not exist, i.e. $k^*_{-i}(\theta_{-i}) = 1$ if and only if

    $$\sum_{j \neq i} \theta_j \geq 0$$

    and 0 otherwise
  - then we define h as

    $$h(\theta_{-i}) = -k^*_{-i}(\theta_{-i}) \left(\sum_{j \neq i} \theta_j \right)$$

Clarke-Groves mechanism (case $c = 0$)

- the Clarke-Groves mechanism uses the Clarke h function leading to

$$t_i(\theta) = k^*(\theta) \left(\sum_{j \neq i} \theta_j \right) - k^*_{-i}(\theta_{-i}) \left(\sum_{j \neq i} \theta_j \right)$$

- Interpretation:
  - What is the transfer of an agent who does not influence the decision (e.g. factory is built with and without this agent)?
  - What is the transfer of an agent who changes the decision? (is *pivotal*)
  - externality transfers

Example (case $c = 0$)

Example (the new factory)

- three agents (Alice, Bob, John) with valuations $\theta_A = 2$, $\theta_B = -3$ and $\theta_J = 2$ for allowing a new factory to be built
- should it be built?
- what are the Groves-Clarke transfers for the three agents?

Budget balance (case $c = 0$)

- note that $\sum_{i=1}^{I} t_i \leq 0$ in the Groves-Clarke mechanism
  - each $t_i(\theta) \leq 0$ for any θ because k^*_{-i} is the efficient decision for agents $j \neq i$

$$t_i(\theta) = k^*(\theta)\left(\sum_{j\neq i}\theta_j\right) - k^*_{-i}(\theta_{-i})\left(\sum_{j\neq i}\theta_j\right)$$

- usually agents have to pay more than what other agents receive!

What if $c > 0$?

- for $c > 0$, we use a little trick
  - define the fictional utility function $\tilde{u}_i(k, \theta) = k\left(\theta_i - \frac{c}{I}\right) + t_i + \bar{m}_i$
  - construct Groves-Clarke payments in the fictional probelm (i.e. where agents have valuation $\theta_i - \frac{c}{I}$)

$$\tilde{t}_i(\theta) = k^*(\theta)\left[\sum_{j \neq i}\left(\theta_j - \frac{c}{I}\right)\right] - k^*_{-i}(\theta_{-i})\left[\sum_{j \neq i}\left(\theta_j - \frac{c}{I}\right)\right]$$

where $k^*_{-i}(\theta_{-i}) = 1$ if and only if

$$\sum_{j \neq i}\left(\theta_j - \frac{c}{I}\right) \geq 0$$

What if $c > 0$?

- now use the following transfers

$$
\begin{aligned}
t_i(\theta) &= \tilde{t}_i(\theta) - k^*(\theta)\frac{c}{I} \\
&= k^*(\theta)\left[-\frac{c}{I} + \sum_{j \neq i}\left(\theta_j - \frac{c}{I}\right)\right] - k^*_{-i}(\theta_{-i})\left[\sum_{j \neq i}\left(\theta_j - \frac{c}{I}\right)\right] \\
&= k^*(\theta)\left[-c + \sum_{j \neq i}\theta_j\right] + h_i(\theta_{-i})
\end{aligned}
$$

What if $c > 0$?

- it is still a dominant strategy to reveal the true type because each agent is a "welfare maximizer"

$$U_i(\hat{\theta}_i, \hat{\theta}_{-i}, \theta_i) = k^*(\hat{\theta}_i, \hat{\theta}_{-i}) \left[\theta_i - c + \sum_{j \neq i} \hat{\theta}_j\right] + h_i(\hat{\theta}_{-i}) + \bar{m}_i$$

- the budget is covered if $k^*(\theta) = 1$:
  - recall that $\tilde{t}_i(\theta) \leq 0$
  - then $t_i(\theta) = \tilde{t}_i(\theta) - \frac{c}{I} \leq -\frac{c}{I}$
- budget is usually more than covered!

Example ($c > 0$)

Example (Should Copenhagen host the Olympics?)

- there are three equally big groups in CPH: "sport fanatics" valuing Olympics $\theta_{sf} = 30$, the non-sporter with valuation $\theta_{ns} = 0$ and the mildly interested with valuation $\theta_{mi} = 5$
- costs of Olympics are 24
- what is the efficient decision?
- what are the fictional transfers $\tilde{t}_i$?
- what are the transfers t_i in the Groves-Clarke mechanism?
- what is the sum of the payments in the Groves-Clarke mechanism?

A small generalization 1

- say there are different levels of k (e.g. how much greenhouse gas emissions are allowed?)
- agents have utility $u_i(k, \theta_i) = v_i(k, \theta_i) + t_i + \bar{m}_i$
- efficiency (we assume $c(k) = 0$ for now): $k^*(\theta)$ is

$$arg \max_k \sum_{i=1}^{I} v_i(k, \theta_i)$$

A small generalization 2

- Groves transfers:

$$t_i(\theta) = \left(\sum_{j \neq i} v_j(k^*, \theta_j) \right) + h_i(\theta_{-i})$$

- it is a dominant strategy to announce true type as each agent is again a "welfare maximizer"

$$U_i(\hat{\theta}_i, \theta_i) = v_i(k^*, \theta_i) + \sum_{j \neq i} v_j(k^*, \hat{\theta}_j) + h_i(\hat{\theta}_{-i})$$

A small generalization 3

- Groves-Clarke mechanism:

$$h_i(\theta_{-i}) = k^*_{-i}(\theta_{-i}) \sum_{j \neq i} v_j(k^*, \theta_j)$$

where $k^*_{-i}(\theta_{-i})$ is defined as

$$arg \max_k \sum_{j \neq i} v_j(k, \theta_j)$$

- if $c(k) > 0$ for some k, we can use the same trick as before:
- fictional valuations $\tilde{v}_i(k, \theta_i) = v_i(k, \theta_i) - \frac{c(k)}{I}$
- leading to some fictional Groves-Clarke transfers $\tilde{t}_i$
- transfers $t_i(\theta) = \tilde{t}_i(\theta) - \frac{c(k^*(\theta))}{I}$

Groves-Clarke mechanism: Take aways and economics

- with quasilinear utility functions dominant strategy implementation can work (to a certain extent)
- public good provision problems can be solved efficiently (but...)

Review questions

- The Groves-Clarke mechanism implements the efficient allocation in dominant stratgies. Why does this not violate the Gibbard-Satterthwaite theorem?
- Write down the Clarke-Groves transfers. Why are these transfers sometimes called "externality transfers"?
- Why is it a dominant strategy in the Clarke-Groves mechanism to reveal one's true type? What is the idea behind the mechanism?

Exercise I

- Our society consists of four people who have the following valuations for building a bridge: $\theta_1 = 3$, $\theta_2 = 4$, $\theta_3 = -5$, $\theta_4 = 0$. The costs of building the bridge are 1. Compute the transfers $\tilde{t}_i$ and t_i in Groves-Clarke mechanism.

Exercise II

- Think of the following allocation problem: There is 1 indivisible good that has to be allocated to one of I agents. Agent i has valuation θ_i for the good. What is the efficient allocation? Apply the Groves-Clarke mechanism to this problem and show that it leads to the same allocation and transfers as a second price sealed bid auction.

(if you have troubles with this, write down an example with three agents and give them some arbitrary valuations; then calculate the Groves-Clarke transfers by (i) trying to make i a welfare maximizer and (ii) determine h by the optimal allocation if i was not there. compare it to the outcome of the auction)

Teaser

Reconsider a variation of last week's teaser:
You rent a furnished flat with a friend of yours. You sit down and discuss whether to buy an espresso machine. You and your friend have a private valuation for the machine of θ_{you} and θ_{friend}. You agree that the machine should be bought if and only if $\theta_{you} + \theta_{friend} \geq c$ where c is the cost of the espresso machine.
Your landlord is willing to supervise your mechanism as a referee. However, the machine will be yours and therefore your landlord will neither accept money nor contribute financially. Is there a (budget balanced!) mechanism that implements the efficient choice function? I.e. is there an efficient mechanism in which the sum of your payments equals c if you buy the machine and 0 if you do not buy the machine? Since you and your flatmate are pretty clever and pretty materialistic, we allow for Bayesian Nash equilibrium implementation and linear utility function.