Midterm “Mechanism Design”

March 21, 2016

1. Assume there are three candidates in an election $X = \{A, B, C\}$ and 5 voters. Preferences are strict and single peaked. This means only the following preference orderings are possible:

   - $A \succ B \succ C$
   - $C \succ B \succ A$
   - $B \succ A \succ C$
   - $B \succ C \succ A$

   (a) Now consider the social choice function $\tilde{f}$ that chooses the median of the top ranked: Think of the candidates being ordered on a line where A is at -1, B is at 0 and C is at 1. Each voter is located at the position of his most preferred candidate. There is a voter i (the median) such that (i) at least 3 voters are located to the left of i or at the same location as i (including i himself) and (ii) at least 3 voters are located to the right of i or at the same location as i (including i himself). [1] $\tilde{f}$ chooses the most preferred alternative of the median.
   Is this social choice function implementable in dominant strategy equilibrium?

[1] To illustrate, say 2 voters prefer candidate A most, 2 voters prefer candidate B most and 1 voter prefers C most. Then the median is one of the voters that like B most: There are 2 voters to their left and 2—including i himself—at the same location (hence (i) is satisfied as $2 + 2 = 4 \geq 3$). There is one voter to the right and 2—including i himself—at the same location and therefore (ii) is satisfied as $2 + 1 = 3$.

(b) If we did not assume that preferences are single peaked, i.e. if we allow all voters to have arbitrary strict preferences over the three candidate, would $\tilde{f}$ be implementable in dominant strategy equilibrium?

2. This exercise deals with the Groves-Clarke mechanism and the expected externality mechanism. Consider the problem of the provision of a public good. There are three people: Alice who values the public good $\theta_A = 5$, Bob whose value is $\theta_B = 10$ and Carol with value $\theta_C = 15$. The costs of the public good are 24.

   (a) Is it efficient to provide the public good?

   (b) What are the Groves-Clarke transfers for the three agents?

   (c) Now assume $\theta_A = 5$ and $\theta_B = \theta_C = 10$. What are the transfers of the three agents in the expected externality mechanism if the prior is the following: each player draws his valuation independently from a distribution that puts probability 1/2 on 5, 1/2 on 10?

   (d) Which main advantage does the expected externality mechanism have over the Groves-Clarke mechanism? Which main advantage does the Groves-Clarke have over the expected externality mechanism mechanism?

3. Take the bilateral trade setup described in lecture 5 (Myerson-Satterthwaite lecture): 1 buyer with valuation v uniformly distributed on $[0, 1]$ and 1 seller with costs c uniformly distributed on $[0, 1]$. v is private information of the buyer, c is private information of the seller. Both buyer and seller have linear utility functions ($yv - t$ and $t - yc$ respectively where y is the probability of trade and t is a transfer payment from buyer to seller). Is there a social choice function that is (i) efficient, that is trade if and only if $v > c$, (ii) budget balanced, i.e. every payment by the buyer is received by the seller and vice versa and (iii) implementable in *dominant strategy equilibrium*? (Note that we do not require voluntary participation in this exercise!)

Either give a social choice function that satisfies all criteria or prove that no such social choice function exists.

4. We turn to Bayesian Nash equilibrium implementation. Take a public good setting: There are 2 agents with private valuation for a public good. Their valuation is either 0 or 1. The probability that the valuation of agent i is 0 is 1/2. The two valuations are independent from another. The costs of the public good are 3/4. A direct revelation mechanism has to decide whether the good is provided or not and has to specify transfer payments. We denote by $t_i(\theta_1, \theta_2)$ the transfer that agent i receives when the valuation of agent 1 is θ_1 and the valuation of agent 2 is θ_2. In this exercise, we will show that there exists no mechanism that satisfies the following four conditions:

   - efficiency: the good is provided if and only if at least one agent has valuation 1
   - interim voluntary participation: the expected utility from participating in the mechanism is at least 0 (for every type of every agent)
   - Bayesian incentive compatibility: each agent maximizes his expected utility by announcing his true type in the direct revelation mechanism
   - budget balance: the sum of the payments has to be 0 if the good is not provided and equal to -3/4 if the good is provided (no matter what the types are)

   We assume that there exists a direct revelation mechanism that satisfies the first three conditions and show that such a mechanism does not satisfy the fourth condition.

   (a) Is it enough to show that there is no direct revelation mechanism satisfying all the conditions above (in order to show that there is no mechanism satisfying the conditions above)?

   (b) Using the interim voluntary participation constraint, show that $t_1(0,1) + t_1(0,0) \geq 0$ and that $t_2(1,0) + t_2(0,0) \geq 0$.

(c) Use incentive compatibility and the previous subquestions to show that $t_1(1,1) + t_1(1,0) \geq -1$.

(d) Use incentive compatibility and the previous subquestions to show $t_2(1,1) + t_2(0,1) \geq -1$.

(e) Combine the results from previous subquestions to show that the mechanism violates budget balance.

(f) What are the implications of this impossibility result for economics/politics? (for this answer you can assume that the result is much more general, i.e. does not depend on the specific valuations, distribution of valuations, number of players etc.)

5. This exercise is about optimal regulation of a monopolist. You can for example think of a private garbage disposal company. The municipality that contracts with the company has to determine how often the garbage is collected per month and what price the company is paid. For simplicity, we assume that the possible numbers of monthly collections q is continuous (i.e. we neglect integer problems and allow q to be any positive real number). Citizens value garbage disposal $u(q) = q$. The company has costs of $c(q, \theta) = (1 + \theta)q^2$ where θ is the company's private information which is distributed uniformly on $[0, 1]$. The municipality maximizes the expected value of consumer valuation minus the price it has to pay. The firm maximizes its profits.

   We will concentrate in this exercise on mechanisms where the municipality announces two functions: $q(\theta)$ and $t(\theta)$. Then the company announces a type $\hat{\theta} \in [0, 1]$ (or decides not to participate which gives it a payoff of 0). It then has to provide $q(\hat{\theta})$ and receives the transfer payment $t(\hat{\theta})$. We will restrict ourselves further by only using mechanisms in which it is optimal for the company to announce its true type θ.

(a) Could the municipality do better by considering more elaborate mechanisms?

(b) Denote the company's rent under the optimal mechanism by $U(\theta) = t(\theta) - (1+\theta)q(\theta)^2$. Show that the following has to hold if announcing its true type is optimal for the company:

- $U(\theta) = U(1) + \int_\theta^1 q(s)^2\, ds$
- $q(\cdot)$ is non-increasing

(c) Show that the company wants to announce its true type in every mechanism in which the two conditions of the previous subquestion are satisfied.

(d) Using the results from the previous subquestions, show that the municipality's objective can be written as $\int_0^1 q(\theta) - (1+2\theta)q(\theta)^2\, d\theta - U(1)$.

(e) What is $U(1)$ in the optimal contract? Show that the optimal $q(\theta)$ is $q(\theta) = 1/(2+4\theta)$.

(f) What is the optimal $t(\theta)$?

(g) What would be the optimal $q(\cdot)$ if the municipality could observe θ ? (this is sometimes called the first best q^{fb})

Compare q^{fb} with the optimal q derived in question (e). Why are they different?