Mechanism Design

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Outline

Lecturer

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- Research interests: contract theory, mechanism design, game theory, (health) insurance

You ?

Course I

- What can you expect?
  - relatively mathematical/technical course
  - I try my best to give you the intuition and economics behind the models
  - you learn methods that are applicable to a **lot of** areas (industrial organization, political economy, taxation, auctions. . .)
  - you will be able to read and understand recent research that applies the tools covered in the course
  - very interesting and intelectually stimulating topic (at least from the lecturer's point of view)
  - see course description for details on what you learn

Course II

What is expected from you?

- revise your math/game theory if necessary
- a little bit of participation in the lecture
- workload in between lectures:
  - **minimal:** read the indicated readings, review until you are able to answer all the "review questions" at the end of the presentation
  - **standard:** minimal + do exercises on last slide + think about "teaser"
  - **get the most out of it:** standard + more exercises in MWG + think about how to model the "teaser" problems
- one week take home midterm assignment (groups allowed and recommended)
- one week take home exam (individual)

Course III: outline (roughly)

- textbook part (MWG ch. 23)
  - dominant strategy mechanism design
  - Groves Clarke mechanism
  - budget balance problem
  - Bayesian mechanism design
  - participation problem: Myerson Satterthwaite theorem
  - optimal Bayesian mechanisms

Course IV: outline (roughly)

- papers
  - externalities on other players
  - correlated types
  - robust mechanism design
- matching mechanisms
  - Gale-Shapley algorithm
  - school choice mechanisms
  - (kidney donation rings)

Course V: Examination

- mandatory midterm:
  - one week take home (you should work in groups of up to 3 students)
  - passing required to participate in the final exam
- final exam:
  - one week take home (individual, no groups)
  - graded on usual scale

Reading

- textbook:
  (MWG) Mas-Colell, Whinston, Green. Microeconomic theory. New York: Oxford University Press, 1995.
  (Tilman Börgers. An introduction to the theory of mechanism design)
- research papers

Course VI: Related courses

- Contract theory and the economics of organization
- (Auctions)
- Other courses in which applications of mechanism design appear:
    - Public finance
    - Industrial organization
    - Political economy
    - Corporate finance
    - Taxation
    - ...

What is Mechanism Design?

- people ("agents") have private information
- which game do you have to let agents play to get the outcome that you want?

The Mechanism Design Problem: Examples

Example: A public project

- I residents of an island decide whether to build a bridge to the main land which costs c
- each resident knows how much he values the bridge himself but does not know the others' valuation
- resident i has valuation θ_i and wealth m_i
- if resident i has to pay t_i his utility is $m_i + \theta_i - t_i$

- When is it efficient to build a bridge?
- How should the decision and payments be organized?

The Mechanism Design Problem: Examples

Example: A public project (continued)

- Egalitarian contribution rule:
  - everyone is asked for his valuation
  - the bridge is built if the sum of the announced valuations is greater than c
  - if the bridge is built, everyone pays c/I

- Does the egalitarian contribution rule lead to efficiency?
- Would you announce your true valuation?

The Mechanism Design Problem: Examples

Example (Trade/Auction)

- one indivisible good (say a house)
- individual 0 owns the house and values it θ_0
- I individuals want to buy the house
- each (potential) buyer knows his valuation θ_i but not the valuation of the others
- the seller wants to sell the house to the buyer j with the highest valuation and get a price equal to θ_j
- can this be done?
- with which mechanism can the seller get the maximum possible revenue?

The Mechanism Design Problem: General formulation

- I agents
- collective choice from a set X of alternatives
- nature assigns to each agent i his type $\theta_i \in \Theta_i$ which is i's private information
- the type vector $\theta = (\theta_1, \ldots, \theta_I)$ is distributed according to a distribution ϕ
- each agent has a utility function $u_i(x, \theta_i)$ that depends on the collective choice and his type

The Mechanism Design Problem: General formulation

Definition (Social choice function)

A *social choice function* is a function $f : \Theta_1 \times \cdots \times \Theta_I \to X$ that assigns to each profile of types $(\theta_1, \ldots, \theta_I)$ a collective choice $f(\theta_1, \ldots, \theta_I) \in X$.

- roughly:
  - social choice function gives a desired outcome as a function of the agents' types

The Mechanism Design Problem: General formulation

Definition (Efficiency)

The social choice function f is *(Pareto) efficient* if for no type profile $\theta = (\theta_1, \ldots, \theta_I)$ there is an $x \in X$ such that $u_i(x, \theta_i) \geq u_i(f(\theta), \theta_i)$ for every i and $u_i(x, \theta_i) > u_i(f(\theta), \theta_i)$ for some i.

- roughly:
    - there is no type profile such that assigning some alternative x instead of $f(\theta)$ makes everyone better off (some strictly better off)

The mechanism design problem: Examples in the general formulation

The Mechanism Design Problem: Towards a definition of "mechanism"

- asking everyone for his θ_i is only one kind of mechanism

Example (Auction (continued))

- individual 0 ("seller") could use a second price sealed bid auction
  - recall: dominant strategy to bid your valuation
  - $y_1(\theta) = 1$ iff $\theta_1 \geq \theta_2$ and $y_2(\theta) = 1$ iff $\theta_2 > \theta_1$
  - $t_1(\theta) = -\theta_2 y_1(\theta)$ and $t_2(\theta) = -\theta_1 y_2(\theta)$ and $t_0(\theta) = -(t_1(\theta) + t_2(\theta))$
- seller could also use a first price auction
- seller could also do the following
  - offer good to agent 1 at price $1/2$
  - if agent 1 rejects the offer, seller offers the good to agent 2 at price $1/2$

The Mechanism Design Problem: "mechanism"

- a mechanism is a game played by the agents
- each agent has a strategy set S_i in this game

Definition (mechanism)

A mechanism $\Gamma = (S_1, \ldots, S_I, g(\cdot))$ is a collection of I strategy sets $(S_1, \ldots, S_I)$ and an outcome function $g : S_1 \times \cdots \times S_I \to X$.

The Mechanism Design Problem: “mechanism”

Example (A public project (continued))

- egalitarian contribution rule is a mechanism
  - $S_i = \Theta_i$
  - g is defined by $g(s_1, \ldots, s_I) = (1, c/I, \ldots, c/I)$ if $\sum_i s_i > c$ and $g(s_1, \ldots, s_I) = (0, 0, \ldots, 0)$ else
- another mechanism is the voluntary contribution mechanism
  - everyone announces a contribution $s_i \in [0, \infty)$
  - bridge is built if the sum of the contributions is higher than c
  - everyone pays $c \frac{s_i}{\sum_j s_j}$ if the bridge is built
  - $S_i = [0, \infty)$ and g is defined by $g(s_1, \ldots, s_I) = (1, c\frac{s_1}{\sum_j s_j}, c\frac{s_2}{\sum_j s_j}, \ldots, c\frac{s_I}{\sum_j s_j})$ if $\sum_j s_j > c$ and $g(s_1, \ldots, s_I) = (0, 0, \ldots, 0)$ else

The Mechanism Design Problem: "mechanism"

- Together with the type sets $(\Theta_1, \ldots, \Theta_I)$, the distribution ϕ and the utility functions $(u_1(\cdot), \ldots, u_I(\cdot))$, a mechanism Γ induces the Bayesian game:

$$\langle I, (S_i)_{i=1,\ldots,I}, (\tilde{u}_i)_{i=1,\ldots,I}, \Theta_1 \times \cdots \times \Theta_I, \phi(\cdot) \rangle$$

  where $\tilde{u}_i(s_1, \ldots, s_I, \theta_i) = u_i(g(s_1, \ldots, s_I), \theta_i)$
- roughly: Bayesian game because
  - agents have to choose a strategy from their strategy set
  - through the mechanism (=rules of the game) their payoff depends on all agents' strategy choices
  - every agent is uncertain about other agents' type

The Mechanism Design Problem: "implementing a social choice function"

Definition (implementing)

The mechanism $\Gamma = (S_1, \ldots, S_I, g(\cdot))$ *implements the social choice function f* if there is an equilibrium strategy profile $(s_1^*, \ldots, s_I^*)$ of the Bayesian game induced by Γ such that $g(s_1^*(\theta_1), \ldots, s_I^*(\theta_I)) = f(\theta_1, \ldots, \theta_I)$ for all $(\theta_1, \ldots, \theta_I) \in \Theta_1 \times \cdots \times \Theta_I$.

- implements: the game has an equilibrium in which we get the desired outcome

The Mechanism Design Problem: "implementing a social choice function"

Example (A public project (continued))

- consider f: build the bridge only if $\sum_i \theta_i > c$ and let $t_i = c/I$ if the bridge is built and 0 otherwise
- we saw above that—in general—the egalitarian contribution rule does not have an equilibrium in which everyone says his true type
  - f is therefore not implementable by the egalitarian contribution mechanism
- Take a simple example
  - 2 agents: agent 1's valuation is 2 and $c = 4$
  - agent 2's valuation is either 1 or 3 (private information of agent 2)
  - check: in this example, the egalitarian contribution mechanism implements f

The Mechanism Design Problem: "direct revelation mechanism"

- in a direct revelation mechanism each player is directly asked to reveal his type

Definition (direct revelation mechanism)

A mechanism Γ is called a *direct revelation mechanism* if $S_i = \Theta_i$ and $g(\theta) = f(\theta)$.

- check in the previous example: the egalitarian contribution mechanism is a direct mechanism for f

The Mechanism Design Problem: "truthful implementation/incentive compatibility"

- a social choice function is **incentive compatible** if truthfully revealing one's type is an equilibrium in the direct revelation mechanism

Definition (truthfully implementable)

A social choice function f is *truthfully implementable* (or *incentive compatible*) if the direct revelation mechanism $\Gamma = (\Theta_1, \ldots, \Theta_I, f)$ has an equilibrium $(s_1^*, \ldots, s_I^*)$ in which $s_i^*(\theta_i) = \theta_i$ for all θ_i and all i.

- "incentive compatible" is sometimes abbreviated with "ic"

The Mechanism Design Problem: “truthful implementation/incentive compatibility”

Example (A public project (continued))

- whether f is incentive compatible depends on the setting
  - if $I = 2$, $\Theta_1 = \{2\}$, $\Theta_2 = \{1, 3\}$ and $c = 4$, then f is ic (no matter what ϕ is) (why?)
  - if $I = 2$, $\Theta_1 = \{2\}$, $\Theta_2 = \{0, 1\}$ and $c = 2.5$, then f is not ic (no matter what ϕ is) (why?)
  - if $I = 2$, $\Theta_1 = \Theta_2 = [0, 1]$, $c = 1$ and ϕ is a uniform distribution, then f is not ic (why?)

Take aways and economics

- we saw a very general and flexible setup (covers a lot of different examples)
- we introduced the vocabulary of mechanism design
- some objectives cannot be realized because the agents have their own private information
- upcoming lectures:
  - we will be more specific
  - we want to know which objectives can be realized by clever mechanisms/institutions/contracts and which objectives cannot be realized

Teaser

There are three possible dates for the exam. We would like to choose the exam date which is preferred by most students.

Is there a direct mechanism where all students announce their true preferences?

Is there a mechanism that achieves the objective?

Review questions

- What is a social choice function?
- Out of which elements does a mechanism exist?
- What is a direct revelation mechanism?
- Explain "truthful implementation"!

Exercises

1. MWG exercise 23.B.1
2. MWG ex 23.B.2