Markets and the First Fundamental Welfare Theorem

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Introduction

- so far
  - how to aggregate preferences
  - Arrow's impossibility theorem
- today: a special aggregation problem
  - exchange economy
  - similar to standard micro model in Bachelor
  - try to make the link:
    - how is this a special case of the social choice model?
    - what additional structure/assumptions are in place?
    - which normative criteria do we use?
    - how do we avoid Arrow's impossibility theorem?

A standard exchange economy

- I consumers
- n goods
- consumer i has initial endowment $e^i = (e^i_1, e^i_2, \dots, e^i_n)$ where $e^i_j \in \Re_+$
  - assumption: each good exists in strictly positive quantities, $\sum_{i=1}^{I} e^i_j > 0$ for all $j = 1, \dots, n$
- consumers preferences over consumption are represented by a utility function $u^i : \Re^n_+ \rightarrow \Re$
  - assumption: u^i is strictly increasing in each component
  - assumption: u^i is strictly quasi-concave
  - assumption: u^i is continuous
- consumers can exchange endowments
  - who should/will eventually consumer what?

Notation

- $e = (e^1, e^2, \ldots, e^I)$ is the vector of endowments
- allocations are denoted by $x = (x^1, x^2, \ldots, x^I)$
  - $x^i \in \Re^n_+$ is agent i's allocation of the n good
- feasible allocations:

$$F(e) = \{x | \sum_{i=1}^{I} x^i = \sum_{i=1}^{I} e^i\}$$

where each $x^i \in \Re^n_+$

Efficiency

Pareto efficiency

An allocation $x \in F(e)$ is Pareto efficient if there is no $y \in F(e)$ such that $u^i(y^i) \geq u^i(x^i)$ for all $i = 1, \ldots, I$ with strict inequality for at least one i.

Comparison

- does Arrow's impossibility theorem apply in this framework?

Prices and the consumer problem

- $p = (p_1, \dots, p_n)$ be a vector of prices (p_j is the price of good j) and assume $p_j > 0$ for all $j = 1, \dots, n$
- assumption: each consumer takes the vector of prices as given
- consumer i's problem:

$$\max_{x^i \in \Re^n_+} u^i(x^i) \qquad s.t.: \quad \sum_{j=1}^{n} p_j x^i_j \leq \sum_{j=1}^{n} p_j e^i_j$$

- think of $m^i(p) = \sum_{j=1}^{n} p_j e^i_j$ as consumer i's income
- given our assumptions, what can we say about the solution of this problem?

Solution to the consumer problem

- solution exists
- solution is unique
- denote the solution to the consumer problem as $x^i(p, m^i(p))$
- $x^i(p, m^i(p))$ is continuous
- demand is homogenous: $x^i(p, m^i(p)) = x^i(\lambda p, m^i(\lambda p))$
- budget constraint holds with equality
- the marginal rate of substitution between any two goods equals the price ratio

$$MRS^i_{j,k} = -\frac{\partial u^i / \partial x^i_j}{\partial u^i / \partial x^i_k} = -\frac{p_j}{p_k}$$

Excess demand

- aggregate excess demand for good j is defined as

$$z_j(p) = \sum_{i=1}^{I} x_j^i(p, m^i(p)) - \sum_{i=1}^{I} e_j^i$$

  - if $z_j(p) > 0$ demand for good j is higher than its supply at price p
  - if $z_j(p) < 0$ demand for good j is lower than its supply at price p

- aggregate excess demand is defined as

$$z(p) = (z_1(p), z_2(p), \ldots, z_n(p))$$

Properties of excess demand

Proposition

Under our assumptions, excess demand satisfies

- continuity: z is continuous at p
- homogeneity: $z(\lambda p) = p$ for all $\lambda \in \Re_{++}$
- Walras' law: $\sum_{j=1}^{n} p_j z_j(p) = 0$

Proof

- continuity:
- homogeneity:
- Walras law:
$\sum_{j=1}^{n} p_j z_j(p) = \sum_{j=1}^{n} p_j \left(\sum_{i=1}^{I} x_j^i(p, m^i(p)) - \sum_{i=1}^{I} e_j^i \right)$
$= \sum_{i=1}^{I} \sum_{j=1}^{n} \left(p_j x_j^i(p, m^i(p)) - p_j e_j^i \right)$
$= \sum_{i=1}^{I} \left[\sum_{j=1}^{n} \left(p_j x_j^i(p, m^i(p)) \right) - m^i(p) \right] = 0$ as budget constraint of each consumer holds with equality

Implications of Walras' law

- suppose we have only 2 goods ($n = 2$) and we have at price vector p excess demand in market 1, $z_1(p) < 0$
  - what can we say about market 2?
- let $n > 2$, if we have excess demand in good 1, $z_1(p) < 0$, what can we say about other markets?
- if $n - 1$ markets are have zero excess demand, i.e. $z_j(p) = 0$ for $j = 1, \ldots, n - 1$, what can we say about the remaining market?

Walrasian equilibrium

Definition: Walrasian equilibrium

A vector $p^* \in \Re^n_{++}$ is called a Walrasian equilibrium if $z(p^*) = 0$.

- all market demands connected
- "general equilibrium"

Walrasian equilibrium: Existence

Existence theorem

A Walrasian equilibrium p^* exists.

Proof existence theorem

somewhat technical, see Jehle and Reny (2011), ch. 5.2.1

Walrasian equilibrium: Efficiency

First fundamental theorem of welfare economics

Let p^* be a Walrasian equilibrium. The equilibrium allocation $x^* = (x^1(p^*), x^2(p^*), \ldots, x^I(p^*))$ is Pareto efficient.

Proof of the first fundamental theorem of welfare economics:

- Suppose, to the contrary, that $y = (y^1, \ldots, y^I)$ Pareto dominates x^*.
  - Then, $\sum_{j=1}^{n} p_j^* y_j^i \geq m^i(p^*)$ for all i with strict inequality for at least one i (Why?)

$$\Rightarrow \sum_{i=1}^{I} \sum_{j=1}^{n} p_j^* y_j^i > \sum_{i=1}^{I} \sum_{j=1}^{n} p_j^* e_j^i$$

  - y must be feasible:

$$\sum_{i=1}^{I} y^i \leq \sum_{i=1}^{I} e^i$$

(note: there are vectors on both sides of the inequality!)

- hence, $p^* \cdot \sum_{i=1}^{I} y^i \leq p^* \cdot \sum_{i=1}^{I} e^i$ as all $p_j^* > 0$ (note: this is a dot/vector product)

$$\Rightarrow \sum_{i=1}^{I} \sum_{j=1}^{n} p_j^* y_j^i \leq \sum_{i=1}^{I} \sum_{j=1}^{n} p_j^* e_j^i$$

Example: 2 agents, 2 goods (Edgeworth box)

First fundamental theorem of welfare economics: comments

- market system leads to efficient allocation
- there are more general versions of this theorem
  - with production, weaker assumptions on consumer preferences, etc.
- decentralized market mechanisms can lead to efficient outcome
  - or: a centralized solution can be implemented in a decentralized way using only prices

Aside: the role of prices I

- the economic problem (putting all resources to their best use) is Herculean at society level
  - what is best use?
    $\rightarrow$ requires knowledge of preferences
  - what are resources?
    $\rightarrow$ requires knowledge of
    - possible production processes
    - natural resources
    - local conditions
    - possible labor supply and preferences concerning labor supply
    - transportation (im-)possibilities
    - ...

Aside: the role of prices II

- planning problem becomes a problem of how to aggregate dispersed information
  - unrealistic to centralize all this information
  - decentralized solution
    - decisions should be made by those that most naturally have most of the necessary information
    - still need enough knowledge of outside world
- prices aggregate all the information a decision maker needs to make the best decision for society
  - consumer knows his own preferences
  - Walrasian price captures opportunity benefit of the resource, i.e. the value of the resource to others
  - each agent can act in interest of society without having to know/understand the interest of society
  - what does an increasing price signal?
- do you know the famous pencil clip?

First fundamental theorem of welfare economics: important assumptions

- all agents are price takers
- complete markets
  - every good that matters for some consumer is traded on its own market
  - guaranteed property rights, i.e. voluntary trade is possible (no theft etc.)
- note:
  - assumptions are sufficient to reach efficiency
  - an efficient equilibrium may still exist if some of the assumptions fail!

Violations of assumptions

- agents are price takers
  - examples of cases where agents are not price takers?
- complete markets assumption
  - a good is not traded on a market:
  - distinct goods are traded on a common market:

The scope for policy: efficiency arguments

- policy within model:
  - guarantee property rights + enforce contracts
- Efficiency reached without policy intervention given our assumptions.
- failure of assumptions is necessary but not sufficient for existence of efficiency enhancing policy
  - outcome may still be efficient
  - efficiency enhancing policy may not be available
- reactions if assumptions fail that are motivated by model
  - competition policy and sector regulation
  - complete/create the market

Aside: The scope for policy: distributional arguments

- second fundamental theorem of welfare economics:
  any efficient allocation is a Walrasian equilibrium for some vector of endowments
- implication
  - realize distributional objectives by redistributing endowments only
  - then let market ensure efficiency
- some caveats to this

Walrasian equilibrium: how to get there?

- how do markets reach a Walrasian equilibrium?
- how do we obtain prices if everyone is price taker?
- metaphor of Walrasian auctioneer

- maybe a good idea to talk about the economics of auctions
  - for auction theory, we need game theory with incomplete information
  - for game theory with incomplete information we need decision making under uncertainty

. . . that's exactly the plan for the coming weeks!